Dynamic Competition in Deceptive Markets

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Abstract

In many deceptive markets, firms design contracts to exploit mistakes of naive consumers. These contracts also attract less profitable sophisticated consumers. I study such markets when firms compete repeatedly and gather usage data about their customers which is informative about the likelihood of a customer being sophisticated. I show in a benchmark model that firms do not benefit from private information in this setting when all consumers are rational. I find that in sharp contrast to a model with only rational consumers, private information on the naiveté of customers mitigates competition and is of great value to its owner despite intense competition. I discuss several implications of the value of customer information on naiveté. Private information on customers' sophistication induces profits that are bell-shaped in the share of naive consumers. Firms prefer an even mix of both customer types. If firms can educate (some) naifs about hidden fees, competition is already mitigated when firms compete for customers with initially symmetric information. Policies that induce symmetric customer information between firms increase consumer surplus. I discuss how the UK governments' midata program might induce crucial aspects of such policies, and illustrate the robustness of results through several extensions.

Keywords: Consumer mistakes, deceptive products, shrouded attributes, big data, targeted pricing, consumer data, add-on pricing, price discrimination, industry dynamics

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1 Introduction

Both intuition and extensive empirical evidence suggest that firms in many markets have a better understanding of consumer behavior and certain product features than consumers themselves. This allows firms to exploit consumer misunderstandings. Examples are markets for credit cards (Ausubel, 1991; Agarwal et al., 2008; Stango and Zinman, 2009, 2014), retail banking (Cruickshank, 2000; OFT, 2008; Alan et al., 2015; CMA, 2016), mortgages (Cruickshank, 2000), insurance (DellaVigna and Malmendier, 2004) or mobile-phone services (Grubb and Osborne, 2014). In most of these markets, firms and consumers interact repeatedly. Yet the existing theoretical work—such as Gabaix and Laibson (2006), Grubb (2009), Armstrong and Vickers (2012), and Heidhues et al. (2016b)—considers non-repeated interactions.[1] In these models, some naive consumers are unaware of shrouded or hidden attributes. I extend this literature by introducing a dynamic model of competition with deceptive products. This allows me to investigate an increasingly relevant aspect of reality: developments in the analysis of large amounts of data allow firms to predict consumer behavior with increasing precision. By evaluating their own customers' usage data, firms have an informational advantage relative to their competitors. The main question I ask is how this informational advantage affects competition when some consumers are more prone to making mistakes than others.

Formally, I study a two-period model with shrouded product attributes. N firms sell a homogeneous good in each period to maximize total profit. They compete for a unit mass of consumers by setting a transparent price and a hidden fee, i.e. the shrouded attribute. There are naive and sophisticated consumers. Naifs pay the hidden fee but do not take it into account.[2] The sophisticates are aware of the hidden fee and avoid it. Thus, both consumers expect the same expenditures for a product—the transparent price. Some are correct—the sophisticates—and others are wrong and pay more than expected—the naifs. Since all consumers choose a product only based on transparent prices, firms cannot discriminate between new customers. The novel feature of the model captures the fact that firms analyze their customers' usage patterns to predict their behavior, and to target offers accordingly: in period 1, firms compete for market shares with symmetric information on customers. After observing which of their first-period customers paid

[1] Gabaix and Laibson (2006) consider an extension where consumers buy a base product once, but an add-on multiple times. But they do not allow for repeated interaction where firms adjust conditions over time.

[2] The hidden fee represents unexpected payments due to costly mistakes. For example, naifs could be unaware of fees or underestimate their own demand for an add-on service such as credit-card borrowing or late payments.

the hidden fee, firms learn to distinguish between naifs and sophisticates within their customer base. Each firm has private information on the naiveté of the consumers in her customer base. In period 2, firms use this information to discriminate between continuing customers based on their level of sophistication. I will later discuss implications when firms can educate naive consumers, i.e. they unshroud hidden fees to make them more transparent to consumers.

Credit cards are an example of a market close to this setting. The market is competitive by conventional measures, i.e the product is quite homogeneous and there are many firms. Consumers are usually aware of maintenance fees, cash rewards, introductory APRs or new-client bonuses. But many consumers ignore overlimit, overdraft or late fees, or underestimate their future borrowing when choosing a credit card. Firms condition offers on customer characteristics, including naiveté, which they can infer from usage data.

My main result is that competing firms benefit from private information on naiveté, because it allows them to reduce the intensity of competition. As in previous articles on deceptive products, naifs will select a suboptimal contract, which they perceive to be optimal.[3] These naifs are more profitable than sophisticated consumers who choose the same contract. But in contrast to the previous literature, firms observe past usage patterns in their customer base, enabling them to distinguish between customers in period 2. Firms use this information on naiveté to target continuing naifs with a smaller transparent price than sophisticates: they target naifs with a transparent discount. But without the discount, sophisticated consumers are more prone to switch to offers made by rivals. Rivals who try to poach profitable naifs are therefore more likely to attract unprofitable sophisticated consumers. To avoid losses from adversely attracting sophisticates, these rivals compete less vigorously. This allows firms to break even on their continuing sophisticates while maintaining positive margins on their naifs. Firms profitably exploit the fact that private information on naiveté allows them to discriminate between continuing customers while competitors lack the information to do so, and naive consumers lack the sophistication to recognize better offers.

Naiveté is crucial for this result. With only rational consumers, competition severely limits the value of information on customers. To establish this, I study a classic analog with only sophisticated consumers. Some purchase only a base product while others also value an add-on. Firms with private information on add-on demand can make targeted consumer-specific offers. But when consumers are aware of their add-

[3] For example, Gabaix and Laibson (2006), Armstrong and Vickers (2012), and Heidhues et al. (2016b), Heidhues and Kőszegi (2017).

on demand, they now recognize offers with a cheaper add-on. This awareness allows uninformed rivals to compete effectively: they can reduce the price of the add-on product to attract add-on consumers *without* adversely attracting consumers who value only the base product. Awareness mitigates adverse selection, and renders private information on preference for add-ons valueless in the absence of naive consumers.

These results suggest that naive consumers render customer usage data more valuable, even in highly competitive markets. While there are many reasons for firms to collect customer data, I find that intense competition severely limits the value of information on consumer preferences when consumers are rational. In contrast, even in highly competitive markets with homogeneous products, private information of firms about consumers' naiveté is highly valuable.

My main result has further important implications. First, in contrast to the previous literature, firms prefer a mix of naive and sophisticated customers. Firms earn positive margins from continuing naifs, but the adverse attraction of unprofitable sophisticates induces rivals to compete less intensely. This is why margins from naifs are proportional to the share of sophisticates, and firms earn the largest profits with a customer base evenly mixed with naifs and sophisticates. This implies that firms can increase profits by trading clients to make customer bases more balanced.

Second, since private information on naiveté is highly profitable despite competition, firms are willing to invest in data and tools that improve predictions on their customers' naiveté. But since these investments do not increase product- or match values, they are profitable but inefficient. Indeed, selling consumer information to third parties is important in the context of online advertising, social networks, credit cards, or loyalty programs in retail markets.[4] My results suggest that even firms in highly competitive markets can profitably (but possibly inefficiently) invest in big-data analysis to improve predictions on consumer naiveté.

Third, my results predict price dispersion when firms are privately informed about the naiveté of their customers. Firms play mixed strategies in period 2. Intuitively, firms try to poach naive consumers. But too much poaching adversely attracts unprofitable sophisticates and induces losses. In equilibrium, firms have to be indifferent between poaching and not poaching. This leads to price dispersion for transparent prices to new- and naive customers. Price dispersion is consistent with evidence on credit-card products (Schoar and

[4]The Economist (2014) reports that leading credit-card networks sell data about their cardholders to advertising companies, and "Bidders for advertising space can go to MasterCard to buy aggregated segments of consumers who are likely to subscribe to particular telecommunications services,[...] or stay at particular hotel chains."

Ru, 2016; Stango and Zinman, 2015). Also consistent with my results, Schoar and Ru (2016) find that firms target lower transparent fees such as annual fees to consumers who are more likely to be naive.

Fourth, the ability to educate consumers about hidden fees mitigates competition for customer bases in period 1 and increases total profits.[5] This holds despite price competition with homogeneous products and initially symmetrically informed firms. Education induces price-coordination in period 1. Intuitively, firms without a customer base have no private information on the naiveté of consumers in period 2 and earn zero profits. These firms unshroud hidden fees to attract consumers, thereby decreasing overall market profits. To prevent unshrouding in period 2, firms coordinate transparent prices in period 1 to make sure that each competitor gets a sufficient customer base. But price coordination mitigates competition for customer bases. Firms do not compete away future profits in period 1. These results are robust when unshrouding affects only an arbitrarily small share of naifs, and when demand is smooth with differentiated products. This suggests that transparency campaigns, capable of educating consumers about hidden fees, can serve as a credible threat to competing rivals and increase profits.

Consumer education or a simplified product design are common policy suggestions for deceptive products. My results suggest the following alternative: induce symmetric information among firms on consumer naiveté. For example, encourage consumers who shop for better offers to share their usage data with rivals. Each rival can then target profitable naifs separately without adversely attracting unprofitable sophisticates. This effectively splits the market and induces marginal-cost prices for all consumers. Sharing customer data restores effective competition when private information on customer naiveté impedes it.

Section 2 introduces the basic setup. Section 3 presents benchmarks, and Section 4 the main results. Section 5 explores policy implications. Section 6 discusses robustness to several extensions, and how the model captures crucial features of important markets such as the U.S. credit-card industry, retail banking, mobile-phone services and others. Section 7 reviews the related literature. Section 8 concludes.

2 The Basic Model

The basic model is based on existing models with shrouded attributes such as Gabaix and Laibson (2006), Armstrong and Vickers (2012), or Heidhues and Kőszegi (2017). The additional feature is that firms and

[5] E.g. Stango and Zinman (2014) and Alan et al. (2015) show that simply mentioning certain fees raises consumer awareness.

consumers interact repeatedly. This allows me to study firms who analyze their customers usage data to distinguish the level of naiveté of their continuing customers.

2.1 Setup

There are two periods $t \in \{1, 2\}$. In each period $N \geq 2$ firms with marginal cost $c \geq 0$ sell a homogeneous product. Each firm n sets transparent and hidden price components f_{nt} and $a_{nt} \in [0, \bar{a}]$, respectively. I follow the literature by assuming a cap $\bar{a}$ on hidden fees.[6] There is a unit mass of consumers with unit demand. In each period consumers value the product at $v > c$ and their outside option at zero.

The two types of consumers differ w.r.t. their perception of the price components. The share $1 - \alpha \in [0, 1]$ is sophisticated. They observe transparent and hidden price components, and can avoid paying the hidden one at no costs.[7] The share α is naive. Also naive consumers take only transparent prices into account, but they end up paying the hidden fee. Thus, all consumers have the same expected expenditures for a product—the transparent price; the hidden fee captures unexpected expenditures of naive consumers. Consumers maximize their perceived utility.[8]

In the basic model, naive consumers do not learn about their naiveté over time, except when educated by a firm. This is consistent with evidence that consumers repeatedly trigger fees they are unaware of.[9]

Firms on the other hand can unshroud hidden fees to naive consumers in each period. Unshrouding educates naive consumers about hidden fees and turns them into sophisticates. This captures the feature that firms can change product design or make fees more transparent to increase awareness and reduce consumer mistakes. For simplicity, the basic model assumes that unshrouding turns all naifs into sophisticates.[10] I also assume that consumers, once educated about hidden fees, remain so for the subsequent period.

[6] Gabaix and Laibson (2006) and Armstrong and Vickers (2012) interpret $\bar{a}$ as a legal or regulatory restrictions on fees, or consumers only noticing a fee when it is sufficiently large. It can also stand for the willingness to pay for an additional service that consumers require unexpectedly after signing the contract.

[7] For example, these consumers avoid expensive credit-card fees by paying back their debt in time. This can also captures precautionary behavior, like avoiding roaming charges by purchasing extra packages or by calling from land-lines. Another interpretation is that firms charge the hidden fee for an add-on service for which sophisticates have no demand.

[8] In each period, sophisticates' utility and naifs' perceived utility of product n is $v - f_n$. Naifs' true utility is $v - f_n - a_n$. Consumers take their perceived continuation utility into account. When hidden fees are shrouded, both types choose a firm $n \in \text{argmax}_{n' \in N}\, v - f_{n'1} + V_{n'2}$, where $V_{n'2}$ denotes the expected continuation utility after consuming from firm n' in period 1.

[9] For evidence, see Cruickshank (2000); Stango and Zinman (2009); OFT (2008). In Supplementary Material C.2 I show that relaxing this simplifying assumption to let some naifs learn about hidden fees on their own does not change results qualitatively.

[10] All results are qualitatively robust to weaker unshrouding. See Appendices A.3 and A.1.

The new feature of this model is that firms analyze their customers' usage history and learn about their level of sophistication over time. I call the set of customers who purchase from a firm in period 1 its customer base. Firms observe consumption patterns in their customer base in period 1, i.e. that naifs pay the hidden fee and sophisticates do not, and learn the types of their continuing consumers. Firms observe only the consumption of their own customers and have private information on their customer base in period 2. This captures that firms can observe their customers' purchase history in more detail and have an informational advantage on their customers relative to rivals.

Firms can use information on their customers to target prices. In period 2, this enables each firm n to charge f_{n2}^{naive} and f_{n2}^{soph} to naive and sophisticated consumers in its customer base, respectively.[11] Competitors of firm n do not observe which of n's customers receives which offer. Naive consumers do not infer from equilibrium offers that they are naive.[12] To attract new customers from competitors, firm n also charges a new-customer price, or poaching price, denoted f_{n2}^{new}. Since all consumers only consider transparent prices, a single new-customer price is not a restriction. In period 1, firms cannot distinguish and therefore discriminate between consumers and set one price f_{n1}.

When consumers are indifferent between all firms, I employ a general tie-breaking rule: each firm gets a market share $s_n > 0$ with $\sum_{n=1}^{N} s_n = 1$. When indifferent between less than N firms, I impose for ease of exposition that a firm n that attracts consumers gets a market share proportional to s_n.

Sorting Assumption: Consumers sort independently of their type among firms that make them indifferent. This simplifies the analysis by guaranteeing that—given shrouding—the distribution of types within a non-empty customer base is the same as in the overall population.

To summarize, the timing of the game is as follows:

Period 1: Competition for a Customer Base

- *Firms* simultaneously choose f_{n1} and a_{n1}, and whether to shroud or unshroud hidden fees.
- *Consumers* buy from the firm they perceive cheapest among the firms preferred to their outside option. When a firm unshrouds hidden prices, naive consumers become sophisticates.

[11] This assumption rules out that in period 2 sophisticated consumers can disguise as new customers of their old firm. In line with the applications in Section 6, firms can prevent this by asking for an ID when consumers sign a new contract.

[12] One interpretation is that naive and sophisticated consumers have non-common priors as in Eliaz and Spiegler (2006). Without this assumption, naifs could be separated as in the Benchmark in Section 3.1.

Period 2: Asymmetric Information on the Firms' Customer Bases

- Firms observe which of their continuing customers paid the hidden fee in period 1, and learn their types. Customer-base information is private: a firm can only identify the type of its own customers.
- *Firms* set prices f_{n2}^{soph} and f_{n2}^{naive} for sophisticated and naive consumers in their customer base, respectively. Additionally, they set a poaching price f_{n2}^{new} to attract customers from competitors. Firms choose hidden fees a_{n2}, and whether to shroud or unshroud hidden fees.
- *Consumers* buy from the firm they perceive as the cheapest. If hidden fees are shrouded, a consumer of type $\theta \in \{soph, naive\}$ who is in firm n's customer base picks the smallest price in $\{f_{n2}^{\theta}, (f_{\hat{n}2}^{new})_{\hat{n}\neq n}\}$ if this price is below v. When a firm unshrouds hidden fees, naifs become sophisticates.

I solve for perfect Bayesian equilibria of the game played between firms. PBE is relatively straightforward here since the Sorting Assumption pins down the beliefs of firms about their rivals' customer base: after unshrouding in period 1, all customers become identical, and type information are obsolete. After shrouding in period 1, beliefs on the composition of customer bases are identical to the distribution of types in the population. This is why I focus on sequential rationality for the rest of this article.

I call an equilibrium where firms shroud hidden fees with probability one a shrouding equilibrium.

The basic model makes some rather particular modeling choices, but this is just one way to model how firms benefit from learning about their consumers' naiveté. Section 6 discusses how the main results are robust to a wide range of alternative modeling choices, and how the model describes important industries such as credit cards, retail banking and others.

3 Benchmarks

To emphasize the impact of consumer naiveté and private customer data, I analyze two benchmark cases. First, a classic analog where 'naive' consumers are aware of their demand for an add-on service. Second, I study the basic model but firms do not learn their continuing customers' types. In both benchmarks, profits are zero and competition maximizes expected consumer surplus.

3.1 Private Customer-Base Information without Naive Consumers

In this benchmark all consumers value the base good with v, but the share α of consumers—called add—might buy an add-on for which they have valuation $\bar{a}$. The remaining consumers only buy the base good and are called $base$. There are two firms A and B, which produce the base good at cost c, and the add-on without additional marginal costs. W.l.o.g., let firm A know all customers' types while firm B knows only their distribution. Firm A can assign prices for each type, f_A^{add}, f_A^{base}, while B can instead offer two products—the base product only, and a product with add-on—at different prices f_B^{add}, f_B^{base}.

Firm A cannot benefit from her information because consumers recognize any better offer. First, note that the uninformed firm B cannot earn positive profits, because the informed firm A could marginally undercut any profitable offer of B. But also the informed firm A earns zero profits. Since consumers are aware of their add-on demand, they recognize and select a cheaper offer of the uninformed firm B. This allows B to compete effectively despite being uninformed. In equilibrium, both firms price at marginal costs and earn zero profits. Sophisticated consumers reveal their private information by their choices. Private information on consumer preferences does not increase profits.

Proposition 1. [Private Customer Information with Sophisticated Consumers only]
When customers are sophisticated and have heterogeneous add-on demand, a firm that is privately informed about add-on-demand types earns zero profits from each type in a competitive market.

Remark: Of course, also in models without naive consumers, firms can have reasons to gather information on their customers that are beyond the scope of this paper. But as I show in Section 4, the rational model strongly underestimates the benefits of information on customers if some consumers are naive.

3.2 No Customer Data

The next benchmark considers the basic model from Section 2 without information on customers. Firms do not learn their customers' types, so they offer only one transparent price in each period.

Proposition 2. [Deceptive Markets without Customer Data]
Shrouding equilibria exist. In each shrouding equilibrium, firms earn zero profits, consumers pay transparent prices $f_{n1} = f_{n2} = c - \alpha\bar{a}$ and naifs additionally pay hidden prices $a_{n1} = a_{n2} = \bar{a}$.

When firms do not learn about their customers level of naiveté, there are no dynamic effects. The equilibrium is simply a repetition of the one-period shrouding equilibrium discussed by Gabaix and Laibson (2006). In each period, shrouding profits from naive consumers are competed away with lower transparent prices. Naive consumers effectively cross-subsidize sophisticated consumers.

The two benchmarks establish that if either all consumers are sophisticated or firms do not have private information about their customers' naiveté, profits are zero.

4 The Benefits of Customer Data in Deceptive Markets

I now discuss the model introduced in Section 2, starting with competition in period 2.

4.1 Exploiting Naiveté with Customer Data

To start, I illustrate why in period 2 of shrouding equilibria, firms earn positive profits and play mixed strategies. Consider period 2 after prices are shrouded in period 1 and all firms have a non-empty customer base. Firms set f_{n2}^{naive} and f_{n2}^{soph} for their own old customers, and f_{n2}^{new} to poach customers from competitors.

Note first that firms exploit naiveté: they charge the largest hidden fee $a_{nt} = \bar{a}$ in each shrouding equilibrium. Sophisticates avoid and do not pay the hidden fee. Naifs ignore the hidden fee but pay it.

To see that second-period profits are positive in a shrouding equilibrium, consider the simple case with two firms A and B, and suppose both firms shroud hidden fees. Firm A charges $f_{A2}^{soph} \geq c$ to avoid losses from continuing sophisticates. But then B, when setting prices $f_{B2}^{new} < c$, attracts all sophisticates of A at a loss. For $f_{B2}^{new} < c$, B needs to attract naifs to break even. Consequently, all $f_{B2}^{new} < c - \alpha\bar{a}$ induce strictly negative profits from new customers, even when B poaches all profitable naifs from firm A. Thus, roughly speaking, prices $f_{A2}^{soph} < c$ and $f_{B2}^{new} < c - \alpha\bar{a}$ cannot occur with positive probability in equilibrium. This has two implications. First, firm B *can* only attract new customers without loss with prices $f_{B2}^{new} \geq c - \alpha\bar{a}$ (see Figure 1). Second, firm A can always deviate to $f_{A2}^{naive} = c - \alpha\bar{a}$ and $f_{A2}^{soph} \geq c$ to achieve profits of at least $s_A\alpha(1-\alpha)\bar{a}$ from consumers in its customer base. Charging $f_{A2}^{new} \geq c$ ensures that these profits are not wasted by unprofitably attracting new customers. The same reasoning holds for B's customer base and can be generalized to any number $N \geq 2$ of firms. Though this is not an equilibrium, it establishes the

Figure 1: Support of prices in period 2.

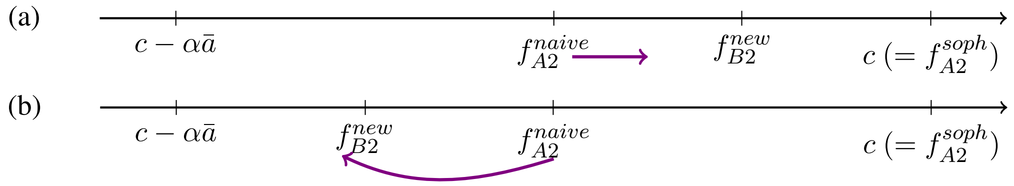

Figure 2: No pure strategy equilibrium in period 2: (a) Firm A can increase profits by moving f_{A2}^{naive} closer to f_{B2}^{new}. (b) Firm A could profitably keep her naive customers by undercutting f_{B2}^{new}.

minimum profits firms can guarantee themselves in each shrouding continuation equilibrium in period 2.

The key reason for positive profits is that poaching rivals never charge new-customer prices below $c-\alpha\bar{a}$, i.e. marginal cost discounted by the hidden fee from an average customer. Using information on their customers, firms can then charge $f_{n2}^{naive} = c - \alpha\bar{a}$ and $f_{n2}^{soph} \geq c$ to earn strictly positive profits. These prices render profitable poaching impossible.

Next, let us see why there is no pure-strategy shrouding equilibrium in period 2. Take again firms A and B, and suppose A charges $f_{A2}^{naive} = c - \alpha\bar{a}$ and $f_{A2}^{soph} = c$. Firm B cannot profitably poach consumers from A and sets $f_{B2}^{new} \geq c$ to avoid losses. But then, A is better off by increasing her naive-customer price to some $f_{A2}^{naive} \in (c - \alpha\bar{a}, c]$. This, however, gives B an incentive to marginally undercut f_{A2}^{naive}. But then A can ensure strictly positive profits by charging $f_{A2}^{naive} = c - \alpha\bar{a}$, and the circular argument starts again (see Figure 2). In equilibrium, B must be indifferent between using f_{B2}^{new} to poach naifs ($f_{B2}^{new} \in (c - \alpha\bar{a}, c)$) and for not-poaching ($f_{B2}^{new} \geq c$). This induces mixed strategies for naive- and new-customer prices.

In the end, after shrouding in period 1 and when all firms have a positive customer base, consumers pay transparent new-customer prices based on the following distribution:

$$F^{new}(f_n^{new}) = \begin{cases} 0, & \text{if} f_n^{new} \in (-\infty, c - \alpha\bar{a}] \\ 1 - \sqrt[N-1]{\frac{(1-\alpha)\bar{a}}{f_n^{new} + \bar{a} - c}}, & \text{if} f_n^{new} \in (c - \alpha\bar{a}, c) \\ 1, & \text{if} f_n^{new} \in [c, \infty) \end{cases}, \quad \forall n. \tag{1}$$

The distribution has a mass point on c of weight $\sqrt[N-1]{1-\alpha}$, so firms use new-customer prices for both poaching and not poaching with strictly positive probability. When rivals do not poach, i.e. set $f_{n2}^{new} = c$, firms who charge c to sophisticated customers keep them with probability 1.

Firms mix naive-customer prices according to

$$F^{naive}(f_n^{naive}) = \begin{cases} 0, & \text{if} f_n^{new} \in (-\infty, c - \alpha\bar{a}] \\ \frac{f_n^{naive} + \alpha\bar{a} - c}{\alpha(f_n^{naive} + \bar{a} - c)}, & \text{if} f_n^{new} \in (c - \alpha\bar{a}, c) \\ 1, & \text{if} f_n^{new} \in [c, \infty) \end{cases}, \quad \forall n. \tag{2}$$

Appendix A.6 illustrates these distributions graphically. The distributions are independent of market shares, implying that firms play identical strategies for the respective prices on $(c - \alpha\bar{a}, c)$. Intuitively, all firms $j \neq n$ mix new-customer prices to make firm n indifferent between all $f_n^{naive} \in (c - \alpha\bar{a}, c)$. This must be true for all n and therefore all new-customer prices must follow the same distribution. The same logic applies to distributions of naive-customer prices.

For $N > 2$ there are other equilibria with $f_{n2}^{soph} \geq c$ for some n, or where $F^{new}(\cdot)$ has less probability weight on the mass point at c and instead charges prices above c. But since these prices are never paid by consumers, all shrouding equilibria lead to the same profits, purchase prices and welfare.

With these mixed strategies at hand, Proposition 3 summarizes the results for period 2.

Proposition 3. [Exploiting Private Information on Customer Data in Period 2]
Shrouding equilibria exist where each firm strictly prefers to shroud hidden fees in period 2 if and only if hidden prices are shrouded in period 1 and each firm has a non-empty customer base.

In such equilibria, each firm n earns profits $\pi_{n2} = s_n\alpha(1-\alpha)\bar{a}$. Hidden prices are $a_{n2} = \bar{a}$. Transparent prices are $f_{n2}^{soph} \geq c$, and consumers pay f_{n2}^{new} and f_{n2}^{naive} based on (1) and (2) respectively.

Firms target naive consumers with transparent discounts. Private information on customer data allows firms to price their continuing customers differently based on their sophistication. Indeed, firms charge $f_{n2}^{naive} < f_{n2}^{soph}$ with probability one. Firms use their informational advantage to offer transparent discounts to their continuing naive customers. Since these customers pay the hidden fee $\bar{a}$, they are still more profitable than sophisticated consumers, i.e $f_{n2}^{soph} < f_{n2}^{naive} + \bar{a}$.

Because of transparent discounts to continuing naifs, rivals adversely attract unprofitable sophisticated consumers. This mitigates competition. Because firms employ usage data to offer lower transparent prices to profitable naifs, unprofitable sophisticated customers are more responsive to poaching offers than profitable naive ones: poaching rivals always attract profitable naifs together with unprofitable sophisticates but sophisticates might come without naifs. Rivals respond to this adverse selection of unprofitable sophisticates by poaching less intensely. As a result of mitigated poaching, firms break even with continuing sophisticates and new customers and earn strictly positive margins from naifs. Indeed, relative to the benchmark without information on naiveté in Proposition 2, prices increase for all consumers.

Naifs make sub-optimal choices. This prevents rivals from poaching only profitable naifs. If naifs were aware of paying the hidden fee, like consumers with add-on demand in Proposition 1, a rival n could set $f_{n2}^{new} = c$ and $a_{n2} = (1-\alpha)\bar{a} - \epsilon > 0$ to undercuts naifs' total price from their old firm and poach naifs without adversely attracting sophisticates. But since naifs do not respond to changes in hidden fees, rivals cannot poach naifs separately from sophisticates.

Shrouding profits are bell-shaped in the share of naive consumers α. Firms do not always want more naive consumers and instead prefer a balanced customer base. Intuitively, a customer base with more sophisticates induces more severe adverse selection of unprofitable sophisticates, and less-intense poaching. Consequently, the expected margin firms earn from naifs—$(1-\alpha)\bar{a}$—increases with the share of sophisticated consumers. Sophisticates have a strategic value to firms. This implies that firms might want to educate some customers about hidden fees but not too many. Moreover, firms with different shares of naive consumers can gain from trading consumer portfolios. I discuss this in more detail in Appendix A.5.

To see that effects on profits are indeed strong, note that the overall market revenue from hidden fees in period 2 is $\alpha\bar{a}$. Of this amount, firms keep the share $(1-\alpha)$ despite competition. For example, if $\alpha = 0.5$, half of the hidden fees paid remain as profits to firms. Indeed, the larger the share of sophisticates, the larger the share of revenues from hidden fees $\alpha\bar{a}$ that is not competed away. Thus, Proposition 3 establishes that firms benefit strongly from their customer data by being able to distinguish naive and sophisticated customers.

Proposition 3 sheds new light on the value of customer data in competitive environments. To see how, compare the results with Propositions 1 and 2. We see in Proposition 1 that when consumers are sophisticated, competition limits the value of firms' *private information on consumer preferences.* In contrast, Proposition 3 shows that *private information on consumer naiveté* is highly valuable despite price competition with perfect substitutes. Proposition 2 highlights that the presence of naive consumers alone does not increase profits when markets are competitive. Profits from naive consumers are competed away with lower transparent prices, and naive consumers cross subsidize sophisticated ones. Proposition 3 shows that private information on the sophistication of customers reduces cross subsidization. Firms increase transparent prices above $c - \alpha\bar{a}$ for *all* consumers and therefore increases the overall price level.

The profitability of identifying naive consumers implies that firms have strong incentives to invest in IT and big-data analysts to improve their targeting abilities. But in the present setting, improved targeting abilities do not increase product- or match values. This renders these investments profitable but inefficient.

The profitability of usage data also implies that firms, especially those active in data intensive industries such as online advertisement or credit cards, can profitably sell consumer data to a market in which they are not active, also if this market is very competitive.[13]

By increasing profits, usage data make shrouding equilibria more stable. As in Proposition 2, naifs avoid unshrouded hidden fees just like sophisticates, and unshrouding firms cannot profitably attract them.[14] But in contrast to Proposition 2, firms earn positive shrouding profits and strictly prefer to keep prices shrouded.

Proposition 3 predicts price dispersion. In line with this, Schoar and Ru (2016) find substantial variation in offers of credit-card companies to new consumers, even after controlling for observable characteristics. Stango and Zinman (2015) observe substantial variation in borrowing costs of credit-card customers, after controlling for observables. Both findings are in line with the mixed strategies (1) and (2) in Proposition 3.

Comparative statics of the price distributions (1) **and** (2)**:** Appendix A.6 shows graphs of the distributions. We see in (1) that as the number of firms N increases, the probability mass of new-customer prices shifts to the right and each firm sets *larger* average prices. More firms in the market reduce the probability to set the lowest new-customer price that poaches naifs. To compensate, firms poach less aggressively.

When the share of naifs α increases, poaching becomes more profitable and firms charge lower new-

[13]For example, the Economist (2014) reports that leading credit-card networks sell data about their cardholders to advertisers, and advertisement space targeted on consumers that are more likely to buy particular products such as telecommunication services.

[14]The most profitable deviation by unshrouding hidden prices is the same as in Proposition 2 and leads to zero profits. In this deviation, a firm unshrouds hidden fees and offers transparent prices equal to marginal costs.

customer prices. We see from (2) that also the naive-customer price decreases in α.[15] Thus, populations with more naifs have lower transparent prices.[16] These comparative statics are in line with findings of Schoar and Ru (2016): credit-card companies target offers with larger annual fees, a rather transparent fee, to credit-card consumers with higher education. That is, consumers who are less likely to be naive.

Remark: All these properties of shrouding continuation equilibria carry over to cases where unshouding is unfeasible or very costly. Without unshrouding, market continuation profits always equal $\alpha(1-\alpha)\bar{a}$. Firms earn a share of this profit equal to their customer base.

4.2 Mitigated Competition for Customer Bases

The profitability of private information on customer naiveté, even when products are perfect substitutes, is the main result of this paper. I now discuss implications of this result on competition for customer bases in period 1. When firms cannot unshroud hidden fees, future profits are competed away in period 1. However, the ability of firms to unshroud hidden fees, instead of helping consumers, softens competition even more.

First, I introduce an equilibrium-selection assumption. The ability of firms to unshroud hidden fees induces multiple continuation equilibria in period 2. To deal with this multiplicity, I make the following equilibrium-selection assumption.[17]

Assumption 1 (Equilibrium-Selection Assumption)**.** *Firms shroud hidden fees in period 2 if and only if all firms earn strictly positive expected profits when hidden fees are shrouded.*

This assumption rules out two types of continuation equilibria. First, firms might miscoordinate on unshrouding in period 2. When at least two firms with a customer base unshroud hidden fees, none of them benefits from unilaterally shrouding hidden fees. The result is a standard Bertrand equilibrium with zero profits. But since each firm strictly prefers the shrouding continuation equilibrium over the Bertrand one, it is plausible that firms coordinate on the equilibrium that is more profitable for each of them.[18]

[15] The average margin of total naive-consumer prices is $\ln(\frac{1}{1-\alpha})\frac{1-\alpha}{\alpha}\bar{a}$, which decreases in α.

[16] Both (1) and (2) also predict a wider range of prices as the share of naive consumers increases.

[17] I could also make the assumption that firms shroud hidden fees in period 2 if and only if all firms strictly prefer the shrouding-continuation equilibrium over unshrouding.

[18] Heidhues et al. (2016b) argue that this is the only reasonable equilibrium. Among other things, the Bertrand-type equilibrium is not robust to positive unshrouding costs.

Second, firms without customer base are indifferent between the shrouding-continuation equilibrium and unshrouding in period 2. They earn zero profits in either case. But deviating from a shrouding equilibrium by unshrouding only earns zero profits because of the simplifying assumption that unshrouding turns all naifs into sophisticates who can avoid hidden fees. These avoiding naifs cannot be profitably attracted by unshrouding firms. As I show in Appendix A.1, shrouding-continuation equilibria where a firm earns zero profits are not robust to an arbitrarily small share of naifs who *cannot* avoid, and therefore pay, unshrouded hidden fees.[19] Firms who deviate from shrouding-continuation equilibria by unshrouding can profitably attract these non-avoiding naifs and earn strictly positive profits from the deviation. This allows me—plausibly—to focus on equilibria in which firms without customer base educate consumers with probability one, since other equilibria are not robust to the presence of non-avoiding naifs.

I now characterize firms' total profits when shrouding occurs. Denote the smallest market share by $s_{min} = \min_n\{s_n\}$ and the set containing all firms that charge the lowest price in period 1 by $M = \{n \in \{1, 2, ..., N\} | f_{n1} = \min_n\{f_{n1}\}\}$. Then we can write down the total profits of firm n given firms shroud in period 1 (Figure 3):

$$\pi_{n1}(f_{11}, ..., f_{N1}) = \begin{cases} \frac{s_n}{\sum_{o \in M} s_o}(f_{n1} + \alpha\bar{a} - c) + 0, & \text{if } f_{n1} = \min_{n'}\{f_{n'1}\} \leq v \ \&\ M < N \\ s_n(f_{n1} + \alpha\bar{a} - c) + s_n\alpha(1-\alpha)\bar{a}, & \text{if } f_{n1} = \min_{n'}\{f_{n'1}\} \leq v \ \&\ M = N \\ 0, & \text{if } f_{n1} > \min\{v, \min_{n'}\{f_{n'1}\}\} \end{cases} \tag{3}$$

Total profits exhibit a new kind of discontinuity that stems from the dynamic nature of the game and the possibility to educate consumers about hidden fees. We saw in Proposition 3 that firms strictly prefer shrouding in period 2 only if they have a positive customer base. If some competitors have no customer base, they have no private information on any consumer's naiveté. These firms deviate from the shrouding continuation equilibrium, and unshroud hidden fees to attract more consumers.[20] But unshrouding turns naifs into sophisticates who avoid hidden fees and induces zero continuation profits for each firm. Therefore,

[19]For example, these non-avoiding naifs might not have liquid funds to pay back credit-card debt to avoid borrowing costs, or they might value an add-on even when it is expensive.

[20]Recall that when an arbitrarily small share of naifs cannot avoid unshrouded hidden fees, firms without customer base strictly prefer unshroding. This is captured by Assumption 1. For details, see the discussion around Assumption 1, and Appendix A.1.

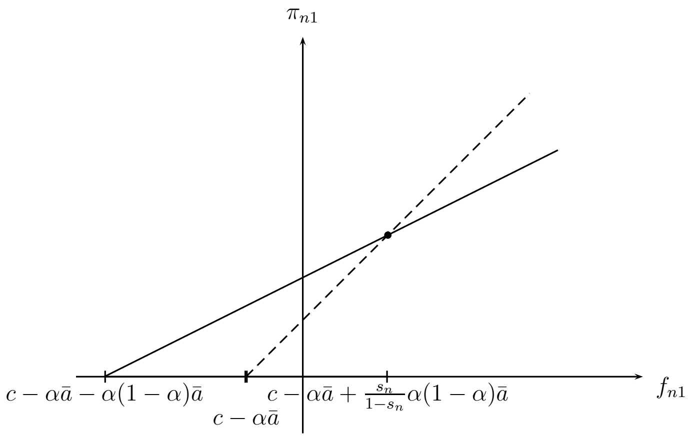

Figure 3: Suppose firms shroud in period 1. The solid line are total profits of a firm when all firms set the same price in period 1. The dashed line are total profits of a firm that undercuts all rivals in period 1. For all $f_1 \in [c - \alpha\bar{a} - \alpha(1-\alpha)\bar{a}, c - \alpha\bar{a} + \frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a}]$, no firm has an incentive to undercut competitors.

to ensure that all firms have a customer base, firms have a strong incentive to coordinate on setting the same transparent price in period 1. This *softens customer-base competition* already in the first period. Future profits are not competed away ex ante, but instead total profits can increase above the second-period level.

Firms have a strong incentive to coordinate on the same transparent price. But they might coordinate on a range of prices, inducing multiple equilibria in period 1 (see Figure 3).[21] But for all firms, shrouding equilibria with a higher transparent price dominate equilibria with a lower transparent price. This makes it plausible to apply the following equilibrium-selection assumption.[22]

Assumption 2 (The Firms' Preferred Shrouding Equilibrium)**.** *In period 1, firms coordinate on the shrouding equilibrium that induces strictly larger total profits than any other shrouding equilibrium.*

Proposition 4. [Mitigated Customer-Base Competition in Shrouding Equilibria]

Shrouding equilibria with shrouding in both periods exist. In each equilibrium satisfying Assumptions 1 and 2, all firms choose hidden fees $a_{n1} = \bar{a}$. All firms set the same transparent price $f_1 = c - \alpha\bar{a} + \frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a}$. Total profits are $\Pi_n = s_n\frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a} + s_n\alpha(1-\alpha)\bar{a}$. Consumers pay second-period prices as

[21] I characterize all these equilibria in detail in the proof of Proposition 4.

[22] The assumption is equivalent to choosing the firms' pareto-dominant equilibrium.

in Proposition 3.

The smallest firm s_{min} features prominently in Proposition 4, because it gains most market shares by deviating from the coordinated price in period 1. Also note that sophisticated consumers do not gain from pretending to be naive. To get a discount in period 2 that is below $\alpha\bar{a}$, they would have to pay $\bar{a}$ in period 1.

These results do not depend qualitatively on the extreme assumption that unshrouding turns all naive consumers into sophisticated ones who avoid hidden fees. Appendix A.3 verifies that results are qualitatively robust even when unshrouding impacts only an arbitrarily small share of naive consumers. Appendix A.1 establishes robustness when some naive consumers do not avoid unshrouded hidden fees. These results are also robust to a smoothly decreasing demand function, as I show in Appendix A.2.

The comparison with Propositions 1 and 2 shows that private information on existing customers has a strong impact on the properties of shrouding equilibria. Total prices increase in both periods for *all* customers. The reason is the following. In period 2, firms use their customers' usage data to target profitable naifs with retention discounts. This causes rivals to adversely attract unprofitable sophisticated consumers, and they poach less intensely. When unshrouding is completely infeasible or very costly, firms compete away future shrouding profits in period 1 and total profits are zero. But if in addition firms can unshroud hidden fees to some naive consumers, they soften competition for customer bases and future shrouding profits are not handed over to consumers in period 1.

The model highlights new and important dynamic effects in markets for deceptive products. While there are no dynamic effects in Proposition 2, they become crucial when firms learn about their customers. The competition for the market in period 1 works very differently from competition within the market in period 2, and the results differ in crucial aspects from known properties of markets for deceptive products. Most importantly, information on customer naiveté is a valuable asset for firms.

5 Policy Implications

When some consumers are naive and make costly mistakes, usage data mitigate competition even in seemingly competitive markets with perfect substitutes. In this section I discuss policies that encourage competition in this setting.[23]

Regulating hidden fees is an obvious intervention to increase consumer surplus. But another policy suggestion derives from the results above. Regulators could try to induce symmetric information of firms on consumers' level of sophistication. One such policy would be to encourage consumers to share their usage data with competitors when shopping for better deals.

Consumption data are usually not only accessible to firms but also to consumers. Since firms have to write a bill to consumers—phone bills depend on how much and which network was called, credit-card bills depend on payments made with the card and the resulting overall balance—many usage data are in principle available to consumers as well and can therefore be given to competing firms. Some alternative policies can also induce more symmetric information on consumers. First, policymakers can disclose each consumer's offer to competitors, i.e. by forcing firms to make public offers. Second, a more extreme intervention would be to force firms to share their customers' usage data with competitors.

To evaluate such a policy in the context of this model, suppose all consumers share their usage data with competitors to hunt for a better deal.[24] This allows each firm to charge different prices to each customer type, whether she is in the firms' customer base or not.

Proposition 5. [Deceptive Markets with Symmetric Consumer Info in Period 2]
Firms earn zero profits in each second-period continuation equilibrium and in period 1. Equilibria exist where shrouding occurs with probability one. In these equilibria, consumers pay total prices equal to marginal costs in period 2 and transparent prices $c - \alpha\bar{a}$ in period 1. Hidden prices are $a_{n1} = a_{n2} = \bar{a}$. If shrouding does not occur with probability one in period 2, it occurs with probability zero.

With symmetric information on customers in period 2, the market is effectively split, and firms compete for naifs and sophisticates separately. This induces marginal-cost pricing and zero profits even in shrouding equilibria. As in Proposition 4, firms target transparent prices equal to marginal cost to sophisticates, but prices for naifs decrease.[25] The policy triggers a rent shift from firms to consumers. These lower profits also

[23] In addition to fostering competition, regulation can also improve efficiency. Heidhues and Kőszegi (2017) argue that transparent prices below marginal costs induce overparticipation of consumers. And Heidhues et al. (2016a) show that large profits can induce inefficient investments in exploitative technologies. Policies that move prices towards marginal costs reduce these inefficiencies.

[24] It is not necessary that all consumer types share their data. Only one type of consumers sharing data has the same implications.

[25] Note that expected payments of sophisticates increase. They no longer benefit from poaching offers intended to attract naifs.

imply larger incentives for firms to unshroud hidden fees. Importantly, the policy induces the same increase in expected consumer surplus also when unshrouding is unfeasible or only affects some naive consumers.

In addition to the regulatory benefits, Propositions 2, 3, and 5 jointly establish that indeed the private information on naiveté causes high profits. This also implies that firms prefer not to become informed about competitors' customers, since this intensifies future competition and decreases shrouding profits.

Changing firms' information on consumer naiveté is an alternative to policies that try to help consumers make better decisions.[26] Even though empirical findings strongly suggest that consumers are not aware of product or contract features in some markets, it is not always clear how exactly they misunderstand these features. This makes it difficult for regulators to design effective simplification or education policies. Such policies require deep regulatory knowledge, a feature they share with well-designed price regulations. In contrast, inducing symmetric access to customer data is much less sensitive to regulatory knowledge.

Naturally, such a policy should not be implemented lightly. One concern is that easily available customer data might induce firms to enter the market just to get the data, and to use them in another market. In addition, partial data sharing can lead to larger profits. Suppose $\alpha \gg 0.5$, no sophisticated consumer shares data, and only few naifs do. Then with data-sharing, firms would have a more balanced customer base of consumers who do not share data. By Proposition 3, a more balanced customer base increases profits and prices.

Similar policies have already been discussed and implemented in practice. Thaler and Sunstein (2008) discuss a policy called RECAP, or smart disclosure, that has been implemented in the UK for consumer financial products as midata. These policies simplify consumer data and make them easily available to consumers to help them make better decisions. Similarly, the new EU General Data Protection Regulation (GDPR) contains data portability requirements that allow consumers to request their personal data from banks without outside authorization. My results suggests a novel mechanism through which smart disclosure, midata, and GDPR can benefit consumers. By making their usage data easily available to consumers, these policies also encourage consumers to take their usage data competitors to shop for better offers. Rivals can then more easily target offers to these consumers.[27]

Banning price discrimination leads to the same outcome as in Proposition 2 and increases consumer

[26]Nudges are one example. Other policies that go in this direction are discussed by Kamenica et al. (2011).

[27]To what extend GDPR will actually enable consumers to shop around with their data will crucially depend on how widely the term ‘personal data’ will be interpreted.

surplus as well. But banning price discrimination in credit-card or retail-banking markets would probably have many adverse consequences. Especially since discriminating consumers along other dimensions, e.g. their risk behavior, can increase welfare.

6 Extensions and Applications

6.1 Extensions

The basic model discussed in Section 4 makes some rather particular assumptions, but this is just one way to model how firms benefit from learning their consumers' naiveté. The main results are robust to a wide range of alternative assumptions.

The basic model assumes that unshrouding turns naive consumers into sophisticates. This is unrealistic for two reasons. First, unshrouding *might only affect some naifs*. Appendix A.3 studies unshrouding that only turns some naifs into sophisticates. Second-period shrouding profits are unaffected, and total shrouding profits decrease but remain positive akin to Proposition 4. As long as unshrouding has some (arbitrarily small) effect on naifs, it reduces continuation profits. Thus, to prevent unshrouding, firms still coordinate transparent prices in period 1 and earn positive total profits. Second, as I discuss when introducing Assumption 1, it is unlikely that all naifs want to or can avoid unshrouded hidden fees. For example, some naifs might not have liquid funds to pay back credit-card debt to avoid borrowing costs, or they might value the add-on. To address these concerns, Appendix A.1 studies *non-avoiding naifs* who pay unsshrouded hidden fees. Non-avoiding naifs make a deviation from the shrouding equilibrium by unshrouding more profitable. Especially firms without market shares then strictly prefer unshrouding, motivating the equilibrium-selection Assumption 1. Non-avoiding naifs also make results robust to positive unshrouding costs.

The basic model assumes homogeneous products, but I analyze an extension with Hotelling-type demand in Appendix A.2. The main results are robust to horizontal product differentiation with a smooth demand function. Shrouding continuation equilibria look very similar to Proposition 3. Again, firms employ information on naiveté to target naive consumers with smaller transparent prices. This causes poaching rivals to adversely attract less-profitable sophisticated consumers and mitigates competition. Product differentiation makes poaching even more difficult and increases shrouding profits further. Similar to Proposition

4, the ability to unshroud hidden fees mitigates competition for customer bases. Allowing again for non-avoiding naive consumers, firms must earn sufficiently large profits in period 2 to keep hidden fees shrouded. But with a decreasing demand function, different first-period prices lead to different sizes of the customer base. This allows firms to coordinate on a first-period price level that gives each firm exactly the right customer base to be indifferent between shrouding and unshrouding in period 2. Any deviation from this price level in period 1 reduces the customer base of one firm enough to cause unshrouding in period 2, thereby reducing continuation profits for everyone. Firms coordinate on this price level. As in Proposition 4, future shrouding profits are not competed away when firms compete for customer bases in period 1.

In the basic model each firm has the same share of naifs in their customer base. Appendix A.5 relaxes this Assumption and allows for different shares of naifs in the firms' customer bases. Firms can profitably trade client portfolios to make customers bases more balanced on average, leading to larger market profits.

The basic model has only 2 priods. The extension in Appendix A.4, looks at a model with two firms and T periods. A shrouding equilibrium exist where in each period $t > 1$, poaching works akin to Proposition 3. Again, firms target continuing naive customers with a lower transparent price.

The Supplementary Material discuss the remaining extensions. In the basic model, naive consumers do not learn about their naiveté over time, except when educated by a firm. This is consistent with empirical evidence of consumers repeatedly triggering fees they are unaware of.[28] But when some naive customers learn about hidden fees on their own, results do not change qualitatively. This is also true when new customers arrive in period 2. In both cases, observing naiveté in period 1 is a noisy but informative signal for behavior in period 2, allowing firms to earn positive shrouding profits.

Entry in period 2 would not reduce shrouding profits to the level of fixed costs of entry. In period 2 an entrant has no customer base and therefore large incentives to unshroud hidden fees. But unshrouding reduces overall market profits, which in turn reduces incentives to enter the market.

6.2 Discussion of Key Modeling Assumptions and Applications

A central assumption of this article is that naive consumers make unexpected payments. These unexpected payments result from consumer mistakes, e.g. consumers might misperceive product- or pricing features, or

[28]For the empirical evidence, see Cruickshank (2000); Stango and Zinman (2009); Shiller (2014); OFT (2008).

misestimate their own demand for an add-on service. This assumption is consistent with empirical evidence in a number of industries, and is made in different ways in many papers in the literature of behavioral industrial organization.[29]

The other key assumption is that firms gather and process information on consumers to design and target offers to naive consumers. Some direct evidence, though not indisputable, is consistent with this assumption. Schoar and Ru (2016) show that credit-card companies target less-educated consumers with more back-loaded payments, i.e. low introductory teaser rates but higher overlimit fees, penalty interest rates, and late-payment fees. However, with their available data they can only check whether offers are conditioned on rather publicly observable information. Gurun et al. (2016) report evidence that mortgage lenders target less-sophisticated populations with more expensive mortgages.

These papers are consistent with firms targeting naive consumers based on publicly observable proxies for naiveté. An additional feature of my model is that firms condition targeted offers also on their private information about their customers, i.e. usage data such as past purchases or browsing data that firms can gather in the context of their exclusive firm-customer relationship. Following Heidhues and Kőszegi (2017), I argue that simple economic reasoning imposes that firms have strong incentives to learn to distinguish their customers by their degree of naiveté, implying that targeting naive customers is or will soon be pervasive. In the settings I look at, naive consumers are more profitable than sophisticated ones with the same initial beliefs and perceived preferences. This implies that i) firms have a strong incentive to use their available information to distinguish their customers' naiveté and ii) firms can use profitability of customers as a proxy for naiveté. Supplementing this argument, much of the empirical literature cited in this paper documents simple correlates of the propensity of consumers to make mistakes, suggesting that also firms have access to at least partial information on naiveté. This is especially likely given recent developments in technologies for gathering, storing, and processing large amounts of data. These arguments suggest that firms *can* acquire information about consumer naiveté *and* firms have a strong incentive to find out which of their consumers are the profitable naive ones. Thus, targeting naive customers is or will likely soon be pervasive.

I now discuss in more detail some applications for this model such as markets for credit cards, retail

[29] I will cite many examples from the literature below in this Subsection.

banking, casinos, (mobile) phone services, and bonus cards in retail markets. Consumer mistakes and targeted pricing occur in these settings, and information collection of customers' purchase patterns is relatively simple and pervasive. For example, in most of these markets detailed information on consumption patterns is needed to write bills to customers.

Credit cards are quite homogeneous products that mainly vary in fee structures. Many consumers pay more than expected because they do not take overlimit, overdraft or late fees into account, or underestimate their tendency to borrow money.[30] The hidden fee captures these unanticipated payments of naive consumers. Examples of rather transparent fees are maintenance fees, cash rewards, introductory APRs or new-client bonuses. Despite the ability of each firm to get credit-scores about new customers, firms have an informational advantage on their existing customers. They have much more detailed information about their clients' past purchasing behavior, e.g. where they buy cloths, how often they visit a pub and which one, and on the credit-card fees that a customer had to pay in the past. Stango and Zinman (2009) provide another example on how firms can learn to distinguish customers based on their naiveté.[31] It is common practice in the credit-card industry to change contract terms of existing contracts and offer bonuses such as miles or cash benefits to existing customers, enabling issuers to target naive consumers with transparent discounts.

Besides these more crucial modeling assumptions, Stango and Zinman (2014) show that simple unshrouding policies are effective. Simply asking consumers about overdraft fees in a survey significantly reduces their probability to pay those fees. Similarly, Alan et al. (2015) find that mentioning overdraft fees to customers of a Turkish retail bank reduces demand for overdraft, even when combined with a discount.

Retail banking is another market close to this setting. Empirical evidence suggests that customers underestimate their likelihood of overdraft when choosing a bank-account.[32] This suggests that fees and interest payments associated with overdraft are hidden fees to many consumers. Account maintenance fees are rather salient and more likely to be transparent fees. Informational advantages are similar to the credit-card example, as is the possibility to change contract terms of existing customers to target transparent discounts.

[30] See Ausubel (1991), Shui and Ausubel (2005), and Meier and Sprenger (2010) for evidence. Agarwal et al. (2008) find that many credit-card consumers seem to not know or forget about some fees. Heidhues and Kőszegi (2017) also discuss the application of the model to credit card markets.

[31] They identify hidden fees as savings in fees that a credit-card customer could have made by shifting liquidity between accounts.

[32] See Alan et al. (2015), OFT (2008), or Cruickshank (2000) for evidence. CMA (2016) report that a quarter of UK retail-bank-account holders use unauthorized overdraft each year, suggesting they do not have the best account for them. Banks earn £1.2 billion a year from unauthorized-overdraft charges.

Gambling markets: Casinos target gamblers with free complimentary goods and services, such as drinks or valet parking, but also free rooms and transportation. This targeting depends on the amount of betting. Indeed, customer data can be highly valuable for casinos.[33]

Mobile-phone services: Grubb (2009), and Grubb and Osborne (2014) show that firms can exploit consumers' overprecise beliefs about their own usage of data and minutes by offering monthly data- and minute packages combined with fees for additional usage. This results in unexpected payments corresponding to hidden fees. The vast amount of data on customers' past calls give firms an informational advantage over rivals. Firms can target transparent discounts like extra minutes or better phones to profitable consumers.

Retailing markets: Johnson (2017) studies retailer competition with unplanned purchases. Consumers visit a shop with a consumption bundle in mind but can engage in unplanned purchases once in the shop. Retailers offer discounts below marginal costs for the planned products and charge positive margins for unplanned purchases. In practice, retailers can use data from loyalty- or bonus-card systems to predict which consumers are more likely to make unplanned purchases, allowing them to target discounts accordingly.

As these applications show, transparent and hidden fees might not exactly correspond to a particular observed fees. A more general interpretation is as anticipated and unanticipated payments. To illustrate, take a credit card with 10€ maintenance fee and 0.10€ for each Euro borrowed. Say a consumer anticipates to borrow 50€, but will actually borrow 100€. Then the anticipated transparent price is 10€ + 5€ = 15€ and the hidden fee is the unanticipated interest of 5€.

7 Related Literature

To the best of my knowledge, I am the first to study the impact of private information on consumer naiveté on targeted pricing and competition, and the first to analyze the impact of this information on naiveté on dynamic competition with shrouded attributes.

Consumer mistakes seem an intuitive explanation for large profits in seemingly competitive industries. Ausubel (1991) suggests that consumer mistakes could be a cause for large profits in the US credit-card industry, even though market fundamentals suggest a highly competitive market. Nonetheless, most papers

[33]For an overview on gambling markets, see Eadington (1999). O'Keeffe (2015) reports in the Wall Street Journal that the most valuable asset in the bankruptcy feud at Caesars Entertainment Corp. in 2015 was not the company's real estate in downtown Las Vegas, but their big-data customer loyalty program, valued at $ 1 billion by creditors.

that investigate how firms take advantage of consumer mistakes and shrouded attributes do not predict extraordinary profits in highly competitive markets. Profits obtained from naive consumers are used to reduce transparent prices to attract more consumers (e.g. see Gabaix and Laibson (2006), Armstrong and Vickers (2012), Murooka (2013), Heidhues et al. (2016b), Heidhues and Kőszegi (2017)). I build on these models and extend them to a dynamic setting. The main novel insight of this dynamic perspective is that over time firms should learn who their profitable naive customers are, and give them retention discounts.

Profits through price floors. Some papers argue that large profits result from price floors. Heidhues et al. (2016b) argue that firms do not want to reduce transparent prices too much because this could attract unprofitable consumers that enjoy a new-client discount without using the product. Miao (2010) analyzes a dynamic model where firms simultaneously offer a base good in a primary market and an add-on on a secondary market. The price on the primary market has to be large enough to prevent consumers from purchasing the base good again, effectively inducing a price floor. This explanation, however, does not apply in settings without a separate secondary market such as the credit-card industry or retail banking. I offer an alternative explanation for large industry profits that does not require a price floor, but rather builds on the interaction of consumer mistakes and private information of firms about their customers.

Heidhues and Kőszegi (2017) study the role of *publicly available seller information on naiveté*, and the impact on third-degree price discrimination. They find impacts on welfare but none on profits. Kosfeld and Schüwer (2017) extend the framework of Gabaix and Laibson (2006) to study welfare implications of consumers' effort to avoid hidden fees. All firms have the same noisy signal on consumers' level of naivete, which firms can use to target unshrouding, and conditional on unshrouding also prices. But since all firms have the same information on consumers, equilibrium profits do not increase with targeted unshrouding. In contrast to these papers, I show that *private* information of firms about their clients' naiveté increases firms' profits. These competition-impairing effects are not present in models with symmetric information.

Ellison (2005) recognizes that adverse selection of less profitable consumers can soften competition. I build on and significantly extent this intuition. In Ellison's paper adverse selection arises because firms are horizontally differentiated and customers who pay a lot on add-on fees are less price sensitive to cuts in base prices. Thus the customers who respond disproportionately to a cut in the base price are cheapskates who are unprofitable because they buy fewer add-ons. This reduces the benefits of cutting prices and leads to larger equilibrium prices. Firms have symmetric information about consumers and charge the same base price to all consumers. Adverse selection arises because less profitable consumers are more sensitive to price changes. In my model, however, all consumers have the same sensitivity to a given price cut. Thus, adverse selection arises from a very different mechanism which is based on firms using private information about customer naiveté to target retention discounts. Also, in Ellison's model consumers have brand preferences and firms derive some market power from horizontally-differentiated products. Yet in my model, adverse selection arises even with perfect substitutes when consumers have no brand preferences. Because private information is not relevant in Ellison's model, but is key in the mechanism worked out in this paper, my results have very different implications for the value of customer data in competitive markets, and for policies that affect information on consumers.

Literature on targeted pricing. Bester and Petrakis (1996), Belleflamme and Vergote (2016), and Montes et al. (2017) study firms who can target prices to consumers with different brand preferences. Armstrong (2006) is most closely related. He studies a Hotelling model where firms can target segments of the market with coupons. Each firm has an informational advantage over some consumers' brand preferences. His results differ in two important ways. First, private information can reduce the informed-firm's profits, because the uninformed rival will compete more intensely. In contrast, I find that private information on naiveté mitigates competition and is highly valuable. Second, when competition becomes more intense, i.e. when brand preferences vanish, all profits disappear. In my model with targeting by naiveté, profits are positive even when products are perfect substitutes.

Literature on switching costs. Switching costs are an alternative explanation for how firms benefit from old customers. For an overview, see, Klemperer (1995). Indeed, the classic incentives to invest in market shares in period 1 and harvest in period 2 are there. However, it is adverse-selection due to private information about customer naiveté, and not switching costs that lead to lock-in and high second-period prices. Additionally, if firms are able to educate naive consumers, the incentive to compete fiercely for market shares is strongly mitigated. Firms price high already in period 1 and earn positive total profits.

The adverse selection of unprofitable sophisticated consumers is reminiscent of *adverse selection and worker poaching in the labor-market literature*. Greenwald (1986) and Katz (1991) study labor markets where current employers let their low-productivity workers go to other firms but keep their high-productivity ones. Results crucially depend on the assumption that firms cannot make offers contingent on ex-post observable private information, i.e. the workers' productivity.[34] While being a reasonable assumption in labor markets, consumption of additional goods or services in consumer markets is frequently easy to verify.[35] Therefore, one would expect adverse-selection effects similar to worker poaching to be less important in retail markets. Nonetheless, I find that adverse selection is important in retail-market settings when some consumers are naive and mistakenly choose the same offers as sophisticates.

8 Conclusion

I investigate the role of customer data in markets in which firms can employ their customers' consumption data to predict the likelihood of customer mistakes. While customer data can also be valuable in rational models, my results suggest that the rational model severely underestimates the firms' benefits of using customer data to target consumers. This article, therefore, gives a novel explanation for high profits in seemingly competitive markets such as the credit-card industry.

These results are particularly important since two key characteristics become increasingly relevant in many markets. First, modern communication technology facilitates targeted offers. Second, big-data analysis becomes increasingly relevant for firms and improves the predictions of their customers' behavior. In particular when big-data allows firms to predict their customers' degree of sophistication, the results of this article shed new light on the role of big data in competitive markets. When firms manage to get hold of their customers' usage data, e.g. via cookies, search histories, or by requiring them to create an online account that facilitates the observation of usage patterns, firms can gather a lot of usage data that they can use to distinguish their customers' sophistication. This article therefore offers a new explanation on how big data related to such services or search engines can be profitably used, or sold even to firms active in competitive markets. In this context, investing in technology that enables firms to target consumers by naiveté is profitable, but reduces welfare.

In addition, big-data analysis has the potential to introduce a novel form of asymmetric understanding to

[34] Katz (1991) argue that even observable productivity might not be verifiable in court. Without this assumption, Mirrlees (1974) and Riordan and Sappington (1988) show that first-best outcomes can be implemented quite generally, even in monopoly settings.

[35] A bank can easily verify whether a customer overdrew on an account, and phone companies can verify the number of calls from customers to any phone number. Contracts that specify prices for such events are standard practice.

market settings when consumers are unaware of the informational traces they leave behind. Shiller (2014), for example, finds that the number of websites visited on Tuesdays and Thursdays predict demand for Netflix accounts while surfing on a Wednesday seems to carry little information. He also simulates that Netflix could have raised profits by only 0.8 percent when using price discrimination based on usual demographic characteristics. By using data on browsing behavior, such as website visits on Tuesdays and Thursdays, profits could have been increased by 13 percent. It seems hard to imagine that many consumers take the effect on prices into account when browsing the internet.

The benefits of private information on customers raises an important follow-up question. If firms benefit from customer data, why do consumers not take their credit-card and telephone bills to competitors to ask for a better deal? As discussed in Section 5, these competitors could make their offers conditional on this information. Indeed, this question applies more generally than in the context of consumer naiveté, and economists and policymakers are concerned with low rates of consumers switching to competitors in many markets.[36] I suggest four reasons why we do not frequently observe consumers taking there billing data to shop for better offers. First, consumers might not be aware of the informational content of their actions. To give an example, credit-card consumers might not be aware of how their purchase decisions correlate with their default risk. Duhigg (2009) gives discusses such correlations in his New York Times article. For example a pub in Canada "where 47 percent of the patrons who used their Canadian Tire card missed four payments over 12 months." Another example is from Shiller (2014) of the previous paragraph. It seems unlikely that consumers are aware that the number of websites they visit on Tuesdays or Thursdays is more informative than their visits on Wednesdays. Related to this is a second reason. Naive consumers who underestimate their expenses also underestimate how profitable they are to firms. A third reason is that billing data are usually not sent to consumers in a format that is standardized across firms and easy to transfer, e.g. with standardized labels in an excel file. This issue is one of the key motivation for policies in Section 5 like RECAP and midata. By allowing consumers to transfer their data to a standardized format, these initiatives aim to facilitate the comparison of products. Fourth, while usage data used for billing are in principle available to consumers, other information that firms have on their customers is not. This is especially true for browsing data which are available to every firm whose consumers use an online account.

[36]For examples, see OFT (2008), CMA (2016), or He and Reiner (2015)

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A Appendix: Extensions

A.1 Non-Avoiding Naifs

I assume in the main text that unshrouding turns naive consumers into sophisticated ones who can avoid hidden fees. This assumption simplifies the analysis but is unrealistic in some cases. One reason is that avoiding previously hidden expenses might be very costly or unfeasible to consumers. Consider a consumer who borrows from a credit-card or retail bank and has no liquid funds available. Without liquid funds, consumers cannot pay back their credit-card debt immediately and cannot avoid high costs of borrowing or late payment fees.[37] Another reason to pay unshrouded hidden fees is a high willingness to pay for an add-on service such as roaming. I develop a model that captures the first case where some naive consumers cannot avoid unshrouded hidden fees. The second interpretation as add-on demand changes the analysis only slightly. I will discuss this case afterwards.

Assume that a share $\eta \in [0, 1)$ of the naifs cannot avoid unshrouded hidden fees, but they take them into account. The remaining $1 - \eta$ naifs avoid unshrouded hidden fees.

The prices and profits of shrouding equilibria do not change in period 2. Both types of naifs are identical when firms shroud hidden fees. But incentives to unshroud hidden fees change and the existence of shrouding equilibria becomes an issue. In shrouding equilibria, non-avoiding naifs pay a total prices above marginal cost. Since they continue to pay unshrouded hidden fees, a firm that deviates and unshrouds hidden fees can profitably attract non-avoiding naifs.

Consider, for example, period 2 of shrouding equilibria as in Proposition 3. A firm can unshroud hidden fees and marginally undercut the smallest total naive-customer price of shrouding competitors. If this total naive-customer price is above v, the unshrouding firm charges v instead. In this way, the unshrouding firm attracts non-avoiding naifs at a total price $\min\{c + (1 - \alpha)\bar{a}, v\}$. By setting the transparent price component above c, the unshrouding firm can make sure not to attract unprofitable sophisticates. Overall, deviating from a shrouding equilibrium by unshrouding hidden fees in period 2 leads to profits of $\alpha\eta \min\{(1 - \alpha)\bar{a}, v - c\}$.

This shows that non-avoiding naifs make deviations from shrouding equilibria more profitable. The following Proposition summarizes the shrouding conditions for the existence of the shrouding equilibria in Sections 3, 4, and 5.

Proposition 6. [Shrouding Conditions with Non-Avoiding Naifs]
Assume the share $\eta \in [0, 1)$ of naive consumers cannot avoid unshrouded hidden fees while the others can avoid them costlessly.

1. *When firms do not learn their customers' types, the shrouding equilibrium as in Proposition 2 exists if and only if*
$$0 \geq \alpha\eta \min\{(1 - \alpha)\bar{a}, v - c\}. \tag{4}$$
When this condition is violated, unshrouding occurs with probability one.

2. *When firms learn their customers' types, shrouding equilibria exist where prices and profits are as in Proposition 3. Each firm strictly prefers to shroud hidden fees in period 2 if and only if hidden prices*

[37] Sophisticated consumers who cannot avoid hidden fees can be screened into another product as in Heidhues et al. (2016b).

are shrouded in period 1, each firm has a non-empty customer base, and

$$s_n\alpha(1-\alpha)\bar{a} \geq \alpha\eta \min\{(1-\alpha)\bar{a}, v-c\}, \ \ \forall n. \tag{5}$$

Otherwise, unshrouding occurs with probability one.

In each pure-strategy equilibrium in period 1, all firms charge $f_1 \in c-\alpha\bar{a}+\left[-\alpha(1-\alpha)\bar{a}, \frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a}\right]$, *giving rise to total profits* $\Pi_n = s_n(f_1+\alpha\bar{a}-c)+s_n\alpha(1-\alpha)\bar{a} \in \left[0, s_n\frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a}+s_n\alpha(1-\alpha)\bar{a}\right]$. *These equilibria exist if only if* (5) *holds and firms strictly prefer shrouding over unshrouding:*

$$s_n(f_1+\alpha\bar{a}-c)+s_n\alpha(1-\alpha)\bar{a} > \eta\alpha\min\{f_1+\bar{a}-c, v-c\} \ \forall n. \tag{6}$$

3. *The results of Proposition 5 remain unchanged.*

For $\eta = 0$, the deviation profits of unshrouding are zero, resulting in the special case depicted in the main body of the paper.

The most important result in this section is the following. With non-avoiding naifs, firms with an empty customer base earn positive profits by educating naifs and strictly prefer to unshroud hidden fees. This motivates the equilibrium-selection Assumption 1. When deviations from shrouding equilibria are profitable, the main results are also robust to positive unshrouding costs.

We see that when $\alpha\eta$ is very 'large', conditions (5) or (6) might be violated and shrouding equilibria do not exist. For example, in the extreme case where $\eta = 1$, unshrouding firms can profitably attract all naifs. This result is similar to Gabaix and Laibson (2006) where shrouding equilibria do not exist when all myopic consumers pay unshrouded hidden fees.[38]

In this model, however, shrouding equilibria can also exist for large η. I show in Proposition 8 that shrouding equilibria exist also for η arbitrarily large when the demand function is smoothly decreasing. In this case, unshrouding reveals to some consumers that the total price is above their willingness to pay. Some non-avoiding naifs no longer buy the product and unshrouding firms can no longer attract the entire market of non-avoiding naifs.

Demand for unshrouded add-ons.

Another reason why naifs might pay unshrouded hidden fees is because they actually value the add-on. To study this case, we interpret $\bar{a}$ as the consumers' valuation for the add-on. Naifs with add-on demand therefore value the base product together with the add-on at $v+\bar{a}$. Thus, the only difference to the model studied so far in this section is that unshrouding increases the valuation for the product by $\bar{a}$. The results from Proposition 3 therefore translate very quickly into this case.

Corollary 1 (Unshrouded Naifs with Add-on Demand)**.** *When the share η of naifs has value $\bar{a}$ for unshrouded hidden fees, the shrouding conditions in Proposition 6 change. Shrouding condition* (4) *translates into*

$$0 \geq \alpha\eta(1-\alpha)\bar{a}.$$

[38] In their model, this is the case when the cost to avoid hidden fees e is equal to or larger than the cap on the hidden fee.

Shrouding condition (5) *turns into*

$$s_n \alpha(1-\alpha)\bar{a} \geq \alpha\eta(1-\alpha)\bar{a}, \ \ \forall n,$$

and shrouding condition (6) *becomes*

$$s_n \alpha(1-\alpha)\bar{a} \geq \alpha\eta(1-\alpha)\bar{a}, \ \ \forall n.$$

The Corollary shows that shrouding conditions are the same as in a special case of the previous model, where non-avoiding naifs cannot avoid unshrouded hidden fees, when v is sufficiently large. When $\bar{a}$ denotes the willingness-to-pay for an add-on, learning the hidden fee does not influence the decision to buy the product or not.

In some settings, it seems reasonable to assume that unshrouding reduces the willingness to pay for an unanticipated add-on. This is the case, for example, if consumers underestimate their willingness to pay for shrouded add-ons. Unshrouding then reduces the willingness to pay to some $v_a < \bar{a}$, and unshrouding firms earn lower deviation profits. For this case, the right-hand-side of the Corollary's shrouding conditions are an upper bound for deviation profits.

In many applications, it is questionable to interpret the hidden fee as an add-on. Heidhues and Kőszegi (2017) and Heidhues et al. (2016b) discuss such applications where the hidden fee should be interpreted as unanticipated expenditures resulting from consumer mistakes, rather than demand for an add-on. Most relevant for this paper, Heidhues and Kőszegi (2017) show that the present model is a reduced-form model of credit-card borrowing with naively present-biased consumers: the transparent fee captures anticipated-, and the hidden fee unanticipated borrowing. In these cases, the previous interpretation of non-avoiding naifs as consumers who cannot avoid the hidden fee seems more appropriate.

Remark: If firms would offer multiple contracts to the same customers, there would be additional equilibria. With multiple offers to the same consumers, firms could make unshrouding unprofitable for competitors in period 2 for any value of η. They could do so by offering an additional contract to their existing naive customers with $(\hat{f}_{n2}, \hat{a}_{n2}) = (c, 0)$. If shrouding occurs, no consumer prefers this product to the one she gets in the equilibrium discussed in Proposition 3. But if unshrouding occurs in period 2, non-avoiding naifs are better off by choosing $(\hat{f}_{n2}, \hat{a}_{n2})$ instead of switching to a competitor. Thus, in the second period and for any $\eta > 0$, firms with zero market shares are indifferent between unshrouding or not. In addition to the shrouding equilibria in Propositions 3 and 4, this would induce equilibria in which firms always shroud in the second period, i.e. also when they have an empty customer base, and total profits are zero. This reasoning, however, relies on the fact that $(\hat{f}_{n2}, \hat{a}_{n2}) = (c, 0)$ is never chosen on the equilibrium path. It is, thus, not robust to consumers wrongly choosing this contract. To illustrate this, suppose naive consumers of firm n wrongly choose this contract with probability $\epsilon > 0$. Since these naifs would also pay a hidden fee, firm n is strictly better off by increasing $\hat{a}_{n2}$ to $\bar{a}$. As a consequence, naifs of firm n pay a total price above c such that competitors of n can unshroud hidden fees and profitably attract these non-avoiding naive consumers. This argument shows that these multiple offers that render unshrouding unprofitable are not robust to being chosen by mistake.

A.2 Smooth Demand with Imperfect Substitutes

In the main text I make the simplifying assumption that consumers are infinitely price sensitive. Marginally undercutting rivals induces a discrete increase in demand. This Section shows that the equilibrium described in Propositions 3 and 4 translates to a setting where firms sell imperfect substitutes and demand decreases smoothly. To do so, I introduce a Hotelling-type model. I show first that poaching works as in Proposition 3. But also the intuition from Proposition 4 carries over. Allowing for non-avoiding naifs who cannot avoid unshrouded hidden fees, firms need to earn sufficiently large profits in shrouding equilibria to keep hidden fees shrouded. Because of the decreasing demand function, different first-period prices induce different sizes of the customer base. Thus, in period 1 firms can coordinate on a price level $\bar{p}$ at which customer bases have just the right size to make firms indifferent between shrouding or not in period 2. Any deviation from this price in period 1 reduces shrouding profits enough to trigger unshrouding in period 2, thereby reducing continuation profits. I prove existence of equilibria where firms collude on $\bar{p}$ in period 1. As in Proposition 4, firms collude on prices in period 1 that do not compete away future shrouding profits.

Consider a Hotelling-type model with a linear city of length one. Two firms $l \in \{0, 1\}$ with identical marginal costs c are located at the endpoints of the linear city. Consumers are uniformly distributed on $[0, 1]$. They are interested in buying at most one unit of the good. "Transportation costs" are linear and given by the product-differentiation parameter τ. Thus, a consumer located at x with value v who buys from firm l at a price f_l gets the perceived utility $v - \tau|x - l| - f_l$. As before, this perceived utility is correct for sophisticated consumers. But the share α of consumers is naive and additionally pay a hidden fee $a_l \leq \bar{a}$. A consumer's outside option has value 0 but is available only at the endpoints of $[0, 1]$. It has net utility $-\tau \min\{x, 1 - x\}$.[39] As in the classic Hotelling model, consumers closer to firm l than the indifferent consumer $\bar{x}_l = \frac{1}{2\tau}(\tau + f_k - f_l)$ walk to firm l and the others to the rival k. But they buy if and only if $v > f_l$.

Consumers who walk to firm l buy if and only if their valuation is larger than the perceived price. Each location $x \in [0, 1]$ has a population of consumers whose valuations $v \in \mathbb{R}$ are distributed according to the cumulative distribution function $H(\cdot)$. For simplicity, I often write $D(f) = 1 - H(f)$. I assume that $D(f)$ is log-concave and twice continuously differentiable. Since $H(f)$ is a CDF, we know that $\lim_{f\to\infty} D(f) = 0$, which will be useful below.

Thus, overall demand of firm l is given by

$$D(f_l)\bar{x}_l.$$

In this specification of the Hotelling model, firms compete for market shares as in a classic Hotelling model. But the market is not automatically fully covered and overall market size depends on the price level. The number of consumers at any location that prefer to buy from l over their outside option is given by $D(f_l)$. I refer to $D(f_l)$ as the *market size of firm l*. The decision to buy or not from l is independent of the transportation cost τ. Thus, τ only affects competition between firms without affecting the market size.[40]

[39] This formulation of the outside option was first introduced by Bénabou and Tirole (2016), and later used by Heidhues and Kőszegi (2017). In a classical Hotelling model, the utility from the outside option is fixed. As a result, τ affects both the level of competition and the attractiveness of the outside option. Bénabou and Tirole's formulation abstracts away from the second effect, implying that a reduction in τ does not automatically increase demand. Demand increases only due to lower prices.

[40] In contrast, in a classic Hotelling model when the market is not fully covered and products become more homogeneous, the

As a result, the market size only increases when the transparent price of firms decreases.

As in the main model from Section 2, there are two periods. In period 1, firms know the distribution of naive and sophisticated consumers. In period 2, firms can distinguish the level of sophistication of consumers in their customer base. The customer base of firm l is again the consumers who bought from firm l in period 1 at first-period price f_{l1}, that is $D(f_{l1})\bar{x}_{1l}$. In both periods v is constant for each consumer. I use the same notation for prices as in the main model.

As in Appendix A.1, the share $\eta \in [0,1]$ of naive consumers *cannot* avoid unshrouded hidden fees.[41] These non-avoiding naive consumers make unshrouding more profitable. But unshrouding also makes non-avoiding naifs aware of the large total price they pay. Thus, unshrouding has two effects on these consumers. They might learn that the rival offers a lower total price (gross transportation costs), and switch. However, they might realize that the total price is above their value v and they stop buying the product. Thus, with a decreasing demand function, unshrouding reduces the overall demand of non-avoiding naifs.

Fudenberg and Tirole (2000) discuss how equilibria in dynamic Hotelling models are sensitive to firms offering short-term or long-term contracts. In short-term contracts, forward-looking consumers might not buy from their ideal firm to get a better poaching offer tomorrow. To abstract away from these issues, I assume i) that in the second period consumers redraw their location on the Hotelling line, and ii) that consumers who do not purchase in period 1 leave the market in period 2. The first Assumption also allows me to isolate learning about naiveté from learning about brand preferences.[42]

The first-period demand of firm l is $D(f_{l1})\bar{x}_{l1}$. I illustrate second-period demand with shrouded hidden fees, using the example of naive customers in l's customer base.

$$\alpha \bar{x}_{l1} D(\max\{f_{l1}, f_l^{naive}\}) \left(\frac{1}{2} + \frac{f_k^{new} - f_l^{naive}}{2\tau}\right).$$

In period 2, the share of naive consumers in l's customer base that prefer to purchase again over their outside option is $\alpha \bar{x}_{l1} D(\max\{f_{l1}, f_l^{naive}\})$. If the second-period transparent price increases ($f_{l1} < f_l^{naive}$), not all naifs in l's customer base purchase again. Because consumers redraw their location on the Hotelling line, the share of naive consumers that prefer to walk to firm l is given by $\frac{1}{2} + \frac{f_k^{new} - f_l^{naive}}{2\tau}$.

I denote the prices and profits that result from a standard symmetric one-shot Hotelling equilibrium with sophisticated consumers, given the firms' customer bases $\bar{x}_{l1} D(f_{l1})$, by f^H and π^H. I give the precise definitions in the proof, but as in a standard Hotelling model, $f^H - c$ and π^H increase in τ and converge to zero as $\tau \to 0$.

Assumption 3.

1. $\alpha \bar{a} \geq \max\{4\tau + (f^H - c), \frac{2\tau}{1-\alpha}\}$
2. $-\frac{D'}{D} \leq \frac{1}{f^H - c + \bar{a}}$.

number of buying consumers increase automatically.

[41] As argued in the second-to-last paragraph of the previous Section, this interpretation fits better the credit-card industry and retail banking.

[42] For learning about brand preferences, see Armstrong (2006). In these models, learning about brand preferences has a negligible effect on profits and prices as $\tau \to 0$.

These Assumptions imply that the Model is not too different from the one described in Section 2. The first one states that τ is small relative to the shrouding benefits $\alpha\bar{a}$, implying that shrouding profits are larger than Hotelling profits π^H. The second Assumption is a regularity Assumption and states that the demand function D does not decrease too fast.

To simplify exposition the following Proposition characterizes shrouding continuation equilibria only after histories with symmetric first-period prices $f_{l1} = f_1 \in (\underline{f}^{naive}, \underline{f}^{naive} + \bar{a} - \tau)$ for all l, where $\underline{f}^{naive} = c - \alpha\bar{a} + \tau + \frac{(1-\alpha)}{2}\frac{D(\max\{f^H, f_1\})}{D(f_1)}(f^H - c)$ is the smallest naive-customer price that a competitor does not want to undercut. Shrouding continuation equilibria also exist after other histories and are described in Lemma 6.

Proposition 7. [Hotelling version of Proposition 3]
Suppose $f_{l1} = f_1 \in (\underline{f}^{naive}, \underline{f}^{naive} + \bar{a} - \tau)$ for all firms l, and that Assumption 3 holds. A Hotelling version of the equilibrium in Propositions 3 exists. Each firm l sets $a_l = \bar{a}$ and $f_l^{soph} = f^H$. Firm l mixes the naive-customer price f_l^{naive} on $\left[\underline{f}^{naive}, \overline{f}^{naive}\right]$, and earns $\frac{1}{2}D(f_1)\alpha\left[\underline{f}^{naive} + \bar{a} - c\right]$ from old naive customers. With the new customer price f_l^{new}, firm l poaches only sophisticated consumers [soft poaching] with some probability $\rho < 1$ and sets $f_l^{new} = f^H$. With probability $1-\rho$, firm l poaches naive and sophisticated customers [aggressive poaching], and randomizes f_l^{new} on $\left[\underline{f}^{new}, \overline{f}^{new}\right]$. With f_l^{new}, firm l earns $(1-\alpha)\pi^H$. $\overline{f}^{new} = \overline{f}^{naive} < c$.

This shrouding continuation equilibrium exists if prices are shrouded in period 1 and the following shrouding condition holds:

$$D(f_1)\alpha(\underline{f}^{naive} + \bar{a} - c) \geq \underbrace{D(\underline{f}^{naive} + \bar{a})\eta\alpha(\underline{f}^{naive} + \bar{a} - c)}_{\text{unshrouding profits from naifs in own customer base}} + \underbrace{D(\underline{f}^{naive} + \bar{a} - \tau)\eta\alpha(\underline{f}^{naive} + \bar{a} - \tau - c)}_{\text{unshrouding profits from naifs in rival's customer base}}. \tag{SC}$$

The shrouding profits and poaching behavior closely match those described in Proposition 3. As before, firms mitigate competition by targeting naive consumers in their customer base with lower transparent prices.

Product differentiation makes sophisticated consumers more profitable. Because of product differentiation, firms can now earn profits π^H, akin to profits in a simple Hotelling equilibrium, from poaching only their rival's sophisticated consumers. I call this 'soft poaching'. Under Assumption 3, these profits are 'small' relative to the profits firms earn from their naive customer base, which is why firms continue to attempt poaching naive consumers. I call this 'aggressive poaching'. Since $\overline{f}^{new} < c \leq f^H$, aggressively-poaching rivals attract sophisticated consumers, and make losses on them. This tradeoff between aggressive and soft poaching is familiar from Proposition 3. So as before, the continuation equilibrium includes mixed strategies for naive-, and new-customer prices.

Product differentiation mitigates poaching for naive customers via two channels. First, since firms earn positive profits from soft poaching, they must also earn these profits when aggressively poaching naive consumers. Second, to poach all naive customers, firms must now undercut their rival's lowest naive-customer price by $\tau > 0$. Both effects make aggressive poaching less profitable, implying that the transparent price at which poaching of any naive customers is not profitable—$\underline{f}^{naive}$—increases in τ. But as $\tau \to 0$, $\underline{f}^{naive} \to c - \alpha\bar{a}$ as in Proposition 3. Also akin to Proposition 3, the profits firms earn at $\underline{f}^{naive}$ pin down shrouding profits. Overall, poaching naive customers is more costly with product differentiation and profits from old naive customers increase.

In contrast to the model in Appendix A.1, shrouding equilibria also exist for arbitrarily large η. The reason is that when $f_1 < \underline{f}^{naive} + \bar{a} - \tau$, unshrouding decreases demand from non-avoiding naifs. Some non-avoiding naifs learn that the product is more expensive than their total willingness to pay. Thus, also for large η deviating from the shrouding equilibrium by unshrouding can be unprofitable when sufficiently many non-avoiding naifs stop buying after unshrouding.

Proposition 7 shows that the equilibrium in Proposition 3 is robust to heterogeneous products and a smooth demand function. What is more, when products become homogeneous and all consumers participate, i.e. have the same valuation $v > c + \tau$, the model converges to the basic model analyzed in the main body of this paper. Also equilibrium prices and profits converge to the ones in Proposition 3.

Corollary 2. *Take the limit where $D' \to 0$ and all probability mass is on a v with $v > c + \tau$, and $\tau \to 0$. In this limit, the profits and prices of the shrouding equilibrium in Proposition 7 converge to the profits and prices of the shrouding equilibrium in Proposition 3, where $\bar{x}_{l1} D(f_{l1})$ replaces the firms' market share s_l. The shrouding condition converges to the second-period shrouding condition with non-avoiding naifs in Proposition 6.*

I now discuss the shrouding condition (SC). The shrouding equilibrium in Proposition 7 exists if the shrouding profits are larger than the profits from deviating by unshrouding hidden fees, i.e. if the shrouding condition (SC) holds. Unshrouding does not affect profits from former sophisticated consumers, so these profits do not appear in (SC). Also recall that unshrouding turns the share $(1-\alpha)\eta$ of naifs into sophisticates. Only the share $\alpha\eta$ of naive consumers continue to pay unshrouded hidden fees. These non-avoiding naifs are the only consumers that firms can profitably attract by unshrouding.

Next, I show that firms coordinate on a first-period price $\bar{p}$ that makes firms indifferent between shrouding and unshrouding in period 2. To do so, I first describe the optimal deviation from the shrouding-continuation equilibrium by unshrouding. Afterwards, I show that unshrouding, by revealing larger total price to consumers, makes demand in period 2 less responsive to the first-period price f_1. This implies that unshrouding-deviation profits in period 2 are non-decreasing in f_1, but shrouding profits in period 2 strictly decrease in f_1. As a result, firms shroud when their customer base is sufficiently large, i.e. f_1 is sufficiently small, and that firms can coordinate on a price $\bar{p}$ at which they are indifferent between shrouding and unshrouding in period 2. To simplify the exposition, I focus again on the case $f_{l1} = f_1 \in (\underline{f}^{naive}, \underline{f}^{naive} + \bar{a} - \tau)$, for all l. This makes the results on coordination of prices in period 1 more visible. But coordination prices $\bar{p}$ can also exist outside of this interval.

First, I describe the optimal deviation from the shrouding-continuation equilibrium by unshrouding hidden fees. Akin to the main model, a firm that deviates from the shrouding-continuation equilibrium by unshrouding sets the transparent price-component to $f^H > c$ to avoid attracting unprofitable sophisticates, and to earn Hotelling profits from continuing sophisticates when the rival does soft poaching. Unshrouding firms set a total naive-customer price $\underline{f}^{naive} + \bar{a}$. This price is below any total price of the poaching rival and ensures that the rival does not poach any profitable non-avoiding naif. With the total new-customer price, the unshrouding firm undercuts its rival's lowest naive-customer price by τ and charges $\underline{f}^{naive} + \bar{a} - \tau$. This way it can profitably attract all its rival's non-avoiding naifs.[43] Thus, to earn the unshrouding profits

[43] Intuitively, in the shrouding-continuation equilibrium lower new-customer prices induce more losses from poached sophisti-

on the right-hand-side of (SC), unshrouding firms charge a total naive-customer price $\underline{f}^{naive} + \bar{a}$, and total new-customer price $\underline{f}^{naive} + \bar{a} - \tau$. Since $f_1 \leq \underline{f}^{naive} + \bar{a} - \tau$, unshrouding increases the perceived total price from period 1 to period 2 and only the share $D(\underline{f}^{naive} + \bar{a} - \tau)\eta\alpha$ of the rival's naifs continues to buy the unshrouded product. Similarly, an unshrouding firm charges $\underline{f}^{naive} + \bar{a}$ to its own naive customers, reducing their market size to $D(\underline{f}^{naive} + \bar{a})\eta\alpha$.

Intuitively, firms who deviate from the shrouding-continuation equilibrium by unshrouding get a larger piece of a smaller pie. They face a tradeoff between increasing their market share from $1/2$ to 1, and reducing market size $D(\cdot)$. Firms can use unshrouding to reveal a total price below the rival's one to non-avoiding naifs in order to poach them. But unshrouding also reveals to some non-avoiding naifs that the total price is larger than their willingness to pay.

Unshrouding reveals a larger total price to non-avoiding naifs. This way, unshrouding makes demand less responsive to the first-period price f_1. We just saw that since $f_1 \leq \underline{f}^{naive} + \bar{a} - \tau$, unshrouding reveals larger total prices in period 2 and some non-avoiding naifs no longer buy. This is why the demand with unshrouded hidden fees is determined by total prices $\underline{f}^{naive} + \bar{a} - \tau$ and $\underline{f}^{naive} + \bar{a}$, and does not directly depend on f_1. Thus, unshrouding profits only depend on f_1 indirectly via $\underline{f}^{naive}$. It is straightforward to show that the right-hand side of (SC) weakly increases in f_1.

Shrouding-continuation profits, on the other hand, strictly decrease in f_1. Since $f_1 > \underline{f}^{naive}$, all naifs in a customer base buy again in period 2 at $\underline{f}^{naive}$. Thus, the first-period price f_1, by determining the size of the firms' customer base, has a direct effect on the shrouding-continuation profits (left-hand-side of (SC)). Assumption 3 implies that this direct effect of f_1 is larger than the indirect effect via $\underline{f}^{naive}$. Thus, shrouding-continuation profits strictly decrease in f_1.

There exists a first-period price $\bar{p}$ that makes firms indifferent between shrouding and unshrouding in period 2. As shown in the previous paragraphs, unshrouding profits (right-hand-side of (SC)) are non-decreasing in f_1. But a larger f_1 strictly decreases the customer base $\frac{1}{2}D(f_1)$ and therefore shrouding-continuation profits (left-hand-side of (SC)). This has two implications. First, shrouding-continuation equilibria exist when the market size $D(f_1)$ is sufficiently large, i.e. f_1 is sufficiently small. Second, if shrouding-continuation equilibria exist, $\lim_{f \to \infty} D(f) = 0$ implies the existence of a first-period transparent price $\bar{p}$ at which both sides of (SC) are equal. If this $\bar{p} \in (\underline{f}^{naive}, \underline{f}^{naive} + \bar{a} - \tau)$, firms can coordinate on a first-period price that makes them indifferent between shrouding or not.[44] Intuitively, non-avoiding naifs make unshrouding more profitable and firms have to earn sufficiently large profits to prefer the shrouding-continuation equilibrium over unshrouding. But because of the decreasing demand function, firms can coordinate on a price $\bar{p}$ that induces a customer base that is just large enough to make firms indifferent

cated consumers. Since poaching profits are constant, firms must earn larger profits from poached naifs at the lower price in the support. But unshrouding firms, by setting the transparent-price component $f^H > c$, do not poach unprofitable sophisticates. Thus, to poach rival's non-avoiding naifs unshrouding firms do not set total prices above the lowest total price in the support of shrouding-equilibrium total prices, i.e. $\underline{f}^{naive} + \bar{a} - \tau$. By Assumption 3, reducing total prices further below $\underline{f}^{naive} + \bar{a} - \tau$ does not increase demand much and is not profitable.

[44] For large enough $\bar{a}$, the interval $(\underline{f}^{naive}, \underline{f}^{naive} + \bar{a} - \tau)$ is large enough such that $\bar{p}$ is always in this interval. For $f_1 \notin (\underline{f}^{naive}, \underline{f}^{naive} + \bar{a})$, the shrouding condition does not directly depend on the first-period price. If $f_1 > \underline{f}^{naive} + \bar{a}$ the market size on both sides of (SC) is $D(f_1)$ and cancels out. For $f_1 < \underline{f}^{naive}$, the market size on both sides of (SC) is fixed and independent of f_1. But since $\underline{f}^{naive}$ depends on f_1, $\bar{p}$ can also exist in these cases. To simplify exposition, I leave out the case where $f_1 \notin (\underline{f}^{naive}, \underline{f}^{naive} + \bar{a} - \tau)$. For more details, see Lemma 6.

between shrouding and unshrouding in period 2.

After firms set the first-period price $\bar{p}$, firms are indifferent between shrouding or not in period 2 and the shrouding-continuation equilibrium is most fragile. But this fragility is exactly why firms collude on $\bar{p}$ in period 1. Any deviation from $\bar{p}$ reduces the market share of one of the firms and causes unshrouding in period 2. Unshrouding makes consumers aware of all prices they pay, which reduces continuation profits by limiting the margins firms can earn from any consumer to $f^H - c$.

The following Proposition summarizes this result. Shrouding equilibria exist where firms collude on $\bar{p}$ in period 1. $\bar{p}$ makes firms indifferent between shrouding and unshrouding in period 2. Any deviation from $\bar{p}$ reduces the market share of one firm and induces unshrouding in period 2, thereby reducing continuation profits. To present this Proposition, denote by $\pi^{dev}(\bar{p})$ the best deviation profits that a firm can achieve when deviating from colluding on $\bar{p}$. I give an example of $\pi^{dev}(\bar{p})$ below and define it precisely in the proof of Proposition 8.

Proposition 8. [Hotelling version of Proposition 4]

Suppose Assumption 3 holds and $\bar{p} \in (\underline{f}^{naive}, \underline{f}^{naive} + \bar{a} - \tau)$. Both firms charge $f_{l1} = \bar{p}$, they earn total profits of

$$\underbrace{\frac{1}{2}D(\bar{p})(\bar{p} + \alpha\bar{a} - c)}_{\textit{period-1 profits}} + \underbrace{\frac{1}{2}D(\bar{p})\alpha\left[\underline{f}^{naive} + \bar{a} - c\right]}_{\textit{period-2 profits from old naifs}} + \underbrace{(1-\alpha)(1+\rho)\pi^H}_{\textit{period-2 profits from poaching, and old sophisticates}} .$$

This is an equilibrium if these profits are larger than $\pi^{dev}(\bar{p})$, and there exist parameters for which this condition holds.

The Proposition parallels Proposition 4. Since $\bar{p} > \underline{f}^{naive} > c - \alpha\bar{a}$, second-period shrouding profits are not competed away in period 1. The reason is the same as in Proposition 4: attracting more consumers in period 1 is unprofitable because it reduces the rival's customer base enough to induce unshrouding in period 2.

Proposition 8 shows that the collusion result of Proposition 4 does not depend on the fierce Bertrand competition where firms loose all market shares if their price is undercut marginally. In Proposition 8 the result obtains as well with a Hotelling-type model with a smooth demand function. Exactly because of the decreasing demand function, firms can collude in period 1 on a price $\bar{p}$ that induces the right size of the customer base to make firms indifferent between shrouding or not in period 2. Because any deviation induces unshrouding in period 2 and reduces profits, this collusion is credible.

To illustrate the existence of these equilibria, consider the limit case where $\tau \to 0$. There are two types of deviations: the deviating firm might shroud and undercut $\bar{p}$, or it might unshroud hidden fees and attract all non-avoiding naifs. Consider first a deviation with shrouding. Suppose for simplicity that the deviating firm marginally undercuts $\bar{p}$.[45] Then the deviation profits are $\pi^{dev}(\bar{p}) = D(\bar{p})(\bar{p} + \alpha\bar{a} - c)$. Marginally undercutting $\bar{p}$ captures the entire market. Firms prefer to collude on $\bar{p}$ if $\bar{p} + \alpha\bar{a} - c < \alpha(1-\alpha)\bar{a}$, that is if the profits a firm earns from deviating are smaller than the profits from the shrouding continuation equilibrium. This is clearly true when $\bar{p}$ is sufficiently small, that is if η is 'large', $\underline{f}^{naive} + \bar{a}$ is 'small', or $-D'/D$ is 'small'.

[45] Marginally undercutting $\bar{p}$ is optimal if $-\frac{D'(\bar{p})}{D(\bar{p})} < (\bar{p} + \alpha\bar{a} - c)^{-1}$

Similarly, unshrouding and attracting all non-avoiding naifs is not profitable if $\frac{1}{2}D(\bar{p})(\bar{p}+\alpha\bar{a}-c)+\frac{1}{2}D(\bar{p})\alpha(1-\alpha)\bar{a} \geq \eta\alpha D(\underline{f}^{naive}+\bar{a})(\bar{p}+\bar{a}-c)$. This is true, for example, if $\eta\alpha D(\underline{f}^{naive}+\bar{a})$ is sufficiently small.

A.3 Partial Unshrouding

When firms unshroud hidden fees, they are unlikely to affect all naive consumers. At the same time, such policies are unlikely to be without any effect at all. Examples for the effectiveness of simple interventions are given by Stango and Zinman (2014) and Alan et al. (2015). In this Section, I show that the result of Proposition 4 on mitigated competition for market shares is robust to partial unshrouding. To this end, I assume that unshrouding turns only the share $\lambda \in [0,1]$ of naifs into sophisticates while the others remain naive.

Proposition 9. [Partial Unshrouding]

- *For $\lambda \in (0,1]$, shrouding equilibria exist where each firm strictly prefers to shroud hidden fees in period 2 if and only if hidden prices are shrouded in period 1, and each firm has a non-empty customer base. Prices and profits under shrouding are as in Proposition 3.*

- *For $\lambda \in (0,1]$, shrouding equilibria with shrouding in both periods exist. In each equilibrium satisfying Assumptions 1 and 2, all firms choose hidden fees $a_{n1}=\bar{a}$. All firm sets the same transparent price $f_1 = c-\alpha\bar{a}+\frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a}-\frac{1-\lambda}{1-s_{min}}\alpha(1-\alpha)\bar{a}$. Total profits are $\Pi_n = s_n[\frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a}-\frac{1-\lambda}{1-s_{min}}\alpha(1-\alpha)\bar{a}]+s_n\alpha(1-\alpha)\bar{a}$. Shrouding occurs with probability one.*

The first bullet point shows that results on the profitability of private customer information are unaffected by partial unshrouding. This is unsurprising since these results are robust to education in the first place.

The second bullet point states that for each $\lambda > 0$, shrouding remains both possible and profitable in period 1. Firms can achieve positive total profits for each $\lambda > 0$. Unshrouding induces market-wide continuation profits $(1-\lambda)\alpha(1-\alpha)\bar{a}$. Even though unshrouding in period 2 may lead to a more balanced customer base, it also makes the targeting of prices to naive and sophisticated customers less precise.[46] Overall, this is why unshrouding in period 2 strictly reduces continuation profits, as long as $\lambda > 0$. Thus, also arbitrarily weak unshrouding lowers the continuation profits of firms, and induces firms to coordinate prices in period 1.

In the extreme case where unshrouding is impossible, i.e. when $\lambda = 0$, second-period market profits remain unchanged at $\alpha(1-\alpha)\bar{a}$. In the absence of unshrouding, firms compete away all future profits in period 1.

The effects of the disclosure policy are unchanged and all profits are competed to zero, though some consumers will always remain naive.

[46]More precisely, firms cannot distinguish in period 2 which continuing consumers turned naive and which one did not. Thus, they also give transparent discounts to former naifs who are now sophisticated an avoid $\bar{a}$.

A.4 T Periods

Take $N = 2$ and $T > 2$ periods. For simplicity, I set $s_n = \frac{1}{2}$ for both firms. Denote by $\delta \in (0, 1)$ the discount factor.

Proposition 10. [Deceptive Markets with Private Information about Customer Bases, $N = 2$ and $T > 2$ Periods]
A profitable shrouding equilibrium with poaching as in Proposition 3 in each period $t > 1$ exists. In each period $t > 1$, both firms have symmetric information about n's consumers if some customers of a firm n were poached in the past. In this case, firm n earns zero profits. Now consider histories after which firm n has private information about its customers' naivete. These histories occur with strictly positive probability. The last period T is identical to Proposition 3 and each firm earns $\frac{1}{2}\alpha(1-\alpha)\bar{a}$. In every period $t \in \{2, ..., T-1\}$, firms play the same strategies. Each firm n sets $f^{soph}_{nt} = \bar{f} \equiv c - \frac{\bar{a}}{2} + \sqrt{1 - 4\delta(1-\alpha)\alpha}\frac{\bar{a}}{2} \in (c - \delta\alpha\bar{a}, c)$. f^{naive}_{nt} and f^{new}_{nt} are mixed on $(c - \alpha\bar{a}, \bar{f})$ and have a mass point at $\bar{f}$. Firms hand back expected profits from the next period when setting $f^{soph}_{nt} < c$, but earn $\frac{1}{2}\alpha(1-\alpha)\bar{a}$ in every period. The first period is identical to Proposition 4.

A T-period extension of Proposition 3 exists. The main characteristics of Proposition 3 persist in all periods $t > 1$ when firms have private information about their customer base. Firms target naive customers with lower transparent prices, and mix naive- and new-customer prices.

The same logic from Proposition 3 applies to each period $t > 1$. Firms offer transparent discounts to naive consumers in their customer base to mitigate poaching and increase profits. This way, firms can ensure shrouding profits of $\frac{1}{2}\alpha(1-\alpha)\bar{a}$ in each period.

Firms earn these shrouding profits from their customer base if and only if they have private information about these customers' types. If either naive or sophisticated consumers of a firm were poached in the past, the rival can target these consumers separately. But the rival also knows that all consumers it never attracted must be of the remaining type. Thus, both firms can distinguish both types in this customer base, and profits are zero.

To maintain the customer base, firms reduce transparent prices for sophisticated consumers. More precisely, in each period $t \in \{2, ...T-1\}$, firms hand back the profits of the next period to sophisticated consumers. Firms lower prices for sophisticated consumers to prevent the rival from learning about its own customers. Take period $T-1$. To earn positive profits in period T, firm n has to keep both customer types. Since firms set larger transparent prices to sophisticated than to naive customers, firm n keeps both customer types if and only if the rival's poaching price $f^{new}_{j,T-1} \geq f^{soph}_{n,T-1}$. Thus, firm n can increase the chance to keep all its customer base by reducing $f^{soph}_{n,T-1}$ to $\bar{f}$. At $\bar{f}$, all expected shrouding profits of the next period are competed away and a further reduction of the price for sophisticated-customers would induce losses. Overall, firms compete away expected future shrouding profits, but earn shrouding profits within the period of $\frac{1}{2}\alpha(1-\alpha)\bar{a}$.

Going backwards from there, the same logic applies in period $T-2$. Expected shrouding profits in T will be handed to consumers in $T-1$, so only the expected shrouding profits in $T-1$ matter for the firm in $T-2$. Thus, $T-2$ works exactly as $T-1$. The same argument continues until period 2. Indeed, firms use the exact same strategies for profitable shrouding in all periods $t \in \{2, ...T-1\}$.

In period 1, continuation profits in the shrouding equilibrium are $\frac{1}{2}\alpha(1-\alpha)\bar{a}$ and zero otherwise. This is the same situation as in Proposition 4 and the same logic applies. Interestingly, in this equilibrium, period 1 is unaffected by additional periods.

Profitable shrouding occurs in each period with strictly positive probability. To see this, note that in each period $t > 1$ firms set a mass point with probability mass of at least $(1 - \alpha)$ at the largest new-customer price $f_{n,2}^{new} = \bar{f}$. At this price, firms poach no customer and the competitor will maintain its customer base in the next period. In each period where they maintain their customer base, firms earn strictly positive profits and strictly prefer shrouding over unshrouding.

A.5 Gains from Trading Customer Portfolios

The results from subsection 4.1 on the benefits of a mixed customer base have important implications. In this section I show that firms with different shares of naive clients, can increase profits by trading parts of their customer base.

For this purpose, I consider a simpler version of the second period of the previous model with two firms that cannot unshroud hidden fees. Assume each firm already has a customer base s_n for $n \in \{1, 2\}$ with $s_1 + s_2 = 1$. Firms have different shares of naifs in their customer bases with $\alpha_1 \neq \alpha_2$. Since firms cannot unshroud hidden fees, they earn at least $s_n \alpha_n (1 - \alpha_n)\bar{a}$. This follows from the same logic leading to Proposition 3.

A potential problem with selling naive customers is that firms might poach them back. Since they know their type, they could target them directly and avoid the adverse attraction of naive consumers. Thus, to gain from trade, firms need to be able to commit not to poach consumers back. Potentially, this could be agreed upon in a contract. Writing such a contract might be problematic due to the anti-competitive nature of such agreements. In many deals, this problem does not occur since firms and banks commonly leave the market and sell their entire (local) credit-card business.[47]

Since second-period profits are strictly concave in α_n, firms with different shares of naifs benefit from trading customer portfolios:

$$s_1\alpha_1(1-\alpha_1)\bar{a} + s_2\alpha_2(1-\alpha_2)\bar{a} \leq \left(s_1\alpha_1 + (1-s_1)\alpha_2\right)\left(1 - \left(s_1\alpha_1 + (1-s_1)\alpha_2\right)\right)\bar{a}.$$

The left-hand side denotes total market profit if both firms keep their initial share of customers. The right-hand side is total market profit if both firms have the same share of naifs, i.e. the share of naifs in the population. The inequality follows directly from the concavity of profits in α. Note that this inequality is strict whenever $s_n \in (0, 1)$ and $\alpha_n \in (0, 1)$ for all $n \in \{1, 2\}$. Thus, firms can increase the profitability of their customers by acquiring customer portfolios that make their own portfolio more balanced. By doing so, they make portfolios more balanced on average and increase total market profits in the second period.

For a closer look, suppose $\alpha_1 < \alpha_2$, i.e. firm 2 has more naifs. Then total market profits maximize when firm 2 sells $s_2(\alpha_2 - \alpha_1)$ naifs and buys $s_1(\alpha_2 - \alpha_1)$ sophisticates in return.

Firms and banks frequently trade credit-card portfolios that are worth several billion dollars. Due to the large number of firms active in the credit-card industry, these deals are unlikely to result from a monopo-

[47] In 2006 the discount department store chain Kohl's sold its credit card business to J.P Morgan Chase for \$1.5 billion (Bloomberg, 2006). Bank of America sold its canadian credit-card business to the TD Bank Group in 2011 for \$8.6 billion (Protess, 2011), and the discount-retailer Target sold its credit-card portfolio to Toronto-Dominion Bank in 2012 for approximately \$5.9 billion (Reuters, 2012).

lization strategy. In contrast to mergers or acquisitions with effects on market power, my model predicts that these deals increase profits for the trading parties but they have no impact on profits of uninvolved rivals. Nonetheless, they are anti-competitive. While there are other explanations for credit-card portfolio deals such as changing business strategies or liquidity constraints, my model suggests an alternative explanation based on the value of data on naive consumers to competing firms.

A.6 Graphs of the Mixed Strategies

This Section depicts graphs of the mixed strategies (1) and (2) in Figure 4 and 5 respectively.

Figure 4 shows that new-customer prices have a mass point at c. In equilibrium, firms are indifferent between poaching naive customers and poaching no one, and they do both with positive probability. Comparing the straight black- and gray lines, we see that a lower share of naive customers shifts the probability mass towards larger prices. With fewer naifs in the population, poaching becomes less profitable. A look at the straight black- and the thin dashed line shows that a larger marginal cost shifts the distribution to the right, but leaves the curvature mostly unaffected. The shift from the straight black line to the dotted one reveals that a lower hidden fee induces firms to poach less intensively. Similarly, comparing the straight black - with the thick dashed line indicates that more competition makes successful poaching of profitable naifs less likely and induces firms to poach less, and charge larger new-customer prices.

Figure 5 depicts the distribution of naive-customer prices. Firms are indifferent between any naive-customer price on $[c-\alpha\bar{a}, c)$. Larger naive-customer prices increase margins from continuing naifs but also increase the probability to loose naifs to poaching rivals. Firms charge this distribution to make poaching rivals indifferent between all prices on $(c-\alpha\bar{a}, c)$. This is also why it has no mass point on this interval. Moving from the straight black- to the straight gray line, we see that fewer naive customers induce larger transparent prices. This is mostly due to the lower end of the support getting larger. The price $c-\alpha\bar{a}$ that poachers do not want to undercut decreases in α. For the same reason, we see a similar shift for lower hidden fees between the straight black line and the dotted one. Finally, Comparing the straight black- with the dashed line, larger marginal costs shift the distribution to the right, just as with new-customer prices.

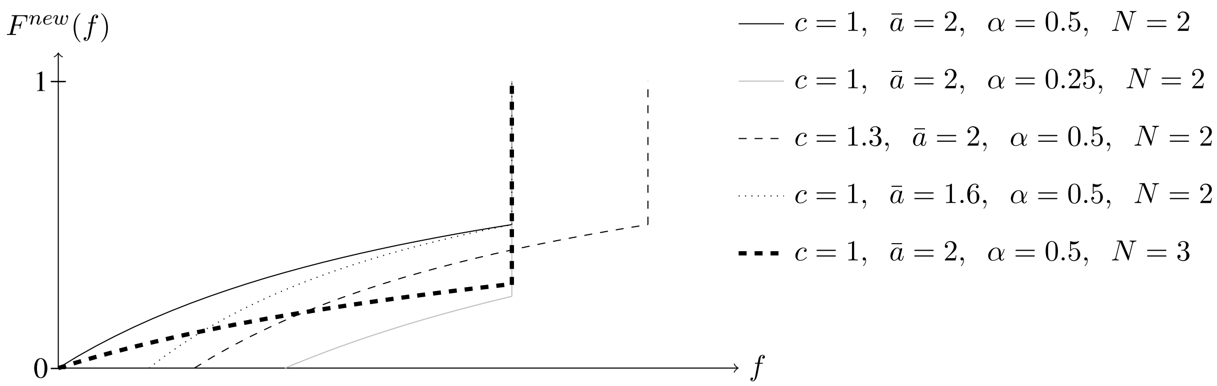

Figure 4: New-customer prices. The support is $(c-\alpha\bar{a}, c]$.

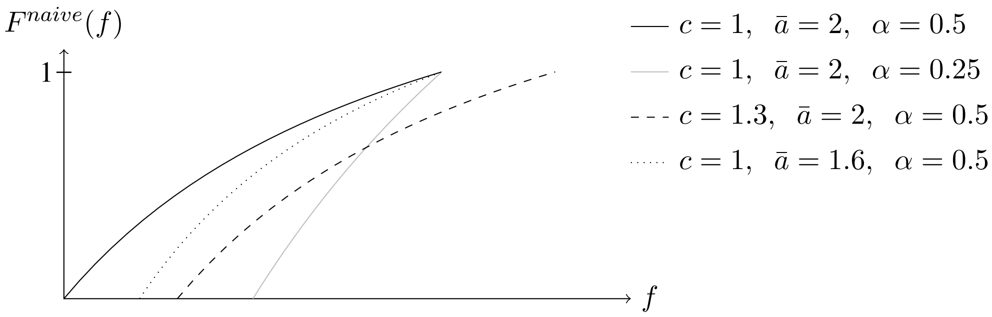

Figure 5: Naive-customer prices. The support is $(c - \alpha\bar{a}, c)$.

B Appendix: Proofs

B.1 Proof of Proposition 1

Proof. Consumers buy at marginal cost in any pure-strategy equilibrium. To see this, note first that firm B cannot earn positive margins from any customer type. Otherwise, firm A—being able to target each customer group—could increase profits by marginally undercutting prices for each customer group. Now suppose towards a contradiction that firm A earns a positive margin from any customer group. Then firm B could offer a separate price for the base product and the add-on, and by offering both prices close enough to marginal cost, B could profitably attract consumers of the informed firm A - a contradiction.

The argument extends to mixed strategies by a standard Bertrand argument as in the proof for Lemma 1, Case (i). $\square$

I prove Propositions 2, 3, and 4 for the more general setup of Proposition 6 with non-avoiding naive consumers. In addition to the basic framework, a share $\eta \in [0, 1)$ of the naive consumers cannot avoid unshrouded hidden fees while the others can avoid them costlessly. I rule out the case $\eta = 1$ to avoid that firms are indifferent between shrouding or not when they only consider their own customer base. After unshrouding hidden fees, the share $\alpha\eta$ of consumers still pays hidden fees while the share $1 - \alpha\eta$ does not. The special case presented in the text is obtained by setting $\eta = 0$.

B.2 Proof of Proposition 2

Since neither firm learns about consumers' types nor consumers about themselves, there is no updating of beliefs from any type; so the equilibrium is a SPNE. The relevant state variables are customer bases, represented by market shares in $t = 1$, and whether shrouding occurred in $t = 1$ or not.

Step 1: Period 2:

In the first step, I determine Nash equilibria of all period-2 subgames, that is for all state variables.

Lemma 1 (Nash Equilibria in Period 2 Subgames)**.**

(i) *After shrouding in period 1, a shrouding equilibrium exists if and only if*

$$0 \geq \eta\alpha \cdot \min\{(1-\alpha)\bar{a}, v-c\}. \tag{7}$$

If shrouding occurs with probability one, Consumers pay hidden fees of $a_{n2} = \bar{a}$, transparent prices $f_{n2} = c - \alpha\bar{a}$, and profits are zero. If $\eta = 0$, profits are zero. If (7) is violated, hidden fees are unshrouded with probability one and consumers pay total prices equal to marginal costs.

(ii) *After unshrouding in period 1, all consumer types pay total prices equal to marginal costs, and hidden fees are zero.*

Proof of Lemma 1. Case (i): In a first step, I derive the strategies of firms given all firms shroud hidden prices. In a second step, I derive conditions under which firms do not deviate from these strategies by unshrouding.

Given all firms shroud, two firms must set $f_{n2} = c - \alpha\bar{a}$ and $a_{n2} = \bar{a}$. Given all firms shroud, all firms with positive market share optimally set $a_{n2} = \bar{a}$ since this does not reduce demand but raises profits. I use a standard Bertrand-type argument to show that $f_{n2} = c - \alpha\bar{a}$ with probability one for at least two firms. One cannot have $f_{n2} \in (c - \alpha\bar{a}, \bar{f}_n]$ with positive probability for all firms for the supremum of transparent prices of firm n of $\bar{f}_n > c - \alpha\bar{a}$. Towards a contradiction, assume $\bar{f}_n > c - \alpha\bar{a}\ \forall n$. First note that $\bar{f}_n = \bar{f}\ \forall n$. Otherwise, a firm setting prices above the lowest supremum, say at $\bar{f}$, earns zero profits whenever these prices occur but could earn strictly positive profits by moving this probability mass to $\bar{f} - \epsilon$ for some $\epsilon > 0$ since $\bar{f} > c - \alpha\bar{a}$. Thus, if all firms have a supremum strictly above $c - \alpha\bar{a}$, they must have the same supremum. If all firms play $\bar{f}_n$ with positive probability, each firm earns non-negative profit when this occurs. But by taking the probability mass from $\bar{f}$ to $\bar{f} - \epsilon$, a firm could win the whole market when all others play $\bar{f}$ and therefore strictly increase her profit. If at least one firm does not play $\bar{f}$ with positive probability, all firms that do so earn zero profit with positive probability and could earn strictly positive profits by moving the probability mass somewhere below $\bar{f}$ instead. Therefore $f_{n2} < \bar{f}\ \forall n$ with probability one. But then profits go to zero as f_{n2} approaches $\bar{f}$ whereas expected profits are strictly positive by playing $c - \alpha\bar{a} + \epsilon$, for some $\epsilon > 0$, since all others play a larger price with positive probability when $\bar{f} > c - \alpha\bar{a}$. Thus, firms could do better by shifting probability mass from marginally below $\bar{f}$ to $c - \alpha\bar{a} + \epsilon$, for some $\epsilon > 0$. This is a contradiction. Hence, we get $\bar{f}_n = c - \alpha\bar{a}$ for at least two firms, since trivially, it is no equilibrium when only one firm sets $\bar{f}_n = c - \alpha\bar{a}$. Thus, firms earn zero profit when shrouding occurs.[48]

Given firms play a candidate shrouding equilibrium in which two firms set $\bar{f}_n = c - \alpha\bar{a}$ and $a_{n2} = \bar{a}$, unshrouding and setting $f_{n2} = c$ and $f_{n2} + a_{n2} = \min\{v, c + (1-\alpha)\bar{a}\}$ attracts all educated naifs that cannot avoid hidden fees. Thus, optimal deviation profits by unshrouding are given by $\alpha\eta \cdot \min\{v - c, (1-\alpha)\bar{a}\}$. When $v - c > (1-\alpha)\bar{a}$ unshrouding is profitable if $\eta > 0$ and a shrouding equilibrium does not exist; if $\eta = 0$, optimal deviation profits by unshrouding are zero and a shrouding equilibrium exists. When $v - c < (1-\alpha)\bar{a}$, shrouding occurs as long as profits in a shrouding equilibrium are larger than profits from unshrouding, that is if $0 \geq \alpha\eta(v - c)$. If $v < c$, optimal deviation profits are negative and a shrouding equilibrium exists. Conversely, a shrouding equilibrium does not exist if $0 < \alpha\eta(v - c)$.

Next, I show that hidden fees are unshrouded with probability one when $\eta\alpha \cdot \min\{(1-\alpha)\bar{a}, v-c\} > 0$ in three steps. Towards a contradiction, assume shrouding occurs with

[48] When I say below that a standard Bertrand type argument applies, I refer to this kind of reasoning.

positive probability and $\eta\alpha \cdot \min\{(1-\alpha)\bar{a}, v-c\} > 0$.

Step (I): *Firms earn positive profits.* When shrouding occurs, firms could unshroud and earn $\eta\alpha \cdot \min\{(1-\alpha)\bar{a}, v-c\} > 0$, but since shrouding occurs with positive probability and firms must be indifferent between shrouding and unshrouding, firms must earn positive profits when shrouding occurs.

Step (II): *Firms earn zero profits whenever shrouding.* Let $\hat{t}$ be the supremum of total prices, i.e. including hidden fees, when unshrouding, paid by educated naifs that cannot avoid hidden fees. Then by playing $\hat{t}$, a firm earns positive profits only if it is the only one that unshrouds and $\hat{t} < f_{n2} + a_{n2}$ with positive probability. Thus, for all total prices above $\hat{t}$, firms earn positive profits only when shrouding occurs and they charge the smallest transparent price. But then, a standard Bertrand-type argument implies that total prices are competed downwards until $f_{n2} = c - \alpha\bar{a}$ for all firms that attract customers and $\hat{t} \leq \min\{(1-\alpha)\bar{a}, v-c\}$. Thus, firms earn weakly less than zero profits whenever shrouding.

Step (III): *Unshrouding occurs with probability one.* Since firms earn zero profits whenever shrouding, they are strictly better of by unshrouding instead since they can then earn $\eta\alpha \cdot \min\{(1-\alpha)\bar{a}, v-c\} > 0$. This implies that firms are better off by unshrouding with probability one, contradicting the assumption that shrouding occurs with positive probability whenever $\eta\alpha \cdot \min\{(1-\alpha)\bar{a}, v-c\} > 0$.

The case depicted in Proposition 3 is for $\eta = 0$. With $\eta = 0$, unshrouding hidden prices can earn a firm maximally zero profits. If this was not so, transparent prices must be above marginal costs with positive probability, which is impossible in equilibrium because of Bertrand competition. I now show that if $\eta = 0$ and shrouding occurs with positive probability, firms must earn zero profits when shrouding. If shrouding occurs with probability one, the result has been shown above. Suppose shrouding occurs with positive probability less then one. We know that unshrouding earns firms maximally zero profits. If *at least one firm* earns strictly positive profits when shrouding occurs, such a firm must have a supremum of transparent prices when shrouding of $\bar{f} > c - \alpha\bar{a}$. But then a competitor could increase profits by shifting all probability mass from unshrouding to shrouding and earn strictly positive profits by setting a transparent price $\bar{f} - \epsilon$ for some $\epsilon > 0$ and hidden fees of $\bar{a}$ - a contradiction. If all firms earn strictly positive profits when shrouding occurs, shrouding would occur with probability one since unshrouding gives zero profits. But then we are in the case from the beginning of this proof which contradicts positive profits. Thus, if $\eta = 0$ and shrouding occurs with positive probability, expected profits must be zero.

Case (ii): The market is effectively split: when unshrouding occurred in t=1, firms compete in transparent prices for sophisticated consumers and in total prices for non-avoiding naifs. By essentially the same Bertrand argument as in Case (i) when all firms shroud, firms that attract consumers charge $f_{n2} = c$ and $a_{n2} = 0$ and earn zero profits. $\square$

Step 2: Period 1:

All consumers face the same price-schedule in period 2, irrespective of the firm they purchase from. Thus, consumers maximize their total payoff by maximizing their first-period payoff. Knowing that firms earn no profits in any second-period subgames, firms simply maximize their per-period profit in period 1. The same Bertrand-type argument as in Case (i) of period 2 applies. $\square$

B.3 Proof of Proposition 3

I look for a Perfect Bayesian Equilibrium and proceed as follows. I prove two preliminary result before characterizing second-period continuation equilibria after all histories. In preliminary 1, I argue that updating of beliefs only matters for the firms' customer base after shrouding in period 1. After such histories, firms learn only their own first period customers' types. In preliminary 2, I derive some properties of transparent prices in Lemma 2. Lemma 3 is the main result of this proof. It characterizes the existence of shrouding continuation equilibria in period 2 as well as consumers' payments and firms' profits.

Preliminary 1: Beliefs after shrouding occurs in period 1.

Assume shrouding occurred in period 1. When consumers are not educated about hidden fees, both consumer types solve the same problem: $\max_{\mathrm{n}} v - f_{n2}, s.t. v - f_{n2} \geq 0$. Hence, both consumer types will always be indifferent between the same set of firms. Therefore the Sorting Assumption implies that the distribution of customers in each customer base is the same as in the population. Hence from observing her own customer base, a firm cannot learn anything about the distribution outside of her own customer base.

Recall that after unshrouding in period 1, all consumers are sophisticated in period 2, and this is known to firms.

Preliminary 2: Properties of Transparent Prices in Period 2.

In a second preliminary step, I establish some characteristics of the firms' second-period price distributions when prices are shrouded.

Lemma 2 (Properties of Transparent Prices in Period 2)**.** *In each equilibrium in which prices are shrouded in period 2 with probability one, the following properties hold.*

(i) $f_{n2}^{naive} \in [c - \alpha\bar{a}, c]$ *with probability one and sophisticates pay a price below c, i.e.* $min\{f_{n2}^{soph}, (f_{\hat{n}2}^{new})_{\hat{n}\neq n}\} \leq c$ $\forall n$ *with probability one.*

(ii) All firms marginally undercut c with the new-customer price with positive probability, i.e. for all $\epsilon > 0$ *and for all* n*,* $f_{n2}^{new} \in (c - \epsilon, c]$ *with positive probability.*

(iii) On each subinterval on $(c - \alpha\bar{a}, c)$ *at least one firm plays naive- and one firm plays new-customer prices with positive probability.*

(iv) $F_n^{new}(.)$ *and* $F_n^{naive}(.)$ *are continuous on* $(c - \alpha\bar{a}, c)$*.*

(v) $F_n^{new}(c - \alpha\bar{a}) = F_n^{new}(c - \alpha\bar{a}) = 0,\ \forall n$*.*

Proof of Lemma 2.

(i) $f_{n2}^{naive} \in [c - \alpha\bar{a}, c]$ *with probability one and sophisticates pay a price below c, i.e.* $min\{f_{n2}^{soph}, (f_{\hat{n}2}^{new})_{\hat{n}\neq n}\} \leq c$ $\forall n$ *with probability one.* I have argued in the main body that in each equilibrium in which prices remain shrouded in the second period, $f_{n2}^{soph} \geq c$, $f_{n2}^{new} \geq c - \alpha\bar{a}$ and $f_{n2}^{naive} \geq c - \alpha\bar{a}$. First, I show that in equilibrium no firm n sets a price $f_{n2}^{naive} > c$ with positive probability. A firm n can guarantee itself strictly positive expected profits from its naive customers by setting $c - \alpha\bar{a}$. Thus, it must earn strictly positive expected profits for almost all prices it charges, and any price it charges with positive probability. Let $\bar{f}_{n2}^{naive}$

be the supremum of those prices and suppose $\bar{f}_{n2}^{naive} > c$ with positive probability. Then, all rivals $\hat{n} \neq n$ must set prices $f_{\hat{n}2}^{new} \geq \bar{f}_{n2}^{naive}$ with positive probability. If all rivals do so, each firm $\hat{n} \neq n$ can deviate and move probability mass from weakly above $\bar{f}_{n2}^{naive}$ to $\bar{f}_{n2}^{naive} - \epsilon$, and for sufficiently small ϵ increase its profits. We conclude that $f_{n2}^{naive} \in [c - \alpha\bar{a}, c] \;\; \forall n$.

To show that $min\{f_{n2}^{soph}, (f_{\hat{n}2}^{new})_{\hat{n}\neq n}\} \leq c \; \forall n$, I first establish that firms earn zero expected profits from new-customers. Towards a contradiction, suppose a firm n earns positive expected profits from new customers and take its supremum of new-customer prices $\bar{f}_{n2}^{new}$. To be profitable at $\bar{f}_{n2}^{new}$, $\bar{f}_{n2}^{new} > c - \alpha\bar{a}$. In addition, there must be a firm $\hat{n} \neq n$ such that $f_{\hat{n}2}^{soph} > \bar{f}_{n2}^{new}$ or $f_{\hat{n}2}^{naive} > \bar{f}_{n2}^{new}$ with positive probability. If $f_{\hat{n}2}^{naive} > \bar{f}_{n2}^{new}$ with positive probability, $\hat{n}$ gets zero profits from naifs whenever playing $f_{\hat{n}2}^{naive} > \bar{f}_{n2}^{new}$. By moving this probability mass to $\bar{f}_{n2}^{new} - \epsilon$ for sufficiently small $\epsilon > 0$ instead, $\hat{n}$ could make strictly positive profits, a contradiction. The same argument applies if $f_{\hat{n}2}^{soph} > \bar{f}_{n2}^{new}$ with positive probability. Hence, new-customer prices earn zero expected profits in equilibrium. This directly implies that firms earn zero profits on their old sophisticates as well: otherwise, by the same reasoning as above, a firm could move the probability mass of its new-customer prices from above the supremum of sophisticates' prices of the positive-profit firm to minimally below it, and thereby increase its profits. It follows that $min\{f_{n2}^{soph}, (f_{\hat{n}2}^{new})_{\hat{n}\neq n}\} \leq c \; \forall n$ with probability one. If $f_{n2}^{soph} > c$ with positive probability, then at least one firm $\hat{n} \neq n$ must set $f_{\hat{n}2}^{new} \leq c$ with probability one, since otherwise a competitor of n would get strictly positive expected profits from new-customer prices. Similarly, if all $(f_{\hat{n}2}^{new})_{\hat{n}\neq n} > c$ with positive probability, then $f_{n2}^{soph} \leq c$ with probability one for $\hat{n}$ not to earn strictly positive profits with new-customer prices. Hence, $min\{f_{n2}^{soph}, (f_{\hat{n}2}^{new})_{\hat{n}\neq n}\} \leq c$ for all n, and we established that the support of f_{n2}^{naive} is $[c - \alpha\bar{a}$,$c]$.

(ii) All firms play new-customer prices arbitrarily close to c with positive probability. (iii)On each subinterval on $(c - \alpha\bar{a}, c)$, at least one firms plays naive-, and at least one firm plays new-customer prices with positive probability. I prove claim (iii) in three steps: first, I establish claim (ii), i.e. that in any arbitrarily small interval $(c - \epsilon, c]$ at least two firms play naive- and all firms play new-customer prices with positive probability. Second, I show the same for any arbitrarily small interval $[c - \alpha\bar{a}, c - \alpha\bar{a} + \epsilon)$ for at least two firms' naive- and two firms' new-customer prices. Third, I prove that on each interval in-between, these prices occur with positive probability.

Step (i): First, I show that for all n and any $\epsilon > 0$, $f_{n2}^{new} \in (c - \epsilon, c]$ with positive probability. Suppose otherwise, i.e. for at least one firm there exists an $\epsilon > 0$ such that $f_{n2}^{new} \in (c - \epsilon, c]$ with probability zero. Of these firms, select a firm n that has the smallest supremum $\bar{f}_{n2}^{new}$. If there are many such firms select one that sets the supremum with probability less than one. Since $\bar{f}_{n2}^{new} < c$, at least one firm $\hat{n} \neq n$ must set $f_{\hat{n}2}^{naive} > \bar{f}_{n2}^{new}$ with positive probability for n to break even. But then, $\hat{n}$ makes zero profit for all $f_{\hat{n}2}^{naive} > \bar{f}_{n2}^{new}$ with probability one, a contradiction. Thus, for any $\epsilon > 0$ all firms set $f_{n2}^{new} \in (c - \epsilon, c]$ with positive probability. It follows that for every $\epsilon > 0$ and every n, some $\hat{n} \neq n$ sets $f_{n2}^{naive} \in (c - \epsilon, c]$ with positive probability: otherwise, firms could not break even when setting $f_{n2}^{new} \in (c - \epsilon, c)$ with positive probability. Since this holds for every n and $\epsilon > 0$, at least two firms set naive-customer prices in any interval $(c - \epsilon, c]$. Thus, for all prices in $(c - \alpha\bar{a}, c)$, every firm sets larger new-customer with positive probability, and at least two firms set larger naive-customer prices with positive probability.

Step (ii): First I show that for every $\epsilon > 0$, at least two firms set $f_{n2}^{naive} \in [c - \alpha\bar{a}, c - \alpha\bar{a} + \epsilon)$ with positive probability. Suppose otherwise and take a firm n and her competitors $\hat{n} \neq n$. Assume towards a contradiction that there exists an $\epsilon > 0$ such that for all $\hat{n}$, $f_{\hat{n}2}^{naive} \in [c - \alpha\bar{a}, c - \alpha\bar{a} + \epsilon)$ with probability zero. Then the infimum of the naive-customer prices of n's competitors $\underline{f}$ satisfies $\underline{f} > c - \alpha\bar{a}$. For naive-customer prices above this infimum to be profitable, all new-customer prices must be larger with positive probability. But then firm n can earn strictly positive profits from new-customers by choosing $f_{n2}^{new} \in (c - \alpha\bar{a}, \underline{f})$ with probability one. But this contradicts the finding that firms earn zero expected profits from new-customers. Since this is true for all n, I conclude that for every $\epsilon > 0$, at least two firms set $f_{n2}^{naive} \in [c-\alpha\bar{a}, c-\alpha\bar{a}+\epsilon)$ with positive probability. To show that the same is true for new-customer prices, suppose towards a contradiction that there exists an $\epsilon > 0$ such that a firm n plays $f_{n2}^{naive} \in [c-\alpha\bar{a}, c-\alpha\bar{a}+\epsilon)$ with positive probability but all $\hat{n} \neq n$ play greater new-customer prices with probability one. But then, n could move its probability mass from below $c - \alpha\bar{a} + \epsilon$ onto this point to strictly increase profits. Thus, we get a contradiction if for any $\epsilon > 0$, less then two firms play $f_{n2}^{new} \in (c - \alpha\bar{a}, c - \alpha\bar{a} + \epsilon)$ with positive probability.

Step (iii): *On each subinterval on $(c - \alpha\bar{a}, c)$, at least one firm sets naive- and at least one other firm sets new-customer prices with positive probability.* Suppose the opposite for some interval $(\widetilde{r}, \widetilde{s})$. Then there are three cases: either no naive- and new-customer price on $(\widetilde{r}, \widetilde{s})$ occurs with positive probability, or only naive-customer prices, or only new-customer prices. Take the largest interval containing $(\widetilde{r}, \widetilde{s})$, in which either no firm sets new- or no firms sets naive-customer prices with positive probability, and denote it by (r, s); i.e., some new- or naive-customer prices are played with positive probability arbitrarily close below r and arbitrarily close above s. Note that due to step (ii), we know that $r > c - \alpha\bar{a}$.

In the first case, no naive- or new-customer price occurs on (r, s) with positive probability. But by construction, some naive- or new-customer price occurs on $(r - \epsilon, r]$ with positive probability. Note that there can be no mass point on r. If more than one firm had a mass-point on r, they could strictly increase profits by shifting probability mass from this mass point to slightly below it. If one firm had a mass point on r, it could shift this mass point upwards into (r, s) and increase margins without affecting expected market shares since (r, s) is empty. But when there is no mass point on r, then for some $\epsilon > 0$ small enough, a firm playing prices in $(r - \epsilon, r]$ with positive probability is strictly better off by shifting this probability mass to slightly below s, a contradiction.

Now consider the second case. Towards contradiction, assume only naive-customer prices are set on (r, s) with positive probability. But by shifting probability mass of naive-customer price from within (r, s) to s, firms can discretely increase margins on naifs while leaving the probability to gain these margins unaffected, a contradiction. Thus, whenever a firm sets a naive-customer price on (r, s) with positive probability, one of her competitors must set a new-customer price on this interval with positive probability.

Third, assume towards a contradiction that only new-customer prices are played on (r, s) with positive probability. If only one firm plays new-prices on (r, s) with positive probability, this firm could strictly increase its profits by moving this probability mass to slightly below s, a contradiction. Now suppose at least two firms play new-customer prices on (r, s) with positive probability. Take a firm n playing price $f \in (r, s)$ and $f' \in (r, s)$ with positive probability where $f \neq f'$. Recall that both prices are the smallest new-customer price with positive probability due to Step (i), and earn zero expected margins in this case, as shown in the beginning of this proof. Since no naive-customer prices occur with positive probability on (r, s), both prices induce exactly the same probability of attracting naifs when being the smallest new-customer price. But since one of these prices is strictly larger, they cannot both have zero expected margins when being the smallest new-customer price, a contradiction. Thus, whenever a firm sets a new-customer price on (r, s) with positive probability, one of her competitors must set a naive-customer price on this interval with positive probability.

(iv) The CDFs are continuous in the interior of the support, i.e. F_n^{new} and F_n^{naive} have no mass point on $(c-\alpha\bar{a}, c)$, $\forall n$. Take F_n^{new} and suppose otherwise. Pick the lowest mass-point of all firms. Say n has this mass point at f. We know from above that larger naive-customer prices occur with positive probability, so that prices at this mass point are paid with positive probability. Then there exists some $\epsilon > 0$ such that no rival $\hat{n} \neq n$ charges a price $f_{\hat{n}2}^{naive}$ in $[f, f+\epsilon)$. For otherwise, a firm $\hat{n}$ that sets $f_{\hat{n}2}^{naive} \in [f, f+\epsilon)$ could charge $f-\epsilon$ instead; as $\epsilon \to 0$, the price difference goes to zero but $\hat{n}$ wins with higher probability. But when no rival charges a naive-customer price in $[f, f+\epsilon)$ and only n sets a mass-point of new-customer prices at f, then n can increase profits by moving the mass point upwards, a contradiction. Alternatively, another firm but n has a mass point on new-customer prices at f as well. Recall that profits from new-customers are zero in expectation. Thus, by shifting the mass point upwards, n looses more often, gaining zero profits in this case; but due to Step (i), n still has the lowest new-customer prices with positive probability and therefore earns a strictly positive margin when attracting customers, a contradiction. This shows that F_n^{new} has no mass point on $(c-\alpha\bar{a}, c)$. A similar argument applies to F_n^{naive}: to see why, suppose otherwise that F_n^{naive} has a mass point on $(c-\alpha\bar{a}, c)$. Pick again the lowest mass point of all firms. Say firm n has this mass point at f. By the same argument as above, there exists some $\epsilon > 0$ such that no rival $\hat{n} \neq n$ sets a price $f_{\hat{n}2}^{new} \in [f, f+\epsilon)$ with positive probability. And since n only competes with these new-customer prices for its naive customers, n can strictly improve profits by shifting the mass point upwards, a contradiction.

(v) $F_n^{new}(c-\alpha\bar{a}) = F_n^{naive}(c-\alpha\bar{a}) = 0, \ \forall n$. Suppose otherwise, i.e. $F_n^{new}(c-\alpha\bar{a}) = p > 0$ for some n. Then no rival $\hat{n} \neq n$ charges $f_{\hat{n}2}^{naive} \in (c-\alpha\bar{a}, c-\alpha\bar{a}+\epsilon)$ for some $\epsilon > 0$, or otherwise $\hat{n}$ could strictly increase profits by moving this probability-mass on $c-\alpha\bar{a}$ instead. But then, by the same argument as in the last paragraph, n can earn strictly positive profits by shifting the mass-point upwards, a contradiction.

Now suppose $F_n^{naive}(c-\alpha\bar{a}) = p > 0$ for some n and take firms $\hat{n} \neq n$ that play new-customer prices on $(c-\alpha\bar{a}, c-\alpha\bar{a}+\epsilon)$ with positive probability. We already know that such firms exist. Then $\hat{n}$'s profits from $f_{\hat{n}2}^{new} = c-\alpha\bar{a}+\epsilon$ converge to some profit-level below $p[s_n(1-\alpha)(c-\alpha\bar{a}-c)+(1-s_n)0]+(1-p)0 = -ps_n\alpha\bar{a} < 0$. This is a contradiction since firms can guarantee themselves at least zero profits from new-customer prices. □

In the next Lemma, I summarize the properties in each shrouding equilibrium in period 2 for each state.

Lemma 3 (Second Period Continuation Equilibria)**.** *There always exists the standard Bertrand equilibrium in which at least two firms unshroud and each consumer pays marginal costs. In addition to this equilibrium, there exist second-period continuation equilibria in which shrouding occurs with positive probability under the following conditions:*

(i) If shrouding occurs in t=1 and all firms have positive customer bases, shrouding occurs with positive probability if and only if

$$s_n\alpha(1-\alpha)\bar{a} \geq \alpha\eta \min\{(1-\alpha)\bar{a}, v-c\}, \ \forall n. \tag{8}$$

In such a shrouding equilibrium, profits are $s_n\alpha(1-\alpha)\bar{a}$ and shrouding occurs with probability one. f_{n2}^{naive} is mixed as in (2). Switching naive-customers of firm n's customer base pay the smallest new-customer prices of n's competitors based on (1). Sophisticated customers in the customer base of firm n pay a price equal to the smallest new-customer price of n's competitors based on (1). When the *above shrouding condition is violated, unshrouding occurs with probability one and all consumers pay a price of c.*

(ii) *If shrouding occurs in t=1 and some firm has an empty customer base, consumers are educated about hidden fees with probability one if and only if $v > c$ and $\eta > 0$. In this case, prices equal marginal costs and firms make zero profits. If $v < c$, shrouding occurs with probability 1 and prices are as in (i), but firms without customer base earn zero profits. If $\eta = 0$ or $v = c$, firms without customer base are indifferent between shrouding or unshrouding.*

Proof of Lemma 3.

Proof of statements (i) I proceed as follows. First, I derive shrouding conditions, i.e. conditions for the existence of shrouding equilibria, and pin down the level of equilibrium profits in a shrouding equilibrium in which firms have a positive customer base. In a second step, I construct the mixed equilibrium strategies for period 2 in the shrouding equilibrium based on (1) and (2).

If $s_n\alpha(1-\alpha)\bar{a} \geq \eta\alpha \min\{(1-\alpha)\bar{a}, v-c\}$ $\forall n$, in all equilibria in which shrouding occurs with positive probability it occurs with probability one. If $s_n\alpha(1-\alpha)\bar{a} < \eta\alpha \min\{(1-\alpha)\bar{a}, v-c\}$ for some n, shrouding occurs with probability zero. If shrouding occurs with probability one, firms earn expected profits of $s_n\alpha(1-\alpha)\bar{a}$ from naifs and zero from sophisticates and new customers. Suppose that shrouding occurs with positive probability. I show that this implies Step (I) - (III) below. Using these facts Step (IV) proves the above.

Step (I): *Firms earn positive profits.* When shrouding occurs, firms can get positive profits of at least $s_n\alpha(1-\alpha)\bar{a}$ by setting $f_{n2}^{soph} = f_{n2}^{new} = c$ and $f_{n2}^{naive} = c - \alpha\bar{a}$. I have established in the text that when shrouding occurs, new-customer prices below $c - \alpha\bar{a}$ are never played as they lead to strictly negative profits for at least one firm. When unshrouding, the share of consumers paying a hidden fee reduces to $\eta\alpha$ and this threshold shifts upwards to $c - \eta\alpha\bar{a}$. Thus, firms can indeed be sure to profitably keep its naive customers when shrouding occurs by setting the above prices. Since shrouding occurs with positive probability, firms earn positive expected profits.

Step (II): *New-customer prices earn zero expected margins in equilibrium conditional on both shrouding or unshrouding occurring. Sophisticated consumers never pay positive margins in equilibrium.* Towards a contradiction, suppose a firm n profitably attracts customers with her new-customer price in expectation. Then firm n must earn positive expected margins with each new-customer price that is played with positive probability. Take the supremum of these prices $\bar{f}_{n2}^{new}$. Then prices that minimally undercut $\bar{f}_{n2}^{new}$, i.e. prices on $(\bar{f}_{n2}^{new} - \epsilon, \bar{f}_{n2}^{new}]$ for some sufficiently small $\epsilon > 0$, profitably attract either sophisticates or naifs from another firm, say $\hat{n} \neq n$. We therefore have to distinguish these two cases.

Suppose n profitably attracts sophisticates conditional on shrouding in any interval of new-customer prices that marginally undercut $\bar{f}_{n2}^{new}$. Then $f_{\hat{n}2}^{soph} \geq \bar{f}_{n2}^{new}$ with positive probability. Note that the inequality must be strict for some $f_{\hat{n}2}^{soph}$ when n sets $\bar{f}_{n2}^{new}$ with positive probability. Then $\hat{n}$ earns zero profits from sophisticates with probability one whenever $f_{\hat{n}2}^{soph} \geq \bar{f}_{n2}^{new}$, though $\hat{n}$ could earn strictly positive profits from sophisticates when shifting this probability mass to $\bar{f}_{n2}^{new} - \epsilon$ for some small enough $\epsilon > 0$, a contradiction. The exact same argument applies conditional on unshrouding occurring.

Now suppose n profitably attracts naifs in any interval of new-customer prices arbitrarily close below $\bar{f}_{n2}^{new}$. They are profitable when shrouding occurs or when unshrouding occurs so that I have to distinguish these two cases. If they are profitably attracted under shrouding, we must have $f_{\hat{n}2}^{naive} \geq \bar{f}_{n2}^{new}$ with positive probability. Note that the inequality must be strict for some $f_{\hat{n}2}^{naive}$ when $\bar{f}_{n2}^{new}$ occurs with positive probability. Then $\hat{n}$ earns zero profits when shrouding occurs on prices $f_{\hat{n}2}^{naive} \geq \bar{f}_{n2}^{new}$ that occur with positive probability. W.l.o.g. let $\bar{f}_{n2}^{new}$ be among the largest such suprema.[49] But then moving probability mass from $[\bar{f}_{n2}^{new}, \bar{f}_{n2}^{new} + \epsilon)$ to $\bar{f}_{n2}^{new} - \epsilon$ increases $\hat{n}$'s profits discretely when shrouding occurs and reduces them by maximally 2ϵ when unshrouding occurs. This is profitable for some small enough $\epsilon > 0$, a contradiction. If n profitably attracts naifs when unshrouding occurs, the same argument can be applied to total prices, i.e. by taking $t_{\hat{n}2}^{naive} = f_{\hat{n}2}^{naive} + a_{\hat{n}2}$ and $t_{n2}^{new} = f_{n2}^{new} + a_{n2}$ with $\bar{t}_{n2}^{new}$ as the supremum to total new-customer prices of firm n.

I conclude that if shrouding occurs with positive probability, new-customer prices earn zero expected profits conditional on shrouding or unshrouding. To show that sophisticated consumers never pay a price $f_{n2}^{soph} > c$, suppose otherwise. Since I have established that sophisticates never pay a new-customer price $f_{\hat{n}2}^{new} > c$, they must pay the positive margin to their old firm, i.e. with $f_{n2}^{soph} > c$. But then, a competitor can earn strictly positive profits with new-customer prices by offering $f_{\hat{n}2}^{new} = f_{n2}^{soph} - \epsilon$ for some $\epsilon > 0$ small enough, a contradiction.

Step (III): *The profits of firms that shroud are weakly smaller than $s_n\alpha(1-\alpha)\bar{a}$ $\forall n$ and zero when unshrouding occurs.* To show that firms' profits are weakly smaller than $s_n\alpha(1-\alpha)\bar{a}$ when shrouding occurs, suppose otherwise, i.e. there exists a firm n that earns strictly larger profits when shrouding occurs. Step (II) shows that firms earn zero profits from new- and sophisticated customers, they therefore earn the positive profits from naive customers from their customer base. Let $\bar{f}_{n2}^{naive}$ be the supremum of n's naive-customer prices that are paid with positive probability. Then all $\tilde{n} \neq n$ must set $f_{\tilde{n}2}^{new} \geq \bar{f}_{n2}^{naive}$ with positive probability. I.e. for all $\epsilon > 0$, some $\hat{n} \neq n$ sets $f_{\hat{n}2}^{new} \in [\bar{f}_{n2}^{naive}, \bar{f}_{n2}^{naive} + \epsilon)$ with positive probability. But by moving probability mass from this interval to $\bar{f}_{n2}^{naive} - \epsilon$, $\hat{n}$ can earn strictly positive profits: if some other firm than $\hat{n}$ sets a smaller new-customer price, $\hat{n}$ earns zero profits from new customers. But since all $\tilde{n} \neq n$, $\tilde{n} \neq \hat{n}$ set $f_{\tilde{n}2}^{new} \geq \bar{f}_{n2}^{naive}$ with positive probability, $f_{\hat{n}2}^{new} = \bar{f}_{n2}^{naive} - \epsilon$ is the smallest new customer price with positive probability. In this case, $\hat{n}$ earns profits strictly above $s_n\alpha(1-\alpha)\bar{a}$ in expectation from n's naifs and looses weakly below $s_n\alpha(1-\alpha)\bar{a}$ from n's sophisticates. Note that we know from Step (I) that $f_{\hat{n}2}^{new} \geq c - \alpha\bar{a}$ and therefore $f_{\hat{n}2}^{naive} \geq c - \alpha\bar{a}$ for all $\hat{n}$, which is why losses from attracting sophisticates from firm n are weakly below $s_n\alpha(1-\alpha)\bar{a}$. From all other sophisticates that $\hat{n}$ attracts with this price, it looses maximally 2ϵ. Thus, for some $\epsilon > 0$ small enough, $\hat{n}$ can discretely increase profits by shifting some probability mass from $f_{\hat{n}2}^{new} \in [\bar{f}_{n2}^{naive}, \bar{f}_{n2}^{naive} + \epsilon)$ to $\bar{f}_{n2}^{naive} - \epsilon$, a contradiction.

To show that shrouding firms earn zero profits conditional on unshrouding, suppose otherwise for at least one firm, say n. Step (II) implies that these profits must be earned from naive customers of firm n's customer base. Thus, n must keep some non-avoiding naifs at a positive total prices $f_{n2}^{naive} + a_{n2} > c$. But then, a competitor $\hat{n} \neq n$ can earn strictly positive profits from new-customer prices conditional by unshrouding and setting $f_{\hat{n}2}^{new} + a_{\hat{n}2} = c + \epsilon$ for some sufficiently small $\epsilon > 0$, which contradicts Step (II), i.e. that new-customer prices earn zero profits. Thus, shrouding firms earn zero profits conditional on unshrouding. Since firms' profits are weakly below $s_n\alpha(1-\alpha)\bar{a}$ when shrouding but by Step (I) they can guarantee themselves these profits when shrouding occurs, we know that firms must earn profits of $s_n\alpha(1-\alpha)\bar{a}$ in expectation when shrouding occurs.

Step (IV): *If $s_n\alpha(1-\alpha)\bar{a} \geq \eta\alpha\min\{(1-\alpha)\bar{a}, v-c\}$ $\forall n$, in all equilibria in which shrouding occurs*

[49] If n was not among the firms with the largest suprema, then another firm would have a larger supremum that earns zero profits for prices that marginally undercut it. But then this firm could do strictly better by shifting this probability mass to $\bar{f}_{n2}^{new}$. Thus $\bar{f}_{n2}^{new}$ can be taken among the largest suprema w.l.o.g..

with positive probability, it occurs with probability one. If this condition is violated, i.e. if $s_n\alpha(1-\alpha)\bar{a} < \eta\alpha\min\{(1-\alpha)\bar{a}, v-c\}$ for at least one n, shrouding occurs with probability zero. Steps (I)-(III) establish that expected profits from new customers are zero, whether shrouding or unshrouding occurs, and whenever shrouding, firms' expected profits are $s_n\alpha(1-\alpha)\bar{a}$ when shrouding occurs and zero when unshrouding occurs. Thus, in any candidate equilibrium in which shrouding occurs with positive probability, it occurs with probability one. Consequently, when $s_n\alpha(1-\alpha)\bar{a} \geq \eta\alpha\min\{(1-\alpha)\bar{a}, v-c\}\ \forall n$, no firm has an incentive to unshroud with probability one and set a total price of $\min\{c+(1-\alpha)\bar{a}, v\}$. But when this condition is violated for at least one firm, this firm has a strict incentive to unshroud with probability one and set the above total price.

Now that I established that in any second-period continuation equilibrium where shrouding occurs, it occurs with probability one, I can use the properties on new- and naive-customer distributions derived in Lemma 2 and the profit levels determined above to construct equilibrium price-distributions.

Mixed strategies for new-customer prices. Recall that firms do not compete for their own old customers with the new-customer price. When a firm n sets her naive-customer price lower than all her competitors' new-customer prices, it keeps her naive customers. Otherwise, it looses them. Thus, expected profits are

$$(1-\prod_{j\neq n}(1-F_j^{new}(f_{n2}^{naive})))\cdot 0 + \prod_{j\neq n}(1-F_j^{new}(f_{n2}^{naive}))\cdot s_n\alpha(f_{n2}^{naive}+\bar{a}-c) = const.\ ,\forall n. \quad (9)$$

We know from Lemma 2 that all new- and naive-customer prices on $(c-\alpha\bar{a}, c)$ occur with positive probability and that $F_j^{new}(c-\alpha\bar{a}) = 0$ for all j. We also know that expected profits from naive-customer prices must be equal to $const. = s_n\alpha(1-\alpha)\bar{a}$ for all prices on the interval. Thus, I can rewrite the above to get

$$\prod_{j\neq n}(1-F_j^{new}(f_{n2}^{naive})) = \frac{(1-\alpha)\bar{a}}{f_{n2}^{naive}+\bar{a}-c}\ ,\forall n \quad (10)$$

In particular, for each $\hat{n}\neq n$ and f_{n2}^{naive} this requires $\prod_{j\neq\hat{n}}(1-F_j^{new}(f^{naive})) = \prod_{j\neq n}(1-F_j^{new}(f^{naive}))$, which implies $F_n^{new}(f^{naive}) = F_{\hat{n}}^{new}(f^{naive}) = F^{new}(f^{naive})$. Using this symmetry in the above equation leads to the expression of (1) on $(c-\alpha\bar{a}, c)$.

Note that the probability mass strictly below c is not equal to one. In fact, we only know from Lemma 2 that $min\{f_{n2}^{soph}, (f_{\hat{n}2}^{new})_{\hat{n}\neq n}\} \leq c\ \forall n$ with probability one. New-customer prices can be strictly larger than c with positive probability, but these prices are never paid by customers and are therefore inconsequential for consumer welfare and firms' profits. Thus, either new-customer prices have a mass point at c and sophisticated customer prices can be strictly larger than c or the other way around. I report the strategy with the mass point on c to ease the exposition of results. This leads to the distribution as in (1).

Mixed strategies for naive-customer prices. Take a firm n that sets f_{n2}^{new} to all consumers that are not in n's customer base. In order to win firm j's customers and break even, it has to offer a new-customer price f_{n2}^{new} such that (i) $f_{n2}^{new} < f_{\hat{n}2}^{new}\forall\hat{n}\neq j$ and (ii) $f_{n2}^{new} < f_{j2}^{naive}$. If f_{n2}^{new} is such that (i) is satisfied, but j's naive-customer price is still smaller, than n attracts only the sophisticated consumers of j, since $f_{j2}^{soph} \geq c$.

Hence, the expected profit of attracting j's customers is

$$(1-F^{new}(f_{n2}^{new}))^{N-2}[(1-F_j^{naive}(f_{n2}^{new}))s_j(f_{n2}^{new}+\alpha\bar{a}-c) \\ +F_j^{naive}(f_{n2}^{new})s_j(1-\alpha)(f_{n2}^{new}-c)] \tag{11}$$

Summing over all $j \neq n$ leads to n's expected profits from new-customer prices:

$$(1-F^{new}(f_{n2}^{new}))^{N-2}[(f_{n2}^{new}+\alpha\bar{a}-c)\sum_{j\neq n}(1-F_j^{naive}(f_{n2}^{new}))s_j \\ +(1-\alpha)(f_{n2}^{new}-c)\sum_{j\neq n}F_j^{naive}(f_{n2}^{new})s_j]=const. \tag{12}$$

Lemma 2 established that all naive-customer prices on $(c-\alpha\bar{a},c)$ occur with positive probability and that $F^{new}(c-\alpha\bar{a})=F_j^{new}(c-\alpha\bar{a})=0$. I have shown above that expected profits from new-customer prices are zero. Now consider $f_{n2}^{naive}\in(c-\alpha\bar{a},c)$. Rewriting the equation gives

$$\sum_{j\neq n}F_j^{naive}(f_{n2}^{new})s_j=(1-s_n)\frac{(f_{n2}^{new}+\alpha\bar{a}-c)}{\alpha(f_{n2}^{new}+\bar{a}-c)},\quad \forall n \tag{13}$$

$$\Leftrightarrow \underbrace{\sum_{j=1}^{N}F_j^{naive}(f_{n2}^{new})s_j}_{\equiv g(f)}=(1-s_n)\underbrace{\frac{(f_{n2}^{new}+\alpha\bar{a}-c)}{\alpha(f_{n2}^{new}+\bar{a}-c)}}_{\equiv\Omega(f_{n2}^{new})}+s_nF_n^{naive}(f_{n2}^{new}),\quad \forall n \tag{14}$$

$$\Leftrightarrow g(f_{n2}^{new})=(1-s_n)\Omega(f_{n2}^{new})+s_nF_n^{naive}(f_{n2}^{new}),\quad \forall n \tag{15}$$

For each n, the condition implies $F_n^{naive}(f_{n2}^{new})=\frac{g(f)}{s_n}-\frac{1-s_n}{s_n}\Omega(f)$. Plugging this into (12) pins down $g(f)=\Omega(f)$ for all f and therefore $F_n^{naive}(f_{n2}^{new})=\Omega(f)$. Hence, in all second-period shrouding equilibria, naive customer prices are mixed symmetrically according to (2).

Proof of statements (ii) I show now that after histories in which shrouding occurs and at least one firm has no customer base and another has one, firms always unshroud hidden fees if $v>c$ and $\eta>0$. Firms earn no profit and consumers pay marginal costs.

Given shrouding occurs with positive probability, the same reasoning as in (i) implies that firms can earn $\tilde{s}_n\alpha(1-\alpha)\bar{a}$ conditional on shrouding from their old naive customers while firms earn zero expected profits from new-customer prices and old sophisticates.[50] Firms without a customer base earn zero total profit since they have no customer base to exploit, and their shrouding condition reduces to $0\geq\eta\alpha\min\{(1-\alpha)\bar{a},v-c\}$. As long as $v>c$ and $\eta>0$, they have a strict incentive to educate customers about hidden fees. When η is equal to zero, profits are zero after unshrouding. Firms without customer base are indifferent between shrouding and unshrouding and there are potentially multiple equilibria. □

[50] $\tilde{s}_n(\geq s_n)$ is the market share a firm gets when not all firms sell to consumers but n does.

B.4 Proof of Proposition 4

I proof the more general statement summarized in the following Lemma. Proposition 4 selects the firms' preferred equilibrium from this Lemma. The following Lemma obtains for $\eta = 0$. I summarize the results for $\eta > 0$ in Proposition 6.

Lemma 4. [Mitigated Customer-Base Competition in Shrouding Equilibria]
Shrouding equilibria with shrouding in both periods exist. In each equilibrium satisfying Assumption 1, all firms choose hidden fees $a_{n1} = \bar{a}$. In equilibria with pure strategies in period 1, all firms set the same transparent price $f_1 \in \left[c - \alpha\bar{a} - \alpha(1-\alpha)\bar{a}, c - \alpha\bar{a} + \frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a}\right]$. Total profits are $\Pi_n = s_n(f_1 + \alpha\bar{a} - c) + s_n\alpha(1-\alpha)\bar{a} \in \left[0, s_n\frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a} + s_n\alpha(1-\alpha)\bar{a}\right]$. For all equilibria in which $\Pi_n > 0$, shrouding occurs with probability one.

The results of Proposition 3 pin down the continuation payoffs after period 1 and can be used to study equilibrium behavior in period 1.

Lemma 3 establishes that when $v > c$ and $\eta > 0$, firms can achieve positive continuation profits if and only if each firm has a positive customer base, i.e. when prices in the first period are identical with positive probability. As I argue in the main text, the multiple continuation equilibria in the extreme case with $\eta = 0$ is not robust, since unshrouding occurs with probability one after such histories if $\eta > 0$. This is why I select this continuation equilibrium when $\eta = 0$.

First, I study equilibria in which firms always set the same transparent price f_1 in the first period. Given the reduced-game profits starting from $t = 1$ specified in (3), the only possible profitable deviations are either (i) shrouding and undercutting competitors or (ii) unshrouding hidden fees and attracting the remaining profitable customers.

(i) is unprofitable if $s_n(f_1 + \alpha\bar{a} - c) + s_n\alpha(1-\alpha)\bar{a} \geq f_1 + \alpha\bar{a} - c$, which is equivalent to $f_{n1} \leq c - \alpha\bar{a} + \frac{s_n}{1-s_n}\alpha(1-\alpha)\bar{a}$.

For $\eta = 0$, unshrouding always induces zero profits, and all $f_1 \in \left[c - \alpha\bar{a} - \alpha(1-\alpha)\bar{a}, c - \alpha\bar{a} + \frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a}\right]$ can be pure-strategy equilibria in period 1.

To check for (ii) when $\eta > 0$, I need to establish the optimal deviation under unshrouding. Given all other firms shroud and play f_1, a firm n can make sure to attract only all profitable customers after unshrouding, i.e. only all non-avoiding naifs and neither educated avoiding naifs nor sophisticates, by setting $\widetilde{f_1} > \max\{c, f_1\}$ and $\widetilde{a_1} < \min\{f_1 + \bar{a}, v\} - \widetilde{f_1}$. The resulting deviation profits are bounded by $\eta\alpha\min\{f_1 + \bar{a} - c, v - c\}$. Note that I do not have to consider the case where $f_1 \geq c$ since the resulting deviation profits of $\eta\alpha(f_1 + \bar{a} - c) \leq f_1 + \alpha\bar{a} - c$ for all $f_1 \geq c$, and therefore deviation (i) is always preferred. Thus, consider $f_1 < c$, in which case only non-avoiding naifs are profitable after unshrouding. Hence, the optimal deviation profits with unshrouding are $\eta\alpha\min\{f_1 + \bar{a} - c, v - c\}$. Deviating in this way is unprofitable if $s_n(f_1 + \alpha\bar{a} - c) + s_n\alpha(1-\alpha)\bar{a} \geq \eta\alpha\min\{f_1 + \bar{a} - c, v - c\}\ \forall n$.

Note that there can be no equilibrium in which firms play mixed strategies in period 1 with a continuous distribution function. When firms mix on some interval with a continuous distribution function, conditional on prices of this interval occurring, the probability of having the same prices is zero and continuation profits are zero as well. Thus, standard Bertrand arguments such as those in the proof of Proposition 2 establish the usual contradiction.

There can, however, be shrouding equilibria in which firms mix over a finite number of prices, each price being played by each firm with positive probability. These prices must be within the range derived above, for otherwise (i) or (ii) above is a profitable deviation. Since continuation profits cannot be larger as when all firms coordinate on the same price with probability one, and the largest such price is given by $f_1 = c - \alpha\bar{a} + \frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a}$, profits must be below $s_n\frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a} + s_n\alpha(1-\alpha)\bar{a}\ \forall n$. At the same time, shrouding profits must be at least $\eta\alpha(v-c) > 0$ if total prices for naifs are larger than v and zero otherwise for each firm. □

B.5 Proof of Proposition 5

In the first step, I establish results on continuation equilibria in Lemma 2. Afterwards, I study the first period.

Step 1: Period 2

Lemma 5 (Period 2 with Disclosure Policy)**.** *An Equilibrium with shrouding in period 2 exists if and only if shrouding occurs in period 1. Shrouding occurs in period 2 either with probability one or with probability zero. When shrouding occurs, both customer types pay a total price of c and naifs a hidden fee $\bar{a}$. Profits are zero in any continuation equilibrium.*

Proof of Lemma 5. First, I analyze continuation equilibria given shrouding occurs in period 1. By the exact same argument as in the proof of Proposition 3, continuation equilibrium profits are zero whenever some firm unshrouded in period 1.

Suppose prices were shrouded in period 1. Then continuation equilibrium profits must be zero conditional on shrouding and unshrouding. Suppose otherwise. Note that whether shrouding or unshrouding occurs, firms have symmetric information on customers and can charge those that were naive and sophisticated in period 1 separately in period 2. The markets for consumers who were naive or sophisticated in period 1 can therefore be treated as separate markets in period 2. For consumers that were sophisticated in period 1, the market is a standard Bertrand market and the results follow immediately. Recall that sophisticates are unaffected by shrouding. For the market for consumers that were naive in period 1, the argument is similar to the one used in the proof on Lemma 3(i) Step (II). Suppose at least one firm earns strictly positive profits conditional on shrouding or unshrouding. Take the firm with the largest profits conditional on either unshrouding or shrouding. If these profits occur conditional on shrouding, take the supremum for which these profits occur and denote it by $\bar{f}$. For positive profits to occur, each competitor must set larger prices with positive probability. I.e., competitors set prices in $[\bar{f}, \bar{f}+\epsilon)$ with positive probability for each $\epsilon > 0$, or $\bar{f}$ would be shifted upwards. But then competitors can increase their profits discretely conditional on shrouding discretely by shifting probability mass from $[\bar{f}, \bar{f}+\epsilon)$ slightly below $\bar{f}$. Since losses conditional on unshrouding are below ϵ, this deviation is strictly profitable for some ϵ small enough, a contradiction. If the largest profits occur conditional on unshrouding the same argument applied to total prices applies. Thus, expected profits are zero for all customers conditional on shrouding and unshrouding. In particular when firms shroud with probability one, a firm's demand is independent of $\bar{a}$ and hence any firm sets $a_{n2} = \bar{a}$, and standard Bertrand arguments applied to each market imply that $f_{n2}^{soph} = c$ and $f_{n2}^{naive} = c - \bar{a}$. When shrouding occurs with probability zero, all consumers pay $f_{n2}^{soph} = f_{n2}^{naive} = c$ since all are aware of hidden fees, whether they can avoid them or not.

I study unshrouding incentives next. When firms shroud with probability one, all consumers pay a total price equal to marginal costs. Unshrouding and undercutting total prices for competitors' non-avoiding naive customers reduces total prices below marginal costs and can therefore not profitably attract these customers. I now establish that shrouding either occurs with probability one or with probability zero. Suppose otherwise. Recall that firms earn zero profits in expectation whether shrouding or unshrouding occurs. When shrouding occurs, customers that were naive in period 1 must pay a transparent price below marginal cost and a hidden fee of $\bar{a}$. If this was not so, a firm could earn strictly positive profits by setting prices for customers that were naive in $t = 1$ of $c - \epsilon$ and $\bar{a}$ for some $\epsilon > 0$ small enough. This would marginally reduce profits on these customers when unshrouding occurs but discretely increase profits when shrouding occurs. Naifs of period 1 therefore purchase at a transparent price below c when shrouding occurs and firms earn zero expected profits from them. But when unshrouding occurs, the share of naive customers in period 2 drops discretely to $\eta\alpha$ and with it the share of naifs of period 1 that pay the hidden fee in period 2. Since these customers pay transparent fees below c and profits are zero when shrouding occurs, firms must earn strictly negative profits with these prices when unshrouding occurs. Thus, these firms are better of by unshrouding with probability one and setting transparent prices to c and hidden fees to zero, a contradiction. □

Step 2: Period 1

By Lemma 5, continuation profits are zero independent of first-period behavior. Hence, the setting is the same as in period 1 of Proposition 2. □

B.6 Proof of Proposition 6

The proof is part of the Proofs of Propositions 3 and 4.

B.7 Proof of Proposition 7

The following Lemma is a generalization of Proposition 7. The Lemma holds for all first-period prices f_{l1}, and resulting customer bases $\bar{x}_{l1}D(f_{l1})$. In this way, the Lemma characterizes shrouding equilibria after any history where shrouding occurs in period 1. Proposition 7 results for symmetric first-period prices $f_{l1} = f_1 \in (\underline{f}_l^{naive}, \underline{f}_l^{naive} + \bar{a} - \tau)$.

Lemma 6. [Hotelling version of Proposition 3] *Suppose Assumption 3 holds. A Hotelling version of the equilibrium in Propositions 3 exists. Each firm l sets $a_l = \bar{a}$, $f_l^{soph} = f^H$. Firm l mixes naive-customer price f_l^{naive} on $\left[\underline{f}_l^{naive}, \overline{f}_l^{naive}\right]$, where $\underline{f}_l^{naive} = c - \alpha\bar{a} + \tau + \frac{(1-\alpha)}{2}\frac{D(\max\{f^H, f_{k1}\})}{D(\max\{\underline{f}_k^{naive} - \tau, f_{k1}\})}(f^H - c)$, and earns $\alpha\bar{x}_{l1}D(\max\{\underline{f}_l^{naive}, f_{l1}\})\left[\underline{f}_l^{naive} + \bar{a} - c\right]$ from old naive customers. With the new customer price f_k^{naive}, firm k poaches only sophisticated consumers [soft poaching] with some probability $\rho < 1$ and sets $f_k^{new} = f^H$. With probability $1 - \rho$, firm k poaches naive and sophisticated customers [aggressive poaching], and randomizes f_k^{new} on $\left[\underline{f}_k^{new}, \overline{f}_k^{new}\right]$. With f_l^{new}, firm l earns $(1-\alpha)\pi_k^H$. $\underline{f}_k^{new} = \underline{f}_l^{naive}$ and $\overline{f}_k^{new} = \overline{f}_l^{naive} = f^H - 2\tau < c$.*

This shrouding continuation equilibrium exists if

$$\bar{x}_{l1} D(\max\{\underline{f}_l^{naive}, f_{l1}\}) \alpha(\underline{f}_l^{naive} + \bar{a} - c) \geq$$
$$\bar{x}_{l1} D(\max\{\underline{f}_l^{naive} + \bar{a}, f_{l1}\}) \eta\alpha(\underline{f}_l^{naive} + \bar{a} - c) + (1 - \bar{x}_{l1})\eta\alpha D(\max\{\underline{f}_k^{naive} + \bar{a} - \tau, f_{k1}\})(\underline{f}_k^{naive} + \bar{a} - \tau - c). \tag{$\widetilde{SC}$}$$

After some preliminary definitions, I prove Lemma 6 in three steps. First, I establish the profits that firms need to earn in a shrouding continuation equilibrium in period 2. Second, I show the existence of a mixed-strategies that induce these profits. Third, I characterize unshrouding incentives ($\widetilde{SC}$) and conclude that the equilibrium exists.

As a preliminary step I characterize f^H and π^H. Suppose in period 2 firm k has a customer base $\bar{x}_{k1} D(f_{k1})$ consisting only of sophisticated consumers, and firm k and l compete for these consumers with prices f_k and f_l, respectively. f_{k1} is k's first-period price and fixed in period 2. Firm k earns for $f_k > f_{k1}$

$$\bar{x}_{k1} D(f_k) \left(\frac{1}{2} + \frac{f_l - f_k}{2\tau} \right) (f_k - c).$$

Since $f_k > f_{k1}$, only $\bar{x}_{k1} D(f_k)$ consumers prefer buying from k over not buying, and since consumers redraw their location, $\left(\frac{1}{2} + \frac{f_l - f_k}{2\tau} \right)$ prefer k over l. Similarly, firm l earns for $f_l > f_{k1}$

$$\bar{x}_{k1} D(f_l) (\frac{1}{2} + \frac{f_k - f_l}{2\tau})(f_l - c).$$

Maximizing both profits leads to the symmetric solution

$$f_l = f_k = \widetilde{f} = c + \frac{\tau}{1 - \tau \cdot \frac{D'(\widetilde{f})}{D(\widetilde{f})}}.$$

If $\widetilde{f} \geq f_{k1}$, not all consumers in k's customer base purchase. But for prices $f < f_{k1}$, all consumers in k's customer base pay less than their valuation and participation does not increase with further price cuts, i.e. $D(f) = D(f_{k1})$. In this case, profit maximization leads to the classic Hotelling prices

$$f_l = f_k = c + \tau.$$

Thus, we get

$$f^H = \begin{cases} \widetilde{f} & \text{if} \widetilde{f} \geq f_{k1} \\ c + \tau & \text{else} \end{cases}$$

and both firms earn profits $\pi_k^H = \bar{x}_{k1} D(\max\{f^H, f_{k1}\}) \frac{1}{2}(f^H - c)$ from k's customer base.

Step 1: Profits in a shrouding continuation equilibrium.

Firm $l (\neq k)$ earns at least $(1 - \alpha)\pi_k^H$ with new-customer prices. Note first that in the candidate equilibrium from Lemma 6, $\overline{f}_l^{new} < f^H - \tau$, implying that firm l poaches all its competitor's sophisticated consumers with prices $f_l^{new} \leq \overline{f}_l^{new}$. Thus, the competitor k earns profits from sophisticated consumers in its customer base if and only if l poaches softly with $f_l^{new} = f^H$. Conditional on soft poaching, k and l compete only for the sophisticated consumers in k's customer base $\bar{x}_{k1}D(f_{k1})$, k using f_k^{soph} and l using f_l^{new}. Consequently, from soft poaching, both firms must earn profits as in a Hotelling equilibrium from the sophisticated consumers in k's customer base. This leads to $f_k^{soph} = f^H$, and when l poaches softly it sets $f_l^{new} = f^H$. We conclude that firm l must earn at least $(1-\alpha)\pi_k^H$ from new-customer prices.

Firm l ($\neq k$) earns exactly $(1-\alpha)\pi_k^H$ with new-customer prices. We know from the last step that l must earn at least $(1-\alpha)\pi_k^H$ from poaching. Thus, firm k can only prevent l from poaching naifs if and only if l earns no more than $(1-\alpha)\pi_k^H$ from poaching naifs. We conclude that l earns exactly $(1-\alpha)\pi_k^H$ from poaching naifs and therefore from poaching new customers.

Firm k earns the profits $\alpha\bar{x}_{k1}D(\max\{\underline{f}_k^{naive}, f_{k1}\})\left[(1-\alpha)\bar{a}+\tau+\frac{1-\alpha}{\bar{x}_{k1}D(\max\{\underline{f}_k^{naive}-\tau, f_{k1}\})}\pi_k^H\right]$ from naive consumers in its customer base. To find the equilibrium profits firms need to earn from naive consumers in their customer base, we determine the smallest naive-customer price $\underline{f}_k^{naive}$ of k for which l no longer wants to poach naive consumers. To do so, suppose firm l poaches all naive consumers of k, that is it sets $f_l^{new} = f_k^{naive} - \tau$. Then l earns

$$\alpha\bar{x}_{k1}D(\max\{f_l^{new}, f_{k1}\})\left(\frac{1}{2}+\frac{f_k^{naive}-f_l^{new}}{2\tau}\right)(f_l^{new}+\bar{a}-c)+(1-\alpha)\bar{x}_{k1}D(\max\{f_l^{new}, f_{k1}\})\left(f_l^{new}-c\right).$$

The first- and second term are the profits from poaching naive and sophisticated consumers respectively. Since $f_l^{new} \leq \overline{f}^{new} < f^H - \tau$, l poaches all sophisticated consumers. The number of buying consumers is $D(\max\{f_l^{new}, f_{k1}\})$. If $\max\{f_l^{new}, f_{k1}\} = f_{k1}$, all consumers in k's customer base buy again at a price f_l^{new}. But if $\max\{f_l^{new}, f_{k1}\} = f_l^{new}$, the product is more expensive in period 2 and some naive consumers no longer buy.

Plugging $f_l^{new} = f_k^{naive} - \tau$ into the last expression simplifies it to

$$\bar{x}_{k1}D(\max\{f_k^{naive}-\tau, f_{k1}\})\left(f_k^{naive}-\tau+\alpha\bar{a}-c\right).$$

Using this and the definition of π_k^H together with the last expression leads to

$$f_k^{naive} \leq \underline{f}_k^{naive} = c-\alpha\bar{a}+\tau+\frac{(1-\alpha)}{2}\frac{D(\max\{f^H, f_{k1}\})}{D(\max\{\underline{f}_k^{naive}-\tau, f_{k1}\})}(f^H-c).$$

Poaching all competitor's customer earns hidden fees $\alpha\bar{a}$ (second term on the right-hand side), but it also requires undercutting the f_k^{naive} by τ (third term) and giving up profits from soft poaching (fourth term). Using Assumption 3, it is straightforward to show that $\underline{f}_k^{naive}$ increases in τ.

Attracting all naive customers is indeed a best response of l. Assumption 3 implies that the marginal revenue of firm l at $f_l^{new} = \underline{f}_k^{naive} - \tau$ is negative, implying that a larger new-customer price leads to lower profits. A lower naive-customer price instead would lower margins without increasing demand.

This shows that firm l does not want to poach k's naive customers if k charges $\underline{f}_k^{naive}$. Firm k therefore needs to earn in equilibrium the profits it earns at $\underline{f}_k^{naive}$ from naive customers in its customer base. In the shrouding continuation equilibrium firm k therefore earns $\alpha\bar{x}_{k1}D(\max\{\underline{f}_k^{naive}, f_{k1}\})\left[\underline{f}_k^{naive}+\bar{a}-c\right]$

from naive consumers in its customer base .

Step 2: Existence of mixed strategies for naive- and new customer prices that induce profts from Step 1.

We pin down shrouding continuation-equilibrium profits in the last step. We show now that mixed strategies exist for naive-, and new-customer prices that lead to the profits derived in the last step.

We first look for a mixed-strategy $G_k^{naive}(\cdot)$ on $[\underline{f}_k^{naive}, \overline{f}_k^{naive}]$ of prices f_k^{naive} that makes firm l indifferent between transparent prices $f_l^{new} \in [\underline{f}_l^{new}, \overline{f}_l^{new} + \tau]$. $\underline{f}_k^{naive}$ is the same as defined previously, $\underline{f}_l^{new} = \underline{f}_k^{naive} + \tau$ and $\overline{f}_k^{naive} = \overline{f}_l^{new} = f^H - 2\tau$. We also denote by $\Phi(f_l^{new})$ the expected share of naive consumers that l poaches from k when setting a price f_l^{new}. By Assumption 3, $\underline{f}_l^{new} < f^H - 2\tau$. For firm l to be indifferent between new-customer prices on this interval, it must hold for all f_l^{new} on $[\underline{f}_l^{new}, \overline{f}_l^{new} + \tau]$ that

$$\bar{x}_{k1} D(\max\{f_{k1}, f_l^{new}\}) \left[\alpha(f_l^{new} + \bar{a} - c)\Phi(f_l^{new}) + (1-\alpha)(f_l^{new} - c)\right] = (1-\alpha)\pi_k^H.$$

At any price f_l^{new} firm l must earn the same profits that it earns when poaching sophisticated consumers of its rival. Since $\overline{f}_l^{new} \leq f_H - \tau$, f_l^{new} attracts all sophisticated consumers of firm k with probability one. Rearranging leads to

$$\Phi(f_l^{new}) = \frac{1-\alpha}{\alpha} \cdot \frac{\frac{\pi^H}{\bar{x}_{k1} D(\max\{f_{k1}, f_l^{new}\})} - (f_l^{new} - c)}{f_l^{new} + \bar{a} - c}.$$

$\Phi(f_l^{new}) \in (0,1)$ for all $f_l^{new} \in [\underline{f}_l^{new}, \overline{f}_l^{new} + \tau]$. Assumption 3 and log-concavity of $D(\cdot)$ imply that $\Phi'(f_l^{new}) < 0$ and $\Phi''(f_l^{new}) > 0$ for all $f_l^{new} \in [\underline{f}_l^{new}, \overline{f}_l^{new} + \tau]$.

We show now that there exist a cumulative distribution function $G_k^{naive}(\cdot)$ on $[\underline{f}_k^{naive}, \overline{f}_k^{naive}]$ of f_k^{naive} that induces $\Phi(f_l^{new})$. To do so, we call $g_k^{naive}(\cdot)$ the probability density function of $G_k^{naive}(\cdot)$

First, note that for $f \in (\overline{f}_k^{naive} - \tau, \overline{f}_k^{naive} + \tau)$, l never poaches all naifs and we have

$$\Phi(f) = \int_{f-\tau}^{\overline{f}_k^{naive}} \left(\frac{1}{2} + \frac{x-f}{2\tau}\right) g_k^{naive}(x) dx.$$

We know that the left-hand side is continuous, so the right-hand side must be continuous as well. We can therefore compute the derivatives on both sides. Doing so and rearranging terms leads to

$$G_k^{naive}(f - \tau) = G_k^{naive}(\overline{f}_k^{naive}) + 2\tau\Phi'(f). \tag{16}$$

Since $\Phi''(f) > 0$, this equation pins down an increasing function G_k^{naive} on the support $(\overline{f}_k^{naive} - \tau, \overline{f}_k^{naive})$. Furthermore, Assumption 3 implies that $2\tau\Phi'(\overline{f}_k^{naive} + \tau) > -1$ so that $G_k^{naive}(f) \leq 1$.

Note that $G_k^{naive}(f)$ can have a mass point at $\overline{f}_k^{naive}$, so we can have $G_k^{naive}(\overline{f}_k^{naive}) - \lim_{f_- \to \overline{f}_k^{naive}} G_k^{naive}(f) > 0$.

We consider now the remaining support $f \in (\underline{f}_k^{naive} + \tau, \overline{f}_k^{naive} - \tau)$. In this case, l might poach all naifs and we get

$$\Phi(f) = 1 - G_k^{naive}(f+\tau) + \int_{f-\tau}^{f+\tau} (\frac{1}{2} + \frac{x-f}{2\tau}) g_k^{naive}(x) dx.$$

Again, the left-hand side is continuous, so the right-hand side must be continuous as well. Computing the derivative and rearranging terms leads to

$$G_k^{naive}(f-\tau) = G_k^{naive}(f+\tau) + 2\tau\Phi'(f). \tag{17}$$

We know from (16) that for $f \in (\overline{f}_k^{naive} - \tau, \overline{f}_k^{naive} + \tau)$, both terms on the right-hand side of (17) are increasing in f. Thus, also $G_k^{naive}(f-\tau)$, is increasing in f on $(\overline{f}_k^{naive} - 2\tau, \overline{f}_k^{naive} + \tau)$. Repeating the argument implies that $G_k^{naive}(f)$ is an increasing function on its support.

Overall we see that (17) determines the slope of $G_k^{naive}(f)$. Equation (16) then determines the probability mass at $G_k^{naive}(\overline{f}_k^{naive})$ such that $G_k^{naive}(\underline{f}_k^{naive}) = 0$. Thus, we conclude that a CDF $G_k^{naive}(f)$ exist that makes firm l indifferent between transparent prices $f_l^{new} \in [\underline{f}_l^{new}, \overline{f}_l^{new} + \tau]$ and it induces the profits $(1-\alpha)\pi_k^H$ from poaching competitor's customers.

It remains to show that $f_l^{new} \notin [\underline{f}_l^{new}, \overline{f}_l^{new} + \tau]$ does not increase profits. Consider first $f_l^{new} > \overline{f}_l^{new} + \tau = f^H - \tau$. This price attracts no naive customers, therefore only $f_l^{new} = f^H$ is a best response and leads to the same profits as $f_l^{new} \notin [\underline{f}_l^{new}, \overline{f}_l^{new} + \tau]$.

Consider next $f_l^{new} < \underline{f}_l^{new}$. Since $G_k^{naive}(\underline{f}_k^{naive}) = 0$, decreasing f_l^{new} below $\underline{f}_l^{new}$ cannot be profitable. To see this, rearranging (17) and plugging in $\underline{f}_l^{new} = \underline{f}_k^{naive} + \tau$ leads to

$$\Phi'(\underline{f}_l^{new}) = -\frac{1}{2\tau}\left[G_k^{naive}(\underline{f}_k^{naive} + 2\tau) - G_k^{naive}(\underline{f}_k^{naive})\right].$$

Since $\Phi'(\underline{f}_l^{new}) < 0$ and $\Phi''(\underline{f}_l^{new}) > 0$, firm l would be indifferent between $f_l^{new} = \underline{f}_l^{new}$ and a marginally lower price if the difference on the right-hand side $G_k^{naive}(\underline{f}_k^{naive} + 2\tau) - G_k^{naive}(\underline{f}_k^{naive})$ would increase. But since $G_k^{naive}(\underline{f}_k^{naive}) = 0$ and $G_k^{naive}(f)$ is increasing, the the right-hand side decreases. Thus, decreasing f_l^{new} below $\underline{f}_l^{new}$ would lead to an insufficient acquisition of new customers and is therefore not profitable. We conclude $G_k^{naive}(f)$ makes it a best response for firm l to play transparent prices $f_l^{new} \in [\underline{f}_l^{new}, \overline{f}_l^{new} + \tau]$.

We now look for a mixed-strategy $G_l^{new}(\cdot)$ on $[\underline{f}_l^{new}, \overline{f}_l^{new}]$ of prices f_l^{new} that makes firm k indifferent between transparent prices $f_k^{naive} \in [\underline{f}_k^{naive}, \overline{f}_k^{naive} + \tau]$.

We follow essentially the same argument that we used to determine the existence of the equilibrium distribution $G_k^{naive}(f)$, so we mostly sketch the argument.

For firm k to be indifferent between naive-customer prices on this interval, it must hold for all f_k^{naive} on

$[\underline{f}_k^{naive}, \overline{f}_k^{naive} + \tau]$ that

$$\alpha \bar{x}_{k1} D(\max\{f_{k1}, f_k^{naive}\})(f_k^{naive} + \bar{a} - c)\Psi(f_k^{naive}) = \alpha \bar{x}_{k1} D(\max\{f_{k1}, \underline{f}_k^{naive}\})(\underline{f}_k^{naive} + \bar{a} - c).$$

Here $\Psi(f_k^{naive})$ is the expected share of naive customers that k keeps when setting price f_k^{naive}. Rearranging terms leads to

$$\Psi(f_k^{naive}) = \frac{D(\max\{f_{k1}, \underline{f}_k^{naive}\})(\underline{f}_k^{naive} + \bar{a} - c)}{D(\max\{f_{k1}, f_k^{naive}\})(f_k^{naive} + \bar{a} - c)}.$$

$\Psi(f_k^{naive}) \in (0,1)$ for all $f_k^{naive} > \underline{f}_k^{naive}$. Assumption 3 and log-concavity of $D(\cdot)$ imply that $\Phi'(f_k^{naive}) < 0$ and $\Phi''(f_k^{naive}) > 0$ for all $f_k^{naive} \in [\underline{f}_k^{naive}, \overline{f}_k^{naive} + \tau]$.

For $f \in (\overline{f}_l^{new} - \tau, \overline{f}_l^{new} + \tau)$, we have

$$G_l^{new}(f - \tau) = G_l^{new}(\overline{f}_l^{new}) + 2\tau\Psi'(f). \tag{18}$$

Since $\Psi''(f) > 0$, this equation pins down an increasing function G_l^{new} on the support $(\overline{f}_l^{new} - \tau, \overline{f}_l^{new})$. Furthermore, Assumption 3 implies that $2\tau\Psi'(\overline{f}_l^{new} + \tau) > -1$ so that $G_l^{new}(f) \leq 1$.

Next, note that for $f \in (\underline{f}_l^{new} + \tau, \overline{f}_l^{new} - \tau)$, we have

$$G_l^{new}(f - \tau) = G_l^{new}(f + \tau) + 2\tau\Psi'(f). \tag{19}$$

We know from (18) that for $f \in (\overline{f}_l^{new} - \tau, \overline{f}_l^{new} + \tau)$ both terms on the right-hand side are increasing in f. Thus, also $G_l^{new}(f - \tau)$, increases in f on $(\overline{f}_l^{new} - 2\tau, \overline{f}_l^{new} + \tau)$. Repeating the argument implies that $G_l^{new}(f)$ is an increasing function on its support.

Overall we see that (19) determines the slope of $G_l^{new}(f)$. Equation (18) then determines the probability mass at $G_l^{new}(\overline{f}_l^{new})$ such that $G_l^{new}(\underline{f}_l^{new}) = 0$. The remaining probability mass $1 - G_l^{new}(\overline{f}_l^{new}) \equiv \rho$ is used for mild poaching of sophisticated consumers. We conclude that a CDF $G_l^{new}(f)$ exist that makes firm k indifferent between transparent prices $f_k^{naive} \in [\underline{f}_k^{naive}, \overline{f}_k^{naive} + \tau]$ and it induces the profits $\alpha\bar{x}_{k1}D(\max\{\underline{f}_k^{naive}, f_{k1}\})\left[(1-\alpha)\bar{a} + \tau + \frac{1-\alpha}{\bar{x}_{l1}D(\max\{\underline{f}_k^{naive} - \tau, f_{k1}\})}\pi^H\right]$ from naive consumers in k's customer base.

It remains to show that prices $f_k^{naive} \notin [\underline{f}_k^{naive}, \overline{f}_k^{naive} + \tau]$ do not increase profits for k. Consider first $f_k^{naive} < \underline{f}_k^{naive}$. Since $\underline{f}_k^{naive} = \underline{f}_l^{new} + \tau$, $f_k^{naive} < \underline{f}_k^{naive}$ reduce margins without increasing demand.

Consider now $f_k^{naive} > \overline{f}_k^{naive} + \tau$. Since $\overline{f}_k^{naive} = \overline{f}_l^{new}$, a price increase above $\overline{f}_k^{naive} + \tau$ reduces demand to zero with probability one.

Step 3: Existence and unshrouding incentives ($\widetilde{SC}$).

We now characterize the profits a firm can earn by unshrouding. Because sophisticated consumers are unaffected by unshrouding, unshrouding leaves the profits from sophisticated consumers unchanged. To do so, an unshrouding firm sets the transparent price-component to $f^H > c$ to avoid attracting unprofitable consumers, and to earn Hotelling profits from continuing sophisticates when the rival does soft poaching. By unshrouding and charging a total price of $\underline{f}_l^{naive} + \bar{a} - \tau$, firm k can profitably attract all naive consumers

$\eta\alpha\bar{x}_{l1}D(\max\{\underline{f}_l^{naive} - \tau + \bar{a}, f_{l1}\})$ of l that still want to buy the product and cannot avoid unshrouded hidden fees. Among k's naive customers, the share $\eta\alpha(1-\bar{x}_{l1})D(\max\{\underline{f}_k^{naive} + \bar{a}, f_{l1}\})$ continues to buy and earns k the same margin as with shrouding. This leads to the shrouding condition ($\widetilde{SC}$).

We now argue that an unshrouding firm k sets $\underline{f}_l^{naive}+\bar{a}-\tau$ to attract all rival's non-avoiding naifs. To do so, we argue first that firms set no price above $\underline{f_l}^{naive}+\bar{a}-\tau$ to poach rival's consumers when unshrouding. In the shrouding equilibrium, poaching firms are indifferent between all new-customer prices in the support. But lower new-customer prices lead to more losses from attracted sophisticated consumers. For poaching firms to be indifferent, the gains from poaching naive customers must compensate for these losses and must be larger for smaller new-customer prices. This means that poaching firms earn more from naive customers at lower new-customer prices in the support. Thus, firms will not set a price above $\underline{f}_l^{naive}+\bar{a}-\tau$ to poach rival's non-avoiding naifs.

Firm k also does not set a price below $\underline{f}_l^{naive}+\bar{a}-\tau$. To see that, note that lower prices reduce profits if $-\frac{D'(\underline{f}_l^{naive}+\bar{a}-\tau)}{D(\underline{f}_l^{naive}+\bar{a}-\tau)} < (\underline{f}_l^{naive}+\bar{a}-\tau-c)^{-1}$ which is true by Assumption 3. We conclude that firms who deviate from the shrouding equilibrium by unshrouding maximize profits from poaching rivals with a total price $\underline{f}_l^{naive}+\bar{a}-\tau$.

Firms k sets $\underline{f}_k^{naive}+\bar{a}$ to naifs in its own customer base with a transparent-price component above c to avoid losses from naifs who turn sophisticated and can avoid unshrouded hidden fees. This is the lowest price that ensures k does not loose any consumers non-avoiding naifs to the poaching rival. The same argument used in the previous paragraph implies that lower prices are not profitable. We conclude that firms who deviate from the srhouding equilibrium by unshrouding sets $\underline{f}_k^{naive}+\bar{a}$ to naifs in its own customer base with a transparent-price component above c to avoid losses from naifs who turn sophisticated and can avoid unshrouded hidden fees.

We conclude that the equilibrium described in Lemma 6 exists.

Remark: Consider the limit case where $D' \to 0$. As $\tau \to 0$, we would expect that this equilibrium converges to the one described in Proposition 3. Indeed, profits clearly converge. But additionally, we can show that the equilibrium strategies converge. Clearly, the supports of $G^{naive}(\cdot)$ and $G^{new}(\cdot)$ converge to $(c-\alpha\bar{a}, c)$. Additionally, rearranging 17 leads to

$$-\Phi'(f) = \frac{\left[G^{naive}(f+\tau) - G^{naive}(f-\tau)\right]}{2\tau}.$$

As $\tau \to 0$, the right-hand side becomes the derivative $g^{naive}(f)$. And the left-hand side converges to the density of (2) for $n=2$. The same argument applied to 19 shows that this is also true for $g^{new}(f)$ and the density of (1). Thus, as products become homogeneous, the shrouding equilibrium in Proposition 7 converges to the equilibrium in Proposition 3. □

B.8 Proof of Proposition 8

First, we define the deviation profits $\pi^{dev}(\bar{p})$. Take $\bar{p}$ as defined before Proposition 4. Recall that any deviation from $\bar{p}$ will lead to unshrouding in period 2. Deviating and shrouding leads to

$$\pi^{shrouding}(\bar{p}) \equiv \max_f \left\{ \left(\frac{1}{2} + \frac{\bar{p} - f}{2\tau}\right) D(f) \left(f + \alpha\bar{a} - c + \frac{f^H - c}{2}\right) + \left(\frac{1}{2} + \frac{f - \bar{p}}{2\tau}\right) D(\bar{p}) \frac{f^H - c}{2} \right\}.$$

From every consumer attracted in period 1, the deviating firm earns $f + \alpha\bar{a} - c$ in period 1, and $\frac{f^H - c}{2}$ in period 2. In period 2, all consumers are aware of all prices and the only symmetric equilibrium leads to Hotelling margins $f^H - c$ that the firm earns from half of its customer base. The other half is poached by the competitor. But in turn, the firm will poach half the consumers of its competitor as well.

Deviating and unshrouding induces

$$\pi^{unshrouding}(\bar{p}) \equiv$$
$$\max_f \left\{ \eta\alpha \left(\frac{1}{2} + \frac{\bar{p} - f}{2\tau}\right) D\left(\underline{f}^{naive} + \bar{a}\right) \left(f + \bar{a} - c + \frac{f^H - c}{2}\right) + \left(\frac{1}{2} + \frac{f - \bar{p}}{2\tau}\right) D\left(\underline{f}^{naive} + \bar{a}\right) \frac{f^H - c}{2} \right\}.$$

Unshrouding allows the firm to attract non-avoiding naive consumers, but it reduces the number of consumers who buy.

Together, we get $\pi^{dev}(\bar{p}) \equiv \max\{\pi^{shrouding}(\bar{p}), \pi^{unshrouding}(\bar{p})\}$.

Thus, the shrouding equilibrium with shrouding in both periods clearly exists if the profits earned from shrouding are larger than $\pi^{dev}(\bar{p})$. The problem has no simple explicit solution, but we argued in the text when it is satisfied for τ in the neighborhood of zero.

It remains to show that in the shrouding equilibrium where firms collude on $\bar{p}$ in period 1, profits from shrouding in period 2 are not competed away in period 1. This follows directly from the fact that $\bar{p} \geq \underline{f}^{naive} > c - \alpha\bar{a}$. $\square$

B.9 Proof of Proposition 9

Relative to Propositions 3 and 4, the incentives to unshroud have changed. First, I derive the shrouding condition for shrouding equilibria in period 2. Note that naifs can avoid hidden fees after unshrouding so that they cannot be profitably attracted by unshrouding. Given shrouding occurred in period 1 and all firms have a positive customer base, the shrouding-equilibrium prices are the same as in Proposition 3. When unshrouding occurs in t=2, a share $(1 - \lambda)$ of the old naifs remain naive. The situation is the same when these consumers learn about hidden fees, i.e. when naiveté in period 1 is not a perfect predictor of naiveté in period 2. But past naiveté remains an informative signal that competitors do not have. Hence, conditional on unshrouding firms can guarantee themselves only profits of $s_n\alpha(1 - \lambda)(1 - \alpha)\bar{a}$ by setting naive-customer prices to $c - (1 - \lambda)\alpha\bar{a}$ and to c for sophisticates and new customers. Since this is strictly smaller than shrouding profits, the argument in the proof of Lemma 3, part (i) still applies accordingly and firms prefer shrouding over unshrouding when shrouding occurs with positive probability.

Second, I identify the most profitable deviations from a shrouding equilibrium path in period 1. There are three candidates: firms could unshroud without changing prices, firms could unshroud and undercut competitors or they could continue to shroud and undercut competitors. At a given price f_1 that is charged by all firms, shrouding in $t = 1$ is profitable if

$$s_n(f_1 + \alpha\bar{a} - c) + s_n\alpha(1-\alpha)\bar{a} \geq s_n(f_1 + (1-\lambda)\alpha\bar{a} - c) + s_n(1-\lambda)\alpha(1-(1-\lambda)\alpha)\bar{a}, \;\; \forall n. \quad (20)$$

For $\lambda = 0$ both sides are identical. For positive λ this condition is equivalent to $\alpha \leq \frac{2}{2-\lambda}$, which holds for all λ. Note that after shrouding in period 1, the second period becomes equivalent to a model with a share of $\widetilde{\alpha} = (1-\lambda)\alpha$ of naive consumers and no option to unshroud. This induces equilibrium profits as in a shrouding equilibrium with a share of naifs of $\widetilde{\alpha}$.

Unshrouding and undercutting a price f_1 is not profitable if

$$s_n(f_1 + \alpha\bar{a} - c) + s_n\alpha(1-\alpha)\bar{a} \geq (f_1 + (1-\lambda)\alpha\bar{a} - c) + (1-\lambda)\alpha(1-(1-\lambda)\alpha)\bar{a}, \;\; \forall n. \quad (21)$$

While simply undercutting is no deviation if

$$s_n(f_1 + \alpha\bar{a} - c) + s_n\alpha(1-\alpha)\bar{a} \geq (f_1 + \alpha\bar{a} - c) + (1-\lambda)\alpha(1-\alpha)\bar{a}, \;\; \forall n \quad (22)$$

it can be easily shown that the last condition is more restrictive for all $\lambda \geq 0$. It follows immediately that this condition is equivalent to

$$f_1 \leq c - \alpha\bar{a} + \frac{s_n}{1-s_n}\alpha(1-\alpha)\bar{a} - \frac{1-\lambda}{1-s_n}\alpha(1-\alpha)\bar{a}, \;\; \forall n. \quad (23)$$

Since the r.h.s. is increasing in s_n, the largest price at which no firm has an incentive to undercut is given by the r.h.s evaluated at the smallest market share s_{min}.

The lower bound of the interval for equilibrium profits is given by the smallest price that earns firms nonnegative profits when all firms play this price with probability one. □

B.10 Proof of Proposition 10

I proceed as follows. In a preliminary step, I show that after histories where at least one customer type of a firm was poached, this firm earns zero profits. The rest of the proof considers histories where no consumer in a firm's customer base was poached and the firm maintains private information about its customers. First, I characterize a profitable shrouding in period $T-1$. Second, I prove that profitable shrouding in all periods $t \in \{2, ..., T-1\}$ looks as in period $T-1$. Third, I show that the first-period is as in Proposition 4 and that first-period competition is unaffected by additional future periods.

Preliminary Step: After histories where at least one customer type of a firm's customer base was poached, firms earn zero profits.

Take a firm n and a period $t > 2$. Whenever either naive or sophisticated customers in n's customer base were poached by n's rival in any period before t, also n's rival can target each customer of n separately, inducing Bertrand competition for each type separately. Thus, firm n earns zero profits in period t.

Step 1: profitable shrouding in period $T-1$

I begin by characterizing expected continuation profits in $T-1$. Period T after histories where none of n's customers has been poached in the past, is the same as period 2 characterized in Proposition 3.

Consider period $T-1$ and suppose no consumer of firm n's customer base of size $\frac{1}{2}$ has been poached in the past. Thus, firm n still has private information about its customer base. The shrouding profits $\frac{1}{2}\alpha(1-\alpha)\bar{a}$ occur in T if none of n's customers is poached in $T-1$. Supposing, for now, that $f^{soph}_{n,T-1} \geq f^{naive}_{n,T-1}$ with probability one and denote the CDF of the rival j's new customer price as $F^{new}_{j,T-1}(\cdot)$, expected continuation profits of n are $(1-F^{new}_{j,T-1}(f^{soph}_{n,T-1}))\delta\frac{1}{2}\alpha(1-\alpha)\bar{a}$.

I now pin down $f^{soph}_{n,T-1} = \bar{f}$. We see that firm n, by reducing $f^{soph}_{n,T-1}$, can reduce the chance of having customers poached away. A lower $f^{soph}_{n,T-1}$ therefore increases continuation profits. The sophisticated-customer price at which firm n breaks even, $\bar{f}$, satisfies

$$\frac{1}{2}(1-\alpha)(\bar{f}-c) + (1-F^{new}_{j,T-1}(\bar{f}))\delta\frac{1}{2}\alpha(1-\alpha)\bar{a} = 0,$$

which simplifies to

$$\bar{f} = c - (1-F^{new}_{j,T-1}(\bar{f}))\delta\alpha\bar{a}. \tag{24}$$

This equation implies that $\bar{f}$ must be between $c-\delta\alpha\bar{a}$ and c.

To pin down $\bar{f}$, I need to first characterize $F^{new}_{j,T-1}(\cdot)$. To do so, suppose $\bar{f}$ as defined above exists and that firm n sets $f^{soph}_{n,T-1} = \bar{f}$. Then the rival j mixes the poaching price $f^{new}_{j,T-1} \in (c-\alpha\bar{a}, \bar{f}]$ to make n indifferent between naive-customer prices on the same support. That is

$$(1-F^{new}_{j,T-1}(f^{naive}_{n,T-1}))\frac{1}{2}\alpha(f^{naive}_{n,T-1} + \bar{a} - c) = \frac{1}{2}\alpha(1-\alpha)\bar{a},$$

which leads to

$$F^{new}_{j,T-1}(f) = \frac{f+\alpha\bar{a}-c}{f+\bar{a}-c},$$

on $(c-\alpha\bar{a}, \bar{f}]$. One can easily check that $F^{new}_{j,T-1}(c-\alpha\bar{a}) = 0$.

Plugging $F^{new}_{j,T-1}(\bar{f})$ into (24) leads to two candidates for $\bar{f}$

$$x_1 = c - \frac{\bar{a}}{2} + \sqrt{1-4\delta(1-\alpha)\alpha}\frac{\bar{a}}{2},$$

and

$$x_2 = c - \frac{\bar{a}}{2} - \sqrt{1-4\delta(1-\alpha)\alpha}\frac{\bar{a}}{2}$$

Some straightforward algebra shows that $x_2 < c-\delta\alpha\bar{a}$ for all $\delta < 1$. x_1 on the other hand is always in $(c-\delta\alpha\bar{a}, c)$. I conclude that $\bar{f} = x_1$.

I next pin down mixed-strategies for $f^{new}_{j,T-1}$ *and* $f^{naive}_{n,T-1}$ *on* $(c-\alpha\bar{a}, \bar{f}]$. We already determined $F^{new}_{j,T-1}(f)$ on $(c-\alpha\bar{a}, \bar{f}]$ above. $F^{new}_{j,T-1}(f) = 1$ for $f < c-\alpha\bar{a}$, and $F^{new}_{j,T-1}(f) = 1$ for $f > \bar{f}$. Thus, $F^{new}_{j,T-1}(f)$ has a mass point at $\bar{f}$. It is straightforward to show that $F^{new}_{j,T-1}(\bar{f}) < \alpha$ and hence the mass point has probability mass larger than $(1-\alpha)$. This pins down $F^{new}_{j,T-1}(\bar{f})$. Note that this mass point is also the probability that no consumer of n is poached and therefore the probability that profitable shrouding equilibria occur in the next period.

I now determine $F^{naive}_{n,T-1}(\cdot)$. As familiar by now, firms n sets $F^{naive}_{n,T-1}(\cdot)$ to make the rival j indifferent between all $f^{new}_{j,T-1} \in (c - \alpha\bar{a}, \bar{f})$.

$$(1 - F^{naive}_{n,T-1}(f^{new}_{j,T-1}))\frac{1}{2}\alpha(f^{new}_{j,T-1} + \bar{a} - c) + (1-\alpha)(f^{new}_{j,T-1} - c) = 0,$$

which leads to

$$F^{naive}_{n,T-1}(f) = \frac{f + \alpha\bar{a} - c}{\alpha(f + \bar{a} - c)},$$

for $f \in (c - \alpha\bar{a}, \bar{f})$). Again, we see that $F^{naive}_{n,T-1}(c - \alpha\bar{a}) = 0$. $F^{naive}_{n,T-1}(f) = 0$ for $f < c - \alpha\bar{a}$ and $F^{naive}_{n,T-1}(f) = 1$ for $f > \bar{f}$. Note $F^{naive}_{n,T-1}(f)$ that there is a mass point at $\bar{f}$. This pins down $F^{naive}_{n,T-1}(f)$.

No profitable deviations in $T-1$. Let us start with the rival j. Firm j earn zero profits from poaching new customers. Setting a lower new-customer price below $c - \alpha\bar{a}$ would poach all of n's customers but j would earn less than zero. Setting a larger new-customer price above $\bar{f}$ attracts no consumer and earns zero profits as well. I conclude that there is no profitable deviation for j's new-customer price.

I now check profitable deviations of firm n. With $f^{soph}_{n,T-1} = \bar{f}$, firm n makes a loss in $T-1$ but breaks even overall because it earns continuation profits in T. With a smaller $f^{soph}_{n,T-1} < \bar{f}$, firm n earns less than zero from sophisticated consumers. For $f^{soph}_{n,T-1} > \bar{f}$, sophisticated consumers are poached with probability one and continuation profits reduce to zero. This reduces profits as well. I conclude n has no profitable deviations for $f^{soph}_{n,T-1}$.

Finally, I check profitable deviations for $f^{naive}_{n,T-1}$. A naive-customer price below $c - \alpha\bar{a}$ reduces margins without increasing demand and does not increase profits. Any $f^{naive}_{n,T-1} > \bar{f}$ is undercut by $f^{new}_{j,T-1}$ with probability one does not increase profits. I conclude that n has no profitable deviations for $f^{naive}_{n,T-1}$.

Step 2: profitable shrouding in period in $t \in \{2, ..., T-2\}$ **exactly as in** $T-1$ We saw in the last step that expected continuation profits from the perspective of period $T-1$ are $(1 - F^{new}_{j,T-1}(f^{soph}_{n,T-1}))\delta\frac{1}{2}\alpha(1-\alpha)\bar{a}$. In period $T-1$, firms hand back these expected continuation profits to sophisticated consumers and earn $\frac{1}{2}\alpha(1-\alpha)\bar{a}$ from their customer base. Thus, in period $T-2$, expected continuation profits are also $(1 - F^{new}_{j,T-2}(f^{soph}_{n,T-2}))\delta\frac{1}{2}\alpha(1-\alpha)\bar{a}$. We can therefore repeat the same argument as in Step 2. I conclude that profitable shrouding in period in $t \in \{2, ..., T-2\}$ is exactly as in $T-1$.

Step 3: period 1 as in Proposition 4 Following the argument in Step 2, continuation profits after period 1 are $\delta\frac{1}{2}\alpha(1-\alpha)\bar{a}$ when shrouding occurs and zero otherwise. Adjusting for δ, these are the same continuation profits that lead to the results in Proposition 4. Thus, the first-period is unchanged by additional future periods. This concludes the proof.

□