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Oligopolistic vs. Monopolistic Competition:  
Do Intersectoral Effects Matter?

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**DISCUSSION PAPER**

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**Oligopolistic vs. Monopolistic Competition:  
Do Intersectoral Effects Matter?**

Claude d'ASPREMONT<sup>1</sup> and Rodolphe DOS SANTOS FERREIRA<sup>2</sup>

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**Abstract**

Recent extensions of the standard Dixit-Stiglitz (1977) model, that go beyond the CES sub-utility assumption, while maintaining monopolistic competition, have mainly emphasized the role of intrasectoral substitutability. We argue that introducing oligopolistic competition can be an alternative extension, still tractable, allowing to restore the role of intersectoral substitutability and reinforcing the general equilibrium dimension of the model. For this purpose, we use the concept of oligopolistic equilibrium and derive a comprehensive formula to characterize the set of potential equilibria with varying competitive toughness. For two particular competitive regimes, price competition and quantity competition, we show how, with strategic interactions, pro-competitive or anti-competitive effects now depend on the elasticity of intersectoral substitution as compared to the elasticity of intrasectoral substitution.

**Keywords:** monopolistic competition, oligopolistic competition, general equilibrium, intersectoral substitution

**JEL Classification:** D43, D51, L13

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# 1 Introduction

It is remarkable how a simple, and now classical, general equilibrium model has been (and still is) useful across several fields in economics. This model, which is the focus of our paper, is the two-sector model introduced by Dixit and Stiglitz (1977), with an imperfectly competitive sector producing differentiated goods under increasing returns, and a perfectly competitive sector producing a homogenous good under constant returns. A main ingredient of Dixit-Stiglitz (DS) model is given by the separability assumption on the representative consumer preferences, allowing to define a sub-utility function on the bundles of differentiated goods, which aggregates the quantities of the differentiated goods into the quantity of a composite good, and implying the analysis "to depend on the intra- and intersector elasticities of substitution" (DS, p. 297). This has the advantage of capturing the two sides of firm competitive behavior: competition within the sector (to gain market share) and competition with the rest of the economy (to increase market size). This model has been fruitfully applied to deal with many issues in industrial organization, international trade and macroeconomics, assuming monopolistic competition in the differentiated goods sector. But this assumption combined with others, such as assuming symmetric additive sub-utility and identical firms (or trivially if assuming only one sector), drastically reduces intersectoral effects. It amounts in fact to confine the analysis of the whole economy to the differentiated goods sector, and to take as sole (or main) ingredients those that are linked to consumer preferences for the different goods within the sector. In particular, preferences across sectors are immaterial to fix the equilibrium price.

It seems therefore that the Dixit-Stiglitz model, as it has been mainly applied, combining monopolistic competition with CES sub-utility and symmetry, is too restrictive. However, the main attempts to go beyond this restrictive version of Dixit-Stiglitz have been either to relax the CES assumption or to relax symmetry, while keeping monopolistic competition. For instance, by relaxing the CES assumption (without relaxing symmetry) to additive but not homothetic preferences (as in section II of Dixit-Stiglitz, 1977) and postulating a decreasing-demand elasticity, Krugman (1979) obtains (in a one-sector model) an elasticity of substitution decreasing in differentiated goods consumption and, as a pro-competitive effect, a decreasing equilibrium price in the number of active firms and in the size of the market. Behrens and Murata (2007) get the same implication by postulating constant absolute risk-averse utility. These results have been recently generalized in a two-sector model by Zhelobodko *et al.* (2012), still keeping additive preferences but allowing for decreasing or increasing intrasectoral elasticity of substitution in consumption and thereby generating both pro- and anti-competitive effects. However, because the sub-utility is still additive, the determination (through the sole intrasectoral elasticity of substitution) of the equilibrium price and quantity of each differentiated good remains independent of the determination of the number of firms. Allowing for non-additive preferences in a multi-sector model, Feenstra (2003) with homothetic translog utility, and Melitz and Ottaviano (2008) with quasi-linear util-

ity and quadratic sub-utility, obtain an elasticity of intrasectoral substitution varying both in consumption and in the number of firms, and generating pro-competitive effects. This is further analyzed in Bertolotti and Epifani (2014).<sup>1</sup> As regards the symmetry assumption, to relax it under monopolistic competition (with or without additive preferences), the only available models seem to be extensions of Melitz (2003), where there is a continuum of potential firms and each firm marginal cost is drawn from a continuous distribution (after paying an entry cost, the same for all). However, fixed costs are identical and the sub-utility has to remain symmetric.

None of these models, though, whether relaxing the CES assumption or the symmetry assumption (or both), really put forward the importance of the relationship between intra- and intersectoral substitution. In one-sector models this is excluded and in multi-sector models with additive sub-utility it is obliterated by the monopolistic competition assumption. But even in models where preferences are non-additive, as emphasized by Parenti et al. (2014), the "primitive" of the models remains the elasticity of intrasectoral substitution and the competitive effects are mainly determined by the properties of the sub-utility.<sup>2</sup>

We argue in this paper that this relationship becomes crucial as soon as strategic interactions are introduced, and that this is true even if we restrict the sub-utility to be CES. To use the CES has the advantage of making the analysis tractable, even in the (fully) non-symmetric case (i.e. a non-symmetric sub-utility and a finite number of firms having individualised unit and fixed costs). But our main argument is not restricted to this case and the model can be made interestingly tractable in several ways. We start with the most general version of DS separable utility function (formula (1) in Dixit-Stiglitz, 1977) and define a comprehensive model of firm oligopolistic behavior, which is shown to nest a multiplicity (a continuum) of competition regimes, including price competition and quantity competition, as well as monopolistic competition as a limit case. This is based on the concept of oligopolistic equilibrium introduced in d'Aspremont *et al.* (2007) and d'Aspremont and Dos Santos Ferreira (2010), integrating the two dimensions of competition mentioned above, competition within the sector for market share and competition with the rest of the economy to increase the sector market size. The main instrument for applications is provided by the markup formula for each firm derived from the first order conditions characterizing the equilibria: each firm equilibrium markup is equal to the weighted harmonic mean of the reciprocals of the intrasectoral and intersectoral elasticities of substitution. The weights involve different elasticities

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<sup>1</sup>In this paper, they argue in favour of the CES but without additive preferences. In section 3, they investigate the effects of strategic interactions when preferences are additive, but in a symmetric one-sector model. An alternative simplifying assumption in order to introduce strategic interactions is to assume a continuum of sectors, each sector having a finite number of identical firms (e.g., Atkeson and Burstein, 2008, and Neary, 2009).

<sup>2</sup>For example, as mentioned by Bertolotti and Epifani concerning the Melitz-Ottaviano model, "the pro-competitive effect of an increase in market size delivered by this type of preferences in monopolistic competition is entirely driven...by the fact that, just as in Krugman (1979), the elasticity of substitution is decreasing in the level of individual consumption". (2014, section 3.1.1).

ties, as well as an essential parameter measuring the competitive toughness of the firm. In this general formula, the two elasticities of substitution are given leading roles. Notice that Dixit-Stiglitz (1977) have a specific case in mind, the one where the differentiated goods are "good substitutes among themselves, but poor substitutes for the other commodities in the economy" (DS, p.297). However, this is only a particular case, since in their model a "sector" is defined by a sub-utility, and one cannot exclude some complementarities among goods in the same sector. Also, if one considers the relations among sectors from a broader perspective (where there could be more sectors with different sub-utilities), some of the composite goods could be good substitutes and other poor substitutes. Our formula shows that accounting for these general equilibrium features is possible.

To illustrate the usefulness of this formula in establishing the importance of the two elasticities of substitution when firms are non negligible within their own sector, and have consequently to take into account the interaction of their sector with the rest of the economy, we take two particular oligopolistic equilibria (with different competitive toughness parameters), one corresponding to the price equilibrium concept, the other to the quantity equilibrium concept. Allowing for non symmetric firms but restricting to the CES sub-utility case, and referring to the Herfindahl index of concentration at equilibrium, we show that less concentration is pro-competitive if and only if intrasectoral substitutability is larger than intersectoral substitutability under both competition regimes. Under price competition, we also show that a more productive firm obtains a higher (lower) budget share for its differentiated good whenever the intra-elasticity of substitution is large (small) enough, in any case larger (smaller) than the elasticity of intersectoral substitution. In the symmetric case, we can also apply our markup formula to monopolistic competition. This is a limit case where all the weight is put on the intrasectoral elasticity of substitution. But there are two ways to reach this limit case. One, traditional, is to make the Chamberlinian large group assumption. The other, still possible with a small number of firms, is to assume that each firm has maximal competitive toughness instead of being insignificant. This gives another view of monopolistic competition, somewhat like Bertrand competition gives another view of perfect competition. Nonetheless, under oligopolistic competition we get very different conclusions regarding competitive effects. The common equilibrium markup and price depend on the number of active firms, even with a CES sub-utility, and they are decreasing (resp. increasing) when this number increases only as long as intrasectoral substitutability is larger (smaller) than intersectoral substitutability. Again the ratio between these two elasticities plays a crucial role. This is also the case when we look at an oligopolistic free-entry equilibrium.

In the following section we define our concept of oligopolistic equilibrium in the two-sector DS model with general separable preferences and derive the markup formula. In section 3, we look at three standard competition regimes as particular cases of oligopolistic equilibria: price competition, quantity competition and monopolistic competition. We then compare them as far as competitive effects are concerned. In section 4, we conclude.

## 2 A general equilibrium model for a two-sector economy

Following Dixit and Stiglitz (1977), we consider a two sector economy, with an imperfectly competitive sector producing under increasing returns a group of differentiated goods and a perfectly competitive sector producing under constant returns a homogeneous numéraire good, which represents the rest of the economy. The technologies of both sectors are described by constant unit labor costs (normalized to 1 in the perfectly competitive sector) plus fixed costs in the imperfectly competitive sector. This simple general equilibrium model allows us to distinguish two types of strategic interactions between producers of the differentiated goods, one intrasectoral, when competing for their market shares, the other intersectoral, when competing for the size of their market relative to the size of the market for the homogeneous good.

### 2.1 A representative consumer with general separable preferences

We suppose a representative consumer inelastically supplying  $L$  units of labor at a wage equal to 1 (the labor productivity in the competitive sector) and receiving a profit  $\Pi$  from the imperfectly competitive sector (the equilibrium profit of the other sector being necessarily zero). He chooses a basket  $\mathbf{x} \in \mathbb{R}_+^n$  of  $n$  differentiated goods (sold at prices  $\mathbf{p} \in \mathbb{R}_{++}^n$ ) and a quantity  $z \in \mathbb{R}_+$  of the numéraire good, so as to maximize, with an income  $Y = L + \Pi$  and under the budget constraint  $\mathbf{p}\mathbf{x} + z \leq Y$ , a separable utility function  $U(X(\mathbf{x}), z)$ . The utility function  $U$  and the sub-utility function  $X$ , aggregating the quantities of the differentiated goods into the quantity of a composite good, are assumed increasing and strongly quasi-concave.

The maximization can be performed in two stages. In the first, the consumer chooses the quantity  $x_i$  of each differentiated good  $i$  given some quantity  $\underline{X}$  of the composite good, by solving the program

$$\min_{\mathbf{x} \in \mathbb{R}_+^n} \{\mathbf{p}\mathbf{x} \mid X(\mathbf{x}) \geq \underline{X}\} \equiv e(\mathbf{p}, \underline{X}), \quad (1)$$

which defines the expenditure function  $e$ . We obtain:<sup>3</sup>

$$p_i = \partial_X e(\mathbf{p}, \underline{X}) \partial_i X(\mathbf{x}) \quad (\text{first order condition}) \quad (2)$$

$$x_i = \partial_{p_i} e(\mathbf{p}, \underline{X}) \equiv H_i(\mathbf{p}, \underline{X}) \quad (\text{Shephard's lemma}), \quad (3)$$

where  $H_i$  is the Hicksian demand function for good  $i$ . By defining the *intra-sectoral elasticity of substitution* of good  $i$  as the elasticity of substitution  $s_i$  of good  $i$  for the composite good, when the bundle of differentiated goods is

<sup>3</sup>We denote  $\partial_{x_i} F(\mathbf{x}, Y) \equiv \partial F(\mathbf{x}, Y) / \partial x_i$ ,  $\partial_Y F(\mathbf{x}, Y) \equiv \partial F(\mathbf{x}, Y) / \partial Y$  and also  $\partial_i F(\mathbf{x}) \equiv \partial F(\mathbf{x}) / \partial x_i$  when there is no ambiguity. Similarly,  $\partial_{ij}^2 F(\mathbf{x}) \equiv \partial^2 F(\mathbf{x}) / \partial x_i \partial x_j$  and  $\epsilon_i F(\mathbf{x}) \equiv \partial_i F(\mathbf{x}) x_i / F(\mathbf{x})$ .

$\mathbf{x}$ , alternatively in terms of quantities and prices, we obtain (as shown in the Appendix):

$$s_i = \frac{1 - \partial_i X(\mathbf{x}) x_i / X(\mathbf{x})}{-\partial_i^2 X(\mathbf{x}) x_i / \partial_i X(\mathbf{x})} = \frac{-\epsilon_{p_i} H_i(\mathbf{p}, \underline{X})}{1 - [\epsilon_i X(\mathbf{x})] [\epsilon_X H_i(\mathbf{p}, \underline{X})]}. \quad (4)$$

In the second stage, the consumer chooses the quantities  $\underline{X}$  and  $z$  of the composite and the numéraire good, respectively, by solving the program

$$\max_{(\underline{X}, z) \in \mathbb{R}_+^2} \{U(\underline{X}, z) \mid e(\mathbf{p}, \underline{X}) + z \leq Y\}. \quad (5)$$

The solution to this program determines the Marshallian demand  $\underline{X} = D(\mathbf{p}, Y)$  for the composite good and the demand  $z = Y - e(\mathbf{p}, D(\mathbf{p}, Y))$  for the numéraire good. The intensity of the response of the consumption  $x_i$  to a change in the price  $p_i$  taking into account the variation of the Marshallian demand  $D$  (rather than through the mere share adjustment as in  $-\epsilon_{p_i} H_i(\mathbf{p}, \underline{X})$ ) can be measured by the demand elasticity computed as follows:

$$\sigma_i \equiv - \left. \frac{dx_i}{dp_i} \right|_{X(\mathbf{x})=D(\mathbf{p}, Y)} \frac{p_i}{x_i} = - \frac{\partial_{p_i} D(\mathbf{p}, Y) p_i}{\partial_i X(\mathbf{x}) x_i} = \frac{-\epsilon_{p_i} D(\mathbf{p}, Y)}{\epsilon_i X(\mathbf{x})}. \quad (6)$$

It expresses an *intersectoral elasticity of substitution* of good  $i$ .

## 2.2 Firms' competitive behavior

We consider competition among  $n$  firms, each firm  $i$  producing a single component of the composite good with a constant positive unit cost  $c_i$  and a positive fixed cost  $\phi_i$ . In order to encompass various competition regimes, we shall analyze the case where firms behave strategically in price-quantity pairs:  $(p_i, x_i) \in \mathbb{R}_+^2$  for each firm  $i = 1, \dots, n$ . These pairs have to satisfy two admissibility constraints.

The first is a *constraint on market share*, focusing on competition within the sector producing the differentiated goods and referring to the first stage of the consumer's utility maximization. It bounds the quantity of good  $i$  by the corresponding Hicksian demand:

$$x_i \leq H_i((p_i, \mathbf{p}_{-i}), X(x_i, \mathbf{x}_{-i})). \quad (7)$$

The second is a *constraint on market size*, focusing on competition of the whole set of producers of the differentiated goods with the sector producing the numéraire good. It refers to the second stage of the consumer's utility maximization, and bounds the size of the market for the differentiated goods by the Marshallian demand:

$$X(x_i, \mathbf{x}_{-i}) \leq D((p_i, \mathbf{p}_{-i}), Y). \quad (8)$$

The constraint on market share emphasizes the conflictual side of competition between the oligopolists, whereas the constraint on market size translates their common interest as a sector.

### 2.3 Oligopolistic equilibria with varying competitive toughness

**Definition 1** An oligopolistic equilibrium is a tuple of pairs  $(p_i^*, x_i^*)_{i=1, \dots, n} \in \mathbb{R}_+^{2n}$  such that, for any  $i$ ,

$$(p_i^*, x_i^*) \in \arg \max_{(p_i, x_i) \in \mathbb{R}_+^2} (p_i - c_i) x_i \quad (9)$$

s.t.  $x_i \leq H_i((p_i, \mathbf{p}_{-i}^*), X(x_i, \mathbf{x}_{-i}^*))$   
and  $X(x_i, \mathbf{x}_{-i}^*) \leq D((p_i, \mathbf{p}_{-i}^*), Y^*)$ ,

and such that  $Y^* = L + \sum_{i=1}^n ((p_i^* - c_i) x_i^* - \phi_i)$ . In addition, we require the profits to be non-negative, namely  $(p_i^* - c_i) x_i^* - \phi_i \geq 0$  for each  $i$ , and the consumer to be non-rationed.<sup>4</sup>

Observe that we are adjusting income  $Y$  parametrically in this definition, and hence supposing that firms neglect the so-called *Ford effects* (the no income feedback effects assumption).<sup>5</sup>

In the symmetric case (with a symmetric sub-utility function  $X$  and identical costs  $c_i = c$  and  $\phi_i = \phi$  for any firm  $i$ ), we shall also use the concept of oligopolistic free-entry equilibrium.

**Definition 2** An oligopolistic free-entry equilibrium is a pair  $(I^*, (p_i^*, x_i^*)_{i \in I^*})$ , formed by a set of active firms  $I^* \subset \mathbb{N}^*$  and an oligopolistic equilibrium  $(p_i^*, x_i^*)_{i \in I^*} \in \mathbb{R}_+^{I^* \times I^*}$ , such that no inactive firm  $i \in \mathbb{N}^* \setminus I^*$  can obtain a positive profit by becoming active (while satisfying the two constraints, for market share and for market size).

We next show that an oligopolistic equilibrium can be characterized by a simple expression for each firm  $i$  (*relative markup* (or Lerner index for the degree of monopoly power) at that equilibrium, that is,  $\mu_i^* = (p_i^* - c_i)/p_i^*$ , derived from the first order conditions of producer  $i$ 's program in Definition 1. To obtain that simple expression, we refer to the intra- and intersectoral elasticities of substitution of good  $i$ ,  $s_i^*$  and  $\sigma_i^*$  respectively (as in equations (4) and (6)), again at the considered equilibrium, and we introduce in addition two simplifying notations. The elasticity  $\alpha_i \equiv \epsilon_i X(\mathbf{x})$  measures the impact of a variation in the quantity of good  $i$  on the volume of the composite good. The elasticity  $\beta_i \equiv \epsilon_X H_i(\mathbf{p}, \underline{X})$  measures the reverse impact of a variation in the

<sup>4</sup>Non-rationing of the consumer implies that both constraints are satisfied as equalities for each  $i$  at equilibrium. It makes the equilibrium compatible with the consumer's program and the resulting demand functions.

<sup>5</sup>Otherwise, each firm  $i$  would take income  $Y$  in its program to be equal to

$$\begin{aligned} & \mathcal{Y}_i((p_i, x_i), (\mathbf{p}_{-i}^*, \mathbf{x}_{-i}^*)) \\ &= L + \sum_{j \neq i} ((p_j^* - c_j) x_j^* - \phi_j) + (p_i - c_i) x_i - \phi_i. \end{aligned}$$

See Hart (1985) and d'Aspremont *et al.* (1989, 1990).

quantity of the composite good on the demand for its component  $i$ , at given prices  $\mathbf{p}$ . The product of the two elasticities, which appears in the multiplier  $1/(1 - \alpha_i \beta_i)$  applied to the elasticity  $-\epsilon_{p_i} H_i(\mathbf{p}, \underline{X})$  of the Hicksian demand in the second expression for  $s_i$  in equation (4), measures the intensity of the feedback originating in a variation in the quantity of good  $i$  and going through the volume of the composite good.

The markup of firm  $i$  at some equilibrium  $(p_i^*, x_i^*)_{i=1, \dots, n}$  (with positive prices and quantities) will be expressed, according to the following proposition, as the weighted harmonic mean of the reciprocals of the two elasticities  $s_i^*$  and  $\sigma_i^*$  at that equilibrium. The corresponding weights will involve, for each firm  $i$ , the elasticities  $\alpha_i^*$  and  $\beta_i^*$  measuring the two reciprocal effects of quantity variations of good  $i$  and of the composite good, as well as a conduct parameter<sup>6</sup>  $\theta_i^* \in [0, 1]$ , stemming from the first order conditions and interpreted as a measure of the *competitive toughness* displayed by firm  $i$  towards its rival oligopolists at the equilibrium  $(p_i^*, x_i^*)_{i=1, \dots, n}$ .

**Proposition 1** *Let  $(p_i^*, x_i^*)_{i=1, \dots, n} \in \mathbb{R}_{++}^{2n}$  be an oligopolistic equilibrium. Then the (relative) markup  $\mu_i^* = (p_i^* - c_i) / p_i^*$  of each firm  $i$  is given by*

$$\mu_i^* = \frac{\theta_i^* (1 - \alpha_i^* \beta_i^*) + (1 - \theta_i^*) \alpha_i^*}{\theta_i^* (1 - \alpha_i^* \beta_i^*) s_i^* + (1 - \theta_i^*) \alpha_i^* \sigma_i^*}, \quad (10)$$

for some  $\theta_i^* \in [0, 1]$ .

**Proof.** We start by making dimensionally homogeneous the two constraints in the program of firm  $i$ , rewriting them in terms of the two ratios:

$$\frac{x_i}{H_i((p_i, \mathbf{p}_{-i}), X(x_i, \mathbf{x}_{-i}))} \leq 1 \text{ and } \frac{X(x_i, \mathbf{x}_{-i})}{D((p_i, \mathbf{p}_{-i}), Y)} \leq 1. \quad (11)$$

The first-order necessary conditions for profit maximization at  $(p_i^*, x_i^*)$  under these two constraints (holding as equalities at equilibrium) can then be expressed, for non-negative Lagrange multipliers  $\lambda_i^*$  and  $\nu_i^*$ , as

$$\begin{aligned} x_i^* &= \lambda_i^* \left( \frac{-\partial_{p_i} H_i(\mathbf{p}^*, D(\mathbf{p}^*, Y^*))}{H_i(\mathbf{p}^*, D(\mathbf{p}^*, Y^*))} \right) + \nu_i^* \left( \frac{-\partial_{p_i} D(\mathbf{p}^*, Y^*)}{D(\mathbf{p}^*, Y^*)} \right) \\ &= \frac{\lambda_i^*}{p_i^*} [-\epsilon_{p_i} H_i(\mathbf{p}^*, D(\mathbf{p}^*, Y^*))] + \frac{\nu_i^*}{p_i^*} [-\epsilon_{p_i} D(\mathbf{p}^*, Y^*)], \end{aligned} \quad (12)$$

and

$$\begin{aligned} (p_i^* - c_i) &= \lambda_i^* \left( \frac{1 - [\partial_X H_i(\mathbf{p}^*, D(\mathbf{p}^*, Y^*))][\partial_i X(\mathbf{x}^*)]}{H_i(\mathbf{p}^*, D(\mathbf{p}^*, Y^*))} \right) + \nu_i^* \left( \frac{\partial_i X(\mathbf{x}^*)}{X(\mathbf{x}^*)} \right) \\ &= \frac{\lambda_i^*}{x_i^*} (1 - [\epsilon_i X(\mathbf{x}^*)][\epsilon_X H_i(\mathbf{p}^*, D(\mathbf{p}^*, Y^*))]) + \frac{\nu_i^*}{x_i^*} [\epsilon_i X(\mathbf{x}^*)]. \end{aligned} \quad (13)$$

<sup>6</sup>To use the terminology of the *New Empirical Industrial Organization* (see Bresnahan, 1989).

We can use these two conditions to compute the markup of firm  $i$  at the equilibrium:

$$\mu_i^* = \frac{p_i^* - c_i}{p_i^*} = \frac{\lambda_i^* (1 - [\epsilon_i X(\mathbf{x}^*)] [\epsilon_X H_i(\mathbf{p}^*, D(\mathbf{p}^*, Y^*))]) + \nu_i^* \epsilon_i X(\mathbf{x}^*)}{\lambda_i^* [-\epsilon_{p_i} H_i(\mathbf{p}^*, D(\mathbf{p}^*, Y^*))] + \nu_i^* [-\epsilon_{p_i} D(\mathbf{p}^*, Y^*)]}. \quad (14)$$

By using the notations  $s_i^*$ ,  $\sigma_i^*$  (as defined by (4) and (6)),  $\alpha_i^* \equiv \epsilon_i X(\mathbf{x}^*)$ ,  $\beta_i^* \equiv \epsilon_X H_i(\mathbf{p}^*, D(\mathbf{p}^*, Y^*))$  and  $\theta_i^* \equiv \lambda_i^* / (\lambda_i^* + \nu_i^*)$ , we finally obtain for  $\mu_i^*$  the expression in equation (10). ■

The normalized Lagrange multiplier  $\theta_i^* \equiv \lambda_i^* / (\lambda_i^* + \nu_i^*)$ , which is the relative shadow price of relaxing the constraint on market share, reflects the weight given to the conflictual side of competition, and hence measures the "competitive toughness" displayed by firm  $i$  towards its competitors in the sector, at the equilibrium  $(p_i^*, x_i^*)_{i=1, \dots, n}$ . The vector  $\theta^* = (\theta_1^*, \dots, \theta_n^*)$  of the degrees of competitive toughness of the different firms thus characterizes this particular equilibrium. Varying this vector in  $[0, 1]^n$  allows us to trace the whole set of potential oligopolistic equilibria. The  $\theta_i$ 's appear as endogenous variables parameterizing the set of equilibria, and refer to different regimes of competition, including in particular standard ones like price or quantity competition, as well as monopolistic competition, as we will see below. Conditions ensuring existence of these standard equilibria naturally entail existence of the corresponding oligopolistic equilibria, as here defined. However, existence of the whole spectrum of potential equilibria (for all values of  $\theta \in [0, 1]^n$ ) is generally not satisfied. The case of a symmetric duopoly has been completely analyzed when sub-utility  $X$  is CES (d'Aspremont *et al.*, 2007) or quadratic (d'Aspremont and Dos Santos Ferreira, 2010). The analysis shows that the range of enforceable values of  $\theta$  becomes more and more restricted as substitutability (resp. complementarity) becomes higher and higher.

Inspection of equation (10) shows that the weight put in the reciprocal of intrasectoral elasticity of substitution  $s_i^*$  (relative to the weight put in its intersectoral homologue  $\sigma_i^*$ ) naturally increases with the competitive toughness  $\theta_i^*$  displayed by firm  $i$  at equilibrium. It decreases with the intensity  $\alpha_i^* \beta_i^*$  of the feedback to the actions of firm  $i$  involving the whole sector.

### 3 Competitive effects under different regimes of competition

We shall now illustrate the usefulness of formula (10) derived in Proposition 1 by considering some standard regimes of competition. We start by the two classical oligopolistic regimes with specific strategic interactions, namely price competition and quantity competition, and then, for comparison sake, the most popular competition regime in applied work, namely monopolistic competition, which explicitly ignores strategic interactions.

### 3.1 Oligopolistic equilibrium under price competition

Let us first consider price competition. It is easy to prove that price equilibria are included in the set of oligopolistic equilibria.

Denoting the Walrasian demand for good  $i$  by  $Q_i(\mathbf{p}, Y) \equiv H_i(\mathbf{p}, D(\mathbf{p}, Y))$ , a *price equilibrium*  $p^*$  is solution to the following program for each firm  $i$

$$\max_{p_i \in [c_i, \infty)} (p_i - c_i) Q_i((p_i, \mathbf{p}_{-i}), Y^*), \quad (15)$$

together with  $Y^* = L + \sum_{i=1}^n ((p_i^* - c_i) x_i^* - \phi_i)$ , where  $x_i^* = Q_i((p_i^*, \mathbf{p}_{-i}^*), Y^*)$  and  $(p_i^* - c_i) x_i^* - \phi_i \geq 0$  for each  $i$ .

**Proposition 2** *Suppose that a price equilibrium  $p^*$  exists. Then, taking  $Y^*$  as just defined,  $(p_i^*, Q_i((p_i^*, \mathbf{p}_{-i}^*), Y^*))_{i=1, \dots, n}$  is an oligopolistic equilibrium.*

**Proof.** Suppose that, for some  $i$  and some  $(p_i, x_i)$  satisfying the two constraints, we have:  $(p_i - c_i) x_i > (p_i^* - c_i) Q_i((p_i^*, \mathbf{p}_{-i}^*), Y^*)$ . Also, by the market size constraint,  $X(x_i, \mathbf{x}_{-i}) \leq D((p_i, \mathbf{p}_{-i}), Y^*)$  and then, by the market share constraint,  $x_i \leq H_i(p_i, \mathbf{p}_{-i}, X(x_i, \mathbf{x}_{-i})) \leq H_i(p_i, \mathbf{p}_{-i}, D((p_i, \mathbf{p}_{-i}), Y^*)) = Q_i((p_i, \mathbf{p}_{-i}), Y^*)$ . Thus,

$$(p_i - c_i) Q_i((p_i, \mathbf{p}_{-i}), Y^*) \geq (p_i - c_i) x_i > (p_i^* - c_i) Q_i((p_i^*, \mathbf{p}_{-i}^*), Y^*), \quad (16)$$

and the result follows by contradiction. ■

We could directly derive the first order conditions for a price equilibrium and write them in such a way that the markup of each firm  $i$  is equal to the harmonic mean of  $1/s_i^*$  and  $1/\sigma_i^*$ , with weights respectively equal to  $1 - \alpha_i^* \beta_i^*$  and  $\alpha_i^* \beta_i^*$ , *i.e.*,

$$\mu_i^* = \frac{1}{(1 - \alpha_i^* \beta_i^*) s_i^* + \alpha_i^* \beta_i^* \sigma_i^*}. \quad (17)$$

But the same expression for the markup  $\mu_i^*$  can be obtained from the general formula (10) obtained in Proposition 1 by taking the competitive toughness to be

$$\theta_i^* = \frac{1}{1 + \beta_i^*}. \quad (18)$$

In terms of our concept of oligopolistic equilibrium, pure price competition requires the degree  $\theta_i^*$  of competitive toughness of firm  $i$  to be the higher the smaller the impact  $\beta_i^*$  of a variation in the quantity of the composite good on the demand for good  $i$ .

In the homothetic case, with  $X$  is homogeneous of degree 1, we have  $\beta_i^* = 1$  for all  $i$ , so that putting all the  $\theta_i$ 's equal to  $1/2$  entails the price equilibrium value of the  $\mu_i$ 's (see d'Aspremont *et al.*, 2007). Moreover  $\sigma_i^* = \sigma^*$  for any  $i$ , in this case. If we further assume a CES sub-utility, we have  $s_i^* = s$  for any  $i$ , which leads to the simplified formula

$$\mu_i^* = \frac{1}{(1 - \alpha_i^*) s + \alpha_i^* \sigma^*}, \quad (19)$$

the weighted harmonic mean of the reciprocals of the elasticities  $s$  and  $\sigma^*$ , with respective weights  $(1 - \alpha_i^*)$  and  $\alpha_i^*$  ( $\alpha_i^*$  being now interpretable as the budget share of good  $i$  at equilibrium). Since it does not suppose equilibrium symmetry (even if sub-utility is symmetric, firms costs may differ), this formula is more general and more transparent than the one derived by Yang & Heijdra (1993), eq. (4), or by d'Aspremont *et al.* (1996), eq. (8).

### 3.2 Competitive effects of price competition

The analysis of the competitive effects of price competition is simple if we concentrate on the (symmetric) CES case (say,  $X(\mathbf{x}) = \left(\sum_{i=1}^n x_i^{(s-1)/s}\right)^{s/(s-1)}$ ,  $s \in (0, 1) \cup (1, \infty)$ ) and base our conclusions on the harmonic mean of all markups weighted by their respective budget shares (the  $\alpha_i$ 's). Using (19), we get the following formula which shows the relation between the *harmonic mean markup* and the *Herfindahl index* of concentration  $\sum_{i=1}^n \alpha_i^{*2}$ :

$$\bar{\mu}^* = \left(\sum_{i=1}^n \frac{\alpha_i^*}{\mu_i^*}\right)^{-1} = \frac{1}{s + (\sigma^* - s) \sum_{i=1}^n \alpha_i^{*2}}. \quad (20)$$

We see that *the pro-competitive effect of less concentration* ( $\bar{\mu}^*$  decreasing when concentration decreases) works only when there is more substitutability among the differentiated goods than between the latter and the numéraire good (i.e. an intrasectoral elasticity of substitution larger than its intersectoral homologue). Otherwise, the effect is anti-competitive. For the symmetric case (assuming  $c_i = c$  and  $\phi_i = \phi$  for any  $i$ ), we obtain a symmetric equilibrium with  $\alpha_i^* = 1/n$  and hence the same equilibrium markup for every firm:

$$\mu^* = \frac{1}{s + (\sigma^* - s)/n}. \quad (21)$$

The equilibrium markup  $\mu^*$  (with  $0 < \mu^* < 1$ ), hence also the equilibrium price

$$p^* = \frac{c}{1 - \mu^*} = \frac{c(ns + \sigma^* - s)}{n(s - 1) + \sigma^* - s}, \quad (22)$$

decreases (resp. increases) when  $n$  increases as long as  $s > \sigma^*$  (resp.  $s < \sigma^*$ ). Hence, the pro-competitive effect (on markup and price) of an increase in the number of firms depends on the dominance of intrasectoral over intersectoral substitutability. This can be summarized as a proposition.

**Proposition 3** *Assume CES sub-utility. As long as  $s > \sigma^*$  (resp.  $s < \sigma^*$ ) the harmonic mean markup increases (resp. decreases) with the Herfindahl index of concentration. If in addition  $c_i = c$  and  $\phi_i = \phi$  for any  $i$ , the equilibrium markups and prices are identical for all firms and decreasing (resp. increasing) when  $n$ , the number of active firms, increases as long as  $s > \sigma^*$  (resp.  $s < \sigma^*$ ).*

Consider now the consequences of cost asymmetries on the relative equilibrium values of the markups, prices and budget shares of each differentiated good. Sticking to the (symmetric) CES case, where the price of the composite good can be written as  $P(\mathbf{p}) = (\sum_i p_i^{1-s})^{1/(1-s)}$ , we can alternatively express the equilibrium budget share of good  $i$  as the elasticity  $\epsilon_i X(\mathbf{x}^*)$  or as the elasticity  $\epsilon_{p_i} e(\mathbf{p}, \underline{X}) = \epsilon_i P(\mathbf{p}^*)$ , hence as

$$\alpha_i^* = \frac{x_i^{*1-1/s}}{\sum_j x_j^{*1-1/s}} = \frac{p_i^{*1-s}}{\sum_j p_j^{*1-s}}. \quad (23)$$

Consequently,

$$\frac{\alpha_i^*}{\alpha_j^*} = \left( \frac{x_i^*}{x_j^*} \right)^{1-1/s} = \left( \frac{p_i^*}{p_j^*} \right)^{1-s}, \quad (24)$$

so that  $p_i^* < p_j^*$  and  $x_i^* > x_j^*$  if and only if  $\alpha_i^* > \alpha_j^*$  (resp.  $\alpha_i^* < \alpha_j^*$ ) whenever  $s > 1$  (resp.  $s < 1$ ). Also, by (19),  $\mu_i^* > \mu_j^*$  if and only if  $\alpha_i^* > \alpha_j^*$  (resp.  $\alpha_i^* < \alpha_j^*$ ) whenever  $s > \sigma^*$  (resp.  $s < \sigma^*$ ). We may deduce the following proposition.

**Proposition 4** *Assume CES sub-utility. Then (i) if  $s > \max\{1, \sigma^*\}$  (differentiated goods are highly substitutable) the more productive firms set lower prices with higher markups, and obtain higher budget shares for their goods, and (ii) if  $s < \min\{1, \sigma^*\}$  (differentiated goods have a low elasticity of substitution) the more productive firms, setting lower prices with higher markups, only obtain lower budget shares for their goods.*

**Proof.** For (i) if  $p_i^* < p_j^*$  and  $\mu_i^* > \mu_j^*$  with  $\alpha_i^* > \alpha_j^*$ , a case which arises when  $s > \max\{1, \sigma^*\}$ , we must have  $c_i < c_j$  (since  $p_i^* = c_i / (1 - \mu_i^*)$ ). By contrast, (ii) we must have  $c_i < c_j$  if  $p_i^* < p_j^*$  and  $\mu_i^* > \mu_j^*$  with  $\alpha_i^* < \alpha_j^*$ , a case arising when  $s < \min\{1, \sigma^*\}$ . ■

Case (i), occurring when  $s$  is high ( $s > \max\{1, \sigma^*\}$ ), can clearly be viewed as pro-competitive, whereas case (ii), occurring when  $s$  is low ( $s < \min\{1, \sigma^*\}$ ), can be viewed as anti-competitive. Notice that case (ii) implies in fact  $s < 1 < \sigma^*$ , as one at least of the two elasticities of substitution must be higher than one, for the markup to be kept smaller than one. By contrast, case (i) decomposes into  $s > 1 \geq \sigma^*$  and  $s > \sigma^* > 1$ .

### 3.3 Oligopolistic free-entry equilibrium under price competition

We have up to now taken as exogenous the number  $n$  of active firms, and analyzed the competitive effects of changes in this number across symmetric price equilibria (see Proposition 3). By assuming free entry, more precisely, by introducing the zero profit condition usually associated to that assumption, we can make the equilibrium number  $n^*$  endogenous, and so consider more deeply the competitive effects of changes in income  $L$ , in the utility weight  $\gamma$  of the

composite good in total consumption, or in productivity  $1/c$  (in the symmetric case).

For simplicity and explicitness, we shall assume symmetry within the oligopolistic sector and CES sub-utility, as well as CES utility:

$$U(\underline{X}, z) = \left( \gamma \underline{X}^{\frac{\sigma-1}{\sigma}} + (1-\gamma) z^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad (25)$$

with constant positive intersectoral elasticity of substitution  $\sigma$  and weight  $\gamma \in (0, 1)$  of the composite good. If we recall that  $Y^* = L$  under zero profits, the expenditure in the oligopolistic sector when all the differentiated goods have the same price  $p$  is

$$p\underline{X} = \frac{\gamma p^{1-\sigma}}{\gamma p^{1-\sigma} + 1 - \gamma} L. \quad (26)$$

The zero-profit condition can then be formulated as

$$n^* \phi = \mu^* p^* \underline{X}^* = \mu^* \frac{\gamma}{\gamma + (1-\gamma)(p^*)^{\sigma-1}} L \quad (27)$$

which, using (21) and (22), leads to

$$(n^* s + \sigma - s) \left( 1 + (1/\gamma - 1) \left( \frac{1}{c} \left( 1 - \frac{1}{s + (\sigma - s)/n^*} \right) \right)^{1-\sigma} \right) = \frac{L}{\phi}, \quad (28)$$

with  $s > \max\{\sigma, 1\}$  or  $s < 1 < \sigma$  (for  $\mu^*$  to be smaller than 1). The elasticity of the function  $N(n^*) \equiv (1 - 1/(s + (\sigma - s)/n^*))^{1-\sigma}$  is equal to

$$\epsilon N(n^*) = (1 - \sigma) \frac{-1/(s + (\sigma - s)/n^*)}{1 - 1/(s + (\sigma - s)/n^*)} \frac{(\sigma - s)/n^*}{s + (\sigma - s)/n^*} \quad (29)$$

which is non-negative if  $(\sigma - 1)(\sigma - s) \geq 0$ , a condition satisfied in the cases  $\sigma \leq 1 < s$  and  $s < 1 < \sigma$ . In both these cases, the LHS of (28) is clearly increasing in  $n^*$ , so that the equilibrium number of active firms is an increasing function of income  $L/\phi$  (normalized in terms of the fixed cost of the individual firm). An increase in the oligopolistic market size, associated with an increase in the utility weight  $\gamma$ , has also a positive impact on  $n^*$ . So has an increase in productivity  $1/c$  if  $\sigma > 1$ . However, if  $\sigma \leq 1$ , an increase in productivity has either no impact or a negative impact on  $n^*$ , which is however not strong enough to reverse its direct impact on the price of differentiated goods.

Once we have determined the sense of variations in  $n^*$ , it is straightforward to apply Proposition 3 to assess the pro- or anti-competitive effects on  $p^*$  (and  $\mu^*$ ) associated with those variations. As to the output  $x^*$  of each differentiated good, it is easy to derive it from the zero-profit condition  $(p^* - c)x^* = \mu^* p^* x^* \approx \phi$ :

$$x^* \approx \frac{1 - \mu^*}{\mu^*} \frac{\phi}{c} = \left( s - 1 + \frac{\sigma - s}{n^*} \right) \frac{\phi}{c}. \quad (30)$$

We can sum up the preceding results in the following proposition.

**Proposition 5** *Assume CES utility and sub-utility, together with symmetry in the oligopolistic sector. The oligopolistic free-entry equilibrium is given by the set of  $n^*$  identical firms selling the same quantity  $x^*$  at the common price  $p^*$ :*

$$x^* = \left( \frac{n^*(s-1) + \sigma - s}{n^*} \right) \frac{\phi}{c} \text{ and } p^* = \frac{n^*s + \sigma - s}{n^*(s-1) + \sigma - s} c. \quad (31)$$

The number of active firms  $n^*$  satisfies the equation (28). If either  $\sigma \leq 1 < s$  or  $s < 1 < \sigma$ , this number is increasing in the (normalized) income  $L/\phi$ , and in the size of the market for the composite good as determined by its utility weight  $\gamma$ . The number  $n^*$  is also increasing (resp. decreasing) in productivity  $1/c$  if  $s < 1 < \sigma$  (resp.  $\sigma < 1 < s$ ). Competitive effects are as follows:  $p^*$  is decreasing and  $x^*$  increasing with  $n^*$  if  $s > \sigma$ , and vary in the opposite sense if  $s < \sigma$ .

### 3.4 Oligopolistic Equilibrium under Quantity Competition and competitive effects

To pursue our review of standard competition regimes, let us next consider quantity competition. Denoting  $Q^{-1}$  the inverse of the Walrasian demand function  $Q(\mathbf{p}, Y) \equiv H(\mathbf{p}, D(\mathbf{p}, Y))$ , a *quantity equilibrium* is a solution  $x^*$  to the following program for each firm  $i$

$$\max_{x_i \in \mathbb{R}_+} \{ Q_i^{-1}(x_i, \mathbf{x}_{-i}^*, Y^*) x_i - c_i x_i \}, \quad (32)$$

where  $Y^* = L + \sum_{i=1}^n ((p_i^* - c_i) x_i^* - \phi_i)$ , where  $p_i^* = Q_i^{-1}(\mathbf{x}^*, Y^*)$  and  $(p_i^* - c_i) x_i^* - \phi_i \geq 0$  for each  $i$ . By a similar argument as in Proposition 2, we get:

**Proposition 6** *Suppose that a quantity equilibrium  $\mathbf{x}^*$  exists. Then, taking  $Y^*$  as just defined,  $(Q^{-1}(\mathbf{x}^*, Y^*), \mathbf{x}^*)$  is an oligopolistic equilibrium.*

**Proof.** Suppose that the quantity equilibrium  $\mathbf{x}^*$  with the associated price vector  $\mathbf{p}^* = Q^{-1}(\mathbf{x}^*, Y^*)$  is not an oligopolistic equilibrium: for some  $i$  and some pair  $(p_i, x_i)$  satisfying the two constraints in (9), *i.e.*

$$x_i \leq H_i(p_i, \mathbf{p}_{-i}^*, X(x_i, \mathbf{x}_{-i}^*)) \text{ and } X(x_i, \mathbf{x}_{-i}^*) \leq D(p_i, \mathbf{p}_{-i}^*, Y^*), \quad (33)$$

we have  $(p_i - c_i) x_i > Q_i^{-1}(\mathbf{x}^*, Y^*) x_i^* - c_i x_i^*$ . First, observe that by duality the two constraints can be reformulated (in terms of prices rather than quantities) as

$$\begin{aligned} p_i &\leq \partial_X e(p_i, \mathbf{p}_{-i}^*, X(x_i, \mathbf{x}_{-i}^*)) \partial_i X(x_i, \mathbf{x}_{-i}^*) \text{ and} \\ \partial_X e(p_i, \mathbf{p}_{-i}^*, X(x_i, \mathbf{x}_{-i}^*)) &\leq \partial_X e(Q^{-1}(x_i, \mathbf{x}_{-i}^*, Y^*), X(x_i, \mathbf{x}_{-i}^*)). \end{aligned} \quad (34)$$

The dual expression of the constraint on market share stems from the first order condition (2). It states that the market price  $p_i$  of good  $i$ , relative to the shadow price  $\partial_X e(p_i, \mathbf{p}_{-i}^*, X(x_i, \mathbf{x}_{-i}^*))$  of the composite good should not exceed the marginal utility  $\partial_i X(x_i, \mathbf{x}_{-i}^*)$ . As to the dual expression of the constraint on

market size, it states that the level of the shadow price of the composite good, in quantity  $X(x_i, \mathbf{x}_{-i}^*)$ , when evaluated at market prices  $(p_i, \mathbf{p}_{-i}^*)$  should not exceed its level when evaluated at the prices corresponding to the inverse demand  $Q^{-1}(x_i, \mathbf{x}_{-i}^*, Y^*)$  (and so reflecting the second stage of utility maximization). Now, the two constraints together imply

$$p_i \leq \partial_X e(Q^{-1}(x_i, \mathbf{x}_{-i}^*, Y^*), X(x_i, \mathbf{x}_{-i}^*)) \partial_i X(x_i, \mathbf{x}_{-i}^*) = Q_i^{-1}(x_i, \mathbf{x}_{-i}^*, Y^*), \quad (35)$$

so that  $x_i$  is a profitable deviation from the quantity equilibrium, in contradiction with  $\mathbf{x}^*$  being a quantity equilibrium. ■

It is easy to derive the first order conditions for the quantity equilibrium if we assume a homothetic sub-utility (implying  $\beta_i^* = 1$  and  $\sigma_i^* = \sigma^*$  for any  $i$ ). We obtain:

$$\mu_i^* = (1 - \alpha_i^*)(1/s_i^*) + \alpha_i^*(1/\sigma^*), \quad (36)$$

The equilibrium markup is the *arithmetic mean* of  $1/s_i^*$  and  $1/\sigma^*$ , with weights respectively equal to  $1 - \alpha_i^*$  and  $\alpha_i^*$  (with  $\alpha_i^*$  interpretable as the budget share of good  $i$ ). Coming back to formula (10) of Proposition 1, this amounts to put

$$\theta_i^* = 1/(1 + s_i^*/\sigma^*), \quad (37)$$

where  $s_i^*/\sigma^*$  is the ratio between *elasticities of intra- and intersectoral substitution* for good  $i$ .

Notice that the quantity equilibrium is less competitive than the price equilibrium (which corresponds to  $\theta_i^* = 1/2$ ) if and only if  $s_i^*/\sigma^* > 1$ . More generally, high substitutability within the oligopolistic sector (relative to the sector of the numéraire good) is so to say compensated by mild competitiveness: pure quantity competition requires the degree  $\theta_i^*$  of competitive toughness of firm  $i$  to decrease as the ratio  $s_i^*/\sigma^*$  increases.

To analyze the competitive effects of quantity competition, it is convenient to use the arithmetic mean of all markups weighted by their respective budget shares. Assuming a CES sub-utility, we get the following alternative formula (to (20)), now establishing the relation between the *arithmetic mean markup* and the *Herfindahl index* of concentration  $\sum_{i=1}^n \alpha_i^{*2}$ :

$$\bar{\mu}^* = \sum_{i=1}^n \alpha_i^* \mu_i^* = \frac{1}{s} + \left( \frac{1}{\sigma^*} - \frac{1}{s} \right) \sum_{i=1}^n \alpha_i^{*2}, \quad (38)$$

which reduces in the symmetric case to the uniform markup

$$\mu^* = \frac{1}{s} + \left( \frac{1}{\sigma^*} - \frac{1}{s} \right) \frac{1}{n}. \quad (39)$$

We thus get a result analogous to Proposition 3.

**Proposition 7** *Assume CES sub-utility. As long as  $s > \sigma^*$  (resp.  $s < \sigma^*$ ) the arithmetic mean markup increases (resp. decreases) with the Herfindahl index of concentration. If in addition  $c_i = c$  and  $\phi_i = \phi$  for any  $i$ , the equilibrium markups and prices of all firms are identical and decreasing (resp. increasing) when  $n$ , the number of active firms, increases as long as  $s > \sigma^*$  (resp.  $s < \sigma^*$ ).*

### 3.5 Monopolistic competition

The last application of Proposition 1 that we will consider is monopolistic competition. This will be a limit case, assuming a symmetric equilibrium (with a symmetric sub-utility function  $X$ , as well as uniform costs  $c_i = c$  and  $\phi_i = \phi$  for any  $i$ ), so that the index  $i$  can be omitted in the following. But this limit case can be reached in two ways. A first way, the traditional one, consists in making the *large group assumption* (in Chamberlin's sense) by supposing  $\alpha^* \approx 0$  in the expression (10). Then we obtain the same

$$\mu^* \approx 1/s^* \tag{40}$$

as the markup for each firm. Also, with symmetry (with  $x_i = x^*$  for all  $i$ ) and  $\alpha^* \approx 0$ , formula (4) boils down to

$$s^* = \frac{\partial_i X(\mathbf{x}^*)}{-\partial_{ii}^2 X(\mathbf{x}^*) x^*} \equiv \frac{1}{r_X(x^*)} \tag{41}$$

where  $r_X(x^*)$  is the value taken by the Arrow-Pratt formula for relative risk aversion when applied to the sub-utility function  $X$  along the diagonal. This value can be interpreted as the *relative love for variety* (see Zhelobodko *et al.*, 2011).

In this case the following facts are well-known about a symmetric monopolistic equilibrium (seen here as the limit of an oligopolistic equilibrium). Since the intersectoral elasticity of substitution vanishes in formula (10) of Proposition 1, the properties of  $U$  are immaterial for the value of the equilibrium price  $p^*$ .

Also well-known is that, when  $X$  is a CES function (so that  $s^*$  is a structural parameter  $s \in (0, 1) \cup (1, \infty)$ ), the equilibrium price of the composite good is constant and equal to

$$p^* = c/(1 - \mu^*) = c/(1 - 1/s) \tag{42}$$

which is independent of the number of active firms. In order to obtain richer results concerning the price variations, the variability of  $s^*$  must consequently be allowed for. Using a translog utility function as the specification of  $X$ , Feenstra (2003) showed that such variability can be secured, in the case of homothetic preferences, so as to obtain pro-competitive effects of an increase in the mass of active monopolists. Zhelobodko *et al.* (2011) showed that both pro-competitive and anti-competitive effects can be obtained, in the case of an additive sub-utility function  $X$  (as introduced by Krugman, 1979, in a one sector model), according to the way the relative love for variety responds to changes in the level of consumption. In particular, the pro-competitive effect of an increase in the number (or the mass) of firms depends on the increasingness of love for variety with the consumption level (rather than on the dominance of intrasectoral over intersectoral substitutability, as in our price and quantity equilibria). They also use the example of quadratic sub-utility to show that additivity can be dispensed with, but they only obtain pro-competitive effects in that example.

Under free entry, still assuming  $X$  to be a CES function, the equilibrium quantity of each differentiated good,

$$x^* \approx (s - 1) \phi / c, \quad (43)$$

is entirely determined by the elasticity of substitution  $s$  and the ratio of fixed to variable costs  $\phi/c$ . Switching to the additive utility function used by Zhelobodko *et al.* (2011) does not considerably change the situation if one sticks to the regime of monopolistic competition, with  $\alpha^* \approx 0$ , and  $\mu^* \approx 1/s^*$  equal to the relative love for variety  $r_X(x^*)$ , a function of  $x^*$  which does not depend upon  $n^*$  (here interpretable as the *mass* of active firms). Thus,  $x^*$  is still determined independently of the equilibrium number  $n^*$  of active firms, which is determined via the utility function  $U$ . For instance, if this function is Cobb-Douglas with exogenous weights  $\gamma$  for the composite good and  $1 - \gamma$  for the numéraire good, we obtain:

$$n^* = \frac{\gamma L}{p^* x^*} = \frac{\gamma L}{s \phi} \quad (44)$$

in the CES case, or  $n^* = \gamma (L/\phi) r_X(x^*)$  in the case examined by Zhelobodko *et al.* (2011). When  $U$  is not Cobb-Douglas, we may still use this expression for  $n^*$ , but with an endogenous equilibrium share  $\gamma^*$  of the composite good in the total budget. More generally, when the sub-utility is not additive, the intrasectoral elasticity of substitution depends (along the diagonal) both on the quantity consumed per differentiated good and on the number of active firms, implying that, at the symmetric free entry equilibrium, prices and quantities are not determined independently from the equilibrium number of firms (see Parenti *et al.*, 2014). Still, the main ingredient remains the intrasectoral elasticity of substitution.

But, as mentioned above, there is another way to obtain  $\mu^* = 1/s^*$  (the formula characterizing monopolistic competition) which has the advantage of allowing to keep a limited number  $n$  of firms, and thus to let  $\alpha^*$  be significantly away from zero: it suffices to take  $\theta^* = 1$ , meaning that firms have maximal competitive toughness, instead of being insignificant.<sup>7</sup> In this case, equation (4) becomes

$$s^* = \frac{1 - \alpha^*}{r_X(x^*)} = \frac{1 - 1/n}{r_X(x^*)}. \quad (45)$$

introducing the dependence on  $n$ . For instance, if the relative love for variety is constant, then the intrasectoral elasticity of substitution becomes an increasing function of  $n$ , and we get the associated results (see Bertoletti and Epifani, 2014). Or, when  $X$  is a CES, the relative love for variety has to be a function of  $n$ , implying that the price and the quantity vary with  $n$ .

Again this means that, at the symmetric free entry equilibrium, prices and quantities are not determined independently from the equilibrium number of

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<sup>7</sup>This is related to the so-called *Bertrand paradox*: tough competition *à la* Bertrand leads to the competitive outcome even in a (symmetric) duopoly, where each firm gets half of the market.

firms. But this is not sufficient to re-introduce the role of intersectoral substitution, as it appears in formula (10) of Proposition 1, and the full general equilibrium dimension of the Dixit-Stiglitz model.

## 4 Conclusion

The purpose of this note is to argue that going beyond the basic version of the Dixit-Stiglitz model can take other paths than relaxing the CES assumption and/or introducing heterogeneity à la Melitz (2003) while maintaining monopolistic competition. One is to introduce oligopolistic competition among a finite number of non negligible firms. This seems realistic, at least for some sectors, and was worth exploring in a general equilibrium perspective with multiple sectors (here only two) and various possible degrees of substitutability-complementarity among goods within the sectors and among composite goods. Symmetry can be straightforwardly relaxed and, by adopting some specific substitutability like the CES, the model can be kept tractable. In our analysis of pro-competitive or anti-competitive effects, the main benefit which we have obtained is to stress the role of intersectoral characteristics, mainly intersectoral substitutability as compared to intrasectoral substitutability, a role which is obliterated under monopolistic competition. Much further work is required of course, but one may hope that Neary (2004) was right when, on the occasion of the twenty-fifth anniversary of Dixit-Stiglitz paper, he considered the relevance (and the obstacles) in going towards a general oligopolistic equilibrium theory but concluded that "the pay-off to even modest progress in this direction would be enormous."

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## Appendix

### The elasticity of substitution

The elasticity of substitution  $s_i$  of good  $i$  for the composite good, when the bundle of differentiated goods is  $\mathbf{x}$ , is defined as the absolute value of the elasticity of the ratio  $x_i/X(\mathbf{x})$  with respect to the marginal rate of substitution  $\partial_i X(\mathbf{x})$

$$s_i \equiv - \frac{\partial (x_i/X(\mathbf{x}))}{\partial (\partial_i X(\mathbf{x}))} \frac{\partial_i X(\mathbf{x})}{x_i/X(\mathbf{x})}. \quad (46)$$

By differentiating  $x_i/X(\mathbf{x})$  and  $\partial_i X(\mathbf{x})$  with respect to  $x_i$ , we obtain

$$s_i = \frac{1 - \partial_i X(\mathbf{x}) x_i/X(\mathbf{x})}{-\partial_{ii}^2 X(\mathbf{x}) x_i/\partial_i X(\mathbf{x})}. \quad (47)$$

The elasticity of substitution  $s_i$  can alternatively be computed in terms of prices. To do that, we fix  $X(\mathbf{x}) = \underline{X}$ , refer to the shadow price of the composite good  $P \equiv \partial_X e(\mathbf{p}, \underline{X})$ , and use the expression of  $x_i$  given by Shephard's lemma (3):

$$s_i = - \frac{\partial (x_i/\underline{X})}{\partial (p_i/P)} \frac{p_i/P}{x_i/\underline{X}} = - \frac{\partial (\partial_{p_i} e(\mathbf{p}, \underline{X}) / \underline{X})}{\partial (p_i/P)} \frac{p_i/P}{\partial_{p_i} e(\mathbf{p}, \underline{X}) / \underline{X}}. \quad (48)$$

By differentiating  $\partial_{p_i} e(\mathbf{p}, \underline{X}) / \underline{X}$  and  $p_i/P$  with respect to  $p_i$ , we obtain

$$s_i = \frac{-\partial_{p_i p_i}^2 e(\mathbf{p}, \underline{X}) p_i / \partial_{p_i} e(\mathbf{p}, \underline{X})}{1 - \partial_{p_i X}^2 e(\mathbf{p}, \underline{X}) p_i / \partial_X e(\mathbf{p}, \underline{X})} \quad (49)$$

and, by using the definition of the Hicksian demand  $H_i$  in (3), together with the first order condition (2), we finally get

$$s_i = \frac{-\epsilon_{p_i} H_i(\mathbf{p}, \underline{X})}{1 - [\epsilon_i X(\mathbf{x})] [\epsilon_X H_i(\mathbf{p}, \underline{X})]}. \quad (50)$$

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