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Trading under Asymmetric Information: Positive and Normative Implications

Andrea Attar* Claude d'Aspremont†

Abstract

We study trading situations in which several principals on one side of the market compete to serve privately informed agents on the other side. In such 'generalized screening' settings, competitors may post mechanisms instead of prices, and the enforceability and the efficiency of the contractual relationships become difficult to evaluate. We revisit these issues, focusing on three applications: bilateral (or multilateral) trade, where all traders have private information, auctions and insurance, where incomplete information is one-sided.

In the first part, as a benchmark, we focus on the standard mechanism design approach with only one principal, the "mechanism designer", and we rely on the revelation principle as a device to characterize equilibrium outcomes. Even then, first-best optimality, combined with Bayesian incentive compatibility and interim individual rationality might be difficult to obtain, as illustrated by Myerson and Satterthwaite (1983) impossibility result, formulated for risk-neutral traders with independent beliefs. In auctions, if the buyers types are correlated à la Crémer and McLean (1985,1988), this impossibility can be bypassed and the seller can extract the whole surplus. In the more general multilateral trade setting, a simple modification of a condition provided by d'Aspremont and Gérard-Varet (1982) allows to implement any distribution of the surplus (Kosenok and Severinov, 2008). However, under risk-aversion, only second-best outcomes can be implemented, as originally shown by Stiglitz (1977) for the monopolistic case, and by Crocker and Snow (1985) for the competitive one.

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In the second part, we consider a class of extensive form games in which several principals (with no private information) compete over mechanisms in the presence of privately informed agents. Applying the standard revelation principle becomes problematic, as first pointed out by Peck (1997): there exist equilibrium outcomes that can be supported by general communication mechanisms, but not by simple direct ones. We revisit a relevant implication of this impossibility, i.e. the recent folk-theorem-like result of Yamashita (2010): if there are at least three agents, a large set of incentive compatible allocations can be supported at equilibrium. For the result to hold, principals have to rely on message spaces that are larger than the corresponding agentsÕ type spaces. In the single agent (or common agency) case, the equilibrium analysis can be further simplified using the delegation principle (Peters, 2001, Martimort and Stole, 2002). In this context, we stress the key role played by the possibility to enforce exclusivity clauses. In standard exclusive competition settings (as Rothchild and Stiglitz, 1976), if a pure strategy equilibrium exists, it is second-best efficient (Crocker and Snow, 1985). This is no longer true under nonexclusive competition. In this case, the possibility to complement his rivals' offers, creates new strategic opportunities for sellers, and crucially modifies equilibrium outcomes: Attar et al. (2014) establish that, in any pure strategy equilibrium, at most one type of agent is actively trading. The impossibility to enforce exclusive trading may further restrict the set of incentive feasible allocations. The recent work of Attar et al. (2016b) characterizes the constraints faced by a planner who does not have access to agents' private information, and cannot prevent agents' from engaging in further trades with sellers. They show that this side-trading opportunity dramatically restricts the set of allocations that are available to a planner. As a matter of fact there is only one incentive compatible allocation that is robust to the possibility of sellers' side trades. This prevents any redistribution between different types of (privately informed) buyers.

Keywords: Mechanism design, Bilateral Trade, Competing Mechanism, Constrained Efficiency, Adverse Selection
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Trading under Asymmetric Information: Positive and Normative Implications

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1 Introduction

Trading under asymmetric information involves traders (buyers and sellers) some (or all) of whom have private information about characteristics that influence their utilities and their beliefs and that are relevant if trading is to be mutually beneficial. However, the terms of trade being fixed through some market institution (with or without public intervention), some traders might have an interest in hiding or distorting their private information. Such behaviors might affect the equilibrium allocation, its existence and its efficiency.

The incentive problems above were traditionally linked to collective issues in public expenditure and taxation theory (Samuelson (1969), Mirrlees (1971)), but, as made clear in the pioneering contributions of Vickrey (1961) and Hurwicz (1973), these issues arise whenever economic decisions are reached through some decentralized process, even when goods are purely private.

The early research on auctions by Vickrey (1961) and others (*e.g.* Griesmer and Shubik (1967), Wilson (1967)) was particularly instructive in this regard. Auction theory uses (without saying) the Bayesian equilibrium concept (formalised by Harsanyi (1967, 1968)) where strategies are functions of the player's type, and auction design has become a major application of

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mechanism design (Myerson (1981)). First-best efficiency is replaced by constrained (or second-best) efficiency and, in the case of one seller, an optimal auction is one for which a Bayesian equilibrium exists and maximizes the seller's expected utility. The non-informed seller acts as a principal (and mechanism designer) and the buyers as agents.

Akerlof (1970) was among the first to point out the potentially dramatic implications of incomplete information in competitive markets. His example features a number of non-informed buyers who compete to attract sellers who are informed about the quality of the product (used cars). Sellers with high quality cars tend to withdraw from the market and at the (competitive) equilibrium price only the "lemons" (or even no car) get traded. Yet, little attention is put on the role of incentive mechanisms to elicit information revelation and (potentially) unfreeze the market. Indeed, in the original Akerlof (1970) example, buyers are restricted to post linear prices, in the spirit of competitive equilibrium theory. The need for a more explicit representation of agents' strategic behavior gave rise to two independent lines of research. The first one, acknowledged as "signalling", develops the analysis of extensive form games in which the informed agents move first. This line of research was initiated by Spence (1973) who introduced the possibility for informed sellers to "signal" the quality of their product (labour) by taking a costly action (education).[1] The second one is usually referred to as competitive "screening". This amounts at considering games in which several players have the power to design contracts to attract privately informed agents. In that line, Rothschild and Stiglitz (1976) were the first to model a non-cooperative game between uninformed insurance companies, acting as principals, and offering exclusive contracts to a buyer who knows her own risk-type and chooses the contract which is best for her.

The purpose of this chapter is to pursue this second line of research. In this respect, we do not aim at revisiting the original approaches to screening, nor to propose an exhaustive survey.[2] We rather start from the remark that situations in which competitors post mechanisms instead of prices are prominent in several modern markets. Examples include: competitive insurance, competing auctions, financial markets (over-the-counter markets,

[1]The possibility of signaling has triggered a lot of game-theoretical research to deal with the multiplicity of equilibria due to the possibility of unanticipated action by the first-movers: e.g. Kreps and Wilson (1982), Cho and Kreps (1987), Kohlberg and Mertens (1986).

[2]Riley (2001) provides an excellent retrospective of signalling and screening models.

interbank market), etc. In some of these markets, the posted mechanisms potentially involve some degree of reciprocity (meet the competition clause) and exclusivity of contractual relationships is not enforceable. A typical example is given by OTC markets where little information on trading volumes is available.[3]

We choose to focus on some selected contributions in the screening approach that may contribute to our understanding of the institutional features described above. As in Rothschild and Stiglitz (1976), we consider settings in which market equilibria may or may not exist and second-best optimality is not guaranteed, depending very much on the specific extensive form and on the contracting assumptions involved.

In this respect, observe that we will only consider optimality and constrained optimality in the typology defined by Holmström and Myerson (1983). For instance we will stick to *ex post* optimal mechanisms in the classical sense of leading to a Pareto-optimal allocation at every state of nature without taking into account the information derived from observing this allocation (*i.e.* we do not deal with the Forges (1994) notion of Posterior Efficiency). Also, we will not review the Walrasian approaches to markets under incomplete information as modeled in Prescott and Townsend (1984)[4].

Specifically, this chapter is divided in two parts. In the first part (Section 2), we adopt the simple mechanism design approach with only one "mechanism designer". When the mechanism designer is an outsider (say a public authority), all traders may have private information and play simultaneously. When the mechanism designer is an insider (a principal, buyer or seller), then he is uninformed and has no private information. Three illustrative applications are introduced: bilateral trade, auctions and insurance. In the second part (Section 3), the model is extended to several principals who are uninformed and have no private information, but compete by designing mechanisms.

[3]Other examples include the U.S. credit card industry (Rysman (2007)), the U.S. life insurance market (Philipson and Cawley (1999)), and the U.K. annuity market (Finkelstein and Poterba (2002, 2004)).

[4]As shown by Rustichini and Siconolfi (2008) such an approach works well if types are publicly known but not under adverse selection. Then prices have to depend on types and to be incentive compatible: each type of consumer should want to buy in the market at the corresponding price.

2 The Mechanism Design Approach

Our first approach to trading rules under incomplete information is based on Mechanism Design. The focus will be on Efficiency and Incentive Efficiency (Holmström and Myerson (1983)). Participation constraints will also be taken into account.

2.1 A trading mechanism

We consider the following scenario. There is a set $\mathcal{I}$ of agents, who are trading: $\mathcal{I} = \{1, \cdots, i, \cdots, I\}$. They are buyers and sellers. The characteristic, or type θ^i, of a trader $i \in \mathcal{I}$, takes values in a set Θ^i. We denote $\Theta = \underset{i \in N}{\times} \Theta^i$ the set of all states of nature (and $\Theta^{-i} = \underset{j \neq i}{\times} \Theta^j$). In this section, we suppose that there is a single Principal (the mechanism designer), who may or may not be an outsider (a planner). An allocation is a vector $x = (x^1, \cdots, x^i, \cdots, x^I)$ in some set X, the set of feasible allocations. Each trader i may decide to participate, by taking the decision $a^i = Y$ (yes), or not to participate and take the decison $a^i = N$ (no). The utility for trader $i \in \mathcal{I}$ is given by the real-valued function $u^i(x, a; \theta)$, defined on the set $X \times \{Y, N\}^{\sharp \mathcal{I}} \times \Theta$. We assume that the utility of a trader i, when not participating, $u^i(x, N, a^{-i}; \theta)$ is given by the utility $u_0^i(\theta^i)$ of some outside option and, without loss of generality, we suppose $u_0^i(\theta^i) \equiv 0$. Observe that, otherwise, the utility of each trader, as defined, might be affected by the types of all others (common value or interdependent values). A particular case, the private value case, is when $u^i(x, a; \theta) \equiv v^i(x, a; \theta^i)$. Each agent $i \in \mathcal{N}$ knows her true type $\theta^i \in \Theta^i$ and we assume that there is a distribution F on the random variable θ which is common knowledge (beliefs $F(\theta^{-i} \mid \theta^i)$ are *consistent*). Beliefs are *free* if $F(\theta^{-i} \mid \theta^i) = F(\theta^{-i} \mid \theta'^i)$, $\forall\, i \in \mathcal{N}$, $\forall\, \theta^i, \theta'^i \in \Theta^i$ (the so-called independent case).

The Principal is supposed to choose a trading mechanism in some set Γ. A mechanism is a pair (M, γ) with $M = \underset{i \in \mathcal{I}}{\times} M^i$, each M^i being the set of messages available to trader i, together with a function $\gamma : M \times \{Y, N\}^{\sharp \mathcal{I}} \to X$. Given a mechanism, each trader $i \in \mathcal{I}$ sends a message m^i to the Principal and chooses a decision $a^i \in \{Y, N\}$. For $m = (m^1, \cdots, m^I) \in M$ and $a \in \{Y, N\}^{\sharp \mathcal{I}}$, the allocation $\gamma(m, a) \in X$ is the resulting allocation. A trading mechanism determines a game with incomplete information. A *Bayesian* *equilibrium* is a vector of strategies $(\tilde{m}, \widetilde{a}) = (\tilde{m}_i, \widetilde{a}_i)_{i\in\mathcal{I}}$ where, for every $i \in \mathcal{I}$, $\tilde{m}^i$ is a measurable function from Θ^i into M^i and $\widetilde{a}^i$ a measurable function from Θ^i into $\{Y, N\}$ such that: $\forall\, i \in \mathcal{I}$, $\forall \theta^i \in \Theta^i$, $\forall\, m^i \in M^i$, $\forall\, a^i \in \{Y, N\}$,

$$\int\limits_{\Theta^{-i}} (u^i(\gamma(\tilde{m}(\theta), \widetilde{a}\,(\theta)), \widetilde{a}\,(\theta)\,; \theta))dF(\theta^{-i} \mid \theta^i)$$
$$\geq \int\limits_{\Theta^{-i}} (u^i(\gamma(m^i, \tilde{m}^{-i}(\theta^{-i}), a^i, \widetilde{a}^{-i}(\theta^{-i})), a^i, \widetilde{a}^{-i}(\theta^{-i}); \theta))dF(\theta^{-i} \mid \theta^i)$$

In addition, this equilibrium should ensure participation, that is: $\widetilde{a}^i\,(\theta) = Y$, $\forall\, i \in \mathcal{I}$, $\forall\, \theta \in \Theta$.

Now, by the Revelation Principle, we can as well consider the associated *direct trading mechanism* $(\Theta, \widetilde{\gamma})$ such that $\widetilde{\gamma}\,(\theta) \equiv \gamma(\tilde{m}(\theta), \widetilde{a}\,(\theta))$ where $\widetilde{a}^i\,(\theta) = Y$, $\forall\, i \in \mathcal{I}$, $\forall\, \theta \in \Theta$, and where the vector of strategies $\tilde{\theta} = (\tilde{\theta}^1, \tilde{\theta}^2, \cdots, \tilde{\theta}^i, \cdots, \tilde{\theta}^I)$ of reporting truthfully its type for each i, *i.e.* $\tilde{\theta}^i\,(\theta^i) = \theta^i$, is a Bayesian equilibrium in the associated game of incomplete information. For simplicity of notation we let, for all $x \in X$ and $\theta \in \Theta$, $U^i\,(x; \theta) \equiv u^i\,(x, Y; \theta)$.

The mechanism $(\Theta, \widetilde{\gamma})$ then satisfies *Bayesian Incentive Compatibility* (BIC)[5]: for all $\theta^i \in \Theta^i$, $\theta'^i \in \Theta^i$ and $i \in \mathcal{N}$,

$$\int\limits_{\Theta^{-i}} \left[U^i\,(\widetilde{\gamma}\,(\theta)\,; \theta)\right] dF(\theta^{-i} \mid \theta^i) \geq \int\limits_{\Theta^{-i}} \left[U^i\left(\widetilde{\gamma}\left(\theta'^i, \theta^{-i}\right); \theta^i, \theta^{-i}\right)\right] dF(\theta^{-i} \mid \theta^i),$$

as well as *interim individual rationality (IIR)*:

$$\int\limits_{\Theta^{-i}} \left[U^i\,(\widetilde{\gamma}\,(\theta)\,; \theta)\right] dF(\theta^{-i} \mid \theta^i) \geq 0,$$

for all $\theta^i \in \Theta^i$ and all $i \in \mathcal{N}$.

When the Principal is an outsider, say a public decision maker, her objective function might be chosen so that some efficiency property be satisfied. For example, following Holmström and Myerson (1983), we may define the

[5]Bayesian Incentive Compatibility (BIC) is the terminology used in d'Aspremont and Gérard-Varet (1979a) (see also Myerson (1982)). In the following, when we say incentive compatible it will mean BIC.

following social welfare function on direct mechanisms

$$W(\tilde{\gamma}) = \sum_{i} \int_{\Theta} \mu^{i}(\theta) \left[U^{i}(\tilde{\gamma}(\theta); \theta) \right] dF(\theta^{-i} \mid \theta^{i}),$$

with every weight $\mu^{i}(\theta)$ nonnegative and some strictly positive. Maximizing W on the set of direct mechanisms leads to *ex post* (Pareto) efficiency. If, for every $i \in \mathcal{I}$, $\mu^{i}(\theta) = \mu^{i}(\theta^{i})$, then we get the stronger property of *interim* efficiency, and if for every $i \in \mathcal{I}$, $\mu^{i}(\theta) = \mu^{i}$, then we get the even stronger property of *ex ante* efficiency. If the maximization is only done over the subset of BIC direct mechanisms, we get respectively, the properties of *ex post*, *interim* and *ex ante* incentive-efficiency.

Example 1: The Quasi-Linear Case

Suppose that $X \subset \mathbb{R}^{(K+1)I}$ is the set of feasible allocations of a finite number K of goods and of the corresponding monetary transfers. An element $(q, t) \in X$ is such that $q = \left(q^{1}, ..., q^{i}, ..., q^{I}\right) \in \mathbb{R}^{KI}$ and $t = \left(t^{1}, ..., t^{i}, ..., t^{I}\right) \in \mathbb{R}^{I}$. Each trader utility function u^{i} is assumed to be separable and transferable in money and can be written as $u^{i}(q, a; \theta) + t^{i}$ (resp. $U^{i}(q; \theta) + t^{i}$). In that case a trading mechanism is a triple (M, χ, τ) where $\chi : M \to X$ is called the *allocation rule* and $\tau : M \to \mathbb{R}^{\mathcal{N}}$ is the *payment scheme*. The *direct trading mechanism* associated to the Bayesian Equilibrium $(\tilde{m}(\theta), \tilde{a}(\theta))$ is the triple (Θ, q, t) where q is the allocation rule $q(\theta) \equiv \chi(\tilde{m}(\theta), \tilde{a}(\theta))$ and t is the payment scheme such that $t^{i}(\theta) \equiv \tau^{i}(\tilde{m}(\theta), \tilde{a}(\theta))$.
The allocation rule q is said to be *(ex post) efficient (EF)* if, for all $q \in Q$ and all $\theta \in \Theta$,

$$\sum_{i \in \mathcal{N}} U^{i}(q(\theta); \theta) \geq \sum_{i \in \mathcal{N}} U^{i}(q; \theta),$$

The payment scheme t is *budget-balancing (BB)*, if for all $\theta \in \Theta$,

$$\sum_{i \in \mathcal{I}} t^{i}(\theta) = 0,$$

These two properties taken together imply that the allocation resulting from the mechanism is *Pareto optimal*. In the private value case, a most well-known *ex post* efficient mechanism, is the Vickrey-Clarke-Groves (VCG) mechanism. Take any *ex post* efficient allocation rule q^{*} and define the payment scheme

$$t^{i}_{VCG}(\theta) = \sum_{j \neq i} U^{j}(q^{*}(\theta); \theta^{i}) + h^{i}\left(\theta^{-i}\right),$$

where $h^i(\theta^{-i})$ is any function independent of θ^i. Because q^* is efficient, agents report truthfully their types (individual and collective objectives coincide) whatever their beliefs. It is a dominant strategy. Choosing high enough h^i's ensures interim individual rationality. The problem is that budget balance is generally not achievable. If $\sum_{i\in\mathcal{I}} t^i_{VCG}(\theta) > 0$ (resp. $\sum_{i\in\mathcal{I}} t^i_{VCG}(\theta) < 0$) for some $\theta \in \Theta$, then the mechanism runs a deficit (resp. a surplus).

Example 2: The Private Value Linear Case

A subcase (extensively used in applications) is to assume linear utilities and private values and that, for every i, $\theta_i \in \Theta_i = [\theta^i_0, \theta^i_1]$, a nondegenerate interval in $\mathbb{R}$, and that i's beliefs are free and represented by a continuous density function $f^i(\theta^{-i})$ with full support. The utility U^i of trader i (assuming participation) is now of the form $U^i(q)\,\theta^i + t^i$. A well-known result[6], characterizing BIC mechanisms, is given by the following lemma:

Lemma 1 *A direct trading mechanism (Θ, q, t) is BIC if and only if*

$$\overline{U^i}\left(\theta^i\right)\theta^i + \overline{t_i}\left(\theta^i\right) = \overline{U}^i\left(\theta^i_0\right)\theta^i_0 + \overline{t}^i\left(\theta^i_0\right) + \int_{\theta^i_0}^{\theta^i} \overline{U}^i\left(\widehat{\theta}^i\right) d\widehat{\theta}^i,$$

where $\overline{t}^i(\theta^i) \equiv \int_{\Theta^{-i}} t^i(\theta^i, \theta^{-i}) f^i(\theta^{-i})\, d\theta^{-i}$ and with $\overline{U}^i(\theta^i) \equiv \int_{\Theta^{-i}} U^i(q(\theta^i, \theta^{-i}))$ $f^i(\theta^{-i})\, d\theta^{-i}$ a nondecreasing function, since by BIC, $\overline{U}^i(\theta^i)\,\theta^i + \overline{t}^i(\theta^i)$ is a convex (a.e. differentiable) function and its derivative is equal to $\overline{U}^i(\theta^i)$.

2.2 Applications

To illustrate this general model, we turn now to three applications.

2.2.1 Bilateral Trade

If we consider the situation where one seller tries to sell an object to several potential buyers, we can further specify Example 2 by assuming (i) that trader 1 is the seller, all other traders being potential buyers, (ii) that an

[6]This is due to Myerson (1981) and Riley and Samuelson (1981). A characterization under efficiency is given in d'Aspremont and Gérard-Varet (1979b).

allocation $q = \left(q^1, q^2, ..., q^I\right) \in Q$ determines the probability $q^i \geq 0$ that trader i will get (or keep) the object (with $\sum_{i \in \mathcal{I}} q^i = 1$), and (iii) that the utility function is simply $U^i(q;\theta) = q^i\theta^i$.

For this context, we can adopt the following specification of the functions h^i's in the definition of the VCG mechanism (see Krishna (2010)), for some efficient trading rule q^*,

$$h^i\left(\theta^{-i}\right) \equiv -\sum_{j \neq i} q^{*j}\left(\theta_0^i, \theta^{-i}\right)\theta^j - q^{*i}\left(\theta_0^i, \theta^{-i}\right)\theta_0^i.$$

Denote VCG^0 the VCG mechanism with this specification. It satisfies EF and BIC, and by Lemma 1 we get IIR, since

$$\overline{U}^i\left(\theta^i\right)\theta^i + \overline{t}^i_{VCG^0}\left(\theta^i\right) =$$
$$\int_{\Theta^{-i}} \left[\sum_{j \in \mathcal{I}} q^{*j}\left(\theta^i, \theta^{-i}\right)\theta^j - \sum_{j \neq i} q^{*j}\left(\theta_0^i, \theta^{-i}\right)\theta^j - q^{*i}\left(\theta_0^i, \theta^{-i}\right)\theta_0^i\right] f^i\left(\theta^{-i}\right) d\theta^{-i},$$

is equal to zero for $\theta^i = \theta_0^i$. Also, by Lemma 1 again, for any other mechanism (Θ, q^*, t) satisfying EF, BIC and IIR, $\overline{t}^i(\theta^i) - \overline{t}^i_{VCG^0}(\theta^i)$ is a nonnegative constant for each i. This observation implies the following:

Proposition 1 *(Myerson and Satterthwaite (1983)): Supposing that $I = 2$, $\theta_0^2 < \theta_1^1$ and $\theta_1^2 \geq \theta_0^1$, there is no direct mechanism satisfying EF, BIC, IIR and, at the same time, balancing the budget (BB).*

Indeed, the VCG^0 mechanism, as just defined, always runs a deficit (with the supposed overlapping intervals $[\theta_0^i, \theta_1^i]$, $i = 1, 2$), and so does any other mechanism satisfying EF, BIC and IIR (see Krishna and Perry (1997) and Krishna (2010)).

There are various ways to escape this impossibility result.

One is to assume[7] that the VCG^0 mechanism, as just defined, runs a surplus (as it would be the case here if $\theta_0^2 \geq \theta_1^1$). Another is to have more

[7]See Krishna and Perry (1997) (Theorem 2) and (under more specific assumptions) Makowski and Mezzetti (1994) (Theorem 3.1). The argument uses a modified "expected externality" mechanism (or AGV mechanism for Arrow (1979) and d'Aspremont and Gérard-Varet (1979a). See also d'Aspremont and Gérard-Varet (1975).

than 2 agents and to vary the ownership shares of the object (Cramton et al. (1987)) or to allow for interdependent beliefs (as we will see below). Also, the budget balance condition can be weakened to Expected Budget Balance (McAfee and Reny (1992)).

2.2.2 Auctions

Auctions are widely used in practice for selling a large variety of objects. Consider again the situation where one seller tries to sell an object to several potential buyers with private values. The seller is now supposed to be the mechanism designer (or the principal) and his type is common knowledge. The set of messages M^i that buyer i sends to the seller is the set of possible bids. Several kinds of auctions are possible. First-Price Sealed-Bid auction (where the winner pays the highest bid), second-price sealed-bid (where the winner pays the second highest bid), English (or open ascending) auction are common examples. The revelation principle still applies and to each equilibrium of an auction mechanism (direct or not) we can associate a BIC direct auction mechanism.

Assume that properties (i), (ii) and (iii) of the previous subsection still hold, that $\theta_1^0 = \theta_1^1 = \theta_0^i = 0$, $i = 2, ..., I$ and that $f(\theta) = \prod_{i=2}^{I} f^i(\theta^i)$. In this case, the mechanism VCG^0 coincides with the second-price direct auction (and with Clarke "pivotal" mechanism). The object is sold to a buyer i reporting the highest valuation (i is a "pivotal" agent) and the amount this buyer i pays to the seller, $-\bar{t}^i_{VCG^0}(\theta^i)$, is equal to $\underset{j \neq i, j>1}{max}\, \theta^j$ the second highest valuation. The other buyers pay nothing. Clearly EF and IIR holds and, since the seller receives $\bar{t}^i_{VCG^0}(\theta^i)$, we get budget balance without affecting incentives since the type of the seller is common knowledge. All buyers report truthfully (it is a dominant strategy) whatever their beliefs. The mechanism is independent of the traders characterics (names, valuations and beliefs) and of the object characteristics: it is anonymous and universal. Also, since values are private and statistically free, the second-price sealed-bid auction is equivalent to the English auction (since the information obtained during the latter auction is irrelevant). The first-price sealed-bid auction (even when formulated as a direct auction mechanism) is different. It satisfies BB and IIR but not, in general, BIC and EF. Vickrey (1961) already mentions the possibility of inefficient allocation in a first-price auction. A simple argument (see Krishna (2010)) is to suppose, with two asymmetric bidders, that the equilibrium bidding strategies are continuous increasing and strictly unequal at some value, say $\tilde{m}^1(\theta) < \tilde{m}^2(\theta)$, for $\theta_0^i < \theta < \theta_1^i$, $i = 1, 2$. Then $\tilde{m}^1(\theta + \varepsilon) < \tilde{m}^2(\theta - \varepsilon)$, for small $\varepsilon > 0$, and bidder 2 still wins although she has a lower value.

However, if we assume symmetry ($\theta_1^i = \theta_1^j$ and $f^i = f^j$, $i, j = 2, ..., I$), the first-price auction is *ex post* efficient and, by the revenue equivalence principle (Riley and Samuelson (1981), Myerson (1981)) the expected revenue of the seller is the same as in any other *ex post* efficient auction, although BIC remains violated (each buyer reports $\theta^i/2$).

More generally, from a mechanism design perspective, one can look for an *ex ante* incentive efficient auction mechanism. In particular, considering that the seller plays the role of a principal (and as such is the mechanism designer), we can look for the direct auction mechanism maximizing the seller expected revenue. Following Myerson (1981), and assuming the virtual valuation function $\psi^i(\theta^i) \equiv \theta^i - (1 - F^i(\theta^i)) / f^i(\theta^i)$ to be increasing, the optimal direct auction is the one attributing the object to the buyer with maximal virtual valuation (if nonnegative) and the winner pays the smallest amount that keeps himself winning. Since the virtual valuation differs from the value and may be negative, the optimal auction is not efficient in general. In the symmetric case $\psi^i \equiv \psi$ for all i and the optimal auction is simply a second-price auction with reserve price equal to $\psi^{-1}(0)$ (see Proposition 5.4 in Krishna (2010)). The optimal auction is obtained under the assumption of statistical independence, and in that case each buyer always benefits from some informational rent.

Myerson (1981) and Crémer and McLean (1985, 1988) show that some correlation between types is necessary and sufficient for the seller to do much better, namely to extract the full surplus. This result holds even if values are interdependent, namely if we assume the more general utility function $U^i(q; \theta) = q^i U^i(\theta)$ (with the normalisation $U_0^i(\theta^i) \equiv 0$). Suppose for simplicity, as in Crémer and McLean (1988), that set of types Θ^i for each i is finite and that the beliefs $F(\theta^{-i} \mid \theta^i)$ are represented by discrete probability distributions with full support and satisfy a very general condition (implying statistical dependence). To extract the whole surplus means that, for each buyer i, $q^i(\theta) = 0$ if $U^i(\theta) < max_j U^j(\theta)$ or if $max_j U^j(\theta) \leq 0$ (recall $\sum_{i \in \mathcal{I}} q^i(\theta) = 1$) and that each buyer's IIR constraints holds as an equality. Crémer-McLean condition simply requires that for each buyer i of type θ^i

there is a lottery $s^i(\theta^{-i};\theta^i)$ defined on Θ^{-i} such that

$$\sum_{\theta^{-i}} s^i\left(\theta^{-i};\theta^i\right) F\left(\theta^{-i} \mid \theta^i\right) > \sum_{\theta^{-i}} s^i\left(\theta^{-i};\theta^i\right) F\left(\theta^{-i} \mid \theta^{\circ i}\right)$$

for every $\theta^{\circ i} \neq \theta^i$. Under this condition, a direct auction mechanism can be constructed to extract the whole surplus, while satisfying BIC and IIR (but not *ex post* individually rationality)[8].

The Crémer-McLean condition can be reinforced and applied in a much larger context (with quasi-linear utilities), in particular to general (direct) trading mechanisms with possibly multiple buyers and sellers, all of multiple types, and with any allocation rule, efficient or not. This is most simply obtained by reinforcing condition B in d'Aspremont and Gérard-Varet (1982) so that IIR can be ensured, in addition to BIC and BB, whenever, for a given allocation rule, the *ex ante* expected surplus is nonnegative:

$$\sum_{i\in\mathcal{I}}\sum_{\theta} U^i\left(q\left(\theta\right);\theta\right) F\left(\theta^{-i} \mid \theta^i\right) F\left(\theta^i\right)) \geq 0.$$

The condition (introduced as condition B^{IIR} in d'Aspremont C. and Crémer (2017)) requires that there exists a budget balanced payment scheme s for all i, all θ^i and $\theta^{\circ i}$, $\theta_i \neq \theta^{\circ i}$, such that

$$\sum_{\theta^{-i}} s^i(\theta^{-i},\theta^i)\, F(\theta^{-i} \mid \theta^i) > \sum_{\theta^{-i}} s^i(\theta^{-i},\theta^{\circ i})\, F(\theta^{-i} \mid \theta^i),$$

and $\sum_{\theta^{-i}} s^i(\theta^{-i},\theta^i) F(\theta^{-i} \mid \theta^i) = 0$.

This condition can be shown to be equivalent to Crémer-McLean condition plus an identifiability condition introduced by Kosenok and Severinov (2008) entailing that, for any allocation rule generating a nonnegative *ex ante* expected surplus, any distribution of this whole surplus (with BB respected) among the traders can be implemented by a BIC and IIR mechanism:

[8] This condition is here stated in its "primal form". It is generic in the finite case. But it implies No-Freeness: For all i, and any $\left(\theta^i,\theta^{\circ i}\right)$, if $\theta^i \neq \theta^{\circ i}$ then $F_i\left(. \mid \theta^i\right) \neq F_i\left(. \mid \theta^{\circ i}\right)$. i.e. no agent has free beliefs over two types. This is called "Belief announcement" in Johnson et al. (1990), "Beliefs Determine Preferences" (BDP) in Heifetz and Neeman (2006).

Proposition 2 *Assume $I \geq 3$ and that the beliefs satisfy condition B^{IIR}. For any allocation rule q and any set $\{\upsilon^i(\theta^i), \theta^i \in \Theta_i$ and $i \in \mathcal{I}\}$ of nonnegative utility levels such that*

$$\sum_{i\in\mathcal{I}}\sum_{\theta^i}\upsilon^i\left(\theta^i\right)F^i\left(\theta^i\right)=\sum_{i\in\mathcal{I}}\sum_{\theta}U^i\left(q\left(\theta\right);\theta\right)F\left(\theta^{-i}\mid\theta^i\right)F\left(\theta^i\right)\geq 0,$$

there is a direct trading mechanism (q,t) satisfying BIC, BB, IIR and such that

$$\sum_{\theta^{-i}}\left[U^i\left(q\left(\theta\right);\theta\right)+t^i\left(\theta\right)\right]F\left(\theta^{-i}\mid\theta^i\right)=\upsilon^i\left(\theta^i\right),$$

for all $\theta^i \in \Theta i$ and $i \in \mathcal{I}$.

This is equivalent to Corollary 1 in Kosenok and Severinov (2008). Of course, this allows for full surplus extraction by a single trader. Matsushima (2007) has a similar result, but under an assumption which is stronger than B^{IIR}. The proof of the proposition is simple. Since

$$\sum_{i\in\mathcal{I}}\sum_{\theta}U^i\left(q\left(\theta\right);\theta\right)F\left(\theta^{-i}\mid\theta^i\right)F\left(\theta^i\right)=\sum_{i\in\mathcal{I}}\sum_{\theta^i}\upsilon^i\left(\theta^i\right)F\left(\theta^i\right),$$

there is a payment scheme τ satisfying BB and such that

$$\sum_{\theta^{-i}}\left[U^i\left(q\left(\theta\right);\theta\right)+\tau^i\left(\theta\right)\right]F\left(\theta^{-i}\mid\theta^i\right)\geq\upsilon^i\left(\theta^i\right),$$

for all θ^i, all i (see lemma 1 in Matsushima, 2007). Now, using the budget-balanced payment scheme s given by condition B^{IIR}, we have a family of budget-balanced payment schemes $t \equiv \tau + Ks$, $K \geq 0$. With K large enough, BIC is satisfied and, since $\sum_{\theta^{-i}} Ks^i(\theta^{-i},\theta^i)F(\theta^{-i} \mid \theta^i) = 0$, we get IIR:

$$\sum_{\theta^{-i}}\left[U^i\left(q\left(\theta\right);\theta\right)+\tau^i\left(\theta\right)\right]F\left(\theta^{-i}\mid\theta^i\right)=\upsilon^i\left(\theta^i\right)\geq 0.$$

The mechanisms which are thus obtained, optimal or not, are interesting from an investigation point of view. As mentioned by Wilson (1985),"it suffices in principle to study direct revelation games in order to find efficient trading rules". But, most importantly, "There often remains a motive, of course, to translate an efficient direct revelation game back into a form of the sort more usually found in practice".[9] Although direct mechanisms use the simplest kind of equilibrium (truthtelling), their rules integrate specific features of the economic environment.[10] They are neither universal, nor anonymous. Although there might be exceptions (*e.g.* a group of advertisers competing on the web for an ad impression), in most standard contexts, trading rules have to be more simple (e.g. independent of the number of participants and of their beliefs), and the complexity is shifted to the equilibrium strategies, to be computed by the participants themselves on the basis of their knowledge of the economic environment.

2.2.3 Insurance

In the first two applications we have developed, traders are restricted to be risk-neutral. All utility functions are assumed to be quasi-linear (or even linear). If one looks at a single insurer selling an insurance policy to a single buyer with private information about the risk she wants to insure (hence of several types), risk-aversion should be an essential ingredient. Our point of departure is the celebrated Rothschild and Stiglitz (1976) insurance economy, as reformulated by Stiglitz (1977) in a monopolistic setting with a single risk-neutral seller (the principal) offering coverage-premium contracts $(q, t) \in \mathbb{R}^2_+$ to ensure BIC and hence having the power to screen different types (the no-trade contract being $(0, 0)$).[11]

We suppose that the buyer (the agent) may be of two types, $\theta \in \{\theta^0, \theta^1\}$, with positive probabilities $F(\theta^1) = \phi$ and $F(\theta^0) = (1 - \phi)$. She has initial wealth W_0 and faces the risk of a loss $L > 0$ with a probability given by her type $\theta \in (0, 1)$ and such that $\theta^1 > \theta^0$ and $L < W_0$. Type θ's preferences over aggregate coverage-premium pairs have an expected-utility representation

$$U(q, t; \theta) \equiv \theta u(W_0 - L + q - t) + (1 - \theta)u(W_0 - t), \tag{1}$$

where u is a twice continuously differentiable, strictly increasing, and strictly concave von Neumann–Morgenstern utility function. It is immediate to check

[9] See Wilson (1985) p. 183.

[10] Note that these features can be very general if the type space is rich enough.

[11] An alternative approach involves the informed agent moving first. This is the signalling problem originally analyzed by Spence (1973) in a labor market context, and further examined by Cho and Kreps (1987). In this class of problems the informed agent is given little, if any, power to design incentive contracts.

that, since $\theta^1 > \theta^0$, type θ's preferences over coverage-premium pairs $(q,t) \in \mathbb{R}^2_+$ are ordered by single crossing. That is, geometrically, in the (q,t) plane, an indifference curve for type θ^0 crosses an indifference curve for type θ^1 only once from below, implying that her willingness to substitute coverage for premium is everywhere higher than type θ^1 's.[12] If the principal provides type θ with coverage q for a premium t, he earns a profit $\pi(q,t;\theta) = t - v(\theta)q$, with $v(\theta^1) > v(\theta^0)$, and we let $v = \phi v(\theta^1) + (1-\phi)v(\theta^0)$ be the average price. Thus, this is a case of *common value*: conditional on a trade taking place, the insurer directly cares about the characteristics, or information, of the insured.

If the probability of loss was common knowledge, the insurer (knowing the type θ) would offer a contract $(q(\theta), t(\theta))$ to each type θ with full coverage $(q(\theta) = L)$ and with the participation constraint binding: $U(q(\theta), t(\theta); \theta) = U_\theta(0,0;\theta)$. This is first-best optimal, the risk-neutral seller bears all the risk and extracts the whole surplus. Note that, if contracts were restricted to be actuarially fair, *i.e.* $t(\theta) - v(\theta)L = 0$, the expected profit of the seller would be zero and the surplus would go to the buyer: $U(q(\theta), t(\theta); \theta) > U(0,0;\theta)$.

With private information, the first-best becomes unfeasible. Given the incentive constraints, only the high-risk type can be fully insured in a monopolistic equilibrium.

If the low-risk type were buying more coverage at better terms, the high-risk would switch to that contract and the seller would lose profit. Therefore, the seller maximizes total expected profit $\phi\pi(q(\theta^1), t(\theta^1); \theta^1) + (1-\phi)\pi(q(\theta^0), t(\theta^0); \theta^0)$ under the BIC and IIR constraints. At the solution the low-risk type gets partial (or zero) insurance coverage, with the BIC constraint being strict,

$$U(q\left(\theta^0\right), t\left(\theta^0\right); \theta^0) > U(q\left(\theta^1\right), t\left(\theta^1\right); \theta^0)$$

and no surplus: $U(q(\theta^0), t(\theta^0); \theta^0) = U(0,0;\theta^0)$. The high-risk type, indeed, gets full coverage, with the BIC constraint binding,

$$U(q\left(\theta^1\right), t\left(\theta^1\right); \theta^1) = U(q\left(\theta^0\right), t\left(\theta^0\right); \theta^1)$$

and positive surplus: $U(q(\theta^1), t(\theta^1); \theta^1) > U(0,0;\theta^1)$.

[12] More formally, the single-crossing assumption states that: For each $(q,t) \in \mathbb{R}^2_+$, $\tau_{\theta^0}(q,t) > \tau_{\theta^1}(q,t)$, where $\tau_\theta \equiv -\frac{\partial U_\theta/\partial q}{\partial U_\theta/\partial t}$ is type θ's marginal rate of substitution of coverage for premium, which is everywhere well defined and strictly decreasing along her indifference curves.

This characterization of the monopoly (second-best) allocation has been recently generalized by Schlee and Chade (2012), in which the set Θ of buyer's types is not restricted to be finite (but with θ_0 the smallest and θ_1 the largest element) and the distribution $F(\theta)$ is arbitrary. The seller's problem can be written in more general terms: $\underset{q(\theta),t(\theta)}{max} \int_\Theta \pi\left(q\left(\theta\right), t\left(\theta\right); \theta\right) dF\left(\theta\right)$, subject to $U(q\left(\theta\right), t\left(\theta\right); \theta) \geq U(q\left(\theta'\right), t\left(\theta'\right); \theta)$ and $U(q\left(\theta\right), t\left(\theta\right); \theta) \geq U(0, 0; \theta)$, for all θ, θ' in Θ. As in Stiglitz (1977), type θ_0 gets no surplus, type θ_1 gets full coverage, all other types get partial insurance. The seller makes positive expected profit. The coverage and premium are nonnegative and co-monotone.

If we now impose that the seller total expected profit be zero, we get the set of second-best contracts as defined by Harris and Townsend (1981) and characterized for this model by Crocker and Snow (1985). These are the *ex post* incentive efficient allocations obtained by maximizing the weighted sum $\mu U(q\left(\theta^1\right), t\left(\theta^1\right); \theta^1) + (1-\mu) U(q\left(\theta^0\right), t\left(\theta^0\right); \theta^0)$ under the constraint that $\phi\pi\left(q\left(\theta^1\right), t\left(\theta^1\right); \theta^1\right) + (1-\phi)\pi\left(q\left(\theta^0\right), t\left(\theta^0\right); \theta^0\right) = 0$, as well as the two BIC constraints, for all values of the nonnegative weights μ and $(1-\mu)$. The resulting allocation depends on the relative weight μ/ϕ of the high-type. If $\phi \geq \mu$ (resp. $\phi \leq \mu$) the high type (resp. low type) receives full coverage and is indifferent to the contract received by the low type (resp. high type): $U(q\left(\theta^1\right), t\left(\theta^1\right); \theta^1) = U(q\left(\theta^0\right), t\left(\theta^0\right); \theta^1)$ (resp. $U(q\left(\theta^1\right), t\left(\theta^1\right); \theta^0) = U(q\left(\theta^0\right), t\left(\theta^0\right); \theta^0)$). To fix an (individually rational) allocation, the surplus can be divided among the two types (under some distributional conditions). Figure 1 depicts the Wilson-Miyazaki-Spence (WMS) second-best allocation $(q_\theta^{WMS}, t_\theta^{WMS})$ for $\theta \in \{\theta^0, \theta^1\}$, in which $\mu = 0$, that is, utility of the low risk type is maximized.

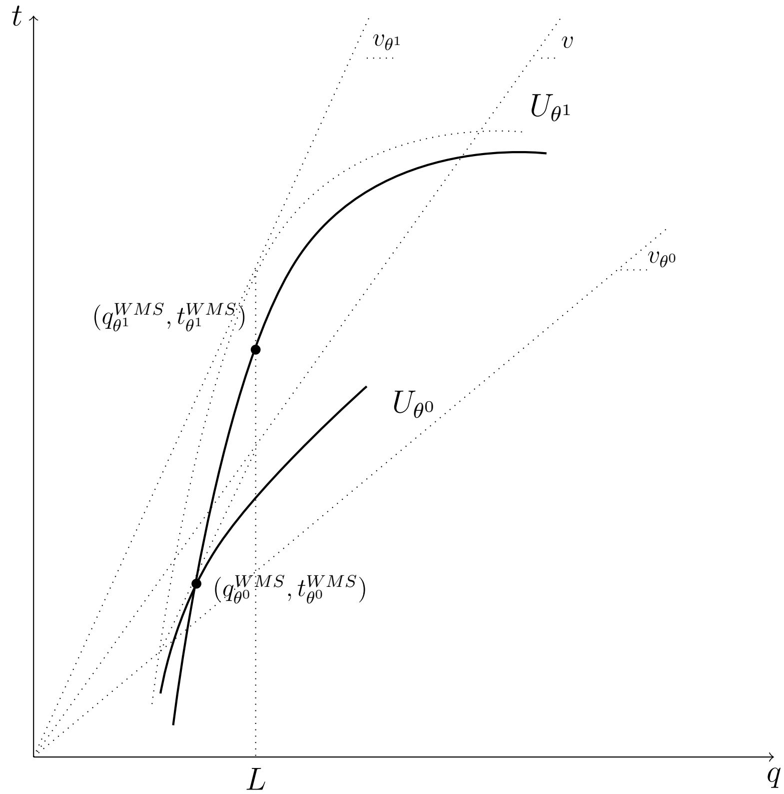

Figure 1: The WMS Allocation

3 The Strategic Approach

This section revisits some recent extensions of the Mechanism Design approach. Specifically, we consider markets subject to incomplete information in which several parties have the power to propose incentive schemes. Our aim is then twofold. First, to investigate to what extent the design of an optimal trading mechanisms by a single principal is affected by the presence of the mechanisms posted by his competitors. Second, to propose a novel approach to the study of such markets, taking into account *both* the relevant informational frictions and the decentralized nature of the contracting process. To this extent, we frame our analysis in the context of an extensive form game in which several principals compete over mechanisms in the presence of several privately informed agents. The next paragraphs introduce a general version of this game and provide a theoretical reference for the economic applications analyzed in the remaining of the section.

3.1 The Model

We refer to a scenario in which several principals (indexed by $j \in \mathcal{J} = \{1, \ldots, J\}$) contract with several agents (indexed by $i \in \mathcal{I} = \{1, \ldots, I\}$). Each agent i has private information about her type $\theta^i \in \Theta^i$ and $\theta = \{\theta^1, \cdots, \theta^I\} \in \Theta = \underset{i \in \mathcal{I}}{\times} \theta^i$ is a random variable with distribution F.

Each principal j may choose an action $x_j \in X_j$. Agents take no actions, except for their participation decisions, with $a^i_j \in \{Y, N\}$ being the decision of agent i to participate with principal j, in which $\{N\}$ stands for not participating, and we let $a^i = (a^i_1, a^i_2, ..., a^i_J)$. We also take $v_j : X \times A \times \Theta \to \mathbb{R}_+$ and $u^i : X \times A \times \Theta \to \mathbb{R}_+$ to be the payoff to principal j and to agent i, respectively, with $X = \underset{j \in \mathcal{J}}{\times} X_j$ and $A = \underset{i \in \mathcal{I}}{\times} A^i$. For a given array of agents' types θ, of actions $a = \left(a^1, a^2, \ldots, a^I\right)$ and of principals' decisions $x = (x_1, x_2, \ldots, x_J)$, the payoffs to agent i and to principal j are $u^i(x, a, \theta)$ and $v_j(x, a, \theta)$, respectively.

Each principal perfectly observes the set of agents who participate with him. Communication is one-sided: each agent i may send a private message $m^i_j \in M^i_j$ to principal j. We let each set M^i_j be sufficiently rich to include the element $\{\emptyset\}$ corresponding to the information "agent i does not communicate with principal j", and to satisfy the standard size restriction $\sharp M^i_j > \sharp \Theta^i$ for every i and j. Principal j takes his decisions contingent on the array of messages m_j he receives, with $m_j = \left(m_j^1, m_j^2, \ldots, m_j^I\right) \in M_j = \underset{i\in\mathcal{I}}{\times} M_j^i$, and on the participation choices of the agents. Formally, we say that a mechanism proposed by principal j is the measurable mapping $\gamma_j : M_j \times \{Y, N\}^{\sharp\mathcal{I}} \to \Delta(X_j)$. We take Γ_j to be the set of mechanisms available to principal j and denote $\Gamma = \underset{j\in\mathcal{J}}{\times} \Gamma_j$. All relevant sets are taken to be compact and measurable with respect to the topology of weak convergence. The competing mechanism game relative to Γ begins when principals publicly and simultaneously commit to mechanisms.

Given the posted mechanisms $(\gamma_1, \gamma_2, \ldots, \gamma_J) \in \Gamma$ and their privately observed types, agents simultaneously take a participation and a communication decision with respect to every principal. In this incomplete information game, a strategy for principal j is a $\gamma_j \in \Gamma_j$, and $\gamma = (\gamma_1, \cdots, \gamma_J) \in \Gamma$ is a profile of strategies for principals.

A strategy for each agent i associates to every profile of posted mechanisms γ a joint participation and communication decision. In a pure strategy, every agent participates with a subset of principals and sends a non-degenerate message only to the principals she participates with. We let $S^i = \left\{s^i \in M^i \times A^i : m_j^i = \emptyset \text{ iff } a_j^i = \{N\}\right\}$ be the strategy set for agent i, with $A^i = \left\{a^i = (a_1^i, \ldots, a_J^i) \in \{Y, N\}^{\sharp\mathcal{J}}\right\}$ and $M^i = \underset{j\in\mathcal{J}}{\times} M_j^i$ representing the sets of participation and communication decision, respectively. Given a profile γ of posted mechanisms, a strategy for agent i is then the measurable mapping $\sigma^i \equiv (\widetilde{m}^i, \widetilde{a}^i) : \Gamma \times \Theta^i \to S^i$, with $\widetilde{m}^i(\gamma, \theta^i) \in M^i$ and $\widetilde{a}^i(\gamma, \theta^i) \in A^i$. Every $\sigma(\gamma, \theta) = \left(\sigma^1(\gamma, \theta^1), \ldots, \sigma^I\left(\gamma, \theta^I\right)\right)$ induces principal j decision $\gamma_j(\sigma(\gamma, \theta)) \in X_j$ and $\gamma(\sigma(\gamma, \theta)) \in X$. The expected payoff to each type θ^i of agent i is:

$$\int_{\theta^{-i}\in\Theta^{-i}} u_i(\gamma(\sigma(\gamma, \theta)), \widetilde{a}(\gamma, \theta), \theta^i, \theta^{-i}) dF(\theta_{-i} \mid \theta_i)$$

with $F(\theta^{-i}|\theta^i)$ being the conditional probability of θ^{-i} given θ^i. The expected payoff to principal j when he plays γ_j against his opponents' strategies γ_{-j} is:

$$V_j(\gamma_j, \gamma_{-j}, \sigma) = \int_{\theta\in\Theta} v_j(\gamma(\sigma(\gamma, \theta)), \widetilde{a}(\gamma, \theta), \theta)\ dF(\theta).$$

The strategies (γ, σ) constitute a Perfect Bayesian equilibrium relative to Γ if σ is a continuation equilibrium for every γ and if, given γ_{-j} and σ, for every $j \in \mathcal{J}$: $\gamma_j \in \underset{\gamma'_j \in \Gamma_j}{argmax}\, V_j\left(\gamma'_j, \gamma_{-j}, \sigma\right)$.

Existence of a (potentially mixed) Perfect Bayesian Equilibrium of an arbitrary game Γ has been established by Page and Monteiro (2003) for the multiple agent case, and by Carmona and Fajardo (2009) for the single agent one. We focus here on characterization results. That is, we investigate how equilibrium outcomes are affected by the set of mechanisms made available to principals. In this respect, the following remarks will be useful in the remaining of the analysis.

Remark 1 *A mechanism available to principal j is direct if agents can only communicate their types to principal j, i.e. if $M_j^i = \Theta^i \cup \{\emptyset\}$ for every i, with $\{\emptyset\}$ representing no communication. We denote a direct mechanism for principal j as $\tilde{\gamma}_j : \underset{i \in \mathcal{I}}{\times} (\Theta^i \cup \{\emptyset\}) \times \{Y, N\}^{\sharp \mathcal{I}} \to \Delta(X_j)$ and the set of direct mechanisms as $\Gamma_j^D \subseteq \Gamma_j$. We let G^Γ be the competing mechanism game induced by a given Γ, and G^D the game in which principals are restricted to direct mechanisms. As in Myerson (1982), a direct mechanism is incentive compatible from the point of view of principal j if, given the mechanisms offered by the other principals, it induces a continuation equilibrium in which agents truthfully reveal their types to him. A direct mechanism $\tilde{\gamma}_j$ can therefore be incentive compatible for a given array $\tilde{\gamma}_{-j}$, but not for some other $\tilde{\gamma}'_{-j} \neq \tilde{\gamma}_{-j}$. An equilibrium is truth-telling if every principal posts an incentive compatible mechanism and agents truthfully reveal their private information to the principals they participate with, whenever this constitutes an equilibrium in their continuation game.*

Remark 2 *The model does not put any specific structure on the agents' message spaces $(M^i)_{i=1}^I$. One is therefore led to ask to what extent the corresponding equilibrium characterization depends on the available modes of communication. Indeed, an agent's report to a given principal may convey information about other principals' mechanisms and this information can be strategically exploited. This suggests that relying on a straightforward application of the Revelation Principle, by restricting agents to only reveal their (exogenous) private information, may involve a loss of generality. In this perspective, Epstein and Peters (1999) are the first to provide a canonical definition of the set of agents' types to which the Revelation Principle should apply. This set includes the agents' physical types and a component of market information, which is rich enough to describe what competitors would do* *under all kinds of different circumstances. Despite its relevance in terms of generality, the result also documents a fundamental difficulty in relying on simple direct mechanisms. These mechanisms indeed turn out to be too complex to be of practical use in applications.*

Remark 3 *The model focuses on "ordinary" contracting games: principals cannot design their mechanism contingent on the proposals of their rivals. An alternative possibility would be to explicitly let them commit to write "contractible" contracts, i.e. contracts that explicitly refer to each other, as done by Peters and Szentes (2012) and Szentes (2015). Such an approach, in turn, would require each of the principals to be able to monitor the entire contracting process, including all relevant off equilibrium threats.*

3.2 Applications

Our general model encompasses several economic approaches to competition in markets subject to incomplete information, as we illustrate below.

Example 3: Trading under Adverse Selection

The simplest application of our setting features the trade between J buyers and I sellers. Each seller is endowed with one unit of a perfectly divisible good. Let q_j^i be the quantity of the good purchased by buyer j from seller i, and t_j^i the transfer he makes in return. The feasible trades $((q_1^1, t_1^1), \ldots, (q_J^1, t_J^1), \ldots, (q_1^I, t_1^I), \ldots, (q_J^I, t_J^I))$ are such that $\sum_j q_j^i \leq 1$ for all i. As in Samuelson (1984) and Myerson (1985), the profit to seller i from trading $(Q^i, T^i) = \left(\sum_j q_j^i, \sum_j t_j^i\right)$ in the aggregate is $T^i - \theta^i Q^i$, where θ^i is seller i's opportunity cost of giving away her endowment. Each buyer j's profit from trading $(\sum_i q_j^i, \sum_i t_j^i)$ is $\sum_i [v(\theta^i) q_j^i - t_j^i]$; thus, he directly cares of the identity of the sellers he is trading with through the common value component $v(\theta)$. Each seller is privately informed of her opportunity cost. As first pointed out by Akerlof (1970), in such circumstances trade is typically threatened by adverse selection whenever $v(\theta)$ increases with θ, since offering to trade at a given price then only attracts the lowest qualities. In this context, a mechanism γ_j for buyer j associates a profile of individual trades to each array of messages he receives from sellers. Mas-Colell et al. (1995) illustrate how this setting can be naturally exploited to model competition in several market scenarios, and Attar et al. (2011) provide a fully strategic formulation of the multiple-buyer multiple-seller game.

Example 4: Competitive Screening

In their canonical analysis of the insurance market, Rothschild and Stiglitz (1976) study strategic competition between intermediaries for the exclusive right to serve a customer facing a binary risk on her endowment $w \in \{w_L, w_H\}$, with probabilities $(\theta, 1-\theta)$ that constitute her private information. Her (expected) payoff is $pu(w_L+d_L)+(1-p)u(w_H+d_H)$, with $(d_L, d_H) \in \mathbb{R}^2$ being the state-contingent transfers issued by the company she trades with. Similarly, market-microstructure models in the tradition of Glosten (1994) consider several market makers who compete to sell shares of a risky asset to a single insider who can trade with any subset of them (Biais et al. (2000) and Back and Baruch (2013)). The private information θ of the buyer is her willingness to trade the asset. When trading an aggregate quantity Q against an aggregate transfer T, the buyer's payoff is $\theta Q - \frac{\sigma^2}{2} Q^2 - T$, with $\sigma > 0$. The sellers are risk-neutral and the cost of selling a share of the asset to type θ is its expected value conditional on the insider's being of type θ. The model of this section can hence be interpreted in terms of competitive screening by letting $I = 1$, $\theta \equiv (p, 1-p)$ and $\tilde{\gamma}_j : \theta \times \{Y, N\} \to \mathbb{R}^2$.

Example 5: Competing Auctions

In a seminal paper, McAfee (1993) analyzes sellers who compete over auctions when buyers' valuation constitute their private information. In these settings, sellers simultaneously and anonymously post their reservation prices and buyers choose at most one auction to participate in. A seller and the buyers who participate in his auction form an isolated corporation. In addition, sellers are restricted to post direct mechanisms, asking each buyer $i \in \mathcal{I}$ to report her valuation $v^i \in [0,1]$. A strategy for seller j is a mechanism $\tilde{\gamma}_j : |I_j| \times [0,1]^{|I_j|} \to \mathbb{R}$, where $I_j \subseteq I$ is the set of buyers that participate in auction j. A pure strategy for buyer i is a mapping $\lambda^i : \Gamma_1 \times ... \times \Gamma_J \times [0,1] \to A^i \times [0,1] \times \mathbb{R}_+$, with Γ_j being set of second-price auctions for $j \in J$. Given her participation decision, it is always a dominant strategy for each of the buyers to truthfully report their private valuations. Specifically, the model of this section adapts to the competing auctions settings of Peters (1997), Peters and Severinov (1997), Burguet and Sakovics (1999), Viràg (2010), Han (2015), Peck (2015).

3.3 Equilibrium Trades

We start by taking a normative perspective. In this respect, the following paragraphs analyze a traditional issue: do second best allocations can be supported as equilibrium outcomes of our competitive setting? A positive answer to this question would provide novel insights towards a reformulation of the second welfare theorem for incomplete information economies.

3.3.1 The multiple agent case

In the recent years, providing a full characterization of the set of equilibrium allocations of games in which several principals have the power to design mechanisms has become a relevant issue in mechanism design. With reference to the class of extensive form games described in Section 3.1, Yamashita (2010) establishes a folk theorem: an allocation is implementable if and only if it is incentive compatible and the payoff of each principal is above a well chosen threshold value.

We illustrate the logic underlying his result in the context of trading under adverse selection (Example 3), assuming that the type of each seller can be either low, $\theta = \theta^1$, or high, $\theta = \theta^0$, for some $\theta^0 > \theta^1 > 0$. To further simplify the exposition, we assume that the quality of the good increases with the type of the seller, that is, $v(\theta^0) > v(\theta^1)$, and that it would be efficient to trade no matter the type of the seller, that is, $v(\theta) > \theta$ for each θ. Finally, to avoid trivial cases, we assume that $x \equiv prob[\theta = \theta^0] \in (0, 1)$. Suppose that sellers communicate with buyers through the message spaces $M^1 = ... = M^i = ... = M^I = \{\theta^1, \theta^0, m\}$. We first show that a monopolistic outcome for buyers can be supported in a pure strategy equilibrium. More precisely, let p^m the price that would be optimally set by a monopsonistic buyer.[13] Suppose now that each buyer commits to buy any quantity from each of the sellers at a constant unit price equal to p^m, unless at least $I - 1$ sellers send him the message m. In this last contingency, and for every profile of sellers' participation decisions, he offers to buy a quantity of one from each of the sellers at a constant unit price $p = v(\theta^0)$ so to maximize the sellers'

[13] As shown by Samuelson (1984) and Myerson (1985), who extensively analyze this trading setting in the case $I = J = 1$, an optimal mechanism for the buyer involves determining a given price p^m at which he stands ready to trade any quantity between 0 and 1. Indeed, given bilateral linearity of preferences, the buyer cannot further increase his profit by designing a direct revelation mechanism $\tilde{\gamma} : [\theta^1, \theta^0] \rightarrow [0, 1] \times \mathbb{R}_+$ which prescribes a quantity and a transfer for each revealed type.

surplus. It is straightforward to check that p^m is supported at equilibrium by having all sellers whose type is $\theta \leq p^m$ trading a quantity of one with the same buyer, and those such that $\theta > p^m$ staying out of the market. Equilibrium are such that each seller, irrespective of her type, sends the message m to each nondeviating buyer she participates with in the subgame following a buyer's unilateral deviation, which constitutes a continuation equilibrium. A similar reasoning guarantees that every incentive compatible allocation can be supported at equilibrium.

Unfortunately, the analysis in Yamashita (2010) does not allow to derive a full equilibrium characterization in general settings. A main drawback of his theorem is that threshold values are not identified in terms of the primitives of the game. Although several works have recently tried to overcome this difficulty, we still lack a general characterization result for the class of incomplete information games analyzed in this section.[14] We do not attempt at filling this gap here, but rather at pointing out two implications of Yamashita's insights that may be relevant for the economic applications of competing mechanism games.

The first implication can be derived from the example above. The reasoning crucially exploits the fact that each buyer uses a message space which is "larger" than each seller's type space $\{\theta^1, \theta^0\}$. Indeed, the message m is used out of equilibrium to deter any profitable deviation by his rivals. Incentive compatible mechanisms, as those identified in Remark 1, are actually not rich enough to reproduce the same threats. That is, if one considers the simpler game in which buyers are only allowed to post direct mechanisms, there always exists a profitable deviation for (at least) one buyer against any equilibrium supporting the price p^m. As already documented in earlier examples (see Peck (1997), Peters (2001), and Martimort and Stole (2002)), restricting buyers to use incentive compatible mechanisms involves a loss of generality: there exist *pure strategy* equilibrium outcomes of a game in which they post indirect mechanisms that cannot be reproduced by incentive compatible ones.[15] The result suggests that the equilibrium predictions of competing mechanism models crucially depend on the set of instruments that are available to competitors. At the same time, but from a more applied standpoint, it calls for the identification of a simple class of mechanisms that

[14]See Szentes (2009), Peters and Troncoso-Valverde (2013), Xiong (2013), and the survey of Peters (2014).

[15]This result is typically acknowledged as a "failure of the revelation principle" in competing mechanism games.

allows to characterize meaningful equilibria. Specifically, we say that a set of mechanisms is "robust" if the corresponding equilibria survive to principals' unilateral deviation towards any indirect mechanism. That is, the corresponding outcomes are supported in an equilibrium of the game in which principals use arbitrary communication mechanisms.

Characterizing the equilibrium outcomes supportable by "robust" mechanisms is relevant for several economic applications. Indeed, economic models of competing mechanisms typically restrict attention to simple incentive compatible mechanisms. That is, principals commit to message-contingent decisions which induce agents to truthfully reveal their exogenous private information. It is therefore natural to investigate in which contexts such incentive compatible mechanisms end up being robust. A case in point is provided by competing auctions settings. In a pioneering work, Peters (1997) shows that when every seller offers a second price auction with reserve price equal to his cost, none of his rivals can improve his profits by deviating to an alternative direct mechanism when the number of sellers gets large. The recent work of Han (2015) extends the result by establishing the robustness of second price auctions against any arbitrary mechanism. Key to his argument is the fact that second price auctions are dominant strategy incentive compatible. This guarantees that a best reply of a single principal to a given profile of mechanisms posted by his opponents can be characterized by an incentive compatible mechanism. One should however appreciate that the result does not hold in general: Attar et al. (2012) show that two-sided communication is needed to obtain a full characterization of principals' best replies. Their analysis stresses the fact that, whenever principals compete in face of privately informed agents, then, from the viewpoint of a single principal, the messages that agents send to his rivals can be seen as hidden actions. Given the profile of mechanisms proposed by his opponents, a principal behaves as if he was interacting with several agents that can take some non-contractible actions, i.e. the messages they send to the other principals. It is hence possible to show, along the lines of Myerson (1982), that he can gain by using mechanisms that induce agents to correlate on the messages they send to his opponents.

The second relevant implication of Yamashita (2010)'s work can be described as follows. His folk-theorem-like result exploits the presence of several, at least three, agents. Yet the strategic settings in which several principals compete to serve a single agent are at the centre-stage of several economic applications. It is therefore natural to investigate to what extent the decentralization of second best allocations may be successfully performed in such a restricted scenario. We perform this task in the following paragraphs.

3.3.2 The single agent case

We analyze here the situation in which principals compete in the presence of a single agent, that is, $I = 1$. It might be useful to analyze this setting in the context of competitive screening (Example 4), in which a risk-averse agent allocates her consumption over two states of nature by purchasing coverage from several risk-neutral sellers. Specifically, we refer to the insurance framework introduced in Section 1.2.3, and we aim at providing a full strategic analysis of the competition between $J \geq 2$ sellers. In this context, the payoff to the single consumer depends on the total coverage she raises from sellers and on the total premium she provides in exchange. It is therefore useful to denote $Q = \sum_{j \in \mathcal{J}} q_j$, and $T = \sum_{j \in \mathcal{J}} t_j$, with (q_j, t_j) being the coverage-premium pair she trades with seller j. Thus, considering again (1), we let $U(Q, T; \theta)$ be the payoff to type $\theta \in \{\theta^0, \theta^1\}$ when purchasing the *aggregate* coverage Q against the *aggregate* premium T, and to refer as the quadruple $\big((Q_{\theta^0}, T_{\theta^0}), (Q_{\theta^1}, T_{\theta^1})\big)$ as an aggregate allocation.

We frame the competitive provision of insurance in the context of a competing mechanism game. Thus, an action x_j, or contract, available to principal j is a coverage-premium pair $(q, t) \in \mathbb{R}^2_+$, and the no-trade contract is $(0, 0)$. The restriction to a single agent setting allows us to simplify the extensive form game described in Section 3.1. In such a scenario, as established by Martimort and Stole (2002) and Peters (2001), every equilibrium outcome of each game Γ can be supported at equilibrium in a "simpler" game in which principals are restricted to post arbitrary menus of contracts, with the agent choosing one item in each menu upon privately observing her type.[16] Our corresponding *menu game* unfolds as follows: first nature selects the consumer's type, then principals simultaneously post menus of coverage-premium pairs. Finally, the agent optimally takes her participation decision and picks one item in the menu of each principal.[17] We make no specific assumption on

[16] The result is often acknowledged as the Delegation Principle.

[17] A simple way to incorporate the consumer's participation decisions in the analysis is to impose that every menu of each intermediary must include the no-trade contract $(0, 0)$. The decision not to participate with principal j therefore corresponds to choosing the item $(0, 0)$ in his menu.

the structure of the sets of available menus. We only require that they are compact sets so that each type's choice problem admits a solution. To cope with standard applications of competitive screening, we restrict attention to pure strategy Perfect Bayesian Equilibria (PBE). Equilibrium trades are threatened by the following conflict. Given single crossing, the riskier type of the consumer is willing to purchase a higher amount of insurance. Yet, given common values, intermediaries are rather willing to sell higher quantities to the less risky type θ^0. The tension between two forces is at the root of adverse selection, and may have destabilizing effects on market equilibria.

Key to our analysis is to specify the agent's participation decisions. Specifically, we say that competition is exclusive if, in a pure strategy, the agent participates with *at most* one principal, in which case she is allowed to pick *at most* one item different from $(0, 0)$, and that competition is nonexclusive otherwise. In single agent contexts, these two market structures deliver different implications for decentralization. We discuss them in the following paragraphs.

Exclusive Competition

If competition is exclusive, our analysis mirrors that originally developed by Rothschild and Stiglitz (1976).[18] Consider the aggregate allocation $RSW = \Big((Q_{\theta^0}, T_{\theta^0}), (Q_{\theta^1}, T_{\theta^1})\Big)$ such that $T_\theta = v_\theta Q_\theta$ for each $\theta \in \{\theta^0, \theta^1\}$ and

$$\tau_\theta(Q_{\theta^1}, T_{\theta^1}) = v_{\theta^1} \quad (2)$$
$$U_{\theta^1}(Q_{\theta^1}, T_{\theta^1}) = U_{\theta^1}(Q_{\theta^0}, T_{\theta^0}). \quad (3)$$

The first condition states that the riskier type θ^1 purchases her first best allocation, and the second one that type θ^0 has to pay a cost to signal her effective quality, which corresponds to a binding IC constraint for θ^1. The allocation is "competitive" in the sense that each intermediary gets a zero profit on each consumer's type. The allocation above is the only candidate to be supported in a pure strategy equilibrium of the exclusive competition game.[19] The allocation is depicted on Figure 2, which represents the buyer's and sellers' indifference curves.

[18]See Attar et al. (2016a) for a general analysis of competing mechanism games under exclusive competition.

[19]The proof of the result is rather standard (see for example Mas-Colell et al. (1995), pp. 460-465), and we do not include it here to ease exposition.

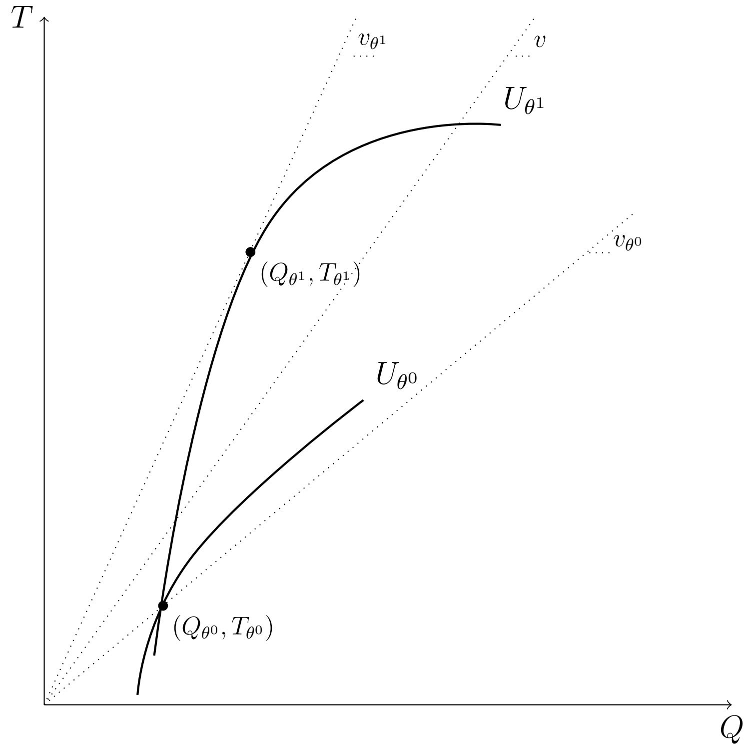

Figure 2: The Exclusivity Outcome

As first acknowledged by Rothschild and Stiglitz (1976), however, pure strategy equilibria fail to exist in a robust number of circumstances. In these cases, a single intermediary can profitably deviate by attracting both types of the consumer in such a way that his gains on the less risky type θ^0 more than offset the losses on the riskier one θ^1.[20]

Nonexclusive Competition

Allowing for nonexclusive competition crucially modifies the strategic behavior of intermediaries. On the one hand, a larger set of deviations becomes available. Indeed, each seller can exploit the offers of his rivals by proposing insurance contracts that the consumer may use to complement her coverage. In principle, this creates new opportunities for undercutting. On the

[20]See Fagart (1996) and Luz (2016) for a full characterization of the conditions needed to guarantee existence of a pure strategy equilibrium.

other hand, each intermediary may exploit the consumer as a coordinating device to possibly prevent his rivals' deviations. This is done by introducing additional threats which take the form of *latent* contracts in one's competitors' menu. The interplay of these two forces dramatically shapes the set of equilibrium allocations with respect to the benchmark scenario in which exclusivity clauses are enforced from the outset.

The recent work of Attar et al. (2014) provides a full equilibrium analysis of the nonexclusive menu game. In general terms, they show that nonexclusivity worsens the impact of adverse selection, and pure strategy equilibria necessarily feature the market breakdown emphasized by Akerlof (1970). In a simple two-type setting, a positive level of trades for one type of the consumer only obtains if the other type does not trade at all. We revisit their arguments in the following paragraphs. We start by establishing the following

Lemma 2 *The RSW allocation cannot be supported at equilibrium in the nonexclusive menu game.*

The intuition for the result can be easily understood in a free entry equilibrium.[21] Consider then an inactive intermediary and suppose that he deviates by offering, together with the null contract (0, 0), the additional contract $(q,t) = (\varepsilon, \varepsilon\chi)$ with ε strictly positive and $\chi \in \Big(v_{\theta^1}, \tau_{\theta^1}(Q_{\theta^0}, T_{\theta^0})\Big)$.[22] It is immediate to check that, since $\chi < \tau_{\theta^0}(Q_{\theta^0}, T_{\theta^0})$, ε can be chosen small enough to guarantee that $U_{\theta^1}\Big(Q_{\theta^1} + \varepsilon, T_{\theta^1} + \varepsilon\chi\Big) > U_{\theta^1}\Big(Q_{\theta^1}, T_{\theta^1}\Big) = U_{\theta^1}\Big(Q_{\theta^0}, T_{\theta^0}\Big)$, which ensures that type θ^1 will be trading the contract (q,t). One should observe that, since $\chi > v_{\theta^1}$, we get $t - v_{\theta^1}q = \varepsilon(\chi - v_{\theta^1}) > 0$. That is, the entrant earns a strictly positive profit on the high risk type θ^1. Given that $v_{\theta^1} > v_{\theta^0}$, the deviation is a fortiori profitable if the deviating contract (q,t) is also traded by the low risk type θ^0. This deviation is illustrated on Figure 3.

[21] See Proposition 1 of Attar et al. (2014) for a general argument.

[22] Since the marginal rate of substitution is decreasing along a given indifference curve, and provided that $Q_{\theta^1} > Q_{\theta^0}$ by single crossing, one gets $\tau_{\theta^1}(Q_{\theta^0}, T_{\theta^0}) > \tau_{\theta^1}(Q_{\theta^1}, T_{\theta^1}) = v(\theta^1)$, which guarantees that the interval is non-empty.

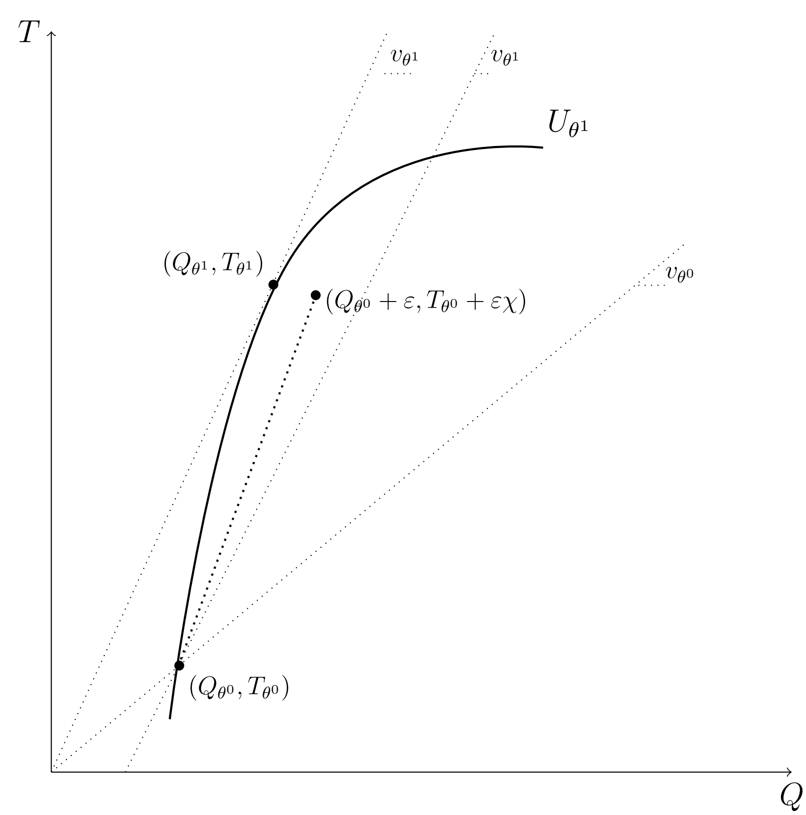

Figure 3: The RSW Allocation is *not* an equilibrium under nonexclusivity

Thus, the deviation exploits the possibility for sellers to attract the *high*-risk type θ^1 by proposing her to trade additional insurance on top of that chosen by the *low*-risk type θ^0. The profitability of any such deviation guarantees that in any separating equilibrium one should have $\frac{T_{\theta^1}-T_{\theta^0}}{Q_{\theta^1}-Q_{\theta^0}} = v_{\theta^1}$. That is, the IC constraint of type θ^1 turns out *not* to be binding. Furthermore, Attar et al. (2014) show that a positive level of trade for type θ^1 obtains at equilibrium only if type θ^0 is left out of the market. It follows that the only candidate to be supported in a pure strategy equilibrium of the nonexclusive menu game is the aggregate allocation $AMS = \Big((Q_{\theta^0}, T_{\theta^0}), (Q_{\theta^1}, T_{\theta^1})\Big)$ with $(Q_{\theta^0}, T_{\theta^0}) = (0, 0)$ and $(Q_{\theta^1}, T_{\theta^1})$ such that $\tau_{\theta^1}(Q_{\theta^1}, T_{\theta^1}) = v_{\theta^1}$, and $T_{\theta^1} = v_{\theta^1} Q_{\theta^1}$. Clearly, this allocation involves a strictly positive trade for the high-risk type θ^1 only if $\tau_{\theta^1}(0, 0) > v_{\theta^1}$. In the specific context of insurance, the AMS allocation is such that the high-risk type achieves her first best level of coverage, so that $Q_{\theta^1} = L$ and $T_{\theta^1} = v_{\theta^1} L$. Theorems 1 and 2 in Attar et al. (2014) show that a necessary and sufficient condition for existence of such an equilibrium is that, starting from the no-trade allocation (0,0), type θ^0 should *not* be willing to purchase insurance issued at the fair price $v = \phi v(\theta^1) + (1-\phi)v(\theta^0)$. More formally, they require that $\tau_{\theta^0}(0,0) \leq v$ which corresponds to the Akerlof's (1970) condition for a market breakdown in which only the worse-quality goods are traded. When this condition is not satisfied, at least one seller can profitably deviate by exploiting the consumer's ability to engage in multiple trades. To clarify this point, consider a candidate separating equilibrium in which aggregate trades are such that $Q_{\theta^0} > 0$. In this case, everything happens as if type θ^0 purchases the aggregate quantity Q_{θ^0}, and type θ^1 purchases it together with the additional insurance $Q_{\theta^1} - Q_{\theta^0}$ priced at the unit price v_{θ^1}.[23] Sellers may therefore engage in a Bertrand-like competition on the first layer, implying that Q_{θ^0} must be priced at v_{θ^0}. Overall, we get $T_{\theta^0} = Q_{\theta^0} v_{\theta^0}$, which guarantees zero profit to each of the sellers even though type θ^0 subsidizes type θ^1 at $(Q_{\theta^0}, T_{\theta^0})$. Now, since no seller is indispensable to provide the consumer with $(Q_{\theta^0}, T_{\theta^0})$, any of them actively trading with type θ^0 has a profitable menu deviation consisting of two nonzero contracts. The first contract, targeted at θ^0, is approximatively the same as the one the consumer trades with θ^0 on the candidate equilibrium path, and makes a profit when traded by type θ^0 only. The second contract, targeted at type θ^1, allows the consumer to purchase the second layer $Q_{\theta^1} - Q_{\theta^0}$ at a unit price slightly less than v_{θ^1}, and makes a small loss when traded by type θ^1. Because the seller now offers the second layer at slightly better terms than his competitors, it is optimal for θ^1 to trade it with him on top of the first layer Q_{θ^0} provided by the other competitors at unit price v. By deviating in this way, the seller almost neutralizes his loss with θ^1, while securing a profit with θ^0. This amounts to dumping bad risks on one's competitors by selling complementary coverage to type θ^1 slightly below the fair premium rate, and basic coverage to type θ^0 significantly above the fair premium rate.

[23] Clearly, this quantity is strictly positive only if $\tau_{\theta^1}(Q_{\theta^0}, T_{\theta^0}) > v_{\theta^1}$.

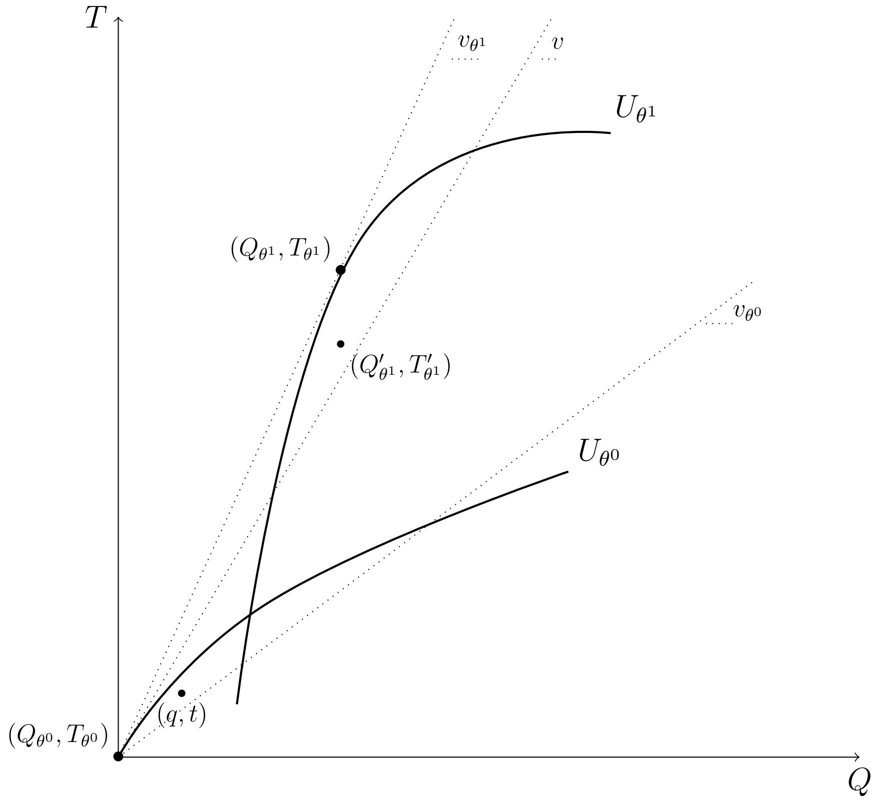

Figure 4: Equilibrium under Nonexclusivity

It remains to show that the AMS aggregate allocation can be supported at equilibrium. This is done in the following

Lemma 3 *If $\tau_{\theta^0}(0,0) \leq v$, then the AMS aggregate allocation can be supported in a pure strategy equilibrium of the nonexclusive menu game.*

An intuition for the proof can be provided along the following lines.[24] Consider the following profile of menus: each seller stands ready to provide any amount Q between 0 and an appropriately chosen upper bound $\overline{Q}$ at a unit price v_{θ^1}.[25] Clearly, trading $(Q_{\theta^0}, T_{\theta^0}) = (0, 0)$ and $(Q_{\theta^1}, T_{\theta^1}) = (L, v_{\theta^1} L)$ is the unique optimal choice for type θ^0 and θ^1, respectively, as shown in Figure

[24] See Attar et al. (2014) for a general analysis.

[25] The upper bounds is only introduced only to make sure that the corresponding menus of contracts are compact, which allows to rely on PBE as a solution concept.

4. Consider now sellers' deviations. Since $\tau_{\theta^0}(0,0) \leq v$, no seller can profitably deviate by attracting both types of the consumer. In addition, since type θ^1 gets her first best level of insurance, the only deviations to be considered are those that cream-skim type θ^0. Specifically, we say that any insurance contract $(q,t) \in CS \equiv \{(q,t) \in \mathbb{R}^2_+ : \frac{t}{q} > v_{\theta^0} \;\; and \;\; U_{\theta^0}(q,t) > U_{\theta^0}(0,0)\}$ constitutes a cream-skimming deviation. Given any such deviation, it is immediate to see that type θ^0 finds it optimal to purchase the corresponding contract (q,t). Yet, given equilibrium menus, also type θ^1 finds optimal to purchase the same contract because she can complement it with some insurance issued by non-deviating intermediaries. Indeed, as depicted in Figure 4, starting from (q,t) type θ^1 can buy additional insurance at price v_{θ^1} so to achieve the allocation $(Q'_{\theta^1}, T'_{\theta^1})$. By doing that, she gets the full insurance quantity $Q'_{\theta^1} = Q_{\theta^1}$ at a smaller unit price $T'_{\theta^1} < T_{\theta^1}$. Overall, the deviation is traded by both types, and it is therefore non profitable. Key to this reasoning is the possibility for type θ^1 to complement, at the deviation stage, any cream-skimming proposal with further trades provided by incumbent intermediaries. These additional opportunities for insurance, i.e. the availability of all quantities between 0 and $Q_{\theta^1} = L$, are usually denoted latent contracts. Despite not being traded at equilibrium, they should be issued to prevent some well chosen deviations, and to guarantee equilibrium existence.

Decentralization with a single agent: a discussion

We have shown in the previous paragraphs that, in standard single agent contexts, the possibility to enforce exclusive contracting has dramatic implications on equilibrium outcomes. We now evaluate the normative implications of this insight.

Recall first that incentive compatibility is the relevant notion of feasibility when the planner fully observe agents' trades. In such a benchmark situation, the planner is allowed to design incentive-compatible mechanisms while perfectly observing, and therefore being able to monitor, aggregate trades (Myerson (1979, 1982)). As documented in Section 2.2.3, restricting attention to the set of budget-balanced and incentive-compatible trading mechanisms, several works have provided a characterization of the second-best efficiency frontier for insurance economies (see Prescott and Townsend (1984) and Crocker and Snow (1985)). The corresponding allocations are regarded as a reference point to evaluate the performances of insurance markets in which intermediaries are able to enforce exclusivity of contracts. Indeed, as first shown by Crocker and Snow (1985), one can identify a set of conditions on agents' preferences guaranteeing that the *RSW* allocation belongs to the second best frontier.[26] Importantly, these conditions are necessary and sufficient for the existence of a pure strategy equilibrium in the Rothschild and Stiglitz (1976) economy. This in turn provides an instance of the first theorem of welfare economics under exclusive competition: any allocation supported in a pure strategy equilibrium is constrained (*second best*) efficient.

Under nonexclusive competition, however, no outside party can monitor the trades between the consumer and any subset of sellers. In general terms, little is known about how the opportunity for privately informed consumers to secretly sign bilateral agreements with sellers further restricts the set of allocations that are feasible to a planner. The recent work of Attar et al. (2016b) provides a first step in this direction. Specifically, they require feasible allocations to be not only incentive-compatible, but also robust to further trading opportunities provided by private sellers. That is, any price-quantity scheme, or tariff, posted by the planner must be entry-proof: no matter the offers subsequently made by an entrant, there is an optimal way for the buyer to combine these offers with the planner's tariff that prevents the entrant from making a profit.

To resume the findings in Attar et al. (2016b), it is useful to refer to an allocation, first identified by Jaynes (1978), Hellwig (1988) and Glosten (1994) which we therefore denote *JHG*. In this allocation, both types purchase the same basic coverage, which type θ^1 complements by purchasing additional coverage. A marginal version of Akerlof (1970) pricing holds: each layer of coverage is fairly priced given the types who purchase it, and the size of each layer is optimally chosen subject to this constraint. Thus, the first layer Q_{θ^1} is optimal for type θ^0 at unit price v,

$$Q_{\theta^0} \equiv \arg\max\{U_{\theta^0}(Q, vQ) : Q \geq 0\}, \tag{4}$$

$$T_{\theta^0} \equiv vQ_{\theta^0}. \tag{5}$$

Then the second layer $Q_{\theta^1} - Q_{\theta^0}$ is optimal for type θ^1 at unit price v_{θ^1}, given that she already purchases the first layer Q_{θ^0} at unit price v,

$$Q_{\theta^1} - Q_{\theta^0} \equiv \arg\max\{U_{\theta^1}(Q_{\theta^0} + Q, T_{\theta^0} + v_{\theta^1}Q) : Q \geq 0\}, \tag{6}$$

$$T_{\theta^1} - T_{\theta^0} \equiv v_{\theta^1}(Q_{\theta^1} - Q_{\theta^0}). \tag{7}$$

[26] See Bisin and Gottardi (2006) for a recent reformulation of these conditions.

The JHG allocation is depicted in Figure 5 below.

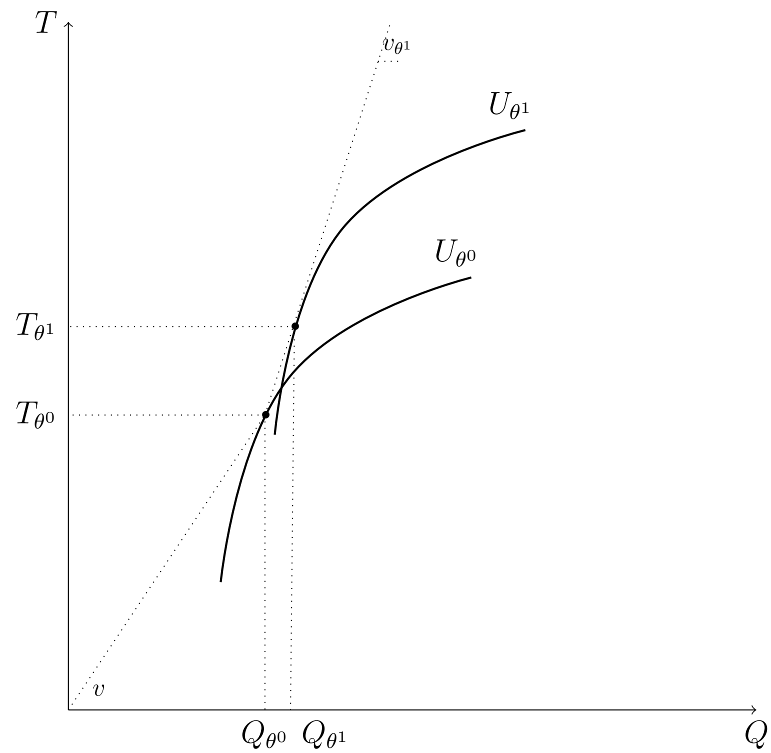

Figure 5: The JHG allocation.

Clearly, the JHG allocation $((Q_{\theta^0}, T_{\theta^0}), (Q_{\theta^1}, T_{\theta^1}))$ makes zero expected profit. However, because the coverage Q_{θ^1} is sold at the average premium rate $v > v_{\theta^0}$, type θ^0 subsidizes type θ^1. This allocation plays a key role in the set of incentive compatible allocations. Specifically, Theorem 1 in Attar et al. (2016b) establish the following result

Lemma 4 *The JHG allocation is the unique budget-balanced allocation implementable by an entry-proof tariff.*

That is, the threat of entry severely limits the scope for redistribution: the planner is constrained by his inability to control the buyer's trades with a potential entrant, as the threat of such trades effectively deprives him of any possibility to transfer utility between the two types. The set of feasible allocations is a singleton. This contrasts with the multiplicity of second-best allocations, which, as discussed earlier, form a nondegenerate frontier. The intuition for the proof of Lemma 4 is as follows. Firstly, to prevent entry, one should have $T_{\theta^0} \leq vQ_{\theta^0}$ and $T_{\theta^1} \leq T_{\theta^0} + v_{\theta^1}(Q_{\theta 1} - Q_{\theta^0})$. Indeed, violating the first inequality would make it profitable for an entrant to profitably attract both types on a contract of unit price above v. Violating the second one would make profitable for an entrant to profitably attract type θ^1 on a contract of unit price above v_{θ^1} which she might combine with $(Q_{\theta^0}, T_{\theta^0})$. Secondly, observe that, given that $T_{\theta^0} - vQ_{\theta^0} + \mu\Big[T_{\theta^1} - T_{\theta^0} - v_{\theta^1}(Q_{\theta^1} - Q_{\theta^0})\Big]$ by budget-balance, the two inequalities above can be satisfied together only when they hold as equalities. This shows that the JHG allocation is the only candidate to be implemented by an entry-proof tariff. In a next step, Attar et al. (2016b) prove existence of such a tariff by considering the convex price-quantity schedule

$$T(q) \equiv 1_{\{q \leq Q_{\theta 0}\}} vq + 1_{\{q > Q_{\theta 0}\}}[vQ_{\theta^0} + v_{\theta^1}(q - Q_{\theta^0})],$$

which is the analogue in our two-type setting of the tariff constructed by Glosten (1994) when demand is continuously distributed.

It is important to clarify the relationship between this allocation and the second best frontier for insurance economies analyzed by Crocker and Snow (1985) among others. In this respect, observe that, in the JHG allocation, only type θ^1 gets fully insured since $\tau_{\theta^1}(Q_{\theta^0}, T_{\theta^0}) > v_{\theta^1}$. In addition, the complementary coverage $Q_{\theta^1} - Q_{\theta^0}$ optimally traded by type θ^1 is strictly positive at the price v_{θ^1}. This in turn implies that her incentive-compatibility constraint is slack: $U_{\theta^1}(Q_{\theta^1}, T_{\theta^1}) > U_{\theta^1}(Q_{\theta^0}, T_{\theta^0})$, which guarantees that the JHG allocation does *not* belong to the second best frontier. To clarify this point, observe that, in a JHG allocation, one also has $\tau_{\theta^0}(Q_{\theta^0}, T_{\theta^0}) > v_{\theta^0}$, i.e. type θ^0 is underinsured. A planner with the ability to fully control trades can then complement the JHG allocation by proposing some additional coverage $(q_{\theta^1}, t_{\theta^1})$ at a premium rate $\frac{t_{\theta 1}}{q_{\theta^1}}$ between v_{θ^0} and $\tau_{\theta^0}(Q_{\theta^0}, T_{\theta^0})$, to be traded by type θ^0 only. As long as this additional amount of coverage is small enough, the relevant incentive constraint of type θ^1 would remain slack, inducing this type not to modify her behavior, and letting the planner achieve a positive (expected) budget surplus. This logic does not extend to the case in which the planner cannot perfectly control trades. In that case, any additional coverage $(q_{\theta^0}, t_{\theta^0})$, designed by the planner to attract type θ^0 alone, would be exploited by an entrant to propose further trades with type θ^1 at a premium rate slightly above v_{θ^1}. This would guarantee a profit to the entrant, and induce a deficit for the planner. Such a reasoning induces Attar et al. (2016b) to conclude that, under nonexclusive competition, the relevant binding incentive constraint for type θ^1 is $U_{\theta^1}(Q_{\theta^1}, T_{\theta^1}) = \max\{U_{\theta^1}(Q_{\theta^0} + Q, T_{\theta^0} + v_{\theta^1}Q) : Q \geq 0\}$, which states that she is indifferent between trading $(Q_{\theta^1}, T_{\theta^1})$ and trading $(Q_{\theta^0}, T_{\theta^0})$ along with contracts issued by an entrant at the fair price v_{θ^1}.

To conclude, we remark that, whenever $\tau_{\theta^0}(0,0) \leq v$, (4) and (5) imply that type θ^0 purchases no insurance in a JHG allocation, that is, $(Q_{\theta^0}, T_{\theta^0}) = (0,0)$. It hence follows from Lemma 3 that, when the nonexclusive menu game has a pure strategy equilibrium, the corresponding allocation is entry proof. A planner who is in the impossibility of controlling aggregate trades cannot therefore improve on such allocation without making profitable the entry of at least one seller. To the extent that this notion is interpreted as a form of constrained efficiency,[27] the result suggests a sense in which market equilibria may achieve efficient outcomes under nonexclusive competition.

References

Akerlof, G. A. (1970): "The Market for Lemons: Quality Uncertainty and the Market Mechanism," *The Quarterly Journal of Economics*, 84, 488–500.

Arrow, K. (1979): "The property rights doctrine and demand revelation under incomplete information," in *Economics and Human Welfare: Essays in Honor of Tibor Scitovsky*, Academic Press, New York.

Attar, A., E. Campioni, and G. Piaser (2012): "Two-sided communication in competing mechanism games," *Journal of Mathematical Economics*, 49, 62–70.

——— (2016a): "On Competing Mechanisms under Exclusive Competition," Mimeo, Toulouse School of Economics.

Attar, A., T. Mariotti, and F. Salanie (2011): "Non Exclusive Competition in the Market for Lemons," *Econometrica*, 79, 1869–1918.

[27] Under complete information, several works have identified the set of *third best* efficient allocations with those allocations implementable by an entry-proof tariff by the planner (see Kahn and Mookherjee (1998) and Bisin and Guaitoli (2004)).

——— (2014): "Nonexclusive Competition under Adverse Selection," *Theoretical Economics*, 9, 1–40.

——— (2016b): "Multiple Contracting in insurance Markets," Mimeo, Toulouse School of Economics.

Back, K. and S. Baruch (2013): "Strategic Liquidity Provision in Limit Order Markets," *Econometrica*, 81, 363–392.

Biais, B., D. Martimort, and J.-C. Rochet (2000): "Competing mechanisms in a common value environment," *Econometrica*, 78, 799–837.

Bisin, A. and P. Gottardi (2006): "Efficient Competitive Equilibria with Adverse Selection," *Journal of Political Economy*, 114, 485–516.

Bisin, A. and D. Guaitoli (2004): "Moral hazard with non-exclusive contracts," *Rand Journal of Economics*, 2, 306–328.

Burguet, R. and J. Sakovics (1999): "Imperfect competition in auction designs," *International Economic Review*, 40, 231–247.

Carmona, G. and J. Fajardo (2009): "Existence of equilibrium in common agency games with adverse selection," *Games and Economic Behavior*, 66, 749–760.

Cho, I.-K. and D. M. Kreps (1987): "Signaling Games and Stable Equilibria," *The Quarterly Journal of Economics*, 102, 179–221.

Cramton, P., R. Gibbons, and P. Klemperer (1987): "Dissolving a Partnership Efficiently," *Econometrica*, 55, 615–32.

Crémer, J. and R. P. McLean (1985): "Optimal Selling Strategies under Uncertainty for a Discriminating Monopolist When Demands Are Interdependent," *Econometrica*, 53, 345–61.

——— (1988): "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions," *Econometrica*, 56, 1247–1257.

Crocker, K. J. and A. Snow (1985): "The efficiency of competitive equilibria in insurance markets with asymmetric information," *Journal of Public Economics*, 26, 207–219.

d'Aspremont, C. and L.-A. Gérard-Varet (1975): "Incentive games with incomplete information: An application to a public input model," in *Colloque International de Théorie des Jeux*, Université Libre de Bruxelles.

——— (1979a): "Incentives and incomplete information," *Journal of Public Economics*, 11, 25–45.

——— (1979b): "On bayesian incentive compatible mechanisms," in *Aggregation and Revelation of preferences*, ed. by J.-J. Laffont, North Holland, Amsterdam.

——— (1982): "Bayesian incentive compatible beliefs," *Journal of Mathematical Economics*, 10, 83–103.

d'Aspremont C. and J. Crémer (2017): "Full surplus Sharing with or without free beliefs," CORE Discussion Paper (forthcoming).

Epstein, L. G. and M. Peters (1999): "A revelation principle for competing mechanisms," *Journal of Economic Theory*, 88, 119–160.

Fagart, M.-C. (1996): "Concurrence en contrats, anti-sélection et structure d'information," *Annals of Economics and Statistics*, 1–27.

Finkelstein, A. and J. Poterba (2002): "Selection Effects in the United Kingdom Individual Annuities Market," *Economic Journal*, 112, 28–50.

——— (2004): "Adverse Selection in Insurance Markets: Policyholder Evidence from the U.K. Annuity Market," *Journal of Political Economy*, 112, 183–208.

Forges, F. (1994): "Posterior Efficiency," *Games and Economic Behavior*, 6, 238–261.

Glosten, L. R. (1994): " Is the Electronic Open Limit Order Book Inevitable?" *Journal of Finance*, 49, 1127–61.

Griesmer, J., R. L. and M. Shubik (1967): "Toward a Study of Bidding Processes, Part IV - Games with Unknown Costs," *Naval Research Logistics Quaterly*, 14, 415–434.

Han, S. (2015): "Robust Competitive Auctions," *Economics Letters*, 137, 207–210.

Harris, M. and R. M. Townsend (1981): "Resource Allocation under Asymmetric Information," *Econometrica*, 49, 33–64.

Harsanyi, J. C. (1967): "Games with Incomplete Information Played by Bayesian Players, I-III Part I. The Basic Model," *Management Science*, 14, 159–182.

——— (1968): "Games with Incomplete Information Played by Bayesian Players Part II. Bayesian Equilibrium Points," *Management Science*, 14, 320–334.

Heifetz, A. and Z. Neeman (2006): "On the Generic (Im)Possibility of Full Surplus Extraction in Mechanism Design," *Econometrica*, 74, 213–233.

Hellwig, M. F. (1988): "A note on the specification of interfirm communication in insurance markets with adverse selection," *Journal of Economic Theory*, 46, 154–163.

Holmström, B. and R. B. Myerson (1983): "Efficient and Durable Decision Rules with Incomplete Information," *Econometrica*, 51, 1799–819.

Hurwicz, L. (1973): "The Design of Mechanisms for Resource Allocation," *American Economic Review*, 63, 1–30.

Jaynes, G. D. (1978): "Equilibria in monopolistically competitive insurance markets," *Journal of Economic Theory*, 19, 394–422.

Johnson, S., J. Pratt, and R. Zeckhauser (1990): "Efficiency Despite Mutually Payoff Relevant Private Information: the Private Value Case," *Econometrica*, 58, 873–900.

Kahn, C. M. and D. Mookherjee (1998): "Competition and Incentives with Nonexclusive Contracts," *RAND Journal of Economics*, 29, 443–465.

Kohlberg, E. and J.-F. Mertens (1986): "On the Strategic Stability of Equilibria," *Econometrica*, 54, 1003–37.

Kosenok, G. and S. Severinov (2008): "Individually rational, budget-balanced mechanisms and allocation of surplus," *Journal of Economic Theory*, 140, 126–161.

Kreps, D. M. and R. Wilson (1982): "Sequential Equilibria," *Econometrica*, 50, 863–94.

Krishna, V. (2010): *Auction Theory*, Academic Press.

Krishna, V. and M. Perry (1997): "Efficient Mechanism Design," Game theory and information, EconWPA.

Luz, V. F. (2016): "Characterization and uniqueness of equilibrium in competitive insurance," Mimeo, University of British Columbia.

Makowski, L. and C. Mezzetti (1994): "Bayesian and Weakly Robust First Best Mechanisms: Characterizations," *Journal of Economic Theory*, 64, 500–519.

Martimort, D. and L. A. Stole (2002): "The revelation and delegation principles in common agency games," *Econometrica*, 70, 1659–1673.

Mas-Colell, A., M. Whiston, and J. Green (1995): *Microeconomic Theory*, Oxford University Press.

McAfee, P. (1993): "Mechanism design by competing sellers," *Econometrica*, 61, 1281–1312.

McAfee, R. P. and P. J. Reny (1992): "Correlated Information and Mechanism Design," *Econometrica*, 60, 395–421.

Mirrlees, J. (1971): "An Exploration in the theory of optimum income taxation," *Review of Economics Studies*, 38, 175–208.

Myerson, R. (1985): "Analysis of Two Bargaining Problems with Incomplete Information," in *Game Theoretic Models of Bargaining*, ed. by A. Roth, Cambridge University Press, 59–69.

Myerson, R. B. (1981): "Optimal auction design," *Mathematics of Operations Research*, 6, 58–73.

——— (1982): "Optimal coordination mechanisms in generalized principal-agent problems," *Journal of Mathematical Economics*, 10, 67–81.

Myerson, R. B. and M. A. Satterthwaite (1983): "Efficient mechanisms for bilateral trading," *Journal of Economic Theory*, 29, 265–281.

Page, F. H. and P. K. Monteiro (2003): "Three principles of competitive nonlinear princing," *Journal of Mathematical Economics*, 39, 63–109.

Peck, J. (1997): "A note on competing mechanisms and the revelation principle," Mimeo, Ohio State University.

——— (2015): "Sky-lift Pricing with Imperfect Competition: An exercise in Competing Mechanisms," Mimeo, Ohio State University.

Peters, M. (1997): "A competitive distribution of auctions," *Review of Economic Studies*, 64, 97–123.

——— (2001): "Common Agency and the Revelation Principle," *Econometrica*, 69, 1349–1372.

——— (2014): "Survey on Competing Mechanisms 2 - The revelation principle," *Canadian Journal of Economics*, 47(2), 373–397.

Peters, M. and S. Severinov (1997): "Competition among sellers who offer auctions instead of prices," *Journal of Economic Theory*, 75, 141–179.

Peters, M. and B. Szentes (2012): "Definable and contractible contracts," *Econometrica*, 80, 363–411.

Peters, M. and C. Troncoso-Valverde (2013): "A Folk Theorem for Competing Mechanisms," *Journal of Economic Theory*, 147, 953–973, mimeo, University of British Columbia.

Philipson, T. and J. Cawley (1999): "An Empirical Examination of Information Barriers to Trade in Insurance," *American Economic Review*, 89, 827–846.

Prescott, E. and R. Townsend (1984): "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard," *Econometrica*, 52, 21–45.

Riley, J. G. (2001): "Silver Signals: Twenty-Five Years of Screening and Signaling," *Journal of Economic Literature*, 39, 432–478.

Riley, J. G. and W. F. Samuelson (1981): "Optimal Auctions," *American Economic Review*, 71, 381–92.

Rothschild, M. and J. Stiglitz (1976): "Equilibrium in Competitive Insurance Markets," *Quarterly Journal of Economics*, 90, 629–649.

Rustichini, A. and P. Siconolfi (2008): "General equilibrium in economies with adverse selection," *Economic Theory*, 37, 1–29.

Rysman, M. (2007): "An Empirical Analysis of Payment Card Usage - super-," *Journal of Industrial Economics*, 55, 1–36.

Samuelson, P. A. (1969): "Contrast between Welfare Conditions for Joint Supply and for Public Goods," *The Review of Economics and Statistics*, 51, 26–30.

Samuelson, W. (1984): "Bargaining under Asymmetric Information," *Econometrica*, 52, 995–1005.

Schlee, E. and H. Chade (2012): "Optimal insurance with adverse selection," *Theoretical Economics*, 7.

Spence, M. (1973): "Job Market Signaling," *The Quarterly Journal of Economics*, 87, 355–374.

Stiglitz, J. E. (1977): "Monopoly, Non-linear Pricing and Imperfect Information: The Insurance Market," *Review of Economic Studies*, 44, 407–430.

Szentes, B. (2009): "A note on "Mechanism games with multiple principals and three or more agents" by T. Yamashita," Mimeo UCL London.

——— (2015): "Contractible Contracts in Common Agency Problems," *Review of Economic Studies*, 82, 391–422.

Vickrey, W. (1961): "Counterspeculation, Auctions, And Competitive Sealed Tenders," *Journal of Finance*, 16, 8–37.

Viràg, G. (2010): "Competing Auctions: Finite Markets and Convergence," *Theoretical Economics*, 5, 241–274.

Wilson, R. B. (1967): "Competitive Bidding with Asymmetric Information," *Management Science*, 13, 816–820.

——— (1985): "Efficient Trading," in *Issues of Contemporary Microeconomics and Welfare*, ed. by G. Feywel, Albany: State University of New York Press, 856–893.

Xiong, S. (2013): "A Folk Theorem for Contract Games with Multiple Principals and Agents," Mimeo Rice University.

Yamashita, T. (2010): "Mechanism games with multiple principals and three or more agents," *Econometrica*, 78, 791–801.