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Family Altruism with Renewable Resource and Population Growth

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In an overlapping-generations model with non-constant population growth, households own a natural renewable resource and have a family-altruism resource bequest motive. The natural resource can be either extracted and sold to firms, or bequeathed to children to increase their adult disposable income. Numerical applications show how family altruism interplays with population growth to shape the whole economy. The role of altruism in the case of two negative demographic shocks shows that the pressure on the natural resource is not necessarily reduced when population size is lower. Transmission mechanisms between generations and general equilibrium effects can yield unexpected outcomes. In particular, family altruism can lead either to preserve or to waste the resource.

Keywords: overlapping generations; population growth; renewable resource

1. INTRODUCTION

Population growth is a critical variable for environment, notably on natural resources. Most of original forests have been lost due to human activities, and future declines in the availability of forests, especially in developing countries, raise major challenges for both bio-diversity conservation and human well-being. Today, more than 2.2 billion people live in 46 countries with less than 0.1 hectare of

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forested land per head, an indicator of critically low levels of forest cover. Based on medium population projections and current deforestation trends, by 2025 the total number of people living in forest-scarce countries could reach 3.2 billion in 54 countries.[1] Avoiding excessive population growth is generally advocated as a prerequisite to sustainability in many developing countries by alleviating too high a pressure on natural resources thanks to a lower extraction rate.

The consequence of population growth on natural resource availability remains disputed. Abernathy (1993), Avise (1993), and Holdren (1992) focus on overpopulation, but empirical evidence does not clearly support that point of view. Li (1991), for example, emphasizes that Chinese forested area changed from 8% in 1949 to 12% in 1984 and to 8% again in 1988, while population grew steadily during that period. By analyzing cross-correlations between national socio-economic indicators, including population growth, and the rate of change of forest cover, FAO (2001) shows that the only variable coming near significance is the proportion of the rural population. Still, it only accounts for 14% of the variation in forest cover at the national level. The deforestation process involves physical, climatic, political, and socio-economic forces. Would a smaller population size necessarily yield a lower resource extraction and, thereby, guarantee resource preservation?

We shall show that this conjecture is too mechanistic because it neglects the effect of demography on the economy and on resources. Different population trends can yield different accumulation processes of man-made and natural forms of capital, leading to unexpected outcomes. These issues require us to use a dynamic general equilibrium model of the economy with an adequate representation of the transmission of resources between generations. We use a general equilibrium overlapping-generations model in which a natural renewable resource (a forest) is used beside man-made capital and labor to satisfy the needs of a growing population.

Amacher et al. (1999) and Ollikainen (1998) study the bequest of timber between generations without modelling a final good production sector. We consider the production process of a consumption-investment good as in Diamond (1965), in natural resource and in man-made capital. Both enter the production function, besides labor. In the forest literature with overlapping generations (OLG), either the production process or the aggregate stock of physical capital beside labor and the extracted resource is ignored (Koskela et al., 2000, 2002;

[1]Source: Population Action International, People in the Balance Update 2006. See also the statistics from FAO (2001).

Olson and Knapp, 1997). We are interested in the transition path, notably in reaction to a demographic shock, whereas most authors focus on steady states. We develop an original intergenerational transmission mechanism of the natural resource. This transmission is usually done either by selling the not yet extracted resource stock or by bequeathing it. When the resource is sold, households are assumed to be selfish (Koskela et al., 2002; Mourmouras, 1991), or households have a resource bequest motive. The assumed bequest motive is based on altruism *à la* Barro (1974), parents care about their children's utility (Amacher et al., 1999). As Becker (1993) admits, this form of intergenerational concern requires human foresight capacities beyond the capacities of the most prescient human being. Alternatively, parents have a joy-of-giving resource bequest motive (Ollikainen, 1998; Bréchet and Lambrecht, 2006). We use family altruism developed without natural resource (Lambrecht et al., 2005; Lambrecht et al., 2006). Parents care about their children's adult income and bequests are made under the form of *numéraire*. Children's adult disposable income also enters the utility function of each family head but the extracted resource is a source of income for young adults and the non-extracted resource stock constitutes the means of the bequest.

We justify the family altruism model (section 2), present the model of population and renewable resource, family altruism resource bequest motive and individuals' and firms' behavior (section 3). We shall define the competitive temporary and inter-temporal equilibrium (section 4). A numerical application and a scenario will highlight its main properties (section 5). The consequences of a temporary drop in population size and a permanent slowdown of fertility are analyzed in section 6.

2. FAMILY ALTRUISM

To study the interplay between population growth and the use of a renewable resource with altruism, we retain neither Barro's (1974) *dynastic altruism* nor Andreoni's (1989) *joy-of-giving*, but *family altruism*.

For the conservation of a natural resource, an infinite-horizon altruism model such as Barro's (1974) is ill suited to understand how private agents interact on market and in their families, when the decision of passing the resource stock to the next generation is at stake. For Barro (1974) and Becker (1991), parents feel concerned about their children through altruistic links, which are operative when parents transfer the *numéraire* good to their children in order to shield them from shocks in well-being. Barro (1974) took a altruistic hypothesis in which altruistic households solve a problem equivalent to the infinitely lived representative agent's. When applied to the effects of public debt, this dynastic altruistic model leads to the conclusion that households can offset any policy of income redistribution between generations through public debt. Barro (1974) revived the Ricardian equivalence and stimulated the debate around the intergenerational effect of public policies. One criticism is that a dynasty is assumed not only to behave as one decision unit but also to have the capacity to foresee future prices and incomes.

However, in the presence of a natural resource, private agents could exhaust the resource because they can fail to foresee the consequences of their present decision. Is there a chance to maintain the stock of the resource in the long run? Under a Barro (1974) altruism the answer is almost always yes: If the chain of bequests in uninterrupted, the dynasty behaves at equilibrium almost as a benevolent planner would do, especially if the discount factor is alike. On the contrary, a finite-horizon altruism leaves the answer open.

Many kinds of finite-horizon altruism have been considered (Michel et al., 2006). For example, the joy-of-giving resource bequest motive of Andreoni (1989) (called *warm glow*) is compatible with a finite time horizon. However, the magnitude of resource transfers is independent of the relative affluence and of the opportunities open to children (Bréchet and Lambrecht, 2006).

Family altruism combines the finite horizon feature and the sensitivity of the bequest decision to changes in the children's economic situation (Lambrecht et al., 2005, who study how pay-as-you-go pensions can foster growth; Lambrecht et al., 2006, who discuss the implication of family altruism for public debt policy). The main assumption of the family model is that the decision unit in which intergenerational links are operative is the family as opposed to the dynasty. This is more realistic than the dynastic model and the joy-of-giving model. We shall focus on the bequest of the natural resource out of an altruistic bequest motive and leave aside bequests of the *numéraire* as well as family investment in the human capital of children under the form of educational expenditures.

Lambrecht et al. (2006), who include *numéraire* bequests and educational expenditures, show that families with a binding bequest constraint under-invest in human capital. Families fail to exhaust all the marginal gains which could otherwise be achieved by freely trading between present personal utility and future income of offspring. This amounts to say that, at the family head's optimum, the return on investment in human capital remains higher than the interest factor. The two returns are not equal to each other. With a natural resource, family heads attaining their non-negativity constraint on *numéraire* bequests would optimally under-invest in their children's income through resource bequest. They would further exploit the resource and sell more of it to the production sector, than in the case of a non-binding constraint.

How can we apply family altruism with a natural renewable resource? We have to redefine the children's adult disposable income cautiously assuming that the natural resource is exploited by each young generation and sold to the firms.

3. THE MODEL

In an OLG economy *à la* Diamond (1965), we first assume that individuals leave bequests of natural resource to their children, apart from the family altruism bequest motive (Lambrecht et al., 2005; Lambrecht, 2006) and that the population growth rate changes over time.

3.1. Family Altruism Bequest Motive and Fertility

Let N_t denote the total number of young individuals at time t. Generations change according to:

$$N_{t+1} = BN_t^{\nu}, \tag{1}$$

with $\nu \in (0, 1)$, $B > 0$ a scale factor and N_{-1} the exogenous total number of old individuals in the initial period. Whatever the initial N_{-1}, the steady-state size of each generation is $N = B^{1/(1-\nu)}$. Consider any time period t on the transition. N_t is the total number of young individuals. The total number of children who will be young adults at time $t+1$ is N_{t+1}; the ratio N_{t+1}/N_t is the "fertility factor," which converges to unity. This is at odd with the standard OLG model *à la* Diamond (1965) in which the ratio between the size of a young and an old generation, $N_{t+1}/N_t := 1 + n$, remains constant, and equal to:

$$1 + n_{t+1} \equiv \frac{N_{t+1}}{N_t} = BN_t^{\nu-1}. \tag{2}$$

Let the size of the initial generation N_{-1} be less (resp. greater) than the steady-state size N. The convergence toward N is monotonically increasing (resp. decreasing). The fertility factor $1 + n_{t+1}$ follows a decreasing (resp. increasing) path toward unity as the size of the generation increases (resp. decreases). As households are homogenous, the factor $1 + n_{t+1}$ is also the total number of children in each household.

We consider the family as a decision unit surviving for two periods. It is composed of a family head and her $1 + n_{t+1}$ children during the first period of their life cycle (adulthood). Each individual is child of the family constituted by her parent in the previous period and constitutes her own family when she is young. Her own family lives for two periods. Altruism is descendant, which means that parents care about their children, but children do not care for their parents.

Families in Lambrecht et al. (2006) are equivalent to the union of Diamond's (1965) households and the next households during their adulthood. During childhood, individuals make no decision. The preferences of a family head are defined over her life cycle consumption, c_t and d_{t+1}, and over her $1 + n_{t+1}$ children's adult disposable income ω_{t+1}. In the dynastic model, preferences are defined over consumption and children's utility, which is formally equivalent to the infinite sum of utilities defined over the whole sequence of consumption of all generations. The sequence of altruistic descendants of a founding father (generation $t = 0$) does not behave as a single dynasty and there is no need to foresee.

The fertility factor $1 + n_{t+1}$ and the size of families $2 + n_{t+1}$ change over time. The dynamics of the generation size is increasing and concave which implies that if the size of generation increases toward the steady state, the family size decreases. In the standard family altruism model with constant population growth, family size and fertility remain constant. Family heads care about the adult disposable incomes of less and less children. The reverse is true for a decreasing population. The population dynamics introduces a trend in the utility function. This means that preferences are time-dependent.

The utility function is additively separable:

$$U_t = (1 - \beta)u_1(c_t) + \beta u_2(d^e_{t+1}) + (1 + n_{t+1})\gamma u_3(\omega^e_{t+1}), \qquad (3)$$

where d^e_{t+1} and ω^e_{t+1} are, respectively, the expected second-period consumption and the expected adult disposable income of each of the $1 + n_{t+1}$ children.

The other extension to the standard family altruism model concerns the expectations formed by the family head on the adult disposable income ω^e_{t+1}. Each young individual works and extracts a renewable resource in the first period of her life. She supplies to firms one unit of labor inelastically on the labor market, for a real wage w_t, and the quantity e_t of the extracted resource on the resource market, for a real price q_t. At each time t, the family head forms expectations about the real wage and resource price at time $t + 1$, she decides about w^e_{t+1} and q^e_{t+1}. She expects her children to extract e^e_{t+1}. The expected adult disposable income of a young individual, as anticipated by the family head at time t, is:

$$w^e_{t+1} + q^e_{t+1} e^e_{t+1} = \omega^e_{t+1}. \tag{4}$$

3.2. The Renewable Resource

We assume that there exists a renewable resource in private property. At any time t, each family head inherits a share z_{t-1} of the family resource stock. This stock has its own natural return, which yields Cz^{ζ}_{t-1}, with $C > 0$ a scale factor, to each family head. In the absence of extraction, this stock Cz^{ζ}_{t-1} is shared among the $1 + n_{t+1}$ children. The dynamics of the resource stock without extraction is:

$$z_t = \frac{Cz^{\zeta}_{t-1}}{1 + n_{t+1}}, \tag{5}$$

with $\zeta \in (0, 1)$. Without extraction, the family head's resource stock converges to a steady state equal to $z = C^{1/(1-\zeta)}$ and $1 + n_{t+1}$ tends to unity.

3.3. The Individuals

To characterize the family head, we assume that her utility is log-linear:

$$U_t = (1 - \beta) \log c_t + \beta \log d^e_{t+1} + (1 + n_{t+1})\gamma \log \omega^e_{t+1}. \tag{6}$$

The income in the first period of a young family head is $\omega_t = w_t + q_t e_t$, it is shared between consumption c_t and saving s_t, giving the first-period budget constraint:

$$w_t + q_t e_t = c_t + s_t. \tag{7}$$

The amount of resource which has not been extracted, $Cz^{\zeta}_{t-1} - e_t$, is bequeathed equally to the $1 + n_{t+1}$ children by the family head. This means that the dynamics of the families' resource stock with extraction is:

$$z_t = \frac{Cz^{\zeta}_{t-1} - e_t}{1 + n_{t+1}}. \tag{8}$$

When old, individuals hold the capital stock through their savings and earn a capital income which they consume entirely:

$$R^e_{t+1} s_t = d^e_{t+1}, \tag{9}$$

where R^e_{t+1} is the expected interest factor on saving s_t (one plus the expected interest factor r^e_{t+1}), and d^e_{t+1} is the expected old-age consumption.

The young family head forms expectations to evaluate the adult disposable income of her children, given by Eq. (4). She can sustain her children's adult disposable income by increasing her resource bequests; we rule out bequest of the *numéraire* as in most altruistic models. Eq. (4) is re-written as:

$$w^e_{t+1} + q^e_{t+1}\left(\left(\frac{Cz^{\zeta}_{t-1} - e_t}{1+n_{t+1}}\right)^{\zeta} - (1+n_{t+2})z^e_{t+1}\right) = \omega^e_{t+1}. \tag{10}$$

The family head maximizes her utility of Eq. (6) under the constraints of Eq. (7), (9), and (10) and taking prices and expectations as given. The solution is characterized by the saving and the resource extraction decisions (after substitution):

$$\max_{s_t,e_t}(1-\beta)\log(w_t + q_t e_t - s_t) + \beta\log(R^e_{t+1}s_t) + (1+n_{t+1})\gamma\log\left(w^e_{t+1} + q^e_{t+1}\left(\left(\frac{Cz^{\zeta}_{t-1} - e_t}{1+n_{t+1}}\right)^{\zeta} - (1+n_{t+2})z^e_{t+1}\right)\right). \tag{11}$$

The first-order conditions are:

$$\frac{1-\beta}{w_t + q_t e_t - s_t} = \frac{\beta}{s_t}, \tag{12}$$

$$\frac{(1-\beta)q_t}{w_t + q_t e_t - s_t} \leq \frac{\gamma q^e_{t+1}\zeta\left(\frac{Cz^{\zeta}_{t-1} - e_t}{1+n_{t+1}}\right)^{\zeta-1}}{w^e_{t+1} + q^e_{t+1}\left(\left(\frac{Cz^{\zeta}_{t-1} - e_t}{1+n_{t+1}}\right)^{\zeta} - (1+n_{t+2})z^e_{t+1}\right)}. \tag{13}$$

The last condition holds true with equality if extraction is positive. It holds true with strict inequality when the optimal extraction is null. The latter appears when, at zero extraction, the marginal benefit from extraction in terms of consumption c_t is larger then the marginal loss in terms of the children's expected adult disposable income ω^e_{t+1}. In the sequel we focus on the case of optimal positive extraction, the case when, at zero extraction, the marginal benefit of extraction is smaller than the marginal gain. Savings can be written as a function of extraction e_t:

$$s_t = \beta(w_t + q_t e_t). \tag{14}$$

and the second condition with equality is written as:

$$(1-\beta)q_t\left(w^e_{t+1}+q^e_{t+1}\left(\left(\frac{Cz^{\zeta}_{t-1}-e_t}{1+n_{t+1}}\right)^{\zeta}-(1+n_{t+2})z^e_{t+1}\right)\right)$$
$$-\gamma q^e_{t+1}\zeta\left(\frac{Cz^{\zeta}_{t-1}-e_t}{1+n_{t+1}}\right)^{\zeta-1}(w_t+q_te_t-s_t)=0. \quad (15)$$

We have a system of two equations in variables s_t and e_t.

In Diamond's (1965) model, the life cycle income and budget constraint are built by adding up income and expenditure of the whole life cycle, in present value. In the family model, we add up income and expenditure over the life cycle plus the adult disposable income of the $1+n_{t+1}$ children, in present value. Formally, we add the present value of Eq. (7), (9), and 4 times $(1+n_{t+1})$. Denote the family intertemporal income by Ω_t, we have:

$$\Omega_t \equiv w_t+q_te_t+\frac{1+n_{t+1}}{R^e_{t+1}}(w^e_{t+1}+q^e_{t+1}e^e_{t+1})=c_t+\frac{d^e_{t+1}}{R^e_{t+1}}+\frac{1+n_{t+1}}{R^e_{t+1}}\omega^e_{t+1}. \quad (16)$$

In the RHS, this family budget displays the three elements of preferences, which are the three items of expenditures of the family head. Any increase in the family income Ω_t is spent over these three items. The buffer used by the family head to transfer income from the c_t to d^e_{t+1} is saving s_t and the one used to transfer income from c_t to ω^e_{t+1} is resource bequest z_t.

3.4. The Firm

The representative firm produces the output Y_t by combining three production factors capital K_t, labor L_t and extracted resource E_t with a Cobb-Douglas technology:

$$Y_t = AK_t^{\alpha_K}L_t^{\alpha_L}E_t^{\alpha_E}. \quad (17)$$

Considering the real interest factor R_t, the real wage w_t and the real resource price q_t as given, the representative firm maximizes its profit in real terms π_t by choosing its demands of capital, labor and resource. We define the real profit as:

$$\pi_t = AK_t^{\alpha_K}L_t^{\alpha_L}E_t^{\alpha_E}-R_tK_t-w_tL_t-q_tE_t. \quad (18)$$

The first-order conditions are:

$$q_t = \alpha_E A \left(\frac{K_t}{L_t}\right)^{\alpha_K} \left(\frac{E_t}{L_t}\right)^{\alpha_E - 1}; \tag{19}$$

$$R_t = \alpha_K A \left(\frac{K_t}{L_t}\right)^{\alpha_K - 1} \left(\frac{E_t}{L_t}\right)^{\alpha_E}; \tag{20}$$

$$w_t = \alpha_L A \left(\frac{K_t}{L_t}\right)^{\alpha_K} \left(\frac{E_t}{L_t}\right)^{\alpha_E}. \tag{21}$$

The firm hires the services of capital, labor, and resource up to the point where their respective marginal productivities equal their respective price.

4. THE COMPETITIVE EQUILIBRIUM

We analyze the temporary equilibrium of period t and then the intertemporal equilibrium.

4.1. Temporary Equilibrium

At t, the aggregate capital stock K_t, which depends on past saving decisions ($K_t = N_{t-1}s_{t-1}$), the family inherited resource stock z_{t-1}, which depends on past extraction decision and past family resource bequest ($z_{t-1} = (Cz_{t-2}^{\zeta} - e_{t-1})/(1 + n_{t+1})$), the young generation size N_t, whose dynamics is $N_t = BN_{t-1}^{\nu}$, and the expectations on the next period real wage, the real wage w_{t+1}^e, and the resource prices q_{t+1}^e are given.

For all t, the temporary equilibrium at t is defined as a vector of prices R_t, w_t, q_t, of individual quantities c_t, s_t, e_t, z_t, d_t, and of aggregate quantities $Y_t, K_t, L_t, E_t, N_{t+1}$, such that all agents, families, and firms maximize their objective function subject to their constraints, and all markets (output, capital, labor, and resource) are clear. We characterize the equilibrium values at time t of these endogenous variables. The conditions of equality between supply and demand of labor, capital, and resource are given by:

- $N_t = L_t$ (exogenous labor supply);
- $K_{t+1} = N_t s_t$;
- $N_t e_t = E_t$.

with the equilibrium prices:

- $R_t = R\left(\frac{K_t}{N_t}, \frac{E_t}{N_t}\right) \equiv \alpha_K A \left(\frac{K_t}{N_t}\right)^{\alpha_K - 1} \left(\frac{E_t}{N_t}\right)^{\alpha_E};$

- $q_t = q\left(\frac{K_t}{N_t}, \frac{E_t}{N_t}\right) \equiv \alpha_E A \left(\frac{K_t}{N_t}\right)^{\alpha_K} \left(\frac{E_t}{N_t}\right)^{\alpha_E - 1}$;
- $w_t = q\left(\frac{K_t}{N_t}, \frac{E_t}{N_t}\right) \equiv \alpha_L A \left(\frac{K_t}{N_t}\right)^{\alpha_K} \left(\frac{E_t}{N_t}\right)^{\alpha_E}$.

At equilibrium, in the two equations in saving s_t and extraction e_t, prices w_t and q_t are superseded by their equilibrium expressions $w(k_t, e_t)$ and $q(k_t, e_t)$ with $k_t = K_t/N_t$ and $e_t = E_t/N_t$. The solutions of this system are function of K_t, z_{t-1}, N_t, w^e_{t+1}, q^e_{t+1}, z^e_{t+1}. The other individual variables at equilibrium are obtained by using the family constraints.

4.2. The Competitive Inter-Temporal Equilibrium

We define the competitive inter-temporal equilibrium as a sequence of temporary equilibria, given the initial conditions K_0, N_{-1}, z_{-1} and a rule of formation of expectations on w^e_{t+1} and q^e_{t+1}.

Perfect foresight implies $w^e_{t+1} = w(k_{t+1}, e_{t+1})$ and $q^e_{t+1} = q(k_{t+1}, e_{t+1})$. The family head at time t computes the next period extraction of her children, whose decision is contemporaneous of the head's second period of life.

However, the children's extraction decision depends on the expectations of their own children's decision and so forth. As the family head organizes her resource bequests decision in a finite entity, expectations are myopic:

$$w^e_{t+1} = \alpha_L A k_t^{\alpha_K} e_t^{\alpha_E} \tag{22}$$

$$q^e_{t+1} = \alpha_E A k_t^{\alpha_K} e_t^{\alpha_E - 1}. \tag{23}$$

The family head expects her children to extract the same as the one she does, $z^e_{t+1} = z_t$.

Given the initial condition K_0, N_{-1}, z_{-1} and the rule of expectations, the competitive inter-temporal equilibrium with myopic foresight is characterized by a sequence $\{k_{t+1}, e_t, z_t\}_{t=0}^{+\infty}$ which satisfies:

$$(1 + n_{t+1})k_{t+1} - \beta(1 - \alpha_K)A k_t^{\alpha_K} e_t^{\alpha_E} = 0, \tag{24}$$

$$\begin{aligned}(1-\beta)\alpha_E A k_t^{\alpha_K} e_t^{\alpha_E - 1}\Bigg(\alpha_L A k_t^{\alpha_K} e_t^{\alpha_E} + \alpha_E A k_t^{\alpha_K} e_t^{\alpha_E - 1}\Bigg(\left(\frac{C z_{t-1}^{\zeta} - e_t}{1 + n_{t+1}}\right)^{\zeta} \\ - (1 + n_{t+2})z_t\Bigg)\Bigg) - \gamma \alpha_E A k_t^{\alpha_K} e_t^{\alpha_E - 1} \zeta \left(\frac{C z_{t-1}^{\zeta} - e_t}{1 + n_{t+1}}\right)^{\zeta - 1} \\ \times ((1 - \alpha_K)A k_t^{\alpha_K} e_t^{\alpha_E} - (1 + n_{t+1})k_{t+1}) = 0\end{aligned} \tag{25}$$

$$z_t = \frac{Cz_{t-1}^{\zeta} - e_t}{1 + n_{t+1}}. \tag{26}$$

5. APPLICATION

5.1. Parameters and Computation

We compute the sequence of temporary equilibria. It consists of pre-determined labor supply N_t and demand L_t; of six simultaneous equations on resource extraction e_t, bequeathed resource stock z_t, savings s_t and real resource price q_t, interest factor R_t and wage rate (w_t); and of post-determined utility and aggregate variables; of expected wage w_{t+1}^e and resource price q_{t+1}^e for the next period.

The initial conditions are K_0, N_{-1}, and z_{-1}. After childhood, individuals live for two periods of 25 years each. The model runs over 20 periods. The implicit equation giving the level of individual extraction is solved through the Newton-Raphson algorithm and the whole model is solved with the Gauss-Seidel algorithm, under the integrated software IODE developed by the Belgian Federal Planning Bureau and publicly available at *www.plan.be*. Table 1 displays the values of the parameters used in the reference scenario. Most of them are conventional. We set up ν and ζ of the population and resource dynamics such that the population steady-state is reached in 15 periods and that the family resource stock without extraction increases over time. Scale parameters were applied to the population and resource dynamics to obtain a level greater than one at the steady state. Two values of γ are considered, a low one at 1.1 and a high one at 1.5.

5.2. Reference Scenario

The main variables are represented on Figure 1, with thin lines for $\gamma = 1.1$ and thick lines for $\gamma = 1.5$.

TABLE 1 Parameter Values

Symbol	Description	Value
α_k	Share of capital in output	0.30
α_l	Share of labor in output	0.60
α_e	Share of natural resource in output	0.10
β	Weight of old-age consumption in utility function	0.25
γ	Degree of family altruism	1.1 or 1.5
ν	Population own's dynamics	0.65
ζ	Natural resource own's dynamics	0.65

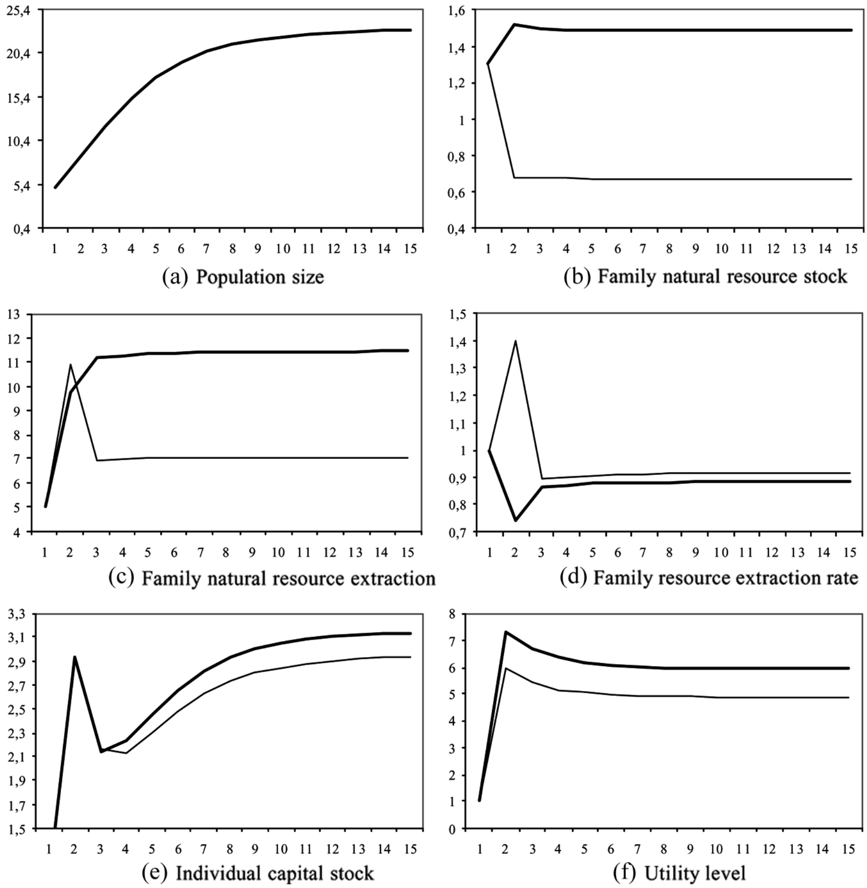

FIGURE 1 Two reference scenarios (thin lines: $\gamma = 1.1$; thick lines: $\gamma = 1.5$).

The solution converges to a steady state. The population grows while decelerating and converges to its steady state after 15 periods. Capital per head and family income Ω_t of Eq. (16) increase during the transition, after an overshoot due to the initial condition. Population, natural resource, and aggregate capital stocks increase over time.

As shown in Figure 1f, utility decreases on the transition path, because fertility and consumption and the old d_{t+1} decrease:

$$d_{t+1} = R_{t+1}s_t = (1 + n_{t+1})(\alpha_k A k_{t+1}^{\alpha_k} e_{t+1}^{\alpha_e})$$

where the first term in parentheses decreases, and the second term increases but slows down. These two effects dominate the increase in consumption of the young and in adult disposable income. This fact depends on the initial conditions, but consumption over the life cycle $c_t + d_{t+1}/\ (R_{t+1})$ increases in this simulation.

Altruism is the only motive for households not to extract and sell the whole resource. If the degree of altruism is too low, the natural resource collapses, carrying away the whole economy. In an OLG model with a joy-of-giving bequest motive, Bréchet and Lambrecht (2006) show this possibility. Here, with family altruism, the lowest value of γ compatible with a positive resource stock is 1.05.

As expected, the higher γ, the higher the family stock of natural resource, as shown in Figure 1b. Extraction is higher, as shown on Figure 1c. This paradox comes from the fact that during the whole transition, families prevent themselves from extracting, leading to more resource, which allows them, at the steady state, to extract more. However, the family extraction rate (Figure 1d) is lower, suggesting that the pressure on the resource is reduced. Capital intensity increases with γ (Figure 1e). Subsequently, family income and utility level increase with γ at the steady state.

6. TWO DEMOGRAPHIC SHOCKS

Figures 2a and 2b show two shocks: a one-third drop due to an epidemic or a war in population size N_3 of the young generation at time $t = 3$, and a decrease in fertility ν from 0.65 to 0.55 from $t = 3$ onwards.

6.1. A Drop in the Population Size

All variables are unchanged in the long run, as Table 2 shows. The shock has only transitory effects.

Let x_t and $\tilde{x}_t$ denote a variable x at time t without and with the shock, respectively. The one-period exogenous shock yields $\widetilde{N}_3 < N_3$

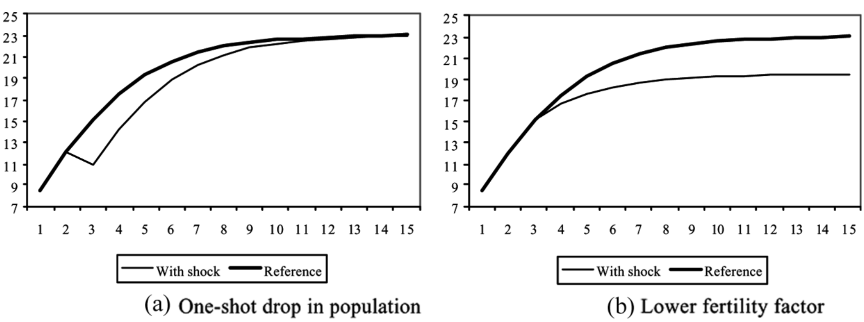

FIGURE 2 The effects of the two demographic shocks on population.

TABLE 2 Effects of a One-Shot Drop in Population Size

	5th time period		20th time period	
	low γ	high γ	low γ	high γ
Individual variables				
Young-age consumption	−2.37	−2.38	0.00	0.00
Old-age consumption	5.03	5.00	0.00	0.00
Savings	−2.37	−2.39	0.00	0.00
Bequest	−2.37	−2.38	0.00	0.00
Renewable natural resource				
Individual extraction	−0.33	−0.42	0.00	0.00
Individual stock	0.14	0.17	0.00	0.00
Individual extraction rate* (point of %)	−0.39	−0.47	0.00	0.00
Total stock	−12.56	−12.54	0.00	0.00
Aggregates				
Population	−12.68	−12.68	0.00	0.00
Capital stock	−19.30	−19.33	0.00	0.00

(% w.r.t. the reference scenario, except*)

and, in period 4, $\widetilde{N}_3 < \widetilde{N}_4 < N_4$. The fertility factor for the $t = 3$ generation is given by $1 + \tilde{n}_4 = B\widetilde{N}_3^{\nu-1}$. It experiences a temporary jump, because $1 + \tilde{n}_4 > 1 + n_4$, then goes back gradually to its steady state. This explains why population size does not change in the long run.

Even though the long run population size remains the same after a shock, does the temporarily lower after-shock population size relieve the pressure on the resource?

Table 2 shows that the response of the aggregate resource stock is negatively affected, a result opposite to our initial guess. The resource extraction rate, defined as e_t/Cz_t^{ζ}, and the natural resource stock per head Z_t/N_t also measure demographic pressure. Table 2 shows that the resource extraction rate falls as well and that the bequeathed family stock increases. Because the total number of children dropped with the demographic shock, each child of generation "young" in $t = 3$ inherits a higher resource stock. What will be the arbitrage of these young family heads at time $t = 3$ between their current consumption $(\tilde{c}_3)$, their consumption when old $(\tilde{d}_4)$ and the income of its heirs $(\tilde{\omega}_4)$? As these goods are normal, young individuals increase the three of them. They do so by increasing savings $\tilde{s}_3$ and the resource bequest $\tilde{z}_3$, with respect to the level they would have reached without the shock.

The effect on extraction $(\tilde{e}_3)$ is ambiguous. One effect is that a higher inherited stock allows a higher extraction and that the induced higher real wage and resource price foster equilibrium extraction.

A second effect is about the higher fertility rate $\tilde{n}_4$ following the negative demographic shock. The RHS of saturated Eq. (13) increases through higher n_{t+1}, which is the marginal utility of the children's disposable income. Family heads react to this shock by a lower extraction $\tilde{e}_3$ ($<e_3$), or equivalently a higher bequest. Decreasing extraction e_t reduces the RHS of Eq. (13) and increases its LHS. For our chosen parameter values this negative effect dominates the positive effect.

In sum, combining a temporary decrease in individual extraction and a temporary increase in the individual family stock yields a decreasing extraction rate, although families are less numerous and the aggregate resource stock lower.

What role does the family altruism bequest motive play? We examine the sensitivity of the variables to the one-shot drop in population size with respect to altruism, γ. How does the transitory decrease in resource extraction combine with altruism? As Table 2 shows, the magnitude of the fall in extraction is higher when family heads have a strong altruistic motive. The parameter γ magnifies the effect of a given change in population size. In this scenario, it magnifies the increase in the marginal utility of ω_{t+1}. A larger increase in bequest, and a larger decrease in extraction, are necessary to re-establish the optimal trade-off between consumption c_t and children's adult disposable income ω_{t+1}. Only the fall in the aggregate stock is lower under strong altruism: to compute the aggregate resource stock under strong altruism, we combine the response of the individual resource stock, increasing in γ, and the response of the total number of families, which is independent of γ. The fall in the aggregate resource stock is lower in absolute value for a high degree of altruism.

6.2. A Lower Fertility Rate

Unlike a one-shot drop in population size, a drop in the fertility parameter ν has a long run effect on population size. As shown in Figure 2b, the population size is lower in the long run. One may expect that the fall in the fertility rate reduces the demographic pressure on the natural resource permanently, leading to a higher long run aggregate resource stock. The parameters of the simulations are such that the resource stock is always positive in the long run, so that we rule out the possibility of resource extinction (Bréchet and Lambrecht, 2006). As shown in Table 3, the answer is no, according to the model.

While population size is reduced in the long run, the aggregate resource stock is smaller as if demographic pressure had increased. In the same way we proceeded with the one-shot fall with population size, we look at the resource extraction rate, e_t/Cz_t^{ζ}, and at the natural

TABLE 3 Effects of a Lower Fertility Rate

	5th time period		20th time period	
	low γ	high γ	low γ	high γ
Individual variables				
Young-age consumption	1.65	1.66	0.00	0.00
Old-age consumption	−2.06	−2.05	0.00	0.00
Savings	1.65	1.66	0.00	0.00
Bequest	1.65	1.66	0.00	0.00
Renewable natural resource				
Individual extraction	0.17	0.23	0.00	0.00
Individual stock	−0.08	−0.12	0.00	0.00
Individual extraction rate* (point of %)	0.20	0.27	0.00	0.00
Total stock	−8.82	−8.86	−15.58	−15.58
Aggregates				
Population	−8.75	−8.75	−15.58	−15.58
Capital stock	−3.70	−3.68	−15.58	−15.58

(% w.r.t. the reference scenario, except*).

resource stock per head Z_t/N_t. In the long run these two indicators are unaffected by the drop in fertility, excluding a relief in demographic pressure. During the transition, these two indicators show a temporary increase of demographic pressure on the resource (the extraction rate is higher and the individual resource stock lower).

This result is because the shock decreases the marginal utility of the children's disposable income, ω_{t+1} (RHS of Eq. (13)), because wage and resource revenues increase due to higher capital accumulation and resource extraction, and because the drop in fertility also decreases the RHS of Eq. (13). Subsequently, the family head reduces her resource bequest.

Table 3 shows that the magnitude of the rise in extraction is actually higher when family heads have a strong altruistic motive. The reason is that the parameter γ magnifies any change in the marginal utility of the children's disposable income, be it positive, as in the one-shot drop, or negative as in this scenario. With respect to the one-shot drop in population size, the effects of the shock in fertility go in the opposite direction, as far as extraction and bequest are concerned. A higher γ magnifies the decrease in the marginal utility of ω_{t+1}. A larger decrease in bequest, and a larger increase in extraction, are needed to re-establish the optimal trade-off between c_t and ω_{t+1}.

At the steady state, all variables per head are the same as in the reference scenario. One could have expected that the degree of altruism influences long run responses, as during the transition.

The population growth rate is actually the same, and equal to zero, in the long run without or with the drop of fertility. The increase in extraction is only temporary and individual extraction converges to its level without the demographic shock, whatever the degree of family altruism. The long run level of the aggregate resource stock depends on the degree of altruism but the magnitude of its long run response to the shock in fertility does not depend on the degree of altruism.

7. CONCLUSION

We developed an OLG model in which the size of generations varies across time and converges to a steady state. A private natural renewable resource, such as a forest, is both extracted and bequeathed out the family altruism bequest motive.

We used this model to study how population growth influences the pressure on the renewable resource and the equilibrium path of the economy. We highlighted the role of family altruism in the case of a one-shot drop in population size and in the case of a lower fertility rate.

In the the one-shot drop, families face a transitory decrease in extraction and an increase in the resource stock simultaneously. This leads to a decrease in the extraction rate. The overall stock is temporarily reduced; stronger family altruism reduces the negative consequences of the shock on the aggregate resource stock. In the long run however, all variables return to their reference values. When considering a slowdown of fertility, the demographic pressure increases during the transition (higher extraction rate and lower family resource stock) because of an endogenously stronger altruism. So, the stronger the degree of family altruism, the higher the pressure of the resource during the transition. In the long run, all individual variables are untouched, but the total resource stock and population size are reduced.

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