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Simon Schopohl

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CORE
Voie du Roman Pays 34, L1.03.01
Tel (32 10) 47 43 04
Fax (32 10) 47 43 01
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Communication Games with Optional Verification

Simon Schopohl*

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Abstract

We analyse a Sender-Receiver game in which the Sender can choose between a costless cheap-talk message and a costly verifiable message. The Sender knows the true state of the world, while the Receiver only learns about the state through the message of the Sender. The utility of both players depends on an action the Receiver chooses. We keep the assumptions about the utility functions and about the messages to a minimum and state conditions for fully revealing equilibria. Under the assumption of "smooth" preferences and utility functions we show that a fully revealing equilibrium in which the Sender uses both her message types can only exist as long as the state space and action space are discrete. We illustrate this result for the classical example of quadratic loss utilities. In a continuous setting we show that there can only exist a fully revealing equilibrium in which the Sender uses different message types in different states if we allow for costless verification in some states of the world or if the utility function of at least one player is discontinuous.

Keywords : cheap-talk, communication, costly disclosure, full revelation, Sender-Receiver game, verifiable information.
JEL Classification : C72, D82.

*CEREC, Université Saint-Louis – Brussels, B-1000 Brussels, Belgium;
CORE, Université catholique de Louvain, B-1348 Louvain-la-Neuve, Belgium
E-mail: simon.schopohl@usaintlouis.be

1 Introduction

This paper studies a Sender-Receiver game in which the Sender can choose between costless cheap-talk messages and a costly verifiable messages that reveals the true state. The Sender knows the true state of the world while the Receiver has a belief about its distribution. Depending on the message of the Sender and his own beliefs the Receiver chooses a given action from an action space. Both players get a payoff depending on the true state of the world and the action the Receiver selects. The message of the Sender can either be a cheap-talk message or a verifiable message. The set of cheap-talk messages correlates to the set of states of the world and sending a cheap-talk message is costless for the Sender. When the Sender uses a cheap-talk message she does not have to tell the truth which yields to the problem that the Receiver might not belief the Sender, depending on the differences in their preferences. On the other hand, if the Sender chooses the verifiable message the Receiver learns the true state of the world. The Sender cannot lie in this message and it reveals the entire truth about the state. For sending the verifiable message the Sender has to pay a cost.

We start our analysis of this model in a discrete setting in which the state space and the action space are finite. In this framework we give conditions for fully revealing equilibria. Those are the equilibria in which the Receiver always learns the true state after reading the message of the Sender. We show that there are three different ways to achieve full revelation. If the preferences of Sender and Receiver are similar, the Sender may have the incentive to send the information about the true state by cheap-talk in each state of the world. In that case the Receiver knows that the Sender has no incentive to lie, because both players have similar preferences and the Receiver can implement the action he prefers most. On the other hand, there can also exist a fully revealing equilibrium in which the Receiver enforces the Sender to use the costly verifiable message in all states. The Receiver can do so by choosing a certain action as a reply to all cheap-talk messages. If the Sender dislikes this action in all states of the world, she always prefers to pay for the verifiable message. The third possibility of full revelation is a combination of both message types. In a subset of states in which the Receiver and Sender have similar interests, the Sender uses different cheap-talk messages, while in all other states the Receiver enforces the usage of the verifiable message. We state detailed conditions for all three cases , illustrate those and give examples.

In a second step we extend our model and allow for a continuous action and state space. The main result is that a fully revealing equilibrium in which the Sender uses different message types in different states of the world cannot exist as long as the utility functions of both players are continuous and the cost for the verifiable message is positive in all states. We illustrate this result for the case that the utility functions follow quadratic loss functions. Furthermore, we show how this result changes if the utility functions are discontinuous or if the verifiable message is costless in some states of the world.

This model can be applied to several classical examples. In Spence (1973) the Receiver of a message is an employer and the Sender an agent who is looking for employment. The interests of both players may differ, but the Sender likes to get an offer from the Receiver. The working effort of the Sender cannot be observed, but she reports it to the Receiver and he chooses an action according to the message. Our model gives the Sender the additional possibility to pay for a certified report of her effort. In the example of a job interview this verifiable message corresponds to a certification of skills by showing credentials or reports of courses and training.

Another well known example is the lemon market by Akerlof (1970). A seller has private information about the quality of the good she is selling. The buyer has to decide whether to buy the product or not. The seller can tell the quality of the good, but her messages are just cheap-talk and the buyer cannot rely on it. In our model the seller can pay to get the quality of her good certified. This allows her to prove the level of quality to the buyer. Obviously, the seller will never pay for the certification if the quality of the product is very low.

One real-life example for this is the market for used-cars. Most often advertisements of sellers just contain information that the seller provides and that the interested party should believe. At the same time there exist many ways for the seller to certify these information, for example by paying an independent consultant. Clearly, this is costly for the seller and she prefers to sell her car without that certificate. For expensive or classic cars where the cost of verification is comparably low to the selling price, we observe the usage of certificates more often.

The literature on cheap-talk goes back to Crawford and Sobel (1982). In their model the content of a message can be whatever the Sender wants it to be. She does not have to tell the truth and so the message may not change the Receiver's beliefs at all. The authors show that there are different types of equilibria. In the babbling equilibrium the Sender uses the same message in each state of the world (or for each of her types). In the informative equilibrium the Receiver learns more about the true state of the world, because the Sender uses different cheap-talk messages in different states. In the setting of Crawford and Sobel (1982) the Sender has no possibility to verify that she tells the truth.

On the other hand, Grossman (1981) and Milgrom (1981) propose very similar models for different applications. The Sender can decide how much information she likes to reveal about an item she wants to sell. She cannot lie or fake these information, but she can choose what information to reveal. In these models the Receiver can enforce full revelation. He assumes that all properties of the object for sale are the worst and he only changes his beliefs if information is revealed by the Sender. This unraveling argument yields to full revelation in equilibrium. In our model the Sender only has one verifiable message that completely reveals the state. We argue that if the Sender could verify as in Grossman (1981) and Milgrom (1981), the unraveling argument would hold and in equilibrium the Sender will reveal the complete information.

To the best of our knowledge there exists only one paper that follows the same idea as this paper and combines the two different strains of communication literature. Eső and Galambos (2013) start with a similar idea, but it is to point out that there are many differences in the settings. Eső and Galambos assume that the players' utility functions are strictly concave and that the players' optimal actions are strictly increasing in the state. Furthermore, they assume that the Sender's utility only depends on the Sender's ideal action and the action the Receiver chooses, but not on the state of the world. Under these assumptions they find that in equilibrium the state space can be split into different intervals and that the Sender uses either the same message for all states of an interval or that she uses the verifiable message in the entire interval. This confirms the result we derive in the continuous setting. Even under their additional assumptions there is no fully revealing equilibrium in which the Sender uses different types of messages in different states of the world. In comparison to Eső and Galambos (2013) this paper starts with less assumptions and focuses more on conditions for full revelation. In addtion, this paper also allows for a finite state space and action space and we show that in this setting there can exist a fully revealing equilibrium in which the Sender uses both message types.

Cheap-talk communication has been added to many different settings and the original model of Crawford and Sobel (1982) has been extended in several directions. Farrell and Gibbons (1989) introduce an additional Receiver. The Sender can choose either to privately speak to one of them or in public to both. The authors observe how the existence of the second Receiver changes the report of the Sender. McGee and Yang (2013) and Ambrus and Lu (2014) do a similar step with multiple Senders. McGee and Yang (2013) focus on two Senders with complementary information. They show that if one Sender reveals more information, the other Sender has incentives to transmit more information as well. Ambrus and Lu (2014) model several Senders who observe a noisy state. They provide conditions under which there exists an equilibrium that is arbitrarily close to a fully revealing equilibrium.

Bull and Watson (2007) and Mookherjee and Tsumagari (2014) deal with communication and mechanism design. While Bull and Watson (2007) focus on costless disclosure, Mookherjee and Tsumagari (2014) add communication cost which prevents full revelation of information. Communication cost is also introduced by Hedlund (2015) in a persuasion game. The author derives two types of equilibria: For high cost there exists a pooling equilibrium, while for not too high cost a separating equilibrium exists.

Other models focus more on disclosure of information and costly communication. Hagenbach, Koessler, and Perez-Richet (2014) analyse a game, where each of several players can tell the truth about his type or can masquerade as some other type. As usual, the player who deviates (from telling the truth) is punished by the other players. If a player masquerades, the other players assume a worst case type and punish him by choosing the action this type of player dislikes. The authors state conditions for full revelation depending on the possible masquerades of each player.

An overview over cheap-talk models and models with verifiable messages can be found in Sobel (2009). The author describes several models and gives some economic examples. Most of these examples can be extended to fit our setting by adding a reasonable verifiable message. Verrecchia (2001) provides an overview over different models of disclosure, which is extended by Dye (2001).

It remains to mention that there are several papers in which the authors have created their own way of modeling communication. Kartik (2009) introduces a model, where the Sender sends a message about her type, but has the incentive to make the Receiver believe that her type is higher than it actually is. If the Sender lies in her message, she has to pay a cost for lying, which depend on the distance between the true type and the type stated in the message.

Dewatripont and Tirole (2005) analyse the communication of Sender and Receiver when both players have to invest effort. The effort of the Sender is to make the message understandable, while the effort of the Receiver corresponds to him paying attention while reading the message. The authors motivate this model by the idea that very unclear messages and reading messages without paying a lot of attention yield to misunderstandings. The probability of understanding the message is influenced by the effort of both players.

Austen-Smith and Banks (2000) introduce the possibility for the Sender to send a costly message with the same content as a costless message. By this way of burning money the Sender has an additional possibility of signaling. The authors show that conditions exist under which both message types are used.

The paper is organized as follows. In Section 2 we introduce the discrete model and state results for this setting. In Section 2.1 we give conditions for fully revealing equilibrium. Section 2.2 shortly discusses partial revelation. In Section 3 we extended our setting to a continuous model and show that the previous results do not hold. We analyze the continuous model where the utility functions of both players are quadratic loss functions in Section 3.1. In Section 3.2 we take a closer look at state dependent cost for the verifiable message. Finally, in Section 4 we conclude. All proofs are relegated to the appendix.

2 Discrete Model

In this part we focus our attention on a model with a finite set of states of the world and a finite set of actions the Receiver can choose from. Let $\Omega = \{\omega_1, \ldots, \omega_L\}$ denote the set of the L different states of the world, where each state ω has the probability $\mathbb{P}[\omega]$.

The timing is as follows: The Sender learns the true state of the world and then she sends a message to the Receiver. We assume that the set of possible cheap-talk messages M corresponds to the states of the world Ω and that the verifiable message v is unique in each state of the world. So, the Sender chooses a message from $M \cup \{v\}$, i.e. she either sends a cheap-talk message or the verifiable message v. There is no possibility for partial disclosure. While sending any cheap-talk message is costless, the Sender has to pay a cost $c > 0$ if she sends the verifiable message. An economic explanation for this cost can be either the payment for a certificate or the investment into effort. For simplicity we assume that the cost is state independent, but the same results hold as long as the cost is positive in all states. The only necessary adjustments are that in the conditions of the following results we would have to replace c by $c(w)$. We deviate from this assumption in Section 3.2.

After reading the message the Receiver chooses an action from the action set $A = \{a_1, \ldots a_N\}$. By $\Delta(A)$ we denote the set of mixed strategies. Both players have preferences about the actions, resulting in different von Neumann-Morgenstern utility functions for both players, depending on the action and state of the world. For the Sender it is given by $\tilde{u}_S : A \times M \cup \{v\} \times \Omega \to \mathbb{R}$ with $\tilde{u}_S(a, m, \omega) = u_S(a, \omega)\ \forall m \in M$, $\tilde{u}_S(a, v, \omega) = u_S(a, \omega) - c$ and $u_S : A \times \Omega \to \mathbb{R}$. So we can split the utility function of the Sender up into two parts: Firstly, a utility function depending on action and state of the world. Secondly, we have to subtract the cost for the message if there is any. For the Receiver the utility function is not depending on the type of the message, but only on the action and state: $u_R : A \times \Omega \to \mathbb{R}$. The utility functions show that there is neither a punishment for lying nor a direct reward for telling the truth. We assume that these utility functions are common knowledge. By $a^*_R(\omega)$ ($a^*_S(\omega)$) denote the action the Receiver (Sender) prefers in the state ω.

Assumption 1. *The Receiver has strict preferences in every state of the world.*

Assumption 1 ensures that $a^*_R(\omega)$ is a single action in all states of the world. This is necessary to avoid the situation that the Receiver is indifferent between two actions.

We denote the Sender's behavior by the function $f : \Omega \to M \cup \{v\}$. This function f maps each state of the world to the message the Sender uses. We assume that the Sender does not mix different messages. The Receiver chooses the action, depending on the message he received. His behavior is characterized by $g : M \cup \Omega \to \Delta(A)$. In equilibria we define the behavior of the

Sender for every state, so $f(\omega)$ and the Receiver's action after each message, i.e. $g(m)\,\forall m \in M$ and $g(v)$. The equilibrium concept we use is Perfect Bayesian Equilibrium.

Definition 1. *A Perfect Bayesian Equilibrium in a dynamic game of incomplete information is a strategy profile (f^*, g^*) and a belief system μ^* for the Receiver such that*

- *The strategy profile (f^*, g^*) is sequentially rational.*
- *The belief system μ^* is consistent whenever possible, given (f^*, g^*).*

In other words, each equilibrium consists of optimal strategies for Sender and Receiver, which are sequentially rational. Furthermore the Receiver has a belief system over the true state of the world depending on the message he receives. This belief system is updated by Bayes rule whenever possible. For Perfect Bayesian Equilibria the actions off the equilibrium path have to be the best actions for the Receiver for at least one belief system. We can neglect this if we limit our attention to actions that are undominated for the Receiver.

We are specially interested in equilibria with full revelation:

Definition 2. *A Perfect Bayesian Equilibrium is fully revealing, if the Receiver knows the true state of the world after reading the Sender's message.*

There is full revelation if the Sender either sends different cheap-talk messages in each state, just verifiable messages, or different cheap-talk messages in some states and verifiable messages in the other states.

Assumption 2. *The function $a_R^* : \Omega \to A$ is injective.*

Assumption 2 assures that in different states of the world the Receiver prefers different actions, i.e. $a_R^*(\omega_i) \neq a_R^*(\omega_j)\ \forall \omega_i \neq \omega_j$. This makes sure that there can be a fully revealing equilibrium, even if the Sender uses cheap-talk messages in several states. Without this assumption there might be two states $\omega_i \neq \omega_j$ with $a_R^*(\omega_i) = a_R^*(\omega_j)$. Then the Sender might say that the state is ω_j if the true state is ω_i (and vice versa), because the Receiver chooses the same action in both states. In that case the equilibrium is not fully revealing.

Assumption 3. *For each action $a_j \in A$ there exists at least one belief system μ such that a_j is the Receiver's best response under the belief system μ.*

By $\hat{\Delta}(A) \subseteq \Delta(A)$ we denote the set of mixed strategies that satisfy this assumption, i.e.

$$\forall \hat{a} \in \hat{\Delta}(A) : \exists \mu : \hat{a} \in \arg\max_a \sum_{\omega \in \Omega} \mu(\omega) \cdot u_R(a, \omega)$$

Assumption 3 requires that each action is optimal for the Receiver under at least one belief system, which means that there are no dominated actions. Our results depend on the idea that the Receiver uses an action as a threat and so enforces the Sender to send verifiable messages. The threat is only credible, if this action is an element of $\hat{\Delta}(A)$.

We can think about different equilibrium refinements as introduced in several papers. The most common ones are the Divinity Criterion by Banks and Sobel (1987) and the Intuitive Criterion by Cho and Kreps (1987). Using one of them adds more conditions for the threat points, so the set $\hat{\Delta}(A)$ gets smaller and the Receiver has less possibilities to make a threat, but the conditions stay the same. In addition such refinements may rule out other equilibria, but this paper deals with the existence of equilibria and not the uniqueness.

2.1 Full revelation

In this part we focus on the existence of fully revealing equilibria. We will state conditions for full revelation, where the Sender uses the cheap-talk messages in all states, conditions where she uses only verifiable messages and conditions where she uses different message types depending on the state. Even if conditions for one of these fully revealing equilibria are satisfied, there may be other equilibria at the same time. We use examples to show that the existence of these different types of full revelation are independent of each other.

Proposition 1 (Full Revelation just by Cheap-Talk Messages)**.**
There is a fully revealing equilibrium with only costless messages sent if and only if:

$$\forall \omega_i \in \Omega : u_S(a_R^*(\omega_i), \omega_i) > u_S(a_R^*(\omega_j), \omega_i) \; \forall \omega_j \neq \omega_i \tag{1}$$

For the case that the Sender just uses the cheap-talk messages and we want to have full revelation, the Sender is not allowed to have any incentive to deviate to another cheap-talk message. Still, it is not necessary that the preferences in all states are the same for Sender and Receiver. It is crucial that the action the Receiver chooses when he knows the true state $a_R^*(\omega_i)$ generates a higher utility for the Sender than the Receiver's most preferred action in any other state $a_R^*(\omega_j)$ with $\omega_j \neq \omega_i$. There is also the possibility that there exists an action the Sender prefers, but which is never included by the Receiver as long as he knows the true state of the world.

If Assumption 2 does not hold, i.e. if there exist two states ω_i, ω_j such that $a_R^*(\omega_i) = a_R^*(\omega_j)$, there might be no fully revealing equilibrium. Still the Receiver can get the highest possible utility in every state, while the Sender just sends cheap-talk messages. If the Receiver learns that the state is ω_i or ω_j his best response is the same action and it generates the highest possible utility for him.

Proposition 2 (Full Revelation just by Verifiable Messages)**.**
There is a fully revealing equilibrium with only verifiable messages sent if and only if:

$$\exists \hat{a} \in \hat{\Delta}(A) : 1.\ \forall \omega_i : \hat{a} \neq a_R^*(\omega_i)$$
$$2.\ \forall \omega_i : u_S(a_R^*(\omega_i), \omega_i) - c > u_S(\hat{a}, \omega_i)$$

The idea behind Proposition 2 is that the Sender replies to cheap-talk messages with an action $\hat{a}$ the Sender really dislikes. With this threat point $\hat{a}$ the Receiver forces the Sender to use the verifiable message. Condition 1 ensures that the threat point $\hat{a}$ is never the action the Receiver chooses if he knows the true state. This condition is necessary because otherwise, there exists a state $\hat{\omega}$ with $\hat{a} = a_R^*(\hat{\omega})$ and in that state the Sender will not use the verifiable message. Independent of the message she chooses, the Receiver implements action $\hat{a}$ and so the Sender prefers the cheap-talk message, because that message is costless for him.

Furthermore, the Sender should have no incentive to use the cheap-talk message in any other state. So the utility she gets from using the verifiable message minus the cost c has to be larger than the utility she would get if the Receiver chooses action $\hat{a}$ (Condition 2). The same idea can be found in several existing papers dealing with verifiable messages, e.g. in Hagenbach, Koessler, and Perez-Richet (2014).

We can combine both propositions and get conditions for full revelation, where the Sender uses both types of messages.

Theorem 1 (Full Revelation by Cheap-Talk and Verifiable Messages)**.**
There is a fully revealing equilibrium with both message types used if and only if there exists $\hat{\Omega} \subsetneq \Omega$ with $\hat{\Omega} \neq \emptyset$ such that

$$1.\ \forall \omega_i \notin \hat{\Omega} : u_S(a_R^*(\omega_i), \omega_i) - c > u_S(a_R^*(\omega_j), \omega_i)\ \forall \omega_j \in \hat{\Omega}$$
$$2.\ \forall \omega_i \in \hat{\Omega} : u_S(a_R^*(\omega_i), \omega_i) > u_S(a_R^*(\omega_j), \omega_i)\ \forall \omega_j \in \hat{\Omega}, \omega_j \neq \omega_i$$

Theorem 1 allows that the Receiver trusts the Sender in some states ($\hat{\Omega}$), but in the other states he enforces the use of the verifiable message as in Proposition 2. To have both message types used, $\hat{\Omega}$ has to be a subset of Ω, not equal to Ω and non-empty. The two conditions in this theorem are similar to those of the previous propositions. Instead of a single threat point $\hat{a}$, each $a_R^*(\omega_j)$ with $\omega_j \in \hat{\Omega}$ has to work as a threat (Condition 1). In addition the Sender is not allowed to have an incentive to deviate to another cheap-talk message if the true state is an element of $\hat{\Omega}$ (Condition 2).

There might be several possibilities for $\hat{\Omega}$. Those sets do not have to be subsets of each other, but also can be disjoint. For the case that there are several subsets we can say that for smaller sets Condition 1 has to hold for more states, but Condition 2 for less states.

For the result of Theorem 1 we require Assumption 2 just for the states in $\hat{\Omega}$. So even if there exist two states $\omega_i, \omega_j \in \Omega/\hat{\Omega}$ with $a_R^*(\omega_i) = a_R^*(\omega_j)$, Theorem 1 still holds. If Assumption 2 does not hold and there exist two states $\omega_i, \omega_j \in \hat{\Omega}$ with $a_R^*(\omega_i) = a_R^*(\omega_j)$, Theorem 1 does not hold, but under the conditions of the theorem, the Receiver still gets the highest possible utility in every state.

One simplification of Theorem 1 can be done to analyse the case of common interest. There might exist certain states ω in which Sender and Receiver share the same interests and prefer the same action, i.e. $a_R^*(\omega) = a_S^*(\omega)$. If we take the set of all those states as $\hat{\Omega}$ in Theorem 1 we do not require the second condition of the Theorem any more. In the remaining states full revelation can be either achieved by cheap-talk, by verifiable message or by a combination of both types. The conditions for the easiest case, in which the Sender uses the verifiable message in all other states in characterized is the following corollary.

Corollary 1 (Full Revelation with Common Interest)**.**
Let $\hat{\Omega}$ denote all states in which Sender and Receiver have common interest, i.e. $\forall \omega \in \hat{\Omega}$: $a_R^*(\omega) = a_S^*(\omega)$. There exists a fully revealing equilibrium in which the Sender uses cheap-talk in all states of common interest and the verifiable message in all other states if:

$$1. \; \forall \omega_i \notin \hat{\Omega} : u_S(a_R^*(\omega_i), \omega_i) - c > u_S(a_R^*(\omega_j), \omega_i) \; \forall \omega_j \in \hat{\Omega}$$

We can also rewrite the first condition of Theorem 1 and focus more on the cost of the verifiable message.

Corollary 2 (Full Revelation by Cheap-Talk and Verifiable Messages)**.**
There is a fully revealing equilibrium with both message types used if there exists $\hat{\Omega} \subsetneq \Omega$ with $\hat{\Omega} \neq \emptyset$ such that

$$1. \; \forall \omega_i \notin \hat{\Omega} : c < \min_{\omega_j \in \hat{\Omega}} u_S(a_R^*(\omega_i), \omega_i) - u_S(a_R^*(\omega_j), \omega_i)$$

$$2. \; \forall \omega_i \in \hat{\Omega} : u_S(a_R^*(\omega_i), \omega_i) > u_S(a_R^*(\omega_j), \omega_i) \; \forall \omega_j \in \hat{\Omega}, \omega_j \neq \omega_i$$

This Corollary rewrites the first condition of Theorem 1 such that we get an upper bound for the cost c that allows for a fully revealing equilibrium in which cheap-talk messages and the verifiable message are used by the Sender. Again, if there are some states of common interest, we can use the union of those sets as $\hat{\Omega}$ as in Corollary 1 and get rid of the second condition.

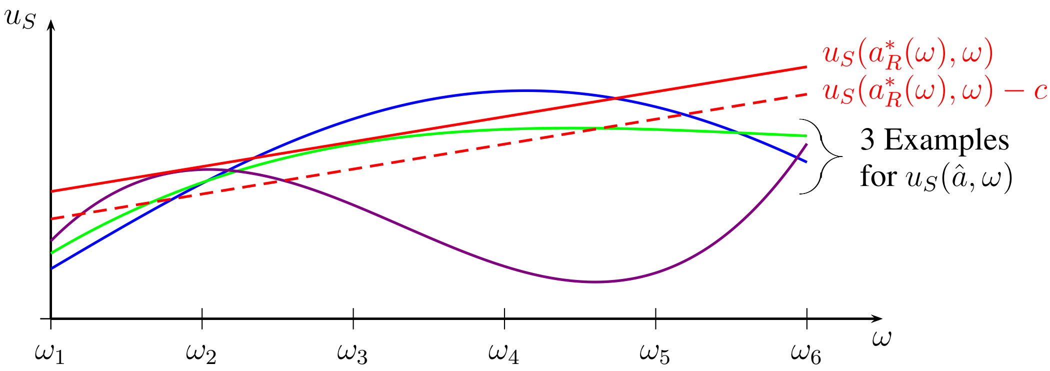

Figure 1: Illustration of Theorem 1.

Figure 1 illustrates the conditions of Theorem 1. We want to point out that we are showing the utility functions for the entire interval $[\omega_1, \omega_6]$, but in this discrete models just the values at $\{\omega_1, \ldots, \omega_6\}$ are of interest. For simplicity we assume that the Receiver responds to all cheap-talk messages with the action $\hat{a}$, i.e. that $\hat{\Omega}$ is just a single state. Since we are looking for a fully revealing equilibrium in which the Sender uses both message types, there has to exist a state $\hat{w} \in \hat{\Omega}$ such that $\hat{a} = a_R^*(\hat{\omega})$. The red line shows $u_S(a_R^*(\omega), \omega)$, the dashed red line is the function $u_S(a_R^*(\omega), \omega) - c$. The blue, green and violet curves are different examples for $u_S(\hat{a}, \omega)$. If $u_S(\hat{a}, \omega)$ follows the blue curve there can be no fully revealing equilibrium in which the Sender uses both message types. In the states ω_3 and ω_4 the action $\hat{a}$ gives a higher utility to the Sender than the action the Receiver prefers most ($a_R^*(\omega)$).

Even if $u_S(\hat{a}, \omega)$ is like the green curve the conditions of Theorem 1 are not satisfied. In this example the Receiver believes the cheap-talk message in ω_3, i.e. $\hat{\omega} = \omega_3$ and the Sender gets no higher utility from $\hat{a}$ compared to $a_R^*(\omega)$ in any other states. Still, the benefit from sending the verifiable message is too low in the states ω_2 and ω_4. If the Sender uses the verifiable message her utility is at the values of the red-dashed line which is lower than the utility the action $\hat{a}$ generates. In this example the cost for verification is too high to achieve a fully revealing equilibrium.

Only in the example of the violet curve there is a fully revealing equilibrium in which the Sender uses different message types in different states of the world. In this example the Receiver beliefs the cheap-talk message in ω_2, i.e. $\hat{\omega} = \omega_2$. In all other states the violet curve is below the red-dashed line, which means that the Sender gets a higher utility from paying for the verifiable message than from sending a cheap-talk message.

We see that the conditions of Theorem 1 are only satisfied if there is no state ω in which $u_S(\hat{a}, \omega)$ is larger than $u_S(a_R^*(\omega), \omega)$. Furthermore, for states in which $\hat{a}$ is not equal to $a_R^*(\omega)$ the utility the Sender gets from action $\hat{a}$ has to be below $u_S(a_R^*(\omega), \omega) - c$.

Propositions 1 and 2 and Theorem 1 give conditions for different types of fully revealing equilibria. It can happen that there is no fully revealing equilibrium just by cheap-talk or just by verifiable messages, but one by a combination of both message types:

Example 1. *Assume that there are two states (ω_1, ω_2) and two actions (a_1, a_2).*

The Receiver prefers a_1 in ω_1 and a_2 in ω_2, while the Sender always prefers a_1. Obviously there is no fully revealing equilibrium with only cheap-talk messages only, because the Sender always wants the action a_1 and so she would lie. Furthermore there is no equilibrium with verifiable messages only, because there is no threat available:

For the mixed strategy that plays a_1 with probability p and a_2 with probability $(1-p)$, we use the notation $pa_1 \oplus (1-p)a_2$. Define $\hat{a} = pa_1 \oplus (1-p)a_2$. For $p = 0$, the Sender will not use the verifiable message in ω_2, because she gets the same action by sending cheap-talk, but verifiable messages are costly. Also for $p > 0$ the Sender will not use the verifiable message in ω_2, because she prefers a_1 over a_2 and so she also prefers $\hat{a}$ over a_2.

Still there is full revelation possible if c is low enough. Let us assume that the cost c is small, i.e. $c < u_S(a_1, \omega_1) - u_S(a_2, \omega_1)$. If the Receiver answers every cheap-talk message by a_2, the Sender will use the verifiable message in ω_1, yielding action a_1. The utility the Sender gets is $u_S(a_1, \omega_1) - c$, while her utility would be $u_S(a_2, \omega_1) < u_S(a_1, \omega_1) - c$ if she sends the cheap-talk message. In the second state ω_2, the Sender will use the cheap-talk message. Both message types will result in action a_2, so the Sender prefers the costless message.

Even though we stated conditions for full revelation, it might happen that there exists no fully revealing equilibrium at all. The easiest example can be done just by two states and two actions:

Example 2. *Assume that the Receiver prefers a_1 in ω_1 and a_2 in ω_2 and the Sender's preferences are the other way round, i.e. she prefers a_2 in ω_1 and a_1 in ω_2. Clearly there is no full revelation just by cheap-talk, because the Sender will always lie. Furthermore the Sender has no incentive to use only the verifiable messages. Assume that the threat point is $\hat{a} = pa_1 \oplus (1-p)a_2$, with the notation used as in the previous example.*

For $p = 0$, the Sender will not use the verifiable message in ω_1, because she prefers a_2 over a_1. The same argument holds even for $p > 0$: Using the cheap-talk message resulting in $\hat{a}$ gives the Sender at least a little chance of a_2. Therefore $u_S(\hat{a}, \omega_1) > u_S(a_1, \omega_1)$ which implies $u_S(\hat{a}, \omega_1) > u_S(a_1, \omega_1) - c$.

The only possibility is to have a fully revealing equilibrium in which both message types are used. Doing the same steps again for Theorem 1 proves that there is no full revelation. So in this example where the preferences of Sender and Receiver differ a lot, the Receiver has no possibility to enforce the full revelation.

2.2 Partial Revelation

In case that full revelation is impossible, there can be either partial revelation or no revelation at all. Under partial revelation the Receiver may learn about the true state in some states of the world or can exclude certain states from further consideration. We distinguish between three different types of partial revelation: First, in some states of the world the Sender uses unique cheap-talk messages and has no incentive to deviate. Second, the Sender may use the verifiable message in some states of the world and so the Receiver knows the true state, but in the other states the Receiver does not learn the complete truth. These types of partial revelation correspond to full revelation on a subset of the state space (as in Proposition 1 and 2). In the third type of partial revelation the Receiver does not learn the true state of the world, but learns that certain states are not the true state of the world. A minimal example works with four states ω_1 to ω_4. The Sender may use the same cheap-talk message m_1 in ω_1 and ω_2 and the cheap-talk message m_2 in ω_3 and ω_4. After receiving m_1 the Receiver learns that the true state is either ω_1 or ω_2, but for sure not ω_3 or ω_4. So the Receiver has gained information through the message of the Sender, but in no state of the world the state is completely revealed to the Receiver.

It is also possible that there is an equilibrium in which some or all types of partial revelation are combined. We leave it to the Reader to rewrite the previous results such that they just hold for some states.

If there is no full revelation, the Receiver can follow different strategies to end up in different partial revealing equilibria. Unlike the case of full revelation, without further assumptions, it is impossible to say which of the many partial revealing equilibria the Receiver prefers.

One may argue that it also might be interesting to see under which conditions the Receiver implements the action the Sender prefers in all states of the world, but in that case of full deception the verifiable message will not be used and the analysis can be carried out in a simpler model.

3 Continuous model

In many settings it is not enough to focus on a finite action or state space. For example at wage negotiations or any discussions concerning prices, we have to deal with continuous intervals. In this section we replace the discrete setting by a continuous model. Without loss of generality we assume $A = \Omega = [0, 1]$. We state different conditions under which there is no possibility for a fully revealing equilibrium. Afterwards we use the example of the quadratic loss function to visualize our results. Proposition 1 and Proposition 2, which give the conditions for fully revealing equilibria with only a single message type, still hold. The conditions in these theorems still have to hold for every state, which is more strict in the continuous model. The following results give us necessary conditions for the existence of different fully revealing equilibria, where the continuity of u_S and a^*_R are the most important factors. Combined with the results from the discrete model we also get the sufficient conditions.

Theorem 2 (Full Revelation under continuous u_S and a^*_R)**.**
Assume that $a^*_R(\omega) : \Omega \to A$ is continuous and that $u_S(a, \omega) : A \times \Omega \to \mathbb{R}$ is continuous in both arguments. Then, in all fully revealing equilibria, the Sender uses only one message type in all states.

Theorem 2 states an important result. Unlike in the discrete setting, a fully revealing equilibrium in which the Sender uses both message types is not possible as long as the utility functions of both players are continuous. In a continuous state and action space with continuous utilities for Sender and Receiver, there exists no action $\hat{a}$ that the Receiver can use as a threat point so that the Sender uses verifiable messages in some states, but cheap-talk in that state in which the Receiver's most preferred action is $\hat{a}$. The reasoning is as follows: Let $\hat{\omega}$ denote the state of the world in which the Receiver wants the action $\hat{a}$, i.e. $a^*_R(\hat{\omega}) = \hat{a}$. At a state $\hat{\omega} + \epsilon_1$ close to $\hat{\omega}$ the Receiver prefers another action, but because of the continuity it is close to $\hat{a}$, i.e. $\hat{a} + \epsilon_2$. If the Receiver uses $\hat{a}$ to reply to cheap-talk, the Sender uses cheap-talk not only in $\hat{\omega}$, but also for states close to $\hat{\omega}$. In that case her utility is $u_S(\hat{a}, \hat{\omega}+\epsilon_1)$ which is larger than $u_S(\hat{a}+\epsilon_2, \hat{\omega}+\epsilon_1)-c$ because of the continuity in the utility functions of Sender and Receiver. Only if there is some discontinuity the Sender can have an incentive to pay for the verifiable message in states close to $\hat{\omega}$.

Proposition 3 (Full Revelation under continuous $u_S(a, \omega)$)**.**
Assume that $u_S(a, \omega)$ is continuous. There can be a fully revealing equilibrium with both message types used if there exists $[\underline{\omega}, \overline{\omega}] \subsetneq [0, 1]$ with

1. $\lim\limits_{\omega \nearrow \underline{\omega}} a^*_R(\omega) \neq a^*_R(\underline{\omega})$ *and*
2. $\lim\limits_{\omega \searrow \overline{\omega}} a^*_R(\omega) \neq a^*_R(\overline{\omega})$.

Proposition 3 states that if u_S is continuous in both arguments, the function a^*_R has to be discontinuous. The interval $[\underline{\omega}, \overline{\omega}]$ gives the interval of states in which the Receiver believes the Sender's cheap-talk message. To achieve that a^*_R has to be neither right-continuous nor left-continuous at a single $\hat{\omega} = \underline{\omega} = \overline{\omega}$ or not right-continuous at $\underline{\omega}$ (Condition 1) and not left-continuous at $\overline{\omega} > \underline{\omega}$ (Condition 2). It is to point out that there may exist several intervals satisfying the conditions of Proposition 3. Furthermore, for $\underline{\omega} = 0$ (1) the first (second) condition is always satisfied.

Corollary 3.

There is a fully revealing equilibrium with both message types used if there exists $[\underline{\omega}, \overline{\omega}] \subsetneq [0, 1]$ *such that:*

1. $[\underline{\omega}, \overline{\omega}]$ *satisfies the conditions of Proposition 3.*

2. *Theorem 1 is satisfied with* $\hat{\Omega} = [\underline{\omega}, \overline{\omega}]$.

Proposition 3 only gives us possible intervals in which the Receiver could trust the cheap-talk messages of the Sender. Still we have to ensure that Theorem 1 holds if we use this interval as $\hat{\Omega}$. This means that all actions the Receiver likes most in the interval $[\underline{\omega}, \overline{\omega}]$ have to work as threat points and in addition the Sender should have no incentive to lie if the true state is that interval.

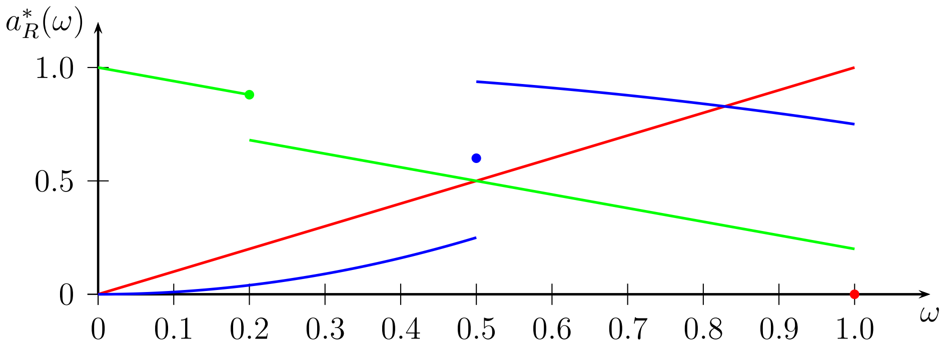

Figure 2: Different examples for $a_R^*(\omega)$.

Figure 2 shows three different discontinuous functions $a_R^*(\omega)$. For the blue graph there can be a fully revealing equilibrium with both message types, where the threat point is $a_R^*(\frac{1}{2})$. The red graph shows a situation where the possible threat point is at the border of the interval, here at $a_R^*(1)$. So Condition 2 of Proposition 3 is satisfied. As the function is discontinuous for $\omega = 1$, Condition 1 also holds. An example where Proposition 3 implies that there can be no full revelation is given by the green graph. The function is continuous coming from below and so it does not satisfy Condition 1.

Proposition 4 (Full Revelation under continuous $a_R^*(\omega)$)**.**

Assume that $a_R^*(\omega)$ *is continuous. Only if* $u_S(a, \omega)$ *is not continuous in at least one argument, there can only be a fully revealing equilibrium with both message types used.*

Proposition 4 states another possibility for discontinuity that allows for full revelation in which the Sender uses different message types in different states of the world. The reasoning is the same as for Proposition 3. We have to combine Proposition 4 with Theorem 1 to get a result similar to Corollary 3.

3.1 Quadratic loss function

For this second part we focus on the quadratic loss utility for the Receiver and a biased quadratic loss utility for the Sender. We show how our general results from the continuous model work and what the intuition behind the missing of the fully revealing equilibria is. The utility functions are $u_R = -(a-\omega)^2$ and $u_S = -(a-\omega-b(\omega))^2$, where $b(\omega) \in \mathbb{R}$ is the continuous state dependent bias function of the Sender. We assume that this bias function is continuous. For positive values of b, the Sender wants to have a higher action than state, while for negative values she wants to have a lower action than state. This is similar to the example Crawford and Sobel (1982) use, but we allow that the bias function is state-dependent.

Clearly we have the problem that $a^*_R(\omega)$ and $u_S(a,\omega)$ are continuous and therefore all fully revealing equilibria just include the usage of one message type. For $A = \Omega = [0,1]$ the function $a^*_R(\omega)$ is bijective and so every action is the best reply for one state, which implies that we can focus on pure strategies. It will happen that we misuse notation a little and denote actions by ω as well. Then we simply mean the action $a = \omega$.

As an immediate conclusion from Theorem 2 we see that there can be no fully revealing equilibrium with both message types used. As long as the bias function $b(\omega)$ is not constant equal to 0, the Sender will not always tell the truth by cheap-talk. In addition it is also impossible to have fully revelation where the Sender just uses the verifiable messages, because every possible threat point $\hat{a}$ is the Receiver's best reply to one state ($\hat{\omega} = \hat{a}$). This means that in $\hat{\omega}$ the Sender will never use the verifiable message, but prefers to save the cost and sends a cheap-talk message.

Corollary 4.
For $A = \Omega = [0,1]$ and quadratic loss utility functions for the players (with $b(w) \not\equiv 0$), no fully revealing equilibrium exists.

This follows immediately from the continuity of u_S and a^*_R and Theorem 2. We can see it in more detail with the help of the following lemma:

Lemma 1.
There is a fully revealing equilibrium if

$$\exists\hat{\omega} : 1.\ \forall\omega > \hat{\omega} : b(\omega) > \frac{\hat{\omega}-\omega}{2} - \frac{c}{2(\hat{\omega}-\omega)}$$
$$2.\ \forall\omega < \hat{\omega} : b(\omega) < \frac{\hat{\omega}-\omega}{2} - \frac{c}{2(\hat{\omega}-\omega)}$$

Lemma 1 states the conditions for a fully revealing equilibrium, where the Sender uses a cheap-talk message in $\hat{\omega}$ and the verifiable messages in all other states. We can state the same result for a set of states with cheap-talk messages, but we use this case to illustrate the problem of the continuous model.

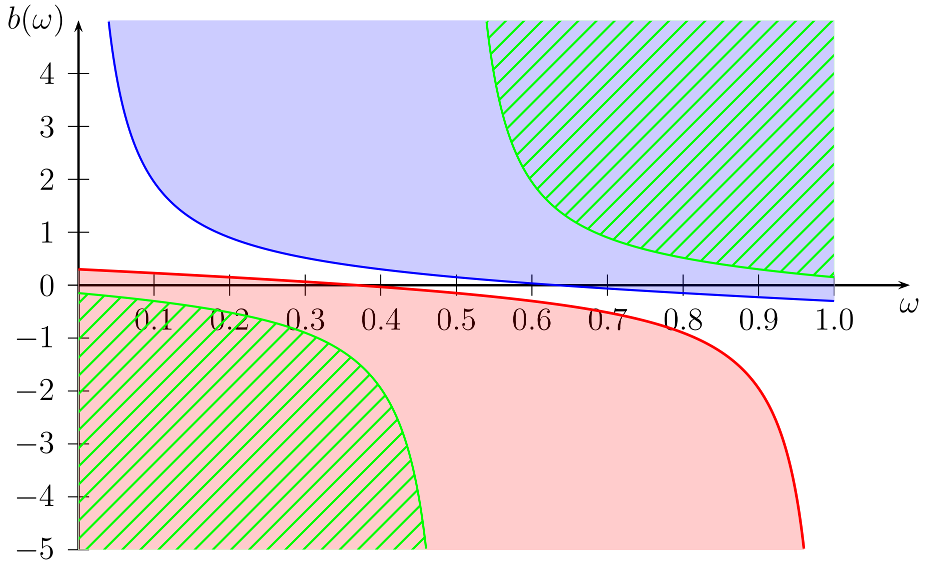

Figure 3: Illustration of Lemma 1 for $\hat{\omega} = 0$, 0.5 and 1 with $c = 0.4$

Figure 3 shows the result from Lemma 1 for three different values of $\hat{\omega}$. For $\hat{\omega} = 0$, the function $b(\omega)$ has to above the blue curve (in the blue area). For $\hat{\omega} = 1$, the function $b(\omega)$ has to be below the red curve (in the red area). For $\hat{\omega} = 0.5$, the function $b(\omega)$ has to be below the green curve for $\omega < 0.5$ and above for $\omega > 0.5$ (in the green shaded area).

This figure already reveals a problem with this setting: No matter the value of $\hat{\omega}$, it is necessary that the bias function $b(\omega)$ has either really high or low values. The problem here is that the bias function has values in the real numbers, but Condition 1 or 2 require $|b(\omega)| = \infty$, for some ω. This means if the Receiver answers every cheap-talk message with $\hat{\omega}$ there is always a neighborhood around $\hat{\omega}$ where the Sender prefers sending the costless cheap-talk message over sending the costly verifiable message. The Sender's utility loss by the quadratic loss function (difference between action and state) is less than the utility loss resulting from the cost c.

3.2 State dependent cost

In the discrete model we could illustrate Theorem 1 as in Figure 1. If $u_S(\hat{a}, \omega)$ is following the violet curve, there exists a fully revealing equilibrium in the discrete setting in which the Sender uses both message types. If we change the state space to the entire interval $[\omega_1, \omega_6]$ this does not hold anymore, because in the states close to ω_2 the violet curve is above the red-dashed line and so the Sender prefers to use a cheap-talk message over the usage of the verifiable message.

In the previous propositions we have discussed several ways of discontinuity which may result in the existence of fully revealing equilibria in the continuous model in which the Sender uses both message types. As long as the cost for the verifiable message is positive in all states we cannot get this type of equilibrium without discontinuity, but if we allow for state dependent cost $c(\omega) \geq 0$ this changes. Going back to the example of the violet curve in Figure 1: If the verifiable message is costless in all the states in which the violet curve is above the red-dashed line, there exists a fully revealing equilibrium in which the Sender uses both message types. Let us denote by $\underline{\omega}$ the state of the world close to ω_1 in which the red-dashed line and the violet curve intersect and by $\overline{\omega}$ the second intersection (between ω_2 and ω_3). If $c(\omega) = 0$ holds for all the states in the intervals $[\underline{\omega}, \omega_2)$ and $(\omega_2, \overline{\omega}]$, then the Sender will use the cheap-talk message in ω_2 and the verifiable message in all other states.

Theorem 1* (Full Revelation by Cheap-Talk and Verifiable Messages)**.**
There is a fully revealing equilibrium with both message types used if and only if there exists $\hat{\Omega} \subsetneq \Omega$ *with* $\hat{\Omega} \neq \emptyset$ *such that*

$$1.\ \forall \omega_i \notin \hat{\Omega} : u_S(a_R^*(\omega_i), \omega_i) - c(\omega) > u_S(a_R^*(\omega_j), \omega_i)\ \forall \omega_j \in \hat{\Omega}$$
$$2.\ \forall \omega_i \in \hat{\Omega} : u_S(a_R^*(\omega_i), \omega_i) > u_S(a_R^*(\omega_j), \omega_i)\ \forall \omega_j \in \hat{\Omega}, \omega_j \neq \omega_i$$

If Ω is continuous, this result clearly requires that for states that are close to $\hat{\Omega}$ the verification has to be costless.

In the case of quadratic loss functions, as discussed before, we can specify the conditions. Let us assume that the Sender uses different cheap-talk messages on the interval $[\underline{\omega}, \overline{\omega}]$. This requires that on that interval the Sender has no incentive to deviate, see Condition 2 of Theorem 1* with $\hat{\Omega} = [\underline{\omega}, \overline{\omega}]$. With state independent cost, in states close to that interval the cost of the verifiable message were to high for the Sender so she would not use it. She only uses the verifiable message in the state $\underline{\omega} - \epsilon$ (analogue for $\overline{\omega} + \epsilon$) if:

$$\begin{aligned} u_S(a_R^*(\underline{\omega} - \epsilon), \underline{\omega} - \epsilon) &\geq u_S(a_R^*(\underline{\omega}), \underline{\omega} - \epsilon) \\ \Leftrightarrow \quad -(b(\underline{\omega} - \epsilon))^2 - c(\underline{\omega} - \epsilon) &\geq -(\epsilon - b(\underline{\omega} - \epsilon))^2 \\ \Leftrightarrow \quad c &\leq \epsilon^2 - 2 \cdot \epsilon \cdot b(\underline{\omega} - \epsilon) \end{aligned}$$

As the bias function has to go to zero as $\underline{\omega} - \epsilon$ goes to $\underline{\omega}$, we can see that the right hand side of the inequality goes to zero if ϵ goes to zero. This implies that for states very close to $\underline{\omega}$ the cost for the verifiable message has to be very close to zero. Moving further away from $\underline{\omega}$ even with higher cost there can be a fully revealing equilibrium in which the Sender uses different cheap-talk messages on the interval $[\underline{\omega}, \overline{\omega}]$ and the verifiable message in the remaining states.

Of course, it is also possible that several disjoint intervals exist on which the Sender uses cheap-talk messages. In that case the cost for the verifiable message has to be close to zero around all those intervals.

4 Conclusion

In this paper we have combined the cheap-talk model of Crawford and Sobel (1982) with the models dealing with verifiable messages. In our Sender-Receiver game the informed Sender can choose between verifiable and non-verifiable messages. While the Receiver only learns the true state for sure after reading a verifiable message, a cheap-talk message will not reveal the true state to him, but let him update his belief system. We stated conditions for a discrete setting under which the Sender reveals the true state to the Receiver. The main idea behind is known from other models as well: The Receiver punishes the Sender for not using the verifiable message by answering every cheap-talk message with an action the Sender dislikes. As we limit our attention to non-dominated action, there always exists a belief system which makes this action best reply and so it makes the threat credible.

If such action does not exist, full revelation can only be achieved by common interests. In that case the Sender has no reason to lie and the Receiver can trust every cheap-talk message. Otherwise there can be only partial revelation or no revelation at all. We briefly discussed the different possibilities for partial revelation. In the case of partial revelation, the Receiver either learns the true state in some states or he can exclude certain states from consideration after reading the message of the Sender.

In a continuous model the enforcement of full revelation is more difficult. If the utility functions of Sender and Receiver are continuous, there is no fully revealing equilibrium where the Sender uses both message types. We have illustrated that with the standard example of the quadratic loss function. If we allow that the cost for the verifiable message is state dependent and that the verifiable message is even costless in some states of the world we can have fully revealing equilibria in which the Sender uses different message types in different states. Otherwise the only possibility to get fully revealing equilibria in the continuous model are discontinuous utility functions.

All in all we stated results that allow to check whether there are fully revealing equilibria or not. Therefore we distinguish between three different types of fully revealing equilibria: The one where both message types are used and those where the Sender always sticks to one kind of message.

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Appendix

Proof. Proposition 1
Only if: Clearly there is a fully revealing equilibrium with only cheap-talk messages if Condition (1) holds: The Receiver will trust every cheap-talk message and the Sender has no incentive to deviate.
If: Proof by contradiction. Let us assume that $\exists \omega_k$ such that $u_S(a_R^*(\omega_k), \omega_k) \not> u_S(a_R^*(\omega_j), \omega_k)$ $\forall \omega_j \neq \omega_k$. This implies that there exists ω_j such that $u_S(a_R^*(\omega_k), \omega_k) < u_S(a_R^*(\omega_j), \omega_k)$ holds. Then the Receiver has an incentive to lie in ω_k and send the cheap-talk message ω_j, so there is no full revelation. $\square$

Proof. Proposition 2
Only if: Follows directly.
If: Proof by contradiction.
Step 1. Let us assume that Condition 2 does not hold. Then there exists a ω_j such that $u_S(a_R^*(\omega_j), \omega_j) - c < u_S(\hat{a}, \omega_j)$ holds. This implies that the Sender prefers sending a cheap-talk message and getting action $\hat{a}$ over sending the verifiable message and action $a_R^*(\omega_j)$. So she will deviate in ω_j and there will be no full revelation.
Step 2. Let us assume that Condition 1 does not hold, then Condition 2 does not hold and we can follow Step 1. $\square$

Proof. Theorem 1
Only if: The equilibrium is as follows: For $\omega \in \hat{\Omega}$ the Receiver trusts the cheap-talk and in all other states the Sender uses the verifiable message.

If: Proof by contradiction.
Step 1. Let us assume that Condition 1 does not hold. This implies that there exist $\omega_i \notin \hat{\Omega}$ and $\omega_j \in \hat{\Omega}$ such that $u_S(a^*_R(\omega_j), \omega_i) > u_S(a^*_R(\omega_i), \omega_i) - c$ holds. So the Sender prefers cheap-talk (and action $a^*_R(\omega_j)$) over the verifiable message (and action $a^*_R(\omega_i)$) and there will be no full revelation, because $a^*_R(\omega_i) \neq a^*_R(\omega_j)$.
Step 2. We assume that Condition 2 does not hold and follow the same steps as in the proof of Proposition 1. □

Proof. Theorem 2
The possible existence of fully revealing equilibria with just one type of message sent follows from the conditions imposed in Proposition 1 and Proposition 2. Assume that the Sender sends a cheap-talk message in $\hat{w}$ and uses the verifiable message in all other states. The argumentation for sending cheap-talk in several states or intervals will be the same. The Sender has an incentive to use the verifiable message if $u_S(a^*_R(\omega), \omega) - c \geq u_S(a^*_R(\hat{\omega}), \omega)$. So for the states close to $\hat{\omega}$ we get:

$$u_S(a^*_R(\hat{\omega} \pm \epsilon), \hat{\omega} \pm \epsilon) - c \geq u_S(a^*_R(\hat{\omega}), \hat{\omega} \pm \epsilon) \tag{2}$$

For $\epsilon \to 0$ and the continuity of u_S and a^*_R this is equivalent to:

$$u_S(a^*_R(\hat{\omega}), \hat{\omega}) - c \geq u_S(a^*_R(\hat{\omega}), \hat{\omega})$$

This leads to $c \leq 0$, which is clearly a contradiction. So under this assumptions it is not possible that there is a fully revealing equilibrium where the Sender uses both message types. □

Proof. Proposition 3
Assume that Condition 1 or 2 do not hold, the problem is the same as described in equation (2), which requires a non-positive cost c. □

Proof. Proposition 4
The proof is analogue to the proof of Proposition 3, using the discontinuity of u_S. □

Proof. Lemma 1
Assume that the Receiver answers every cheap-talk message with $\hat{\omega}$.
The utility of the Sender for any state ω is given by:
u_S("verifiable message") $= -(-b(\omega))^2 - c$
u_S("cheap-talk message") $= -(\hat{\omega} - \omega - b(\omega))^2 = -\left[(\hat{\omega} - \omega)^2 - 2(\hat{\omega} - \omega) \cdot b(\omega) + (b(\omega))^2\right]$

So the Sender will use the verifiable message if and only if:

$$\begin{aligned} -(-b(\omega))^2 - c &> -\left[(\hat{\omega}) - \omega)^2 - 2(\hat{\omega} - \omega) \cdot b(\omega) + (b(\omega))^2\right] \\ \Leftrightarrow \qquad -2b(\hat{\omega} - \omega) &> -(\hat{\omega} - \omega)^2 + c \end{aligned}$$

Case 1: $\omega > \hat{\omega}$

$$\begin{aligned} \Leftrightarrow \qquad -2b &< -(\hat{\omega} - \omega) + \tfrac{c}{\hat{\omega} - \omega} \\ \Leftrightarrow \qquad b &> \tfrac{\hat{\omega} - \omega}{2} - \tfrac{c}{2(\hat{\omega} - \omega)} \end{aligned}$$

Case 2: $\hat{\omega} > \omega$

$$\begin{aligned} \Leftrightarrow \qquad -2b &> -(\hat{\omega} - \omega) + \tfrac{c}{\hat{\omega} - \omega} \\ \Leftrightarrow \qquad b &< \tfrac{\hat{\omega} - \omega}{2} - \tfrac{c}{2(\hat{\omega} - \omega)} \end{aligned}$$

□

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