2017/33

DP

Ana Mauleon, Simon Schopohl and Vincent Vannetelbosch

Competition for leadership in teams

CORE
CENTER FOR OPERATIONS RESEARCH AND ECONOMETRICS

CORE
Voie du Roman Pays 34, L1.03.01
Tel (32 10) 47 43 04
Fax (32 10) 47 43 01
Email: immaq-library@uclouvain.be
https://uclouvain.be/en/research-institutes/immaq/core/discussion-papers.html

Competition for leadership in teams

Ana Mauleon[*] Simon Schopohl[†] Vincent Vannetelbosch[‡]

November 2017

Abstract

We analyze a model of information centralization in teams where players can only exchange information through an endogenous network. Over several periods each player can either pass or not pass her information to her neighbors. Once one player has centralized all the information, all players receive some payoff. The winner who collects all the information gets an additional reward. Since each player discounts payoffs over time, she faces the dilemma of either letting another player centralizing all the information fast, or trying to collect all the information by herself and to overtake the leadership. We find that there is always a single winner who centralizes the information at equilibrium and that only minimally connected networks can be pairwise stable. We also characterize the winner and the duration for any network and for any discount factor. We show that the star network is always pairwise stable. More surprisingly, we find that even networks in which the centralization takes a long time can be pairwise stable.

Keywords : communication network; dynamic network game; information transmission; leadership; pairwise stability; team project.
JEL Classification : C72, C73, D83, D85.

[*]CEREC, Saint-Louis University – Brussels; CORE, University of Louvain, Louvain-la-Neuve, Belgium. E-mail: ana.mauleon@usaintlouis.be

[†]CEREC, Saint-Louis University – Brussels; CORE, University of Louvain, Louvain-la-Neuve, Belgium. E-mail: simon.schopohl@usaintlouis.be

[‡]CORE, University of Louvain, Louvain-la-Neuve; CEREC, Saint-Louis University – Brussels, Belgium. E-mail: vincent.vannetelbosch@uclouvain.be

1 Introduction

The objective of the paper is to develop a model of information centralization in teams where team members can only exchange information through a communication network and to predict which communication networks could emerge in the long run. There are n players arranged in a connected network and each player belonging to the team has a unique piece of information. The task is to centralize all the information. The team must centralize all the information facing some exogenously given deadline. In each period, each player decides either to hold on her information item or to pass on to her neighbors in the communication network. Players take their decisions sequentially following some decision order or hierarchical ranking. The timing in each period is as follows. Player 1 takes her decision first. Next, each player chooses his action knowing the choices made by the players preceding him in the ranking. After all players have selected an action, pass on or hold on, the information is passed according to their decisions. The decision order or hierarchical ranking can be interpreted as the leadership structure within the team. Hence, player 1 is the current leader, and so players, beside collaborating for collecting all the information, compete for becoming the winner and the new team leader and getting the reward. If there is no winner after the items have been transmitted, the next period starts. As soon as at least one player has centralized all the information the game ends and all players receive some positive payoff. The player who collects all the information gets an additional reward. If several players centralize all the information at the same time they split the reward equally. The players have time preferences and so the utility of each player depends on the time it takes until one player centralizes all the information and whether she gets the reward or not.

We find that there is always a single winner who centralizes the information at equilibrium. When the common discount factor is low, the player with the lowest eccentricity is the winner, where the eccentricity of a player is the geodesic distance from this player to the player furthest away from her in the communication network. All players prefer to centralize the information as fast as possible. The player with the lowest eccentricity can centralize the information fastest and so she wins. The duration is given by the number of periods it takes the winner to centralize the information and is then equal to the minimal eccentricity (i.e. the radius) of the network. When the discount factor becomes large, players care less about the duration and focus more on becoming the new leader and getting the additional reward. Obviously, the leader has an advantage over the other players since she takes her decision first. She can then enforce the other players to pass on their information since otherwise they will not be able to centralize the information at all. However, the leader can decide not to centralize all the information if it is beneficial for her that another player collects all items faster. Another reason why the leader may not end up being the player who centralizes all the information is that some player blocks her from winning. Some players may have incentives to hold on their items long enough making it impossible for the leader to win. For instance, some player may hold on his information first to avoid that the leader, who has a much higher eccentricity than him, wins. As soon as the leader cannot win anymore, she passes on his information. This is the only type of situations where, at equilibrium, the duration is not equal to the eccentricity of the winner.

Beside characterizing the winner and the duration for any network and for any discount factor, we also predict which communication networks could emerge in the long run. We assume there is an infinitesimally small cost to form a link. Under this assumption only minimally connected networks can be pairwise stable[1] and players will add a link to each other if the duration for centralizing all the information becomes shorter. We also find conditions that exclude some networks from being pairwise stable without having to check all possible additional links. For example, networks in which two players, in equilibrium, play such that they block each other from winning are never pairwise stable. In addition, we provide more information about which network structures are always (star or symmetric star-like networks) or never (line networks with an even number of players) pairwise stable. One interesting result is that even with endogenous network formation it can take the winner a long time to centralize all information. Even though the star network is the network that emerges most often, there are other pairwise stable networks that have a long duration, e.g. some line networks with an odd number of players. Finally, we show that all our results are robust if we instead require that players only need to collect at least $n-1$ items. We do further robustness checks and analyze closely related settings. We show that we can replicate most of our results even if two players have the same information item or if a single player is more patient or impatient than all other players.

One motivation for our model is the process of finding a spokesman, a promoter or a coordinator for a research project. All players contribute a part to the success of the project, but in the end only one player coordinates and promotes the project to the outside world. The additional reward can be either some additional funding the spokesman of a project gets or some benefits in terms of scientific reputation. Other examples include R&D joint ventures, political agreements, international agreements for fighting terrorism or climate change negotiations. Another example are the Panama Papers[2] investigations in 2015 where many different journalists and newspapers worked together, but the project is now promoted by only a small subset of the people who worked on it. The German journalist Bastian Obermayer was first to obtain all the raw data. He and his newspaper (Süddeutsche Zeitung) realized they could not analyze all the data by themselves. They contacted the International Consortium of Investigative Journalists and started the team project to retrieve information from the data. So, in our model Bastian Obermayer would be designated as the first player or leader, while the process of working through all the documents represents the centralization of (useful) information. In this example we can find all our assumptions satisfied. Obviously, the entire team had a strong incentive to

[1] See Jackson and Wolinsky (1996).

[2] All information available at https://panamapapers.icij.org/

finish the project early as they wanted to stop the damage that was dealt by the firms and people they investigated. This corresponds to our assumption that the players discount over time. In this application, all newspapers and journalists were rewarded with the Pulitzer Prize and so they all got the same payoff. At the same time Bastian Obermayer was the one that represented the project in many interviews and on television, which gave him additional publicity. Clearly, the International Consortium of Investigative Journalists can be represented as a network. Even though, all newspapers might be able to share information with each other, this does not hold for all journalists in the newspapers and especially not for the external specialists that were hired directly from individual newspapers. These specialists worked on certain tasks and sent their results to the newspaper who hired them. Then, the information was shared by this newspaper with other journalists. Even the assumption that all the players have unique information is closely approximated in this example. Everyone had access to the more than 2.6 Terabyte of data, but the tasks were split between the players. The people working on this project completed their aim to go through all the information, because they did not want anyone to get away with what they had done.

We now turn to the related literature. A similar setting was first introduced in Hagenbach (2011) but with an exogenously given communication network. The players are arranged in a network and want to centralize the information, but they decide simultaneously whether to pass or to hold on their information. This game of information transmission has multiple equilibrium outcomes. We rather adopt a sequential decision order yielding a unique equilibrium outcome. This decision order or ranking allows us not only to make more precise statements about the winner and the duration, but also to endogenize the communication network. Radner (1993) studies the efficiency of hierarchies in a model where players process information. He compares different structures and shows which hierarchical structures are efficient for a given set of variables. Another approach of information centralization is done by Jehiel (1999). In his setting a decision maker needs to gather some information to decide about a project. The decision maker's future employment depends on the outcome of the project. The decision maker gets fired if he selects a bad project. All other players get a certain share of the surplus of the project. Jehiel (1999) states conditions for optimal communication structures from the players and from the decision maker perspective. Closer to Hagenbach (2011) is the work of Schopohl (2017), who starts with a similar setting, but focuses on exogenous networks in which a given player wants to centralize the information. All other players are arranged in a hierarchy and compete for a reward. Schopohl (2017) compares different network structures with the focus on the time it takes until all the information is centralized.[3] Bonacich (1990) conducted two experiments on communication in networks where the participants

[3]For strategic information transmission networks and its related literature we refer to Galeotti, Ghiglino and Squintani (2013). They study a model of cheap-talk on networks and show how the players' welfare increases with more truthful messages. In addition, they find that in larger communities the communication decreases.

had to find a quotation. Each player receives a different subset of letters and could pass on the information to her neighbors or hold on the information private. Similar to our setting, the first player who completes the quotation receives a reward. One difference between Bonacich experiments and our model comes up with the predictability of a missing piece of information. While in the experiments the players could make guesses about information they could not centralize, this cannot happen in our setting. However, our results are still valid if we instead require that players only need to collect at least $n-1$ items.

The idea of leadership has been neglected in economics for a long time. Hermalin (1998) builds a model and asks 'why players should follow a leader?'. Hermalin (1998) shows that the leader has two ways to convince her fellow agents. Either by leading by example or by making a sacrifice. The first is associated with long working times of the leader to motivate the agents, while the second represents the idea that the leader gives small gifts to show that the work of her agents is valuable. Komai, Stegeman and Hermalin (2007) focus on a different aspect of leadership. They compare the collective decisions an organization makes in two settings and provide an answer to the question 'why there should be a leader?'. If all players have full information players can free ride on the decision making. However, if only the leader has full information and reveals only a part of it to the other players, the players have to invest into effort and cannot free ride, improving the efficiency of the decision making process. Dewan and Squintani (2017) propose a model where a leader is selected and then this leader receives cheap-talk messages from the agents. To trust the messages the leader must first build a network of trustworthy associates. Dewan and Squintani (2017) show that the quality of the leadership depends on the judgment and wisdom of the people surrounding the leader.[4]

In economics the literature on teams deals mainly with the comparison of individual decision making and team decision making and the positive effects of teams for individuals. Cooper and Kagel (2005) show that two-player teams behave more strategically in a signaling game than individuals. They show that in three different experiments, which vary in the difficulty of learning to play strategically, teams learn to play strategically faster than individuals. Kocher, Strauss and Sutter (2006) let participants of an experiment choose between team decision making and individual decision making. They show that even though that teams win more often, players may decide not to join a team to avoid discussions or compromises. Maciejovsky, Sutter, Budescu and Bernau (2013) show in an experiment how individuals learn from team decisions. In their experiment players make better individual decisions because of knowledge spillover from previous decisions they made as part of a team. Sutter and Strassmair (2009) focus on an experiment with communication between teams and between team members. Two teams compete and within a team coordination is required. The addition of communication between teams

[4]Dewan and Myatt (2008) model the influence of a leader on a mass. For a survey on leadership, see Ahlquist and Levi (2011).

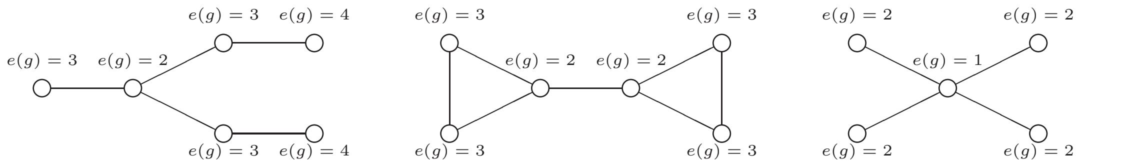
may lead to collusion. On the other hand, communication between team members reduces the free riding problem and leads to higher efforts. Holmstrom (1982) points out that there is large problem of moral hazard in teams and players may have high incentives for free riding.

The paper is organized as follows. In Section 2 we introduce the model and some definitions. In Section 3 we show that there is always a single winner and that only minimally connected networks can be pairwise stable. In Section 4 we characterize the winner and the duration for any network and for any discount factor, and we also predict which communication networks could emerge in the long run. In Section 5 we discuss the robustness of our main results by relaxing some main conditions. Finally, in Section 6 we conclude.

2 Model

Let $N = \{1, \ldots, n\}$ be a set of finite players, which are arranged in an undirected network g. A link in g between player i and player j is denoted by $ij \in g$. Let $N(g) = \{i \in N \mid \exists\, j \in N \text{ such that } ij \in g\}$ be the set of players who have at least one link in g. Let $N_i(g) = \{j \in N \mid ij \in g\}$ be the set of neighbors of player i in g. A player i who has only one neighbor, $|N_i(g)| = 1$, is called a loose end. A path in g from i to j is a sequence of distinct links, which connect player i and player j. The length of a path is equal to the number of links on the path. A network g is said to be connected if there exists at least one path between all players $i, j \in N(g)$ with $i \neq j$. A network g is minimally connected if for all players $i, j \in N(g)$ with $i \neq j$, there exists exactly one path that connects player i and player j. Given a connected network g, the geodesic or shortest distance between player i and player j is the length of the shortest path between them and is denoted by $d_{ij}(g)$.[5]

Definition 1 (eccentricity and radius)**.**
Let g be connected. The eccentricity of player i is $e_i(g) = \max_{j \in N} d_{ij}(g)$. The radius of the graph g is $r(g) = \min_{i \in N} e_i(g)$.



(a) min. connected, $r(g) = 2$

(b) not min. connected, $r(g) = 2$

(c) star, min. connected, $r(g) = 1$

Figure 1: Eccentricity and radius.

[5]Throughout the paper if we do not mention $N(g)$ for a connected network g, we implicitly presume that $N(g) = N$.

The eccentricity of player i is the geodesic distance from player i to the player furthest away from her. Given a connected network g, the radius is the minimal eccentricity. Figure 1 illustrates the notions of eccentricity and radius for three different networks.

At the beginning of the game each player belonging to the team is in the possession of a unique piece of information. We label the information items such that player i has item i in her possession, while player j has item j in his possession. There is a finite number of periods T. In each period the players can either pass on (P) all their information to their neighbors or hold on (H) the information private. When player i passes on her information, she passes copies of all the information items in her possession to all her neighbors $j \in N_i(g)$. The information set of each player is non-decreasing over time t. Players only pass on copies of their items to their neighbors, but they also keep the information in their possession.[6] Once a player has collected all the information, the game ends and the players stop passing information. The payoff of each player equals 1, except for the player who centralizes all the information. This player receives an additional reward R. If there are several players who have collected all information in the same period, those players share the reward equally. We call the player(s) who get a share of the reward the winner(s). For every network g, the set of winners of the game with deadline T is denoted by $W(g,T)$. The duration of the game is given by the number of periods it takes the winner(s) to centralize the information, and is denoted by $\tau(g,T)$. If no player centralizes all the information, all players get a payoff of 0. Players have time preferences with constant discount factor, $\delta \in (0,1)$, and the utility function of player i is given by

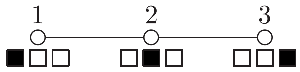
$$u_i(g,T) = \begin{cases} 0 & \text{if } W(g,T) = \emptyset \\ \delta^{\tau(g,T)-1} \cdot 1 & \text{if } W(g,T) \neq \emptyset \text{ and } i \notin W(g,T) \\ \delta^{\tau(g,T)-1} \cdot (1 + R \cdot (1/|W(g,T)|)) & \text{if } i \in W(g,T). \end{cases}$$

The effects of the reward R and the discount factor δ are similar in terms of players' incentives to pass or hold on their information. If the discount factor or the reward is large, then each player focuses on getting the reward for herself and cares less about the duration it takes for centralizing all the information. On the contrary, if the discount factor or the reward is small, then each player tends to prefer that all information is centralized as soon as possible. For the rest of the paper, we fix the reward $R = 1$.

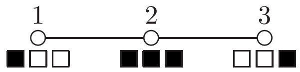
In each period t, the players choose sequentially their action $a_i^t \in \{P, H\}$, $i \in N$. There is a fixed decision order or current leadership ranking $\{1, 2, ..., n\}$ that determines the order of who is making a choice along the sequence. The timing in each period t is as follows. The current leader, player 1, starts and takes her decision, which is then observed by all other players. Afterwards, player 2 decides, already knowing what player 1 did.

[6] Each player either passes on all her information to her neighbors or holds on. That is, she is not able to split her information into different information items. This is consistent with R&D collaborations in chemistry or biology where after adding item A to item B it is only possible to give information about the resulting item AB, but not about both components.

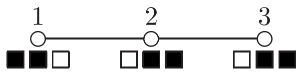
Then, player 3 chooses his action knowing the choices made by both players preceding him in the ranking. When player n has to pick an action, he takes into account the decisions of all other players. After all players have selected an action, pass on (P) or hold on (H), the information is passed according to their decisions. If there is no winner after the items have been forwarded, the next period starts. If a player is indifferent between passing and holding on, we simply assume that she passes on the information. The game is solved backwards looking for subgame perfect Nash equilibria.



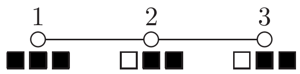
(a) Players and their information at $t = 1$



(b) Players and their information after $t = 1$, with $a_1^1 = a_3^1 = P$ and $a_2^1 = H$



(c) Players and their information after $t = 1$, with $a_1^1 = H$ and $a_2^1 = a_3^1 = P$



(d) Players and their information after $t = 2$, with $a_1^1 = a_1^2 = H$ and $a_2^1 = a_2^2 = a_3^1 = P$

Figure 2: A three-player line network

In Figure 2 we look at one possible way to arrange three players in a line. Each box represents a piece of information. A box is filled if the player has that piece of information in her possession. If we face a short deadline of $T = 1$, the only equilibrium outcome is that player 1 and player 3 pass on their information and player 2 is the unique winner as displayed in (b). If player 1 or player 3 would deviate, all players would obtain an utility of 0, so they prefer to pass on all their information. The action of player 2 makes no difference for the utility, because the other players cannot get all items within one period. As soon as we extend the deadline to $T = 2$, the analysis becomes more complex and the equilibrium now depends on δ. If players are sufficiently impatient, $\delta \leq 1/2$, the equilibrium is as before: player 1 and player 3 pass on their information in the first period and player 2 wins. But, if players are patient, $\delta > 1/2$, player 1 has incentives to hold on her information during both periods forcing the other two players to pass on their information to her (see (c) and (d)). The intuition works as follows. For low values of δ, the players rather focus more on centralizing all the information, while for higher values of δ, the reward becomes more important than the duration for centralizing all the information. If player 1 holds on her information in the first period, player 3 is indifferent and so he passes on his information to player 2. In the second period, player 1 can enforce player 2 to pass on his information by holding on her information once again.

If the network g is not connected, it is impossible for any player to centralize all the information. Otherwise, the shortest time for player i to centralize all the information in case every other player passes on his information in every period is equal to player i's eccentricity, $e_i(g)$. Only if the deadline T is larger or equal to the eccentricity of player i, player i can win the game. From now on, we set $T = |N| - 1$ and we write $W(g)$ for $W(g, |N| - 1)$ and $u_i(g)$ for $u_i(g, |N| - 1)$. It implies that each player can potentially be the winner of the game. If we would set a shorter deadline, some players could not ex-ante win the game. For any deadline larger than $|N| - 1$, the current leader, namely player 1, would even have a stronger advantage over the other players. Player 1 takes her decision first and can enforce the rest of the players to pass on their information, if she can threat that there would be no winner in case they hold on. In fact, we will show later on that player 1 does not win either if she prefers that another player wins or if she is blocked from winning. In the later case, at least one other player holds on his information long enough so that player 1 cannot become the winner anymore. With a deadline greater than $T = |N| - 1$, the incentives for the other players to block player 1 decrease.

3 One winner and minimally connected networks

Our objective is to characterize the winner(s), the duration and the networks that emerge in the long run. We first provide some general results that turn to be helpful for the characterization carried out further on. We show that, given any connected network, only a single player will get the reward at equilibrium. This result does not depend on the discount factor δ nor on the network g as long as g is connected. In addition, we show that only minimally connected networks can emerge in the long run. Since no player can centralize all the information if the network g is not connected, we mostly consider connected networks and we set $\tau(g) = \infty$ if g is not connected.

Proposition 1. *There is a unique winner at equilibrium, i.e.* $|W(g)| = 1$*.*

Proof. Since it is a game with perfect information, it cannot be optimal for the players that no one centralizes all the information. The players prefer to pass on without getting the reward over failing to centralize all the information. Hence, there is at least one winner.

Suppose that player i and j are both winning and share the reward after τ periods. Without loss of generality let $i < j$. To end up in a situation where both i and j win, both players need to pass on their information at least once, so that i can obtain the item of j and vice versa. The decision order in each period is such that player i always decides before player j. Hence, player i can decide to hold on her information in any period and be the unique winner, because she enforces player j to still pass on his information. If player j would also hold on his information, his utility would be 0 and so player j prefers passing on over holding on his information in any period. Solving the game backwards, we know that player j passes on his information at the latest in the last period, so that player i wins. If player i holds on in the period before, player j anticipates that he will pass on his information in the last period and make player i the winner. Since players have time preferences, player j prefers to pass on in the second to last period so that the game ends earlier. Repeating this argument yields to the conclusion that if player i holds on her information in all periods, player j passes on his information so that player i is the unique winner. $\square$

The easiest way to understand the intuition behind Proposition 1 is to consider the game with just two connected players. No matter the deadline $T > 1$, player 1 always wins after a single period. If both players hold on their information before the last period T, player 1 still holds on her information in T forcing player 2 to pass on his information. Otherwise, both players would be worst off (ending with a payoff of 0). In the second to last period $T-1$, player 2 has incentives to pass on his information since he anticipates that if he holds on, he will pass on in period T, and he ends up worse off because of time preferences. Repeating the above argument until we reach period $t = 1$, we get that player 2 already passes on his information during the first period, and player 1 is the single winner. We denote by w the player who is the single winner.

Beside looking for who is going to be the winner, we are also interested in understanding which communication networks are likely to arise in teams when players need to centralize all the information. To do so, we need to define a notion which captures the stability of a network. We restrict our analysis to pairwise deviations and we suppose that there are positive but infinitesimally small costs to forming links. In other words, the cost of any link is always less important than any payoff from centralizing all the information. Hence, as in Goyal and Joshi (2003) or Mauleon, Sempere-Monerris and Vannetelbosch (2014), we use a strict version of Jackson and Wolinsky's (1996) notion of pairwise stability.[7] A network is pairwise stable if no player has an incentive to delete one of her links and no two players strictly benefit from adding a link between them.

Definition 2 (pairwise stability)**.**
A network g is pairwise stable if (i) for all $ij \in g$, $u_i(g) > u_i(g-ij)$ and $u_j(g) > u_j(g-ij)$, and (ii) for all $ij \notin g$, if $u_i(g) < u_i(g+ij)$ then $u_j(g) \geq u_j(g+ij)$.

Corollary 1. *Take any g such that $ij \notin g$. Player i and player j will add the link ij if and only if the network $g+ij$ has a shorter duration than g, i.e. $\tau(g+ij) < \tau(g)$.*

Corollary 1 follows directly from Proposition 1. Both players have to pay for the additional link ij and they only do so, if they both benefit from the link. Since at equilibrium only one player is the winner, the only possibility for both players to benefit from an additional link, is that this link decreases the time it takes until the information is centralized.

Proposition 2. *Only minimally connected networks can be pairwise stable.*

All the proofs not in the main text can be found in the appendix. The intuition behind Proposition 2 is as follows. As soon as a network is not minimally connected, it

[7]See Mauleon and Vannetelbosch (2016) for an overview on network formation games.

cannot be pairwise stable, because there is always at least one link that can be deleted while the duration remains constant. Even if players are connected by two or more paths, the information just flows through one path. Since links are costly, the players prefer to delete links until the network is minimally connected. Proposition 2 restricts our search for pairwise stable networks to minimally connected networks. Given any minimally connected network, we only have to consider additional links that players may want to form. Players will never delete a link and move to a non-connected network since it would be impossible to centralize all the information, and all players would end up with zero utility. It then follows directly that a network that differs from a pairwise stable network by just one link cannot have a shorter duration than the duration of the pairwise stable network.

4 Winner, duration and stability

In this section we characterize the winner, the time it takes for the winner to centralize all the information and the stable networks. From the preceding section we already know that the winner and the duration for collecting all the information are affected by the value of the discount factor δ. For low values of δ, the players care less about the reward and more about the time. We show that, for $\delta \leq 1/2$, the duration is the shortest possible and the eccentricity determines who is going to be the winner. For high values of δ, player 1 has an advantage over all other players thanks to the decision order. Nevertheless, we show that player 1 is not guaranteed to be always the winner. We first consider low discount factors. This analysis conveys the main intuition behind the results. We next provide general results for any value of δ. Moreover, we provide corollaries that simplify the search for the winner for high discount factors.

4.1 Low discount factor

For $\delta \leq 1/2$ the players prefer to decrease the duration for centralizing all the information by one period over becoming the winner at a later time.

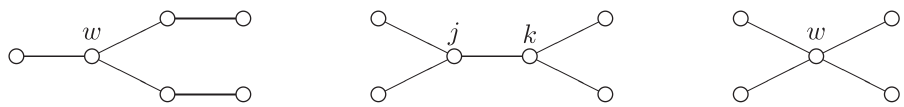
Proposition 3. *Assume $\delta \leq 1/2$ holds. The winner is $w = \min_{e_i=r(g)} i$ and the duration is $\tau(g) = r(g) = e_w(g)$.*

Proof. For $\delta < 1/2$ all players prefer centralizing all the information in a given period and not winning over winning one period later. If player i can end the centralization in period τ by passing on, but then another player wins, player i's utility is equal to $\delta^{\tau-1}$. If she holds on her information and then wins in period $\tau + 1$ her utility is equal to $\delta^{\tau} \cdot 2$ but less than $\delta^{\tau-1}$. For $\delta = 1/2$ she is indifferent and passes on by assumption. Since this reasoning holds for all players it follows that the player who can centralize all the information fastest wins and the duration is minimal and equal to the radius of g. If there are several players with the same minimal eccentricity, then the one who is first in the decision order wins. If she holds on her information in all periods, she becomes the unique winner and gets an utility of $\delta^{e_i(g)-1} \cdot 2$, while if she passes on at least once her utility is at most $\delta^{e_i(g)-1}$. The other players with the same eccentricity, anticipating the behavior of the player ranked before them in the decision order, have no incentives to hold on their information as shown in the first part of the proof. $\square$

Proposition 3 tells us that, for low discount factors ($\delta \leq 1/2$), the player with the lowest eccentricity is the winner. All players prefer to centralize the information as fast as possible. The player with the lowest eccentricity can centralize the information fastest and so she wins. If there are several players with the lowest eccentricity, then the one who is first in the decision order wins. This player can hold on her information to avoid that other players win. Since all the other players want to centralize the information as fast as possible, they will pass on their information to her. The duration is then equal to the radius of the network.

Proposition 4. *Assume $\delta \leq 1/2$ holds. A minimally connected network g is pairwise stable if and only if there is no link $ij \notin g$ that decreases the radius, i.e. $\forall ij \notin g$: $r(g + ij) = r(g)$.*

Proof. From Proposition 3 we know that the duration equals the radius. Proposition 1 states that there is always a unique winner at equilibrium. Combining these two results with Proposition 2, it follows that two players can only have incentives to form a link if this link decreases the radius and by that decreases the duration. $\square$

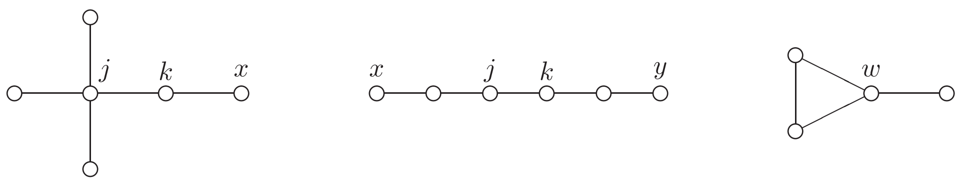


(a) $r(g) = 2$, Winner: w

(b) $r(g) = 2$, Winner: $\min j, k$

(c) $r(g) = 1$, Winner: w

Figure 3: Pairwise stable networks for $\delta \leq 1/2$.



(a) $r(g) = 2$, Winner: $\min j, k$

(b) $r(g) = 3$, Winner: $\min j, k$

(c) $r(g) = 1$, Winner: w

Figure 4: Non pairwise stable networks for $\delta \leq 1/2$.

The duration for centralizing all the information is equal to the radius. Two players will only form an additional link, if they both benefit from this new link. But, only one of them could be the winner. Hence, this new link must decrease the duration in order to offset the infinitesimally small cost for forming links. Figure 3 shows us three different network structures that are pairwise stable. In (a) and (c) there is a single player with the lowest eccentricity, while in (b) two players have the lowest eccentricity. In none of the networks it is possible to add a single link that decreases the radius. Hence, these three networks are pairwise stable. On the contrary, in network (a) of Figure 4, players have incentives to form links. In this network, depending on the decision order, either player j or k wins. Player j has an incentive to link with player x, because it decreases the duration by one period and player j would become the winner even if he makes his decision after player k. In network (b) the two players (j and k) who are most central have incentives to form links with the players who are the most far away (j to y or k to x). With such additional link the radius decreases to two and so the duration decreases by one period. The network (c) is not minimally connected and so it cannot be pairwise stable.

4.2 Low and high discount factor

When the discount factor becomes large, players care less about the duration and focus more on becoming the new leader and getting the additional reward. Obviously, player 1 who is the current leader has an advantage over the other players since she takes her decision first. She can then enforce the other players to pass on their information since otherwise they will not be able to centralize the information at all. However, player 1 can decide not to centralize all the information if it is beneficial for her that another player i collects all items faster. This can happen only if the discount factor δ is not too high and there is a significant difference in the eccentricity of the players 1 and i. The other reason, why player 1 may not end up being the player who centralizes all the information is that some player(s) block(s) her from winning. Some player(s) may have incentives to hold on their items long enough making it impossible for player 1 to win. For instance, player i may hold on his information first, to avoid that player 1 who has a much higher eccentricity than him wins. As soon as player 1 cannot win anymore, player i passes on his information. This is the only type of situations where, at equilibrium, the duration is not equal to the eccentricity of the winner.

Given the network g, the deadline T and the actions in period $s = 1, \ldots, t-1$, $(a_j^s)_{j \in N}$, we denote by $\Delta_i\left(g, t, T, (a_j^s)_{j \in N}^{s=1,\ldots,t-1}\right)$ the remaining time needed from period t so that player i can centralize all the information. At the beginning of the game, the minimum time needed for player i to centralize all the information is equal to her eccentricity, that is $\Delta_i(g, 1, T, \emptyset) = e_i(g)$. In any minimally connected network, a player who is a loose end plays no role once she has passed on her information. We can remove such player from the network and look at the reduced network instead. For a minimally connected g, the remaining time needed from period t so that player i can centralize all the information, $\Delta_i(g, t, T, (a_j))$, is equal to the eccentricity of player i on the reduced network $\widehat{g}$ with deadline $T-(t+1)$. It means that $\Delta_i(g, t, T, (a_j)_{j \in N}^{s=1,\ldots t-1}) = \Delta_i(\widehat{g}, 1, T-(t+1), \emptyset) = e_i(\widehat{g})$, and $\Delta_i(g, 1, T, (a_j)_{j \in N}^{s=1,\ldots t-1}) = t - 1 + \Delta_i(\widehat{g}, 1, T - (t+1), \emptyset) = t - 1 + e_i(\widehat{g})$.

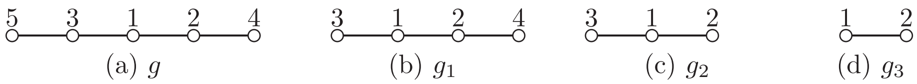


Figure 5: A line-network g and its reduced networks g_1, g_2, g_3.

In Figure 5 we illustrate the notion of remaining time and reduced network, and how we can remove players who are loose ends from the network once they have passed on their information. Starting from network g we obtain that the remaining time so that player 1 can centralize all the information is equal to $\Delta_1(g, 1, 4, \emptyset) = e_1(g) = 2$. Suppose first that $a_5^1 = P$ and $a_4^1 = H$. We can remove player 5 from g and we get the reduced network g_1. Then, $\Delta_1(g, 2, 4, a_5^1 = P, a_4^1 = H) = \Delta_1(g_1, 1, 3, \emptyset) = e_1(g_1) = 2$. Suppose instead that $a_4^1 = a_5^1 = P$, then we can remove both players 4 and 5 from g and we get the reduced network g_2 with $\Delta_1(g, 2, 4, a_4^1 = a_5^1 = P) = \Delta_1(g_2, 1, 3, \emptyset) = e_1(g_2) = 1$. Suppose next that $a_4^1 = a_5^1 = P$ and $a_3^2 = P$. Then, we can first remove both players 4 and 5 and next player 3 from g and we get the reduced network g_3 with $\Delta_1(g, 3, 4, a_4^1 = a_5^1 = P, a_3^2 = P) = \Delta_1(g_3, 1, 2, \emptyset) = e_1(g_3) = 1$.

Definition 3 (blocking)**.** We say player k has the power and incentive to block player j in favor of player i at time t_k^j if there exists $\tau_k^j \in \{1, \ldots, T\}$ such that

(i) $\Delta_j(g, t_k^j, T, (a_l)) + \tau_k^j > T - t_k^j$ and

(ii) for $i \neq k$ we have $\Delta_i(g, t_k^j, T, (a_l)) + \tau_k^j < \Delta_j(g, t_k^j, T, (\widehat{a}_l))$

with $a_k^t = H \ \forall t < t_k^j$ and $\widehat{a}_k^t \neq a_k^t$ for some $t < t_k^j$.

Definition 3 tells us that player k blocks player j if this decreases the duration it takes to centralize all the information. The first condition (i) ensures that if player k holds on his information for τ_k^j periods, player j cannot win. The second condition (ii) gives the incentives for player k to block player j in favor of player i. The time it takes for player i to collect all the information once player k holds on his information during τ_k^j periods (a_k^t) is less than the time it would take for player j in case that k passes on $(\widehat{a}_k^t \neq a_k^t)$.

Definition 4 (mutual blocking)**.**
Let player k have the power and incentive to block player j and let player j have the power and incentive to block player k. There is mutual blocking if the periods in which player j and player k block each other are identical, i.e. $t_k^j = t_j^k$ and $\tau_k^j = \tau_j^k$.

Figure 6: Blocking and mutual blocking.

Figure 6 illustrates the notions of blocking and mutual blocking. Suppose that $\delta > (1/2)^{1/2}$. In the left-hand network (a) player 2 wins after two periods at equilibrium.

In the first period player 1 and player 4 pass on their information, while in the second period player 3 and player 5 pass on. The other actions have no impact on the equilibrium outcome. The reason that player 1 passes on in the first period is that, if she would deviate and hold on her information, player 4 would have an incentive to block her. Indeed, player 4 would hold on in the first period and by that he could achieve that player 1 is unable to win and so player 2 wins after three periods. The first period of blocking would increase the duration by one, so player 1 does not deviate and no blocking happens at equilibrium. In the right-hand network (b), player 2 has incentives to block player 1 from winning and player 1 has incentives to block player 2 from winning at equilibrium. This leads to a duration of three periods with player 3 as the winner. In the first period player 1 and player 2 hold on their information and hence prevent that one of them wins. If player 1 or player 2 would win, the duration has to be four periods. After that period of mutual blocking, both player 1 and player 2 pass on their information and player 3 wins after three periods. Both players who block each other have incentives to do so since they will end up with an utility of δ^2, while if they deviate their utility would be only δ^3. However, if we increase the deadline T by an additional period, player 2 would have to hold on two periods to avoid that player 1 wins. Then, player 2's utility would be the same whether player 1 wins after four periods or whether player 3 wins after $2 + 2$ periods. By assumption he would pass on, making player 1 the winner. This last example emphasizes that increasing the deadline gives even more advantage to the current leader, namely player 1.

The next two propositions characterize the winner and the duration for any value of the discount factor, $\delta \in (0, 1)$.

Proposition 5. *Player $i \in N$ wins if and only if*

1. *$\forall j < i$ we have $e_j(g) > e_i(g)$, and either*
   - 1.1. *$\delta \leq (1/2)^{1/(e_j(g)-e_i(g))}$ or*
   - 1.2. *there is some $k \in N$ who has the power and incentive to block j in favor of i,*
2. *$\forall j > i$ either*
   - 2.1. *$e_j(g) \geq e_i(g)$ or*
   - 2.2. *$e_j(g) < e_i(g)$, $\delta > (1/2)^{1/(e_i(g)-e_j(g))}$ and there is no $k \in N$ who has the power and incentive to block i in favor of j.*

Proposition 5 tells us that several conditions have to be satisfied for making player i the winner. First, it must take more time for all players who decide before i to centralize all the information (Condition 1) and these players must either prefer that i wins, because the discount factor is small (Condition 1.1) or they must be blocked from winning (Condition 1.2). Second, all players who decide after player i either cannot centralize all the information faster (Condition 2.1) or can centralize all the information faster but the discount factor is such that i prefers to win and i is not blocked from winning (Condition 2.2).

Proposition 6. *Let player $i \in N$ be the winner. The duration $\tau(g,T)$ is given by $e_i(g) + \tau_{mutual}$ where τ_{mutual} is the number of mutual blocking periods, i.e.*

$$\tau_{mutual} = \left| \bigcup_{x=1}^{|N|} \left(\bigcup_{y=1}^{|N|} \left(\{t_x^y, \ldots, t_x^y + \tau_x^y - 1\} \cap \{t_y^x, \ldots, t_y^x + \tau_y^x - 1\} \right) \right) \right|$$

with t_x^y and τ_x^y as in Definition 3.

Proposition 6 tells us that the duration is equal to the eccentricity of the winner plus the number of mutual blocking periods, where the number of mutual blocking periods is the union of all mutual blocking periods of player x with other players, which is given by the intersection of blocking periods of player x and player y. In the left-hand network (a) of Figure 6, player 1 is blocked from winning, but there is no mutual blocking period, so that player 2 wins after two periods. In the right-hand network (b) there is a mutual blocking period, which stops player 1 and player 2 from winning and leaves player 3 as the winner after three periods.

For small discount factors, Proposition 3 already helps us to find the winner easily. For larger discount factors, Proposition 5 enables us to get some corollaries that simplify searching for the winner.

Corollary 2. *Assume $\delta > 1/2$ holds. If player 1's eccentricity is lower or at most $(i-1)$ higher than player i's eccentricity, i.e. $\forall i \neq 1 : e_i(g) + i - 1 \geq e_1(g)$, then player 1 is the unique winner at equilibrium. The duration is equal to the eccentricity of player 1, $\tau(g) = e_1(g)$.*

(a) (b)

Figure 7: Two networks with player 1 as winner.

When the discount factor is high, the advantage of taking the decision first is large. As long as player 1's eccentricity is just slightly larger than the eccentricities of the other players, player 1 centralizes all the information. For each player ranked further away in the decision order, the difference between the eccentricity of this player and the eccentricity of player 1 can be larger, while player 1 still remains the winner. Figure 7 looks at two networks where player 1 is the winner. In both networks, the eccentricity of player 2 is $e_1(g) - 1$ and the eccentricity of player 3 is $e_1(g) - 2$, but it is still player 1 who wins at equilibrium because the difference between the eccentricities is relatively small.

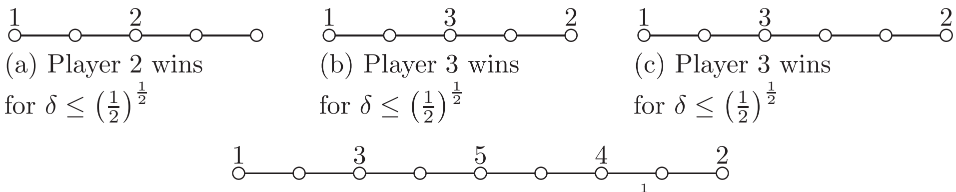
Corollary 3. *Assume $\delta > 1/2$ holds. Player i wins after e_i periods if*

1. $\forall j < i : e_i(g) < e_j(g) - 1$ *and* $\delta \leq (1/2)^{1/(e_j(g)-e_i(g))}$, *and*

2. $\forall j > i$ *either*

   2.1. $e_j(g) \geq e_i(g) - 1$ *or*

   2.2. $e_j(g) < e_i(g) - 1$ *and* $\delta > (1/2)^{1/(e_i(g)-e_j(g))}$.



(a) Player 2 wins for $\delta \leq \left(\frac{1}{2}\right)^{\frac{1}{2}}$

(b) Player 3 wins for $\delta \leq \left(\frac{1}{2}\right)^{\frac{1}{2}}$

(c) Player 3 wins for $\delta \leq \left(\frac{1}{2}\right)^{\frac{1}{2}}$

(d) Player 5 wins for $\delta \leq \left(\frac{1}{2}\right)^{\frac{1}{2}}$

Figure 8: Four networks where player 1 is not the winner.

For intermediate values of the discount factor δ, results are similar. If δ is low enough and $e_i(g) + 1$ is smaller than the eccentricity of all the players that take their decision before player i and the same does not hold for a player who makes his decision after player i, then player i is the winner who centralizes all the information. In Figure 8 we look at four networks where Corollary 3 holds. In the network (a) player 1 prefers that player 2 wins since the discount factor is too low. Even though the neighbors of player 2 would prefer to win, they cannot win, because player 2 makes his decision before them. The analysis from networks (b) and (c) is similar. Player 1 and player 2 prefer that player 3 wins, since player 3 can win in two periods while player 1 or player 2 cannot. In network (d), even though player 3 can win faster than player 1 and player 2, there is another player, namely player 5, who can centralize all the information even faster. Indeed, Condition 2.1 of Corollary 3 holds for player 3 but Condition 2.2 does not hold because of player 5.

Beside characterizing the winner and the duration for any network, our objective is to predict which networks could emerge in the long run. We already know that two players will add a link to each other if the duration for centralizing all the information becomes shorter. Moreover, only minimally connected networks can be pairwise stable. We now provide some propositions that help us to exclude some networks for being pairwise stable without having to check all possible additional links.

Lemma 1. *There is no network in which three or more players want to block each other.*

There are many different network structures where three (or more) players have a long geodesic distance to each other. It cannot occur that those players can block each other from winning. The main reason behind this result is the modeling of the deadline $T = |N| - 1$.

Proposition 7. *No network in which there occurs a mutual blocking period at equilibrium is pairwise stable.*

Using Proposition 7 we get directly that the right-hand network (b) in Figure 6 cannot be pairwise stable for $\delta > (1/2)^{1/2}$. On the other hand, the left-hand network (a) is pairwise stable for all values of δ. There exists no way to decrease the duration from two to one period, because no additional single link can change the radius to one. Even though player 1 would like to form an additional link to become the winner, the other players have no incentive to do so.

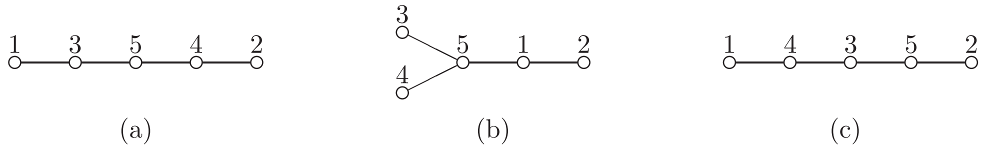


Figure 9: Pairwise stable networks depending on the discount factor δ.

Remark 1. The set of pairwise stable networks is neither increasing nor decreasing in δ.

The stability of a network depends on the discount factor. Some networks are pairwise stable for all values of δ, some are only pairwise stable for low values of δ, while others are only pairwise stable for high values of δ. The left-hand network (a) in Figure 9 is pairwise stable for $\delta \leq 1/2$ since no additional link can decrease the duration (and the radius) from two periods to one period. For $\delta > 1/2$, player 1 wins after four periods. In that case all players except player 3 have an incentive to link with player 1 and by that decrease the time it takes for player 1 to centralize all the information. Thus, there are networks that are pairwise stable only for low values of δ. In the center network (b), player 1 wins after two periods when $\delta > 1/2$. The players have no incentive to add links, because even if player 2 and player 5 add the link 25, player 1 would remains the winner after two periods. In addition, it is not possible for player 1 to win after one period by just adding a single link. For $\delta \leq 1/2$, it is player 5 who wins after two periods. In that case player 2 and player 5 have incentives to form the link 25 so that the duration decreases by one period. Thus, there are networks that are pairwise stable only for high values of δ. The right-hand network (c) is pairwise stable for $\delta \leq (1/2)^{1/2}$. In that case player 3 wins after two periods. For $\delta > (1/2)^{1/2}$, player 1 and player 2 hold on in the first period, but block each other from winning. After that mutual blocking period, player 3 wins, but it takes him three periods to centralize all the information. Player 1 and player 2 or player 1 and player 5 have an incentive to link since then player 1 would win after two periods. Thus, even for $\delta > 1/2$, there are networks which are pairwise stable only for some δ, namely $\delta \in \left(1/2, (1/2)^{1/2}\right]$.

Despite these previous results, it is hard to find the set of pairwise stable networks. We know that we can limit our attention to minimally connected networks, but even then there are many different network structures and for each structure there are several ways how to arrange the players. The next proposition give us additional information about which network structures are always or never pairwise stable.

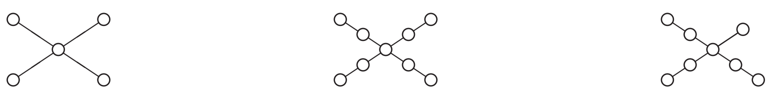
Proposition 8.

1. *If $n > 3$ then any star network is pairwise stable;*
2. *Any symmetric star-like network with at least three arms*[8] *is pairwise stable;*
3. *If $n > 2$ is even then no line network is pairwise stable.*

Proof. Part 1. Take $n > 3$. In any star network, the player who wins is either the center or player 1. If $\delta \leq 1/2$, the center is the unique winner since all players want to centralize all the information as fast as possible. There is no incentive for an additional (costly) link since the players already centralize all the information in a single period. If $\delta > 1/2$, then player 1 is the single winner. Suppose that player 1 is not the center of the star. Her optimal strategy is to hold on during all periods and forcing the other players to pass on their information to her. The duration is two periods. Player 1 cannot add just a single link to decrease her eccentricity from 2 to 1 since there are more than three players, $n > 3$. Thus, player 1 has no incentive to form a single additional link. The other players who are not the center cannot decrease the duration since an additional link would not change their eccentricity and player 1 (who is the first player to take her decision in each period) can stick to her optimal strategy. Using Corollary 1 yields the result. Suppose now that player 1 is the center of the star. Then, we can use the same arguments as for $\delta \leq 1/2$.

Part 2. The proof is analogue to the one of part 1. The center cannot form any link and the remaining players cannot reduce their eccentricity by adding a link. In any symmetric star-like network with at least three arms, there always exist at least two players for any given player to whom she has her maximal geodesic distance. Thus, a single additional link cannot decrease the duration. From Lemma 1 it follows that at equilibrium there is no blocking in any symmetric star-like network with at least three arms.

Part 3. Take $n > 2$ even. For each player i in any line network g there exists exactly one player j with the same eccentricity, $e_i(g) = e_j(g)$. When player i is the winner, it implies that $i < j$ for j such that $e_i(g) = e_j(g)$. Suppose that i and j are such that $e_i(g) = e_j(g)$, $i < j$ and $ij \notin g$. Both i and j have incentives to add the link ij since it does not change the winner but it decreases the duration for centralizing all the information. Suppose now i and j are such that $e_i(g) = e_j(g)$, $i < j$ and $ij \in g$. Then, there exists a single player k to whom the winner w has her maximal geodesic distance. Both w and k want to form a link since it decreases the duration, and moreover, this new link does not change who is the winner. □



(a) symmetric (b) symmetric (c) asymmetric

Figure 10: Star-like networks.

[8]An arm of a star-like network is a line network with the center of the star at one end of the line.

In any star network where one central player is connected to all other players, only the players who are not the center of the star can form a link, but no link can decrease the duration for centralizing all the information. Thus, the star network is pairwise stable. A similar argument holds for symmetric star-like networks with radius $r \geq 2$. In a star-like network the length of the arms can be longer. A star-like network is symmetric if all arms of the star have the same length. Figure 10 provides some examples of symmetric and asymmetric star-like networks. While stars and symmetric star-like networks are pairwise stable, most line networks are not pairwise stable. The reason is that, if the number of players is even ($n > 2$), there always exists an additional link that decreases the duration. Thus, a line network can only be pairwise stable if the number of players is two or odd. But, if the discount factor is low ($\delta \leq 1/2$), using Proposition 4, we conclude that all line networks with more than five players are not pairwise stable.[9]

Many minimally connected network structures can be pairwise stable. One could wonder whether some of them are more likely to emerge in the long run than others. If we start from some randomly selected connected network and we assign the same probability to all links the players want to delete or to form, we will follow improving paths leading towards some pairwise stable network. For instance, start from a connected network that is not pairwise stable and suppose that there are two links that players want to cut and three links that the players want to form. We select each of these five changes with equal probability, but depending on which link gets selected the improving paths may lead to completely different pairwise stable networks. The probability for ending up in some pairwise stable network varies widely and this probability does not only depend on the network structure but also on the position of the players within the network. Take $\delta > 1/2$ and $n = 5$. The star network with player 1 as the center arises with a probability of 27.9%, while the star networks with a different center have a probability of approximately 4.3% each. In addition, the probability for ending up in any other pairwise stable network lies between 0.8% and 2.5%. When $\delta \leq 1/2$, only the network structure influences the probability.

As already mentioned earlier, the competition for the reward can be interpreted as a competition for the team leadership when the task of the team is to centralize all the information. The decision order represents the current hierarchical ranking within the team with player 1 being the current leader. One can argue that the fight for the leadership not only happens once, but gets repeated over and over again. Once a player has centralized all the information, she becomes the new leader of the team and moves to the first spot in the ranking, and she heads the team to execute a new task of centralization. Then, she has a better position in the decision order, but the same position in the network. From Proposition 5 it follows directly that this player will stay the winner as long as the network structure does not change. The pairwise stable network of the previous round is

[9]For $n > 2$ odd and $\delta > 1/2$, line networks are pairwise stable since the unique centered player cannot decrease her eccentricity by forming a single link.

still pairwise stable under the new decision order. For instance, if the star network with the current leader as the center emerges, the current leader is the one who centralizes the information. She will keep the leadership and in the next rounds this star network with her as the center will remain pairwise stable. However, in some situations, the players may start from a different network in the next round. For instance, if one player leaves the team and is replaced by another player, the new player may not have the same links as the old one. Then, another pairwise stable network can emerge and this might change the leader.

5 Robustness

What happens if players only need to collect at least $n-1$ items?
In some situations it may be reasonable to assume that the players do not need to centralize all information items. For instance, it is often the case that if we collect $n-1$ pieces of a puzzle we are able to guess the missing piece. We now show that most of our results are robust to such change.

Assume that players whose information get centralized obtain the same utility as before: 1. The player who first centralizes at least $n-1$ items gets an additional reward $R = 1$. If a single winner centralizes more than $n-1$ items, she also obtains $1+R$. It remains to point out two important features. First, a player whose information is not centralized by some winner gets 0 since he does not contribute to the team task. Second, if there are two (or more) players who first centralize at least $n-1$ items, they share the reward equally, and all players whose information is centralized receive a payoff of 1. For instance, consider a team consisting of five players and let the reward R be equal to 1. If after two periods player 1 has collected the items $1, 2, 3, 4$ and player 2 has collected the items $2, 3, 4, 5$, player 1 and player 2 obtain an utility of $\delta \cdot 3/2$, while player 3, player 4 and player 5 obtain an utility of δ.

Proposition 9. *Only minimally connected networks g such that $|N(g)| = n-1$ can be pairwise stable.*

Proposition 9 tells us that, if players need only to centralize at least $n-1$ items, then all pairwise stable networks consist of one isolated player and one minimally connected network connecting the remaining $n-1$ players. Obviously, for such network g connecting $n-1$ players, the results about the winner and the duration from the previous sections hold. However, even though the network g could be pairwise stable if we would restrict the analysis to $n-1$ players, the network g may not be pairwise stable since some players may have incentives to link with the isolated player. On the other hand, if the network g is not pairwise stable when we restrict the analysis to $n-1$ players, then the network g is not pairwise stable in general.

In Figure 11 we look at the stability of networks when only at least $n-1$ items need to be centralized. Proposition 9 is not satisfied in network (a). The center has no incentive

Figure 11: Centralizing $n-1$ items with $\delta > (1/2)^{1/2}$.

to be linked to all other players, because she only needs the information of $n-2$ other players, so she cuts a link and we end up in (b). In network (b), one player is isolated (or disconnected) from all other players, who form a star. In both (a) and (b) player 1 is the only winner and the duration is one period. The network (c) illustrates the case in which a star would be pairwise stable if the team would be composed of only four players, but it is not pairwise stable once the team consists of five players. Player 1 wins after two periods but player 2 has an incentive to link with the isolated player, because he then can centralize the information in one period. In network (d) the line network is even not pairwise stable if the team would be composed of only four players. Player 1 is the winner after three periods. The duration can be decreased with an additional link between player 1 and player 3 or between player 1 and player 5 or between player 4 and player 5. Player 1 has an incentive to link with player 2, because it would decrease the duration as well. Moreover, player 3 wants to form a link with player 2, so that he can centralize all information within one period.

What happens if two players have the same information?

In other situations players may have the same information in the beginning. While in some cases this setting is similar to the one previously described, there are other cases in which this new setting yields to new results. Assume that two players have the same information and that the players only need this information once for a successful project. Furthermore, assume that all players connected in the network get a payoff of 1 if the information is centralized by another player.

Then, the pairwise stable networks consist either of one minimally connected network of n players or of one isolated player and one minimally connected component of size $n-1$. In the second case the isolated player is one of the players with a copy.

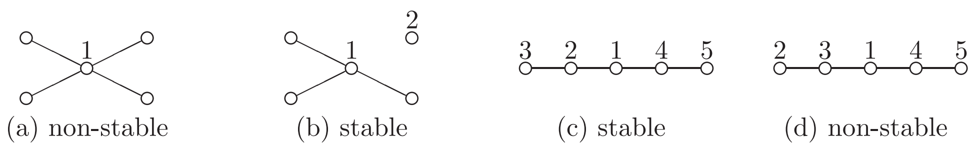


Figure 12: Examples with $\delta > (1/2)^{1/2}$, where players 1 and 2 have the same items.

Figure 12 shows some examples of stable and non-stable networks under the assumption that players 1 and 2 have the same information. In network (a) player 1 would cut the link to player 2, because she already has the same information as player 2. Therefore, she has no incentive to keep the link with player 2. In general, no star network of n players is pairwise stable. If two players on the periphery have the same information the center cuts the link with one of them. On the other hand, if the center shares the information with another player, as in network (a), the center cuts the link to that player. Removing the link between player 1 and player 2 yields to the network displayed in example (b), which is pairwise stable. Player 1 still centralizes the information of the other players within one period.

In this setting, networks that consist of one minimally connected component of all players can be pairwise stable as well. One example is shown in network (c). Even though player 1 and player 2 have the same information, they have to keep their connection, because otherwise no player will be able to centralize all information. In this example player 1 centralizes the information after two periods. For that she does not need the information of player 2, but she needs player 2 to pass on the information from player 3 to her. One additional link cannot decrease the duration and so the network is pairwise stable. This is not the case in example (d). Player 2 is a loose end and therefore he does not spread the information of other players. So, player 3 has an incentive to cut the link with him.

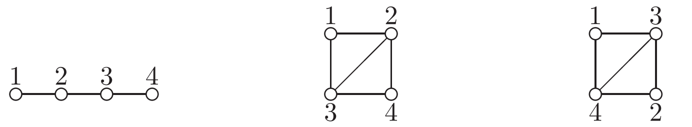
What happens if one player is more patient or impatient than the others?

In all previous results we rely on the assumption that the players have the same discount factor. However, in situations where firms cooperate with each other and work together on a joint project this assumption might not hold. As soon as one player has a different discount factor we cannot make statements like Proposition 3 or Proposition 4. Still those results hold if $\delta_i \leq 1/2$ holds for all players $i \in N$, where δ_i denotes the discount factor of player i. Furthermore, we can generalize Proposition 5:

Proposition 10. *Player $i \in N$ wins if and only if*

1. *$\forall j < i$ we have $e_j(g) > e_i(g)$, and either*
   - *1.1. $\delta_j \leq (1/2)^{1/(e_j(g)-e_i(g))}$ or*
   - *1.2. there is some $k \in N$ who has the power and incentive to block j in favor of i,*
2. *$\forall j > i$ either*
   - *2.1. $e_j(g) \geq e_i(g)$ or*
   - *2.2. $e_j(g) < e_i(g)$ and either*
     - *2.2.1 $\delta_i > (1/2)^{1/(e_i(g)-e_j(g))}$ and there is no $k \in N$ who has the power and incentive to block i in favor of j or*
     - *2.2.2 $\delta_i \leq (1/2)^{1/(e_i(g)-e_j(g))}$, but $\exists k \in \{i+1, \dots j-1\}$ such that $e_k(g) > e_j(g)$, $\delta_k > (1/2)^{1/(e_k(g)-e_j(g))}$ and there is no $\ell \in N$ who has the power and incentive to block k in favor of j.*

As described in Section 4.2 Condition 1.1. ensures that player $j < i$ does not want to win. Of course, this condition depends on the discount factor of player j. On the other hand we need to modify Condition 2.2. If player j can centralize the information faster than player i (Condition 2.2), then either player i wants to win herself because her discount factor δ_i is large enough (2.2.1) or player i wants player j to win, but there exists another player k who decides in between those players that centralizes the information slower than j (i.e. $e_k(g) > e_j(g)$) and this player does not want player j to win (2.2.2). With this modified proposition we can find the winner even if different players have different discount factors.



(a) Winner: 2
Duration: 2

(b) Winner: 2
Duration: 1

(c) Winner: 1
Duration: 2

Figure 13: Examples with $\delta_1 \leq (1/2)^{1/2}$ and $\delta_{2,3,4} > (1/2)^{1/2}$

In Figure 13 we look at the winner and the duration of networks when the discount factor of player 1 is lower than that of the other players. In network (a) player 2 wins after 2 rounds. If all players had the same discount factor $\delta > (1/2)^{1/2}$, player 1 would win after 3 rounds, but because of her low discount factor player 1 prefers to pass on her information in the first round. The network displayed in example (b) has player 2 as the winner and a duration of one period. If $\delta_1 > (1/2)^{1/2}$ holds, player 1 wins after two rounds. As in example (a), player 1 prefers to let player 2 win, because he has a lower eccentricity. This corresponds to the new Condition 1.1 of Proposition 10. On the other hand, in example (c) we see a network that has the same winner and the same duration independently of the discount factor of player 1. If player 1 would pass on her information item in the first round player 2 becomes the winner after two rounds and so she prefers to hold on which makes her the winner after two rounds. In this example the new Condition 2.2.2 is crucial, because only player 1 prefers that player 3 wins after a single period, but this does not hold for player 2 because he has a different discount factor.

6 Conclusion

We have proposed a model of information centralization in teams where players can only exchange information through an endogenous communication network. Players face a trade-off between cooperating for centralizing all the information fast and competing for being the one who collects all the information and becomes the team leader. Over several periods each player can either pass or not pass her information to her neighbors. Once one player has centralized all the information, all players receive some payoff. The winner who collects all the information gets an additional reward. Since each player discounts payoffs over time, she faces the dilemma of either letting another player centralizing all the information fast, or trying to collect all the information by herself and overtaking the leadership.

We have found that there is always a single winner who centralizes the information at equilibrium and that only minimally connected networks can be pairwise stable. We have characterized the winner and the duration for any network and for any discount factor, and we have predicted which networks emerge in the long run. Whether some player becomes the winner or not depends on her position in the network, her rank in the decision order and the discount factor. If the discount factor is low, the player with the lowest eccentricity wins. On the other hand, if the discount factor is high, a player first ranked in the decision order can outweigh a player with a low eccentricity. In addition, at equilibrium some players might hold on their information just to ensure that no player with an excessive eccentricity wins. This confirms our observations regarding the Panama Papers. Bastian Obermayer, who might not has been the player with the lowest eccentricity, but definitely the first player in the decision order, represents the project to the outside. Furthermore, only minimally connected networks can be pairwise stable. It follows that additional channels of communications between two players either do not benefit both players or lead to a breakdown of other links afterwards. The information only flows through a single path between the players. Again, whether some minimally connected network is pairwise stable or not depends on the decision order and the discount factor. For instance, switching two players in a pairwise stable network may destabilize the network, because of the crucial role played by the decision order or current leadership ranking.

Finally, we have shown that our results are robust. For instance, if players only need to collect at least $n-1$ items, then all pairwise stable networks consist of 1 isolated player and one minimally connected network connecting the remaining $n-1$ players. Similarly, most our results can be modified easily if several players have the same information or if players have different discount factors.

Acknowledgements

Ana Mauleon and Vincent Vannetelbosch are Senior Research Associates of the National Fund for Scientific Research (FNRS). Financial support from the Spanish Ministry of Economy and Competition, FEDER and European Union under the project ECO2015-64467-R, from the MSCA ITN *Expectations and Social Influence Dynamics in Economics* (ExSIDE), Grant No721846 (1/9/2017-31/8/2020), and from the Belgian French speaking community ARC project n°15/20-072 of Saint-Louis University - Brussels are gratefully acknowledged. Simon Schopohl is thankful for the financial support of EDEEM (European Doctorate in Economics Erasmus Mundus) and the DFH-UFA.

Appendix

Proof of Proposition 2. From Proposition 1 we know that there is a single winner at equilibrium. Hence, two players (neighbors or not) will never share the reward at equilibrium. We now show that networks which are not minimally connected networks cannot be pairwise stable.

Suppose that g is not minimally connected and player i is the single winner. Hence, there is a cycle in g and player i receives all information items from the other players, but she just needs to receive them once.

1. Player i is part of the cycle. Suppose that player j and player k are also part of the cycle. The equilibrium outcome is independent of the number of items that player j or k has. The information flows either from j over k to i, or from k over j to i, or from j to i and from k to i. In the first (second) case, the link(s) between j (k) and i are useless and will be deleted to save costs. In the third case, player j and player k have no incentive to be linked, except than being connected through player i. If the information reaches player i from two different paths, the slower path has no purpose and can be deleted.

2. Player i is not part of the cycle. Suppose that player j, player k and player l are part of the cycle. At equilibrium the winner i just needs to receive all the information once. Since player i is not part of the cycle, she can receive the information from players j, k and l only through one of them. If (without loss of generality) player ℓ collects the items of players j and k, and then passes on the information items to player i, at least one link between them can be deleted to save costs.

(a) Winner i is part of the cycle

(b) Winner i is not part of the cycle

Figure 14: Illustration of the proof

□

Proof of Proposition 5. Suppose that player i is the winner. Then: (i) all players j who decide before i (i.e. $j < i$) must need more time than player i for centralizing all the information and either want that player i wins or they are blocked from winning; (ii) all players j who decide after i (i.e. $j > i$) either must need more time than player i to centralize all the information or if they can centralize all the information faster than player i, they want i to win and nobody can block her from winning.

□

Proof of Proposition 6. We show by contradiction that the duration can be neither larger nor smaller than the sum of the eccentricity of the winner and the number of mutual blocking periods.

$\not>$ Given that the strategies of the players are optimal, the duration cannot be longer.

$\not<$ Let τ_k be the number of mutual blocking periods in total, starting from period t_k. The same argument holds if several mutual blocking periods take place not successively. By the definition of mutual blocking, we get

$$\forall i \in N : \Delta_i(g, t_k, T, (a_l)) = \Delta_i(g, t_k + \tau_k, T, (a_l)).$$

This implies that the duration cannot be lower than the eccentricity of the winner plus the number of mutual blocking periods.

□

Proof of Lemma 1. We show that three players have no incentive to mutually block each other. Without loss of generality, take player 1, player 2 and player 3. We first show that in a star-like network g with three arms, where player 1, player 2 and player 3 are the loose ends (i.e. the node at the end of each arm), they will not mutually block each other. A star-like network is one type of network architecture, but if the players do not mutually block each other in such type of network, they still do not block once we add additional links to the network.

Suppose that player 1, player 2 and player 3 block each other and that they all have the same eccentricity $e_1(g) = e_2(g) = e_3(g)$. Let β be the number of blocking periods and let player c be the center of the star-like network g. It follows that the duration $\tau(g)$ is equal to the sum of $e_c(g)$ and β. In addition, $\beta + e_1(g) = T + 1 = |N|$ has to hold since the number of blocking periods has to be large enough to prevent player 1, player 2 and player 3 from winning. Replacing β by $|N| - e_1(g)$ in the expression for the duration yields: $\tau(g) = e_c(g) + |N| - e_1(g)$. Players only block each other if the duration including blocking is lower than their eccentricity: $\tau(g) < e_1(g)$. Merging both equations gives us $|N| + e_c(g) < 2 \cdot e_1(g)$. In a symmetric star-like network, the eccentricity of the loose ends is twice the eccentricity of the center, and so we get $|N| + e_c(g) < 4 \cdot e_c(g)$ which reverts to $|N| < 3 \cdot e_c(g)$. Since the eccentricity of the center of a symmetric star-like network with x arms is $e_c(g) = (|N| - 1)/x$, we get $|N| < (3/x) \cdot (|N| - 1)$. This inequation never holds if there are at least three arms, $x \geq 3$. Thus, in a symmetric star-like network with at least three arms, the loose ends will never block each other. One can consider all other networks as symmetric star-like networks with additional players and links and then the same arguments hold.

□

Proof of Proposition 7. From Lemma 1 we know that only two players j, k can block each other. Without loss of generality, assume $j < k$. The shortest number of blocking periods is one, which happens only on the line network. The loose ends block each other for one period and hence they achieve that a player with a lower eccentricity wins. In case of a line the eccentricity of player j and player k is $e_j(g) = e_k(g) = |N| - 1$ and the eccentricity of the winner is $e_w(g) = |N|/2$ if $|N|$ is even or $e_w(g) = (|N| - 1)/2$ if $|N|$ is odd. The time it takes the winner to centralize all the information is equal to the sum of her eccentricity plus the number of blocking periods, $e_w(g) + 1$. It follows that the network cannot be pairwise stable, because player j and player k would prefer to add the link jk. If g is a line, we get $e_j(g + jk) = e_k(g + jk) = |N|/2$ if $|N|$ is even or $e_j(g + jk) = e_k(g + jk) = (|N| - 1)/2$ if $|N|$ is odd. Then player j is the winner after $e_j(g + jk)$ periods which is strictly less than $e_w(g) + 1$. All other networks can be considered as adding players and links to the line network. For each player we add to the line, player j and player k have to block each other for an additional period. Let g' be the network where we have added x players to the line with $|N'|$ players, and so we have a tree with $|N'| + x$ players in total. The smallest eccentricity is $e_w(g') = |N'|/2$ if $|N'|$ is even and $(|N'| - 1)/2$ if $|N'|$ is odd. Therefore, we get that the duration of g' is $\tau(g') = e_w(g') + 1 + x$. Again player j and player k will link. The furthest the additional players can be away is if they are all linked to the center of the line. By this we get $e_j(g' + jk) \leq |N'|/2 + x$ if $|N'|$ is even and $e_j(g' + jk) \leq (|N'| - 1)/2 + x$ if $|N'|$ is odd. It follows that $e_j(g' + jk) \leq e_w(g') + 1 + x$ and so we have shown that the network g' cannot be pairwise stable. Notice that these arguments hold since it is assumed that in case of indifference the players pass on their information.

□

Proof of Proposition 9. Notice that any connected network that is not minimally connected cannot be pairwise stable following the same arguments as in the proof of Proposition 2. We have to show that only minimally connected networks g such that $|N(g)| = n - 1$ can be pairwise stable. We first show that a minimally connected network g such that $|N(g)| = n$ cannot be pairwise stable. There always exists at least one player k who has the highest geodesic distance to the winner. Obviously, player k is a loose end. Let player j be the player who is linked to k ($jk \in g$). Player j has an incentive to cut the link jk, because even without that link the winner can centralize the information of $n - 1$ players in the same time, and player j saves the infinitesimally small cost for having link jk. Therefore, any minimally connected network g with $|N(g)| = n$ is never pairwise stable.

□

Proof of Proposition 10. Analogue to the proof of Proposition 5.

□

References

[1] Ahlquist, J. S., and M. Levi (2011) Leadership: what it means, what it does, and what we want to know about it, *Annual Review of Political Science* 14, 1-24.

[2] Bonacich, P. (1990) Communication dilemmas in social networks: an Experimental study, *American Sociological Review* 55, 448-459.

[3] Cooper, D., and J. Kagel (2005) Are two heads better than one? Team versus individual play in signaling games, *The American Economic Review* 95(3), 477-509

[4] Dewan, T., and D. P. Myatt (2008) The qualities of leadership: direction, communication, and obfuscation, *The American Political Science Review* 102, 351-368.

[5] Dewan, T., and F. Squintani (2017) Leadership with trustworthy associates, Working Paper, Warwick University.

[6] Galeotti, A., C. Ghiglino, and F. Squintani (2013) Strategic information transmission networks, *Journal of Economic Theory* 148, 1751-1769.

[7] Goyal, S., and S. Joshi (2003) Networks of collaboration in oligopoly, *Games and Economic Behavior* 43, 57-85.

[8] Hagenbach, J. (2011) Centralizing information in networks, *Games and Economic Behavior* 72, 149-162.

[9] Hermalin, B. E. (1998) Toward an economic theory of leadership: leading by example, *The American Economic Review* 88, 1188-1206.

[10] Holmstrom, B. (1982) Moral hazard in teams, *The Bell Journal of Economics*, 324-340.

[11] Jackson, M., and A. Wolinsky (1996) A strategic model of social and economic networks, *Journal of Economic Theory* 71, 44-74.

[12] Jehiel, P. (1999) Information aggregation and communication in organizations, *Management Science* 45, 659-669.

[13] Kocher M., S. Strauss and M. Sutter (2006) Individual or team decision-making - Causes and consequences of self-selection, *Games and Economic Behavior* 56, 259-270.

[14] Komai, M., M. Stegeman, and B. E. Hermalin (2007) Leadership and information, *The American Economic Review* 97, 944-947.

[15] Maciejovsky B, M. Sutter, D. V. Budescu and P. Bernau (2013) Teams make you smarter: How exposure to teams improves individual decisions in probability and reasoning tasks, *Management Science* 59, 1255-1270.

[16] Mauleon, A., J.J. Sempere-Monerris, and V. Vannetelbosch (2014) Farsighted R&D networks, *Economics Letters* 125, 340-342.

[17] Mauleon, A., and V. Vannetelbosch (2016) Network formation games. In *The Oxford Handbook of the Economics of Networks.* (A. Galeotti, Y. Bramoullé and B. Rogers, eds.), Oxford University Press, UK.

[18] Radner, R. (1993) The organization of decentralized information processing, *Econometrica* 61, 1109-46.

[19] Schopohl, S. (2017) Information transmission in hierarchies, Center for Mathematical Economics Working Paper 570, University of Bielefeld.

[20] Sutter, M. and C. Strassmair (2009) Communication, cooperation and collusion in team tournaments - An experimental study, *Games and Economic Behavior* 66, 506-525.