On the Impact of Innovation on the Marginal Abatement Cost Curve

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Abstract

When modeling the effects of innovation on the marginal abatement cost (MAC) curve, many studies in environmental economics have posited, implicitly or explicitly, a uniform downward shift. The purpose of this paper is to thoroughly investigate this claim in a simple theoretical framework by introducing innovation in the production function of a price-taking, polluting firm in four economically meaningful ways. We establish that the effects of innovation on the MAC curve depend critically on the specific type of innovation, and that only innovation in end-of-pipe technology leads to a *uniform downward* shift of the MAC curve. A second class of results points to the fact that for other types of innovation in the overall production process, the scope for an upward shift of the MAC curve in response to innovation is easier to justify theoretically. These results call for a

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re-appraisal of various results in environmental policy obtained in theoretical work relying on this postulate.

1. Introduction

The marginal abatement cost (MAC) curve is a ubiquitous concept in various strands of the environmental economics literature, ranging from purely theoretical studies to policy-oriented debates. In the process of building theoretical frameworks of analysis, studies dealing with firms' incentives for innovation and/or technology adoption have faced the modeling issue of determining how a given (exogenous) level of innovation would affect the MAC curve. Many such studies have postulated that the proper way to model innovation or technological adoption in a convenient black-box manner is that innovation results in a uniform downward shift of a firm's MAC curve.

The purpose of the present paper is to assess the theoretical scope of validity of this widespread modeling assertion. We consider a stylized model that is sufficiently rich for a succinct treatment of this issue allowing for different notions of innovation. A price-taking firm produces a single output from two inputs available in competitive markets, capital and energy, with the latter being a polluting input. We rigorously derive the firm's MAC curve, or MAC, which associates to every upper bound on the level of emissions the cost of reducing emissions by one additional unit. In other words, the MAC curve captures a firm's burden of responding to a unit tightening of its emissions constraint. Then, using standard comparative statics reasoning, we determine the impact of technological progress on the MAC curve.

On intuitive grounds, this postulate appears reasonable in cases where the firm's output is essentially independent of innovation, as would be the case for instance if production and pollution control were separate activities (e.g., when using scrubbers to control SO_2 emissions). By contrast, as the present paper will confirm, the postulate seems significantly less justified in cases where innovation directly affects the firm's output, following for instance a change in the production process that reduces the use of certain polluting inputs, such as fossil fuels generating emissions of greenhouse gases.

While the diversity of theoretical models used in these studies is such that the level of validity of the postulate is likely to vary substantially across models, it seems fair to say that to the best of our knowledge the related literature has not provided a thorough theoretical justification for this postulate.[1] Furthermore, without referring to specific theoretical models, part of the policy-oriented literature, which uses the MAC as a convenient tool of reasoning, has also typically adopted the postulate that environmental innovation shifts the MAC curve downwards in their argumentation (see Palmer et al., 1995, or Jaffe et al., 2002 in a critique of the Porter hypothesis).

[1]The studies that discuss the postulate at hand include the following: Downing and White (1986), Milliman and Prince (1989), Jung et al. (1996), and Goulder and Mathai (2000), among others. See Jaffe et al. (2002) for a survey.

Since the claim that innovation shifts the MAC curve downwards appears in different theoretical settings as well as in policy discussions without a specified theoretical framework, it is important to allow for a broad variety of ways to model innovation.[2] We propose four of them, two of which are commonly found in some related form in the theoretical environmental economics literature.[3] In the first of these (Type I below), innovation amounts to an increase in the ratio of energy use over emissions.[4] The next two forms of innovation, Type II and Type III below, are standard forms of innovation in industrial organization (broadly known as process innovation/research and development) and in macroeconomics (known as factor-augmenting innovation) respectively, but not in environmental economics. Nonetheless, in the present setting, such innovation forms necessarily encompass an environmental dimension to the extent that they facilitate substitution away from polluting factors such as fossil fuels. This, along with our goal of a comprehensive investigation, motivates their inclusion in our analysis.

For Type I-III innovation, a firm can reduce pollution only by reducing output or by changing its input mix (i.e., substituting capital for energy). For the fourth form of innovation (type IV below), any excess emissions beyond the limit imposed will get abated, with the objective of the firm reflecting abatement costs explicitly. Innovation is then postulated to correspond to a uniform (parametric) shift in the abatement cost function. This approach thus captures end-of-pipe environmental innovation. This model allows the firm the option of separating the choice of cleaning up its pollution from the choice of its input mix and level of production, while keeping input substitution and variability in the firm's output.

In all four cases, the level of innovation or technology adoption is taken to be exogenous for the firm. This is fully in line with our primary motivation, which is of a purely modeling nature and consists of assessing the effects of innovation on the MAC curve. In other words, our main concern here is to determine the extent of validity of the black box manner in which innovation is often modeled in the environmental literature: as a downward shift in the MAC curve. A clear understanding of this issue is obviously a pre-requisite for other studies that would deal with the incentives firms have for (endogenous) innovation or technology adoption levels in various market settings.

The postulate under consideration here is the direct analog of the way innovation or technological progress is generally modeled in microeconomic theory: As downward shifts in the production cost function as well as in the corresponding marginal cost function (see e.g., Brander and Spencer, 1983).

[2]Since innovation is taken to be exogenous to the firm throughout, it would be more appropriate to refer to it as technology adoption. With this point clarified, we will use the two notions interchangeably.

[3]We also point out that there are several notions of innovation/technology adoption that appear in the literature, with some of the policy-oriented papers leaving unspecified what is meant by innovation.

[4]While this ratio is taken to be constant in the paper for simplicity, a generalization to any monotonic and concave relationship between energy use and emissions is equally valid.

We prove below that the postulate of downward shift is fully justified in the case of ordinary cost functions, thus vindicating how industrial economists have been modeling process research and development.

More specifically, we derive three classes of results. The first is that Type I innovation will always shift the MAC curve downwards for lax emissions constraints, but upwards for tight emissions constraints. In addition, for a Cobb-Douglas production function, it causes a clockwise rotation of the MAC curve about a point on its graph, a property that need not extend to other production functions.[5] Thus, this type of innovation cannot possibly lead to a uniform downward shift of the MAC curve. The second result is that for two types of innovation in the overall production process, while a uniform downward shift of the MAC curve is theoretically possible, it entails a very strong substitutability property on the interaction between the two factors of production (energy and capital), which is a rather restrictive assumption in the present context. By contrast, for these types of innovation, a uniform upward shift of the MAC curve entails only a complementarity assumption on the factors of production, which constitutes a rather natural assumption in this context. The third result is that end-of-pipe innovation (Type IV) does indeed lead to a downward shift in the MAC curve as commonly postulated in the literature.

The present analysis thus reveals a mixed picture for a theoretical foundation in support of this common assertion in environmental economics. One consequence is that both theoretical and policy economists should always be more specific as to the innovation form they have in mind when discussing pollution control. This also raises the possibility that this presumption of a downward shift of the MAC curve most likely originated from an unfortunate direct analogy with the effects of process innovation on ordinary cost functions in industrial organization.

Two recent studies, directly related to the present paper, also share our main motivation of drawing attention to the lack of theoretical justification for the postulate at hand. Bauman et al. (2008) construct a one-input model and consider various forms of innovation and their effects on the MAC curve. Baker et al. (2007) provide an extensive review of the literature that relies on some version of the postulate at hand, and a convenient classification of the underlying models into two broad classes defined in part according to whether innovation is postulated to shift the MAC curve down or not.[6] Based on numerical simulations applied to alternative real-life examples, the authors also show that innovation can lead to an upward shift of the MAC curve. While leading to similar results to those obtained by these studies,

[5]In other words, the postinnovation MAC curve may intersect the old one at several points instead of just one, so that one cannot strictly speak of a rotation.

[6]Studies not covered by Baker et al. (2007) include Requate and Unold (2003) and the survey by Jaffe et al. (2002), who note on p. 54 : "Because technology diffusion presumably lowers the aggregate marginal cost function, it results in a change in the efficient level of control."

our framework is somewhat more general in that it allows for general two-dimensional production and abatement cost functions, and our arguments are fully analytical.

Both these studies go on to argue in some detail that the manner in which innovation is modeled to affect the MAC curve plays a decisive role in determining the nature of the results in the literature dealing with the comparative merits of different regulatory instruments in providing firms with incentives to innovate in environmental technology. Hence a proper understanding of the impact of innovation on the MAC is a pre-requisite to a sound analysis of the policy issues where the use of the MAC as an analytical tool is widespread. We refer the reader to the extensive treatment of this important issue offered by these two studies.

This paper is organised as follows. In Section 2, we present our general framework and show how to derive the MAC curve. Sections 3–4 contain our results on the effects of innovation on the MAC curve. Section 5 contains an illustrative example using a Cobb-Douglas specification. Section 6 provides some concluding remarks.

2. The Marginal Abatement Cost Curve

In this section, we present the simple framework used throughout this paper, postulating a price-taking single-output profit-maximizing firm that is subject to environmental regulation in the form of a binding emissions constraint. As the two-input production function is taken to be of a general form, satisfying only the standard properties of production theory, we also summarize general results about the direct impact of the emissions constraint on the behavior of the firm. For the sake of a self-contained presentation, we also derive elementary properties of MAC curves.

2.1. The General Framework

Consider a profit-maximizing firm producing a single output y by means of two inputs, x, which represents energy, and k, which stands for capital (or a bundle of all non-energy inputs). Denote the firm's production function by f, so that

$$y = f(k, x). \tag{1}$$

The use of energy in the production process generates pollution in the form of emissions e, the level of which we postulate to be linearly related to the amount of energy used by the firm[7], i.e., with a being a positive parameter,

$$e = \frac{x}{a}. \tag{2}$$

[7]Our results are actually robust to a general specification of the relationship between energy usage and emissions level, so we elect this common specification only for convenience. Among others, Klepper and Peterson (2006) use the same formulation with x as fossil fuel and e as CO_2 emissions.

We assume throughout that f satisfies the standard properties of a neoclassical production function, as reported in the following assumptions.[8]

ASSUMPTION A: *The function* $f : R_+^2 \longrightarrow R_+$ *satisfies the following conditions:*

(i) $f(0, 0) = 0$.

(ii) *f is twice continuously differentiable.*

(iii) $f_1(k, x) > 0$ *and* $f_2(k, x) > 0$, *for all k,x.*

(iv) $f_2(k, 0) = +\infty$, *for all* $k > 0$.

(v) *f is differentiably strictly concave jointly in (k,x), i.e.*

$$f_{11} < 0,\ f_{22} < 0,\ \text{and}\ f_{11} f_{22} - f_{12}^2 > 0.$$

All components of Assumption A are standard in production theory and have well known economic interpretations. Furthermore, under Assumption A, the firm's profit maximization problem always has a unique optimal solution and the Implicit Function Theorem may be applied to the first-order conditions, as we shall do repeatedly for the purpose of comparative statics. At this point, these assumptions do not include specific restrictions on the sign of the cross partial $f_{12}(k, x)$, so that *a priori* the two inputs may be complements (if $f_{12}(k, x) \geq 0$) or substitutes (if $f_{12}(k, x) \leq 0$).

Since we do not restrict $f(k, 0)$ to be zero for $k > 0$, positive production is allowed under zero emissions in this model. This will obviously not be the case for the convenient Cobb-Douglas illustration covered in Section 3.5.

We shall consider four different types of innovation. The first, to be referred to as Type I innovation, corresponds to technological progress that has the property of reducing the emissions/input ratio (e.g., a technology that reduces carbon emissions by using natural gas instead of coal). This innovation is simply modeled as an increase in the parameter a.

The second, to be referred to as Type II innovation, is the familiar increase in overall production efficiency. It is generally modeled as a uniform upward shift in the production function and is broadly referred to as process innovation in industrial organization.

The third form of innovation (or type III) considered in this paper is factor-augmenting for capital, a form commonly used in the macroeconomic literature on innovation and growth (Aghion and Howitt, 1997).

The fourth form of innovation (or type IV) is modeled as a decrease in the unit cost of abatement, thus capturing end-of-pipe technological progress, in

[8]Throughout the paper, partial differentiation will be denoted by a subscript corresponding to the relevant variable (e.g $f_1 = \partial f/\partial k,\ f_{11} = \partial^2 f/\partial k^2,\ f_2 = \partial f/\partial x, f_{12} = \partial^2\ f/\partial k \partial x, \ldots$).

a model allowing for endogenous levels of abatement. As this requires an extension to the basic model, it is covered separately in Section 4.

While Types I and IV are standard forms of innovation in the environmental economics literature, Types II and III are not. They are included for the sake of completeness, as they indirectly embed technological progress of an environmental nature. As these various forms of innovation impact the firm's production process in different ways, there is no *a priori* reason to expect that they will have the same effect on the *MAC* curve in the present setting. In fact, we show below that Types I essentially leads to a clockwise rotation of the MAC curve (with at least one, but possibly many, points remaining invariant), Types II and III imply an upward shift in the MAC curve under some minor restrictions, and Type IV results in a downward shift in the MAC curve.[9] Thus, the last result is the only one that fully confirms the common postulate of a downward jump in the MAC curve.

2.2. Derivation and Structure of the MAC Curve

In order to derive the MAC curve, we follow the standard technique (see e.g., Montgomery, 1972 or McKitrick, 1999) consisting of computing the total abatement costs by subtracting the optimal profit level under a given binding constraint on emissions from the optimal profit level at the laissez-faire outcome (with no constraint on emissions).[10] To this end, we first characterise the unconstrained situation, i.e., the baseline. We may, in view of (2), think of the firm as choosing emissions instead of energy. With the output price normalized to unity and r and q denoting the market factor prices for capital and energy, respectively, the profit maximization problem of the firm is

$$\max_{k,e\geq 0} \ \Pi(k, e) = f(k, ae) - rk - qae. \tag{3}$$

In view of Assumption A, there is a unique optimal solution (k^*, e^*) satisfying

$$f_1(k^*, ae^*) - r = 0 \tag{4}$$

$$f_2(k^*, ae^*) - q = 0. \tag{5}$$

Denote the corresponding optimal profit, our baseline, by $\Pi^* = \Pi(k^*, e^*)$.

Consider the problem of the firm when its level of emissions is constrained by

$$e \leq \hat{e}.$$

[9]Throughout this paper, a downward (resp., upward) shift of a curve will mean that the curve weakly decreases (resp., increases) at every point of its domain, and strictly decreases (resp., increases) for some points of its domain.

[10]Klepper and Peterson (2006) derive a MAC curve at the micro or firm level as well as at the macro level. Their aim is to investigate the effects of the (exogenous) world energy price. Van Soest, List, and Jeppesen (2006) develop a method for quantifying the stringency of environmental regulation.

In order to avoid the uninteresting case where environmental regulation does not affect the firm's behavior, we make the following assumption throughout.

ASSUMPTION B: *The emissions constraint is binding on the firm, i.e.,* $\hat{e} < e^*$.

The corresponding (constrained) levels of energy and capital use are

$$\hat{x} = a\hat{e}$$

and, noting that the constrained-optimal k will depend only on the product $a\hat{e}$,

$$k(a\hat{e}) = \arg\max_k \Pi(k, \hat{e}) = \arg\max_k \{f(k, a\hat{e}) - rk - qa\hat{e}\}.$$

By Assumption A, $k(a\hat{e})$ is implicitly defined as a single-valued function by

$$f_1(k(a\hat{e}), a\hat{e}) - r = 0. \tag{6}$$

The corresponding constrained profit level is thus

$$\Pi(k(a\hat{e}), \hat{e}) = f(k(a\hat{e}), a\hat{e}) - rk(a\hat{e}) - qa\hat{e}. \tag{7}$$

Following common practice, we define the Total Abatement Cost (TAC) curve as the difference between constrained and baseline profits as the emission constraint varies:

$$TAC(\hat{e}) = \Pi(k(a\hat{e}), \hat{e}) - \Pi(k^*, e^*). \tag{8}$$

Observe that the TAC is negative-valued, with a maximal value of 0 at $e = e^*$ (see e.g., McKitrick, 1999). The MAC curve is defined as the total derivative of the TAC curve with respect to the level of the constraint $\hat{e}$, or as $\Pi\ (k^*, e^*)$ is constant,

$$MAC(\hat{e}) \triangleq \frac{d\Pi(k(a\hat{e}), \hat{e})}{d\hat{e}}. \tag{9}$$

The common effective domain of the functions TAC and MAC is the interval $[0, e^*]$. Nonetheless, we may view their domain as being all of $[0, \infty)$ upon observing that by definition, $TAC(\hat{e}) = MAC(\hat{e}) = 0$, for all $\hat{e} \geq e^*$. Indeed, if the emissions constraint is non-binding, the firm incurs no cost in complying with $\hat{e}$ since it will just use e^*.

We now derive some basic properties of the MAC curve that will be useful later.

Taking the derivative on the RHS of (9) yields

$$MAC = af_1(k(a\hat{e}), a\hat{e})k'(a\hat{e}) + af_2(k(a\hat{e}), a\hat{e}) - rak'(a\hat{e}) - qa.$$

Using the first-order condition (6), this reduces to

$$MAC = a[f_2(k(a\hat{e}), a\hat{e}) - q]. \tag{10}$$

From the first-order condition (5), Assumptions A and B, it follows that MAC is a nonnegative function of $\hat{e}$ on $[0, e^*]$.

Next, to determine how the MAC curve behaves when $\hat{e}$ changes, we first need to evaluate $k'(a\hat{e})$. To this end, differentiate (6) and collect terms to obtain

$$k'(a\hat{e}) = -\frac{f_{21}(k(a\hat{e}), a\hat{e})}{f_{11}(k(a\hat{e}), a\hat{e})}. \tag{11}$$

Then taking the total derivative in (10) with respect to $\hat{e}$

$$\begin{aligned}\frac{dMAC}{d\hat{e}} &= a[af_{21}(k(a\hat{e}), a\hat{e})k'(a\hat{e}) + af_{22}(ak(a\hat{e}), a\hat{e})] \\ &= a^2\left[-\frac{f_{21}^2(k(a\hat{e}), a\hat{e})}{f_{11}(k(a\hat{e}), a\hat{e})} + f_{22}(k(a\hat{e}), a\hat{e})\right] \text{by (11)} \\ &< 0 \text{ by Assumption } A(v).\end{aligned}$$

Hence, as $\hat{e}$ increases in $[0, e^*]$, MAC is globally decreasing from its maximal value $MAC(0)$ to $MAC(e^*) = 0$.

In words, $MAC(\hat{e})$ stands for the marginal foregone profit by the firm when responding (by re-adjusting its capital and output levels) to a unit decrease of the emissions limit, starting from $\hat{e}$. An alternative way of describing it is as the incremental cost for the firm of being subject to a unit tightening of the emissions constraint. It follows that the proper way of reading the graph of the MAC curve would proceed backwards along the horizontal axis so as to see marginal cost increasing as $\hat{e}$ declines.

For an unambiguous understanding of the effects of imposing or tightening the emissions constraint on the firm in the present general setting, we provide a full summary of the associated results. Note that all the proofs of this paper are given in the Appendix.

PROPOSITION 1: *Under Assumptions A and B, a decrease in the value of $\hat{e}$ will lead to:*

(a) *a decrease (resp. increase) in the constrained-optimal level of capital $k(a\hat{e})$ if $f_{21}(k(a\hat{e}), a\hat{e}) \geq$ (resp. $\leq$)0.*

(b) *a decrease (resp. increase) in the optimal level of output if*

$$f_1(k(a\hat{e}), a\hat{e})f_{21}(k(a\hat{e}), a\hat{e}) - f_2(k(a\hat{e}), a\hat{e}) \times f_{11}(k(a\hat{e}), a\hat{e}) \geq \text{(resp.)} \leq 0.$$

(c) *a decrease in the firm's optimal profit.*

Thus, when constrained to reduce emissions, a firm will reduce its use of capital whenever energy and capital are complements but will increase its demand for capital if the two inputs are substitutes. While this finding is fully intuitive, the fact that the firm need not lower its output production level is perhaps less so. The interpretation of the condition given in Proposition 1(b) will be discussed in the next section. For now, we simply observe that in case

f_{21} is strongly <0, as $\hat{e}$ falls, capital may be substituted for energy at such a high rate as to cause final output to go up. Finally, with Part (c) being obvious, we observe that the TAC curve measures the costs—in the form of foregone profits—associated with both the underlying capital and output (upward or downward) re-adjustment processes by the firm.

3. Impact of Innovation on the MAC Curve

This section investigates, in this emissions-constrained setting, the impact of three different types of innovation on the MAC curve. We begin with one of the two innovation forms most closely associated with abatement technology, namely an increase in the energy-emissions ratio.

3.1. Effects of Type I Innovation

Here we investigate the effects on the MAC curve of any environmental innovation that takes the form of an increase in the constant a. This would for instance amount to substituting a cleaner input for a polluting one, a typical example of which is fuel switching. In view of the central role played by this form of innovation and by Type IV below, the associated results clearly form the main part of this paper.[11] We derive a general result that covers the class of production functions used in neo-classical production theory (see Assumption A), allowing for input substitution and output changes.

PROPOSITION 2: *Under Assumptions A and B, any environmental innovation that increases a causes*

- *(i)* *the effective domain of the MAC curve,* $[0, e^*]$, *to shrink at the upper end, i.e.,* $de^*/da < 0$, *and*
- *(ii)* *the MAC curve to remain unchanged for at least one* $\hat{e}_0 \in (0, e^*)$, *shift up for* $\hat{e}$ *sufficiently close to 0 and shift down for* $\hat{e}$ *sufficiently close to* e^*.

Thus the impact of an innovation in abatement technology on the *MAC* curve will always take the form of a nonuniform shift. At this level of generality, all that can be said is that the shift is upwards for very stringent emissions constraints (i.e., low enough $\hat{e}$) and downwards for very lax emissions constraints (i.e., high enough $\hat{e}$). For intermediate values of $\hat{e}$, either outcome may prevail. However, for the specific but central case of a Cobb-Douglas production function (see Section 4), we will see that there is a unique point $\hat{e}_0 \in (0, e^*)$ with the property that the MAC curve shifts up for $\hat{e} \in [0, \hat{e}_0)$ and down for $\hat{e} \in (\hat{e}_0, e^*]$.[12]

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[11] The analysis of Type IV innovation allows for an endogenous level of emissions abatement and thus requires a significant extension of the model, whence its treatment in a separate section.

[12] We now explain why this finding for the Cobb-Douglas specification does not necessarily extend to the general case. Referring to (21), observe that for the effect to be a rotation

It is easy to see that the two effects reported in Proposition 2(i)–(ii) are consistent with each other[13]. More precisely, given the conclusion of Part (ii), Part (i) becomes a necessary implication, which does indeed hold, as shown.

The economic intuition behind Proposition 2 is as follows. Equation (10) indicates that the MAC is the product of two terms. An increase in a has opposite effects on these two terms. On the one hand, given $\hat{e}$, an increase in a translates into the opportunity to consume more energy, and thus to produce more output (as $d\hat{x} = \hat{e}\,da$ follows from $\hat{x} = \hat{e}a$). Hence, the marginal productivity of energy decreases since f is concave, and the term in brackets in (10) also decreases. Other things equal, this effect lowers the MAC. On the other hand, a second effect follows from the fact that energy use engenders less pollution upon an innovation. In other words, given a, a higher cutback on energy is needed to reach a given level of reduction in emissions. Thus a one unit reduction in the limit $\hat{e}$ is more binding on energy consumption if a is higher (as $d\hat{x} = a d\hat{e}$). This second effect, captured by the first term in (10), leads to an upward shift in the MAC curve. The overall outcome balances these two effects out, resulting in the non-uniform change described in the result.

A uniform downward shift, as posited in the literature, makes the MAC curve a convenient tool for a graphical analysis of the effects of innovation. This is often the way policy discussions on the effects of innovation and associated policy measures are discussed, as for instance in Palmer et al. (1995). While a rotation (around a unique pivot point) would lower the attractiveness of the MAC concept, reasoning on the basis of stringent (i.e., upward shift of the MAC) versus lax (i.e., downward shift of the MAC) emissions standards remains possible. However, the presence of multiple crossing points between the pre and postinnovation MAC curves would suppress even the latter possibility, and would make such policy analysis much more complicated.

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about a unique point $\hat{e}_0$, the following condition is needed (see proof of Proposition 2 in Appendix): $\frac{d^2 MAC}{da^2}\Big|_{\frac{dMAC}{da}=0} < 0$. A long computation (using (18) and (10)) shows that

$$\frac{d^2 MAC}{da^2}\bigg|_{\frac{dMAC}{da}=0} = \frac{2}{a\hat{e}}[f_2 - q] + \frac{a\hat{e}}{f_{11}^2}\left[-\frac{f_{21}^3}{f_{11}} f_{111} + f_{11}^2 f_{222} - 3 f_{21} f_{212} f_{11} + \frac{f_{21}^2}{f_{11}} f_{211}(f_{11} + 2)\right]$$

As this expression involves first, second, and third derivatives of f, and since no standard restrictions on the latter are generally postulated in production theory, there is no reason to expect the sign at hand to be globally negative.

We conclude that the pre and the postinnovation MAC curves may, *a priori*, intersect several times, although we do not have an explicit closed-form example to that effect.

[13]An alternative way to think about the shorter effective domain is that if $\hat{e}$ is only slightly less than e^*, an innovation that increases a will make x^* feasible, so that the emissions constraint becomes non-binding and the firm thus incurs no cost in complying with it.

3.2. Effects of Type II Innovation

In this subsection, we investigate the effects on the MAC curve of any general innovation that takes the form of a uniform shift in the production function. While such an innovation has not been specifically considered in the environmental economics literature, it may be viewed as incorporating an environmental dimension in that the same pre-innovation output level can be produced with lower emissions (and the same capital) after the innovation. We will return to this point at the end of Section 3.4.

For this type of innovation, the production function can be written as $\alpha f(k, ae)$, with the preinnovation situation corresponding to $\alpha = 1$ and the post-innovation situation to $\alpha > 1$.

The corresponding constrained profit level is thus

$$\Pi(k(\hat{e}), \hat{e}) = \alpha f(k(\hat{e}), a\hat{e}) - rk(\hat{e}) - qa\hat{e}.$$

With the MAC curve defined by (9), our aim here is to study the effects of an increase in α, starting from a value of 1. Our result for this type of innovation provides respective necessary and sufficient conditions for an increase in α to lead to local upward and downward shifts of the MAC curve (i.e. around a particular point). It follows from this characterization that there will be a global shift upwards as long as energy and capital are complements in production or the innovation is sufficiently drastic.

PROPOSITION 3: *Under Assumptions A-B, any innovation that increases α will cause*

(a) *the MAC curve to shift up locally at a point $\hat{e} \in [0, e^*]$ if and only if*

$$f_1(k(\hat{e}, \alpha), a\hat{e}) f_{21}(k(\hat{e}, \alpha), a\hat{e}) - f_2(k(\hat{e}, \alpha), a\hat{e}) f_{11}(k(\hat{e}, \alpha), a\hat{e}) \geq 0. \tag{12}$$

(b) *the MAC curve to shift up globally and its effective domain $[0, e^*]$ to expand if either of the following conditions hold:*
   (i) $f_1 f_{21} - f_2 f_{11} \geq 0$ *for all $k, x \geq 0$, or*
   (ii) *α is sufficiently large.*

We now provide a detailed discussion and economic interpretation of Condition (12) or the property that $f_1 f_{21} - f_2 f_{11} \geq 0$. This is clearly equivalent to the property that

$$\text{the technical rate of substitution} \frac{f_2(k, x)}{f_1(k, x)} \text{is increasing in } k \tag{13}$$

and is implied by the property that the two inputs are (weak) complements in production, i.e., $f_{21} \geq 0$, or the marginal product of capital increases with higher levels of energy use, which is quite a natural property in the present setting. Nonetheless, we refrain from imposing this plausible restriction *a priori* since our ultimate aim is to investigate the effects of environmental innovation on the curve in the fullest generality possible. Thus allowing for the possibility that f_{21} might be <0, *at least locally*, is desirable.

There are two other inter-related perspectives that shed further light on the meaning of the property (13), which may be summarized as follows. The first of these is that, as shown in Proposition 1(b), (13) is the minimally sufficient condition that guarantees that the firm's output will fall as a consequence of a tighter limit on allowable emissions. Thus the essence of Proposition 3 is that for production processes that have the property that tightening environmental regulation lowers output, a process innovation (or a higher α) will always result in an upward shift of the MAC curve. In view of the fact that much of the policy-oriented debate about the industrial effects of pollution control seem to often take for granted that emissions limits do indeed lead to lower production levels by firms, the assumption validating an upward shift in the MAC curve seems to be widely accepted by policy makers, at least tacitly.

The second perspective is that (13) is also the minimally sufficient condition that ensures that the firm will demand more of the energy input when the price of the output goes up in the firm's profit maximization problem with unconstrained emissions. (This fact is easily proved with the standard techniques used in this paper, and is thus left to the interested reader.) As the firm always chooses to produce more output when the output price goes up, (13) may be viewed as ensuring that energy is a normal – in the sense of non-inferior – input in production.

The economic intuition is the following. As is easily verified, an increase in α always leads to an increase in the output produced by the constrained firm. Since (13) implies that energy is a normal good for the unconstrained firm, innovation will then induce the firm to ideally demand more energy, and the ensuing tightening of the emissions constraint will induce an upward shift in the MAC curve for reasons similar to those behind the second effect described following Proposition 2.

Bauman et al. (2008) also addresses the effects of this type of innovation on the MAC curve. However, in their treatment, they also assume that the firm is not a price-taker in the output market, but rather a monopolist facing a downward-sloping demand (in the firm's output). In such a framework, similar results as ours can be obtained, but it is also possible for innovation to lead to a higher MAC curve at low levels of abatement and to a lower curve at high levels of abatement (in other words, to a counter-clockwise rotation of the MAC curve around a unique pivoting point). This last result is only possible in a price-maker setting, i.e., it disappears under a price-taking assumption.

3.3. On Modeling Standard Process R&D

Since Type II innovation corresponds precisely to what is commonly referred to as process R&D in the industrial organization literature, it is worthwhile to confirm that, in contrast to the present setting, the standard way of modeling process R&D in industrial organization, as downward shifts in both the firm's cost and marginal cost functions (see e.g., Brander and Spencer, 1983), is justified in full generality.

Indeed, consider a firm with a production function f using an n -dimensional input vector z at (constant) input price vector w. With the preinnovation cost function defined as usual as $C(y) \triangleq \min w.z$ subject to $y = f(z)$, the postinnovation cost function will be

$$\begin{aligned}\hat{C}(y) &= \min w.z \text{ subject to } y = \alpha f(z)\\ &= \min w.z \text{ subject to } y/\alpha = f(z)\\ &= C(y/\alpha)\end{aligned}$$

As $\alpha > 1$, $C(y/\alpha)$ is indeed a downward shift of $C(y)$, with the marginal cost function $MC(y) = \frac{1}{\alpha} C'(y/\alpha)$ having also shifted down relative to its preinnovation level $C'(y)$.

It is clear that the concepts of MAC curve and the standard marginal cost curve are defined in fundamentally different ways. It is worth noting that, if the MAC were defined at constant output, i.e., without allowing the firm to re-adjust its input mix and its output, then the postulate of a downward shift in response to innovation would generally be justified.[14]

3.4. Effects of Type III Innovation

Here we investigate the effects on the MAC curve of a capital-augmenting innovation. Again, although such an innovation has not been specifically considered in the environmental economics literature, it may be viewed as incorporating an environmental dimension in that the same preinnovation output level can be produced with lower emissions (and the same capital) after the innovation (more on this point at the end of this subsection).

In this case, the preinnovation and the postinnovation production functions can be written as $f(k, ae)$ and $f(Ak, ae)$, respectively, with $A > 1$. The corresponding constrained profit level is thus

$$\Pi(k(\hat{e}), \hat{e}) = f(Ak(\hat{e}), a\hat{e}) - rk(\hat{e}) - qa\hat{e}.$$

With the MAC curve still defined by (9), our aim here is to study the effects of an increase in A, starting from a value of 1, on the MAC curve. Our result for this type of innovation shows that an increase in A leads to a global upward

[14]A study by Goulder et al. (1999) reports that 98% of NO_x emissions reductions come from firms reducing emissions per unit of output and only 2% from reductions in industry output. This would indicate that the theoretical effects that drive our conclusions may have quite a limited impact in some specific real-life settings.

(resp. downward) shift of the MAC curve if and only if energy and capital are complements (resp. substitutes) in production at the optimal input choice.

PROPOSITION 4: *Under Assumptions A and B, any general innovation that increases A will cause the MAC curve to shift upwards [resp. downwards] globally if and only if* $f_{21}(Ak(\hat{e}), a\hat{e}) \geq 0$ [*resp.* ≤ 0], *at the firm's constrained optimal choice.*

It is quite plausible to postulate that capital and energy constitute complimentary inputs in the production process for most settings that would be realistically captured by our framework. If so, the impact of a capital-augmenting innovation on the *MAC* curve will be an upward shift. Nonetheless, since we do not wish to exclude the possibility that the two inputs may be substitutes over a limited range of operations, we cannot exclude a downward shift of the *MAC* curve following such an innovation.

The economic intuition for this result is quite similar to that of Proposition 3. An increase in A implies an increase in output, which implies a higher demand for energy by the firm given that the two inputs are complements, which in turn induces an upward shift in the MAC curve for reasons similar to those described following Proposition 3.

As the adoption of the type II or type III innovation will typically lead to an upward shift of the MAC curve, such innovation will lead to an increase in total abatement costs. It is then natural to ask whether it is plausible for an environmentally regulated firm to adopt such innovations.[15] We now argue that the answer is affirmative, provided the adoption cost is low enough. Indeed, the decision to adopt or not the new technology should be based on comparing the change in profits, which is clearly positive despite the increase in total abatement costs, and the cost of adopting the new technology.

Let $\hat{\Pi} = \Pi(k(a\hat{e}), \hat{e})$ and $\Pi^* = \Pi\ (k^*, e^*)$ be the profit of the firm with and without a constraint on emissions, respectively. By definition, the total abatement cost is $TAC = \hat{\Pi} - \Pi^*(< 0)$ (recall (8)). Denote these quantities before and after the adoption of the innovation by subscripts 0 and 1, respectively and let I be the adoption cost.

It is in the interest of the firm to adopt the innovation iff :

$$\begin{aligned} \hat{\Pi}_1 - I > \ & \hat{\Pi}_0 \\ & \Leftrightarrow \Pi_1^* + TAC_1 - I > \Pi_0^* + TAC_0 \\ & \Leftrightarrow \Pi_1^* - \Pi_0^* > TAC_0 - TAC_1 + I \end{aligned} \tag{14}$$

Hence the firm will adopt the innovation if the resulting increase in profits is larger than the sum of the adoption cost and the increase in total abatement costs.

[15]We are indebted to a referee for having drawn our attention to this issue.

In the special case where the innovation has no impact on the unconstrained profit ($\Pi_1^* = \Pi_0^*$), then (14) reduces to

$$TAC_1 - TAC_0 > I \tag{15}$$

and the firm would adopt the new technology if the gain in abatement costs were larger than the adoption cost. However, for any $\hat{e}$, type II and type III innovations lead to $TAC_1 < TAC_0 (<0)$. Hence the criterion (15) for the adoption of the new technology is not the appropriate one. Since type II or type III innovations do have a positive impact on the unconstrained profit ($\Pi_1^* > \Pi_0^*$), one must rely on criterion (14) to determine the adoption decision.

It is also important to notice that type II and type III innovations could be adopted in the absence of any environmental policy (i.e., for $\hat{e} \geq e^*$). In such a case, $\Pi^* = \hat{\Pi}$ (thus $TAC = 0$) and the criterion (15) for the adoption of the new technology reduces to $\Pi_1^* - \Pi_0^* > I$. However, type I innovation will never be adopted in the absence of any environmental policy because the unconstrained profit is not affected by such an innovation ($\Pi_1^* = \Pi_0^*$). Hence, type I innovation, as opposed to type II and type III innovations, is specifically an environmental one.[16]

3.5. A Cobb-Douglas Example

This section contains an illustrative example of our general analysis, using a Cobb-Douglas production function. In addition to the concrete insights that a closed-form example provides, there will be an added benefit here in that the conclusion of Proposition 2 will be strengthened. Unlike our general model though, this specification imposes that there be no possible production with zero emissions.[17]

Consider the production function $y = Ak^{\gamma} x^{\beta}$, where A, γ and β are positive parameters with $\gamma + \beta < 1$ so that production has decreasing returns to scale. Let $e = \frac{x}{a}$.

We consider only two types of innovation, Type I and Type II, as Type II and Type III innovations are equivalent with a Cobb-Douglas production function.

We first characterise the unconstrained problem (the baseline)

$$\max_{k,x \geq 0} \Pi(k, x) = Ak^{\gamma} x^{\beta} - rk - qx$$

[16]The same comment applies to the fourth type of innovation, namely the ‘end-of-pipe’ innovation, that is tackled in Section 4.

[17]Baker and Shittu (2006) use a CES production function in a different setting. This would allow the study of the effect of the elasticity of substitution parameter.

The solution is

$$k^* = \left[\frac{\gamma A x^{\beta}}{r}\right]^{\frac{1}{1-\gamma}} \quad \text{and} \quad x^* = ae^* = \left[\frac{\beta A^{\frac{1}{1-\gamma}}\left[\frac{\gamma}{r}\right]^{\frac{\gamma}{1-\gamma}}}{q}\right]^{\frac{1-\gamma}{1-[\gamma+\beta]}}.$$

Let us now consider the problem of the firm when its level of emissions is constrained in a binding way by $e \leq \hat{e} < e^*$. Upon maximizing with respect to k, we have

$$\Pi(k(a\hat{e}), \hat{e}) = [1-\gamma]\, A^{\frac{1}{1-\gamma}} \left[\frac{\gamma}{r}\right]^{\frac{\gamma}{1-\gamma}} (a\hat{e})^{\frac{\beta}{1-\gamma}} - qa\hat{e}$$

The MAC curve is then

$$MAC = \frac{\partial \Pi(k(a\hat{e}), \hat{e})}{\partial \hat{e}} = \beta A^{\frac{1}{1-\gamma}} \left[\frac{\gamma}{r}\right]^{\frac{\gamma}{1-\gamma}} a\,(a\hat{e})^{\frac{\gamma+\beta-1}{1-\gamma}} - qa \tag{16}$$

To see how the MAC function reacts to changes in a, consider

$$\frac{\partial MAC}{\partial a} = \frac{\beta}{1-\gamma} \beta A^{\frac{1}{1-\gamma}} \left[\frac{\gamma}{r}\right]^{\frac{\gamma}{1-\gamma}} (a\hat{e})^{\frac{\gamma+\beta-1}{1-\gamma}} - q \tag{17}$$

Using the first order condition for the unconstrained maximization of Π (k, ae) with respect to e, together with Assumption A and the fact that $\hat{e} < e^*$, we conclude that

$$\beta A^{\frac{1}{1-\gamma}} \left[\frac{\gamma}{r}\right]^{\frac{\gamma}{1-\gamma}} (a\hat{e})^{\frac{\gamma+\beta-1}{1-\gamma}} > q. \tag{18}$$

Since $\frac{\beta}{1-\gamma} < 1$ because of decreasing returns of scale, it follows from (18) that $\partial\, MAC/\partial\, a$ can be positive or negative. In the left neighbourhood of e^*, MAC is close to 0, so that $\beta A^{\frac{1}{1-\gamma}}[\frac{\gamma}{r}]^{\frac{\gamma}{1-\gamma}}(a\hat{e})^{\frac{\gamma+\beta-1}{1-\gamma}} \approx q$, which gives $\partial\, MAC/\partial\, a < 0$. This is consistent with the fact that $\partial\, e^*/\partial\, a > 0$ (see Proposition 2). Moreover, as $\hat{e} \downarrow 0$, $\partial MAC/\partial a$ tends to infinity (since the exponent of $\hat{e}$ in (18) is <0). So for $\hat{e}$ sufficiently small, $\partial\, MAC/\partial\, a$ must be positive. Since the LHS of (18) is decreasing in $\hat{e}$, there must be one and only one value of $\hat{e}$, call it e_0, at which $\partial\, MAC/\partial\, a = 0$. The way MAC evolves with a is illustrated in Figure 1-1 hereafter (MAC and $MAC\prime$ are the MAC curves before and after the change of a respectively).

As seen in the proof of Proposition 2, the uniqueness of the point e_0 (at which the MAC curve remains invariant to the increase in a) for the Cobb-Douglas example is a property that need not extend to more general productions functions.

Let us now consider an increase in A (Type II or Type III innovation). From (16), it is clear that $\forall \hat{e},\ \partial MAC/\partial A > 0$. Furthermore, it is also clear that $\partial e^*/\partial A > 0$. So an increase in the productivity parameter A translates into an unambiguous upward shift of the MAC curve, along with an extension of its domain. This is depicted in Figure 1-2. Since the Cobb-Douglas production function features complementary inputs, this finding is fully conform with the conclusions of both Propositions 3 and 4.

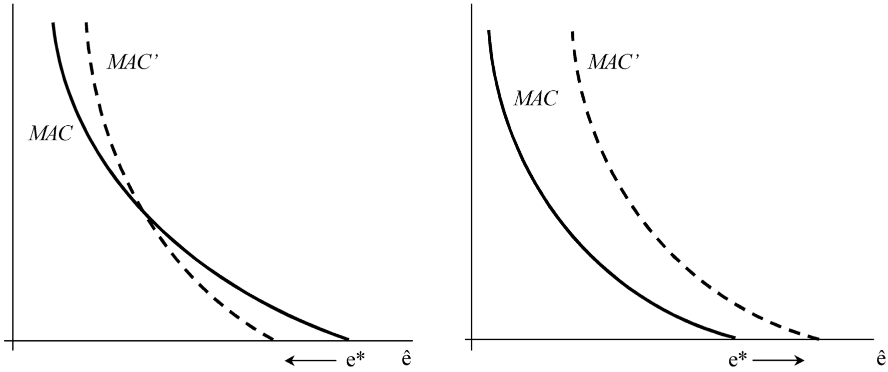

Figure 1: (1) Variation of MAC with a (left); (2) Variation of MAC with A (right).

Using a single-input production function, Bauman et al. (2008) reports similar versions of these results for the innovation forms reported here, in particular a clockwise rotation of the MAC curve for Type 1 innovation. Using mostly CES formulations and numerical simulations, Baker et al. (2007) provide many examples of how innovation impacts the MAC curve, including cases of a clockwise rotational shift such as presented here and cases of a downward shift in the MAC curve.

4. The Case of End-of-pipe Abatement

In this section, the basic model is extended in a natural way to allow for end-of-pipe abatement. As in earlier sections, we investigate the effects of exogenous innovation in this type of abatement technology on the MAC curve. Unlike previous forms of innovation though, the main result in this case vindicates the standard postulate of a downward shift in the MAC curve.

In this setting, the firm may choose any capital-energy combination (k, x), knowing that it has access to an abatement technology that can reduce the resulting pre-abatement or gross emissions level, $e = x/a$, to the permitted emissions limit of $\hat{e}$, whenever needed (i.e., when $x/a > \hat{e}$). The associated cost of abatement is then given by a general function $C(e - \hat{e}, e) = C(x/a - \hat{e}, x/a)$, which depends on both the quantity abated $(e - \hat{e})$ and the initial level of gross emissions (e).

The following assumptions on C are in effect for this section.

Assumption C: *Given a fixed $\hat{e}$, the function $C : [\hat{e}, \infty) \longrightarrow R_+$ satisfies the following conditions:*

(i) $C(0, e) = 0$.

(ii) *C is twice continuously differentiable.*

(iii) $C_1(e - \hat{e}, e) > 0$ *and* $C_2(e - \hat{e}, e) < 0$, *for all* $e > \hat{e}$.

(iv) *$C(e-\hat{e}, e)$ is strictly increasing in e, i.e.* $\frac{dC(e-\hat{e},e)}{de} = C_1(e-\hat{e}, e) + C_2(e-\hat{e}, e) > 0$.

(v) *$C(e-\hat{e}, e)$ is differentiably strictly convex in e, i.e.* $C_{11}(e-\hat{e}, e) + 2C_{12}(e-\hat{e}, e) + C_{22}(e-\hat{e}, e) > 0$.

To provide some economic justification for this set of assumptions, note first that the fact that the cost of abatement depends both on the quantity abated $(e-\hat{e})$ as well as directly on gross emissions (e) makes sense for many polluting processes: Abating say 10 units of emissions out of a gross amount of 20 units should be more costly than abating 10 units out of 100 units.[18] This also justifies the second part of (iii), the first being obvious. Part (iv) says that abatement cost increases faster with the quantity abated than it decreases with the level of gross emissions. Thus, given a fixed $\hat{e}$, higher gross emissions always lead to higher abatement costs, which is a very natural property for such a cost function. Finally, Part(v) says that the rate of increase of abatement costs is increasing in the level of gross emissions, which reflects natural decreasing returns in the abatement technology.[19]

The firm's objective is to maximize its profit function given by

$$\max_{k,x\geq 0} \hat{\Pi}(k, x, \hat{e}) = f(k, x) - rk - qx - \theta C\left(\frac{x}{a} - \hat{e}, \frac{x}{a}\right) \qquad (19)$$

where θ is a shift parameter that decreases with end-of-pipe abatement innovation.

The main result in this section confirms the usual postulate that environmental innovation results in a global downward shift of the MAC curve.

PROPOSITION 5: *Under Assumptions A-C, end-of-pipe abatement innovation will always lead to*

(a) *a higher level of gross emissions e, and*

(b) *a downward shift of the MAC curve.*

In this setting, the firm has the option of continuing to use the optimal input mix (k^*, x^*) corresponding to the unconstrained emissions situation (thus producing the optimal output y^*), and then abating the resulting excess emissions $e^* - \hat{e}$. However, except in degenerate cases, the firm will actually react to the imposition of an emissions limit by changing its input mix and thus also the level of output it produces.

[18]This explicit dependence of abatement costs on the level of gross emissions is often ignored in the literature on end-of-pipe depollution, as for instance in Bauman et al. (2008).

[19]Observe that Part (v) reflects a one-dimensional notion of convexity, which is thus weaker than joint convexity of C in its two arguments. Furthermore, as will be seen below, we need not restrict the sign of the cross-partial derivative C_{12} in the analysis below, other than through the upper limit on its absolute value implied by Assumption C(v).

One should note that Proposition 5 rests on the assumption that the abatement cost curve C (not to be confused with the MAC curve) is shifts downwards uniformly following the adoption of an innovation through a decrease in θ. Some "end-of-pipe" innovations might not satisfy this assumption. On this issue, Baker et al. (2007) take the example of dry and wet scrubber technologies to reduce sulfur dioxide emissions and state that: "Wet scrubbers are more expensive but remove a high amount of SO2, more than 95%. Dry scrubbers are less expensive, but remove only about 80–90% of SO2. Technological change that resulted in less costly dry scrubbers would decrease the marginal cost of abatement levels in the range 80–90%, but would increase the marginal cost of achieving abatement levels in the 95% + range.' (Baker et al., 2007, p.17)." In the present paper, this example can be seen as a non-uniform variation of θ as a function of $\hat{e}$. For instance, we would observe an increase of θ on $\{0, c\}$ and a decrease of θ on $\{c, e\}$ (with $c < e$). In such a case the MAC curve will shift upwards on $\{c, e\}$ as can be seen from the end of the proof of Proposition 5.

5. Conclusion

This paper has established that the postulate that innovation leads to a downward shift of the MAC curve does not have as broad a scope of theoretical validity as often asserted. The approach used introduces exogenous innovation in the production function of a price-taking emissions-constrained polluting firm in four economically meaningful ways, and deriving the effects of increasing innovation on the MAC curve, when the firm is allowed to adjust its output and input mix in response to innovation.

For one form of innovation in abatement technology, the result is that the MAC curve shifts up for low levels of the emissions constraint (i.e., stringent regulation) and down for high levels of the emissions constraint (i.e., lax regulation). For general-process or factor-augmenting innovation, the impact of innovation on the MAC curve is an upward shift provided the energy and capital inputs are complements or mild substitutes in the production process, a reasonable assumption in this setting. Finally, for innovation in end-of-pipe technology, the effects is a downward shift in the MAC curve, in conformity with the standard belief.

These findings call for a re-examination of some of the policy-oriented theoretical results in environmental economics that were derived using frameworks of analysis including the postulate at hand as a building block. This is done in an extensive manner by recent work by Bauman et al. (2008) and Baker et al. (2007), which also recognizes that the prevailing way of modeling the impact of innovation on the MAC curve is often theoretically inadequate.

Given the mixed nature of the results presented here, one should refrain from making general claims about an innovation-induced downward shift of the MAC curve, and rather restrict such claims to end-of-pipe and related innovations in an explicit manner.

Appendix

This appendix provides the proofs of all the results reported in the paper.

Proof of Proposition 1: The proof of Part (a) follows directly from (11) and Assumption *A*.

For Part (b), differentiate the equation $y = f(k(a\hat{e}), a\hat{e})$ with respect to $\hat{e}$ and obtain

$$\frac{\partial y}{\partial \hat{e}} = a[f_1(k(a\hat{e}), a\hat{e})k'(a\hat{e}) + f_2(k(a\hat{e}), a\hat{e})]$$

Substituting (11) in gives

$$\frac{\partial y}{\partial \hat{e}} = a\left[-f_1(k(a\hat{e}), a\hat{e})\frac{f_{12}(k(a\hat{e}), a\hat{e})}{f_{11}(k(a\hat{e}), a\hat{e})} + f_2(k(a\hat{e}), a\hat{e})\right]$$

and the desired conclusion follows.

For Part (c), simply observe that $\Pi(k^*, e^*) > \Pi(k(a\hat{e}), \hat{e})$ since (k^*, e^*) is the unique argmax of $\Pi\ (k, e)$, by Assumption *A* and $\hat{e} < e^*$ by Assumption *B*. ■

Proof of Proposition 2: To prove part (i), consider the firm's unconstrained problem and differentiate (4) and (5) with respect to *a* to obtain

$$f_{11}(k, ae)\frac{\partial k}{\partial a} + af_{12}(k, ae)\frac{\partial e}{\partial a} = -ef_{12}(k, ae)$$
$$f_{21}(k, ae)\frac{\partial k}{\partial a} + af_{22}(k, ae)\frac{\partial e}{\partial a} = -ef_{22}(k, ae).$$

Solving for $\partial e^*/\partial a$, say by Cramer's rule, we get

$$\frac{\partial e^*}{\partial a} = -\frac{e^*}{a} < 0.$$

Hence the upper bound of the domain $[0, e^*]$ shrinks as *a* increases.

To prove part (ii), totally differentiating (10) with respect to *a* yields

$$\frac{dMAC}{da} = f_2(k(a\hat{e}), a\hat{e}) - q + a[\hat{e}\,f_{21}(k(a\hat{e}), a\hat{e})k'(a\hat{e}) + \hat{e}\,f_{22}(k(a\hat{e}), a\hat{e})] \tag{A1}$$

Substituting (11) into (20) yields

$$\frac{dMAC}{da} = [f_2(k(a\hat{e}), a\hat{e}) - q] + a\hat{e}\left[-\frac{f_{21}^2(k(a\hat{e}), a\hat{e})}{f_{11}(k(a\hat{e}), a\hat{e})} + f_{22}(k(a\hat{e}), a\hat{e})\right] \tag{A2}$$

The first term in brackets on the RHS is >0 in view of Assumptions A and B, and (5). On the other hand, the second term in brackets on the RHS is <0 by concavity of f. Let us next check the endpoints $\hat{e} = 0$ and $\hat{e} = e^*$. We have

$$\left.\frac{dMAC}{da}\right|_{\hat{e}=0} = +\infty \text{ by Assumptions } A(ii) \text{ and } A(iv) \tag{A3}$$

and

$$\left.\frac{dMAC}{da}\right|_{\hat{e}=e^*} < 0 \text{ by (5) and Assumption } A(v) \tag{A4}$$

As $dMAC/da$ is continuous in $\hat{e}$ (by Assumption A), it follows from (A3), (A4) and the Intermediate Value Theorem that $dMAC/da = 0$ for at least one value of $\hat{e}$, call it $e_0 \in (0, e^*)$. This conclusion of Proposition 2 then follows directly from (A3) and (A4). ■

Proof of Proposition 3: For Part (a), following the same steps as in the previous proof, but replacing the production function by $\alpha f(k, a\hat{e})$, we arrive in place of (10) at (here $k(\hat{e}, \alpha)$ denotes the emission-constrained firm's optimal choice of capital)

$$MAC = a[\alpha f_2(k(\hat{e}, \alpha), a\hat{e}) - q].$$

Differentiating with respect to α yields

$$\frac{dMAC}{d\alpha} = a[f_2(k(\hat{e}, \alpha), a\hat{e}) + \alpha f_{21}(k(\hat{e}, \alpha), a\hat{e}) k_2(\hat{e}, \alpha)]. \tag{A5}$$

To evaluate $k_2(\hat{e}, \alpha)$, differentiate w.r.t. α the first order condition for in k, i.e.

$$\alpha f_1(k(\hat{e}, \alpha), a\hat{e}) - r = 0 \tag{A6}$$

and collect terms to obtain

$$k_2(\hat{e}, \alpha) = -\frac{f_1(k(\hat{e}, \alpha), a\hat{e})}{\alpha f_{11}(k(\hat{e}, \alpha), a\hat{e})}. \tag{A7}$$

Substituting (A7) in (A5)

$$\frac{dMAC}{d\alpha} = a\left[f_2(k_2(\hat{e}, \alpha), a\hat{e}) - f_{21}(k_2(\hat{e}, \alpha), a\hat{e}) \frac{f_1(k_2(\hat{e}, \alpha), a\hat{e})}{f_{11}(k_2(\hat{e}, \alpha), a\hat{e})}\right]. \tag{A8}$$

Then $dMAC/d\alpha \geq 0$ if and only if (12) holds, which settles Part (a).

For Part (b)(i), the first claim follows directly from the fact that the assumption for this part is that (12) holds at all $\hat{e}$ for all $k, x \geq 0$. We now prove the second claim, that the domain $[0, e^*]$ expands if (12) holds. To this end, first differentiate the first order conditions for the firm's *unconstrained problem* with respect to α to obtain

$$\alpha f_{11}(k, ae)\frac{\partial k}{\partial \alpha} + a\alpha f_{12}(k, ae)\frac{\partial e}{\partial \alpha} = -f_1(k, ae)$$
$$\alpha f_{21}(k, ae)\frac{\partial k}{\partial \alpha} + a\alpha f_{22}(k, ae)\frac{\partial e}{\partial \alpha} = -f_2(k, ae)$$

Solving for $\partial e^*/\partial\alpha$, say by Cramer's rule, we get

$$\frac{\partial e^*}{\partial \alpha} = -\begin{vmatrix} f_1 & \alpha f_{11} \\ f_2 & \alpha f_{21} \end{vmatrix} \Big/ \begin{vmatrix} \alpha f_{11} & a\alpha f_{12} \\ \alpha f_{21} & a\alpha f_{22} \end{vmatrix}.$$

By Assumption A, the denominator is >0 and thus $\frac{\partial e^*}{\partial \alpha}$ has the same sign as the numerator, which has the sign of (12).

For Part (b) (ii), to see that α large enough is also sufficient for (12), use (A6) to rewrite (A8) as

$$\frac{dMAC}{d\alpha} = a\left[f_2(k_2(\hat{e},\alpha), a\hat{e}) - \frac{r f_{21}(k_2(\hat{e},\alpha), a\hat{e})}{\alpha f_{11}(k_2(\hat{e},\alpha), a\hat{e})}\right].$$

In view of Assumption A, choosing α large enough will make the ratio $r f_{21}/\alpha\ f_{11}$ as small as desired. This conclusion then follows from the fact that $f_2 > 0$. ■

Proof of Proposition 4: Following the same steps as the previous proofs, but with the production function being $f(Ak, a\hat{e})$, we arrive in place of (10) at

$$MAC = a[f_2(Ak(\hat{e}, A), a\hat{e}) - q]$$

Differentiating with respect to A yields

$$\frac{dMAC}{dA} = af_{21}(Ak(\hat{e}, A), a\hat{e})\ [k + Ak_2(\hat{e}, A)] \tag{A9}$$

To evaluate $\partial k(\hat{e}, A)/\partial A$, differentiate w.r.t. A the first order condition in k, i.e.

$$Af_1(Ak(\hat{e}, A), a\hat{e}) - r = 0$$

and collect terms to obtain

$$k_2(\hat{e}, A) = -\frac{f_1(Ak(\hat{e}, A), a\hat{e})}{A^2 f_{11}(Ak(\hat{e}, A), a\hat{e})} - \frac{k}{A},$$

which, upon substitution in (A9) yields

$$\frac{dMAC}{dA} = -\frac{af_1(Ak(\hat{e}, A), a\hat{e})\, f_{21}(Ak(\hat{e}, A), a\hat{e})}{Af_{11}(Ak(\hat{e}, A), a\hat{e})},$$

which says that $dMAC/dA$ has the same sign as $f_{21}(Ak(\hat{e}, A), a\hat{e})$ in view of Assumption A, which is the desired conclusion. ■

Proof of Proposition 5: Applying the envelope Theorem to (19) yields

$$MAC(\hat{e}, \theta) = \frac{\partial \hat{\Pi}(k, x, \hat{e})}{\partial \hat{e}} = \theta C_1 \left[\frac{1}{a} x(\hat{e}, \theta) - \hat{e}, \frac{1}{a} x(\hat{e}, \theta) \right].$$

To determine the direction of shift of the MAC curve, consider (with the arguments of the various partials of C and f often dropped for clarity)

$$\frac{\partial MAC(\hat{e}, \theta)}{\partial \theta} = C_1 + \frac{\theta}{a}(C_{11} + C_{12}) \frac{\partial x(\hat{e}, \theta)}{\partial \theta}. \qquad \text{(A10)}$$

To find the sign of $\partial x(\hat{e}, \theta)/\partial \theta$, consider the first order conditions

$$f_1(k, x) - r = 0$$

and

$$f_2(k, x) - q - \frac{\theta}{a} \left[C_1 \left(\frac{x}{a} - \hat{e}, \frac{x}{a} \right) + C_2 \left(\frac{x}{a} - \hat{e}, \frac{x}{a} \right) \right] = 0.$$

Differentiating w.r.t. θ, we get a system of two equations, with unknowns $\frac{\partial k(\hat{e}, \theta)}{\partial \theta}$ and $\frac{\partial x(\hat{e}, \theta)}{\partial \theta}$,

$$f_{11}(k, x) \frac{\partial k(\hat{e}, \theta)}{\partial \theta} + f_{12}(k, x) \frac{\partial x(\hat{e}, \theta)}{\partial \theta} = 0$$

and

$$f_{12}(k, x) \frac{\partial k(\hat{e}, \theta)}{\partial \theta} + \left\{ f_{22}(k, x) - \frac{\theta}{a^2}(C_{11} + 2C_{12} + C_{22}) \right\} \frac{\partial x(\hat{e}, \theta)}{\partial \theta} - \frac{1}{a}(C_1 + C_2) = 0.$$

Using Cramer's rule on this system, we obtain

$$\frac{\partial x(\hat{e}, \theta)}{\partial \theta} = \frac{1}{a} \frac{f_{11}(C_1 + C_2)}{f_{11}\left\{ f_{22} - \frac{\theta}{a^2}(C_{11} + 2C_{12} + C_{22}) \right\} - f_{12}^2} \qquad \text{(A11)}$$

To prove (a), use Assumptions A(v) and C(v) to conclude that the denominator Δ in (A11) is >0. Then Assumption C(iv) yields $\frac{\partial x(\hat{e}, \theta)}{\partial \theta} < 0$.

Since end-of-pipe innovation amounts to a decrease (rather than an increase) in θ, innovation increases gross emissions.

To prove (b), plug (A11) back into (A1) and simplify to obtain

$$\begin{aligned}\frac{\partial MAC(\hat{e},\theta)}{\partial\theta} &= \frac{1}{\Delta}C_1\left(f_{11}f_{22}-f_{12}^2\right)\\ &\quad -\frac{\theta}{\Delta a^2}f_{11}\{C_1(C_{12}+C_{22})-C_2(C_{11}+C_{12})\}\\ &> \frac{1}{\Delta}C_1\left(f_{11}f_{22}-f_{12}^2\right)\\ &\quad -\frac{\theta}{\Delta a^2}f_{11}\{-C_2(C_{12}+C_{22})-C_2(C_{11}+C_{12})\}\\ &= \frac{1}{\Delta}C_1\left(f_{11}f_{22}-f_{12}^2\right)+\frac{\theta}{\Delta a^2}f_{11}C_2\{C_{11}+2C_{12}+C_{22}\}>0\end{aligned}$$

where the first inequality is due to Assumption C(iv) and the last one to Assumptions A(v), C(iii) and C(v).

Hence, end-of-pipe innovation causes a downward shift in the MAC curve. ■

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