



2018 / 11

DP

Justina Klimaviciute, Pierre Pestieau
and Jérôme Schoenmaeckers

Long-term care insurance with
family altruism: Theory and empirics



CORE

Voie du Roman Pays 34, L1.03.01

Tel (32 10) 47 43 04

Fax (32 10) 47 43 01

Email: immaq-library@uclouvain.be

[https://uclouvain.be/en/research-institutes/
immaq/core/discussion-papers.html](https://uclouvain.be/en/research-institutes/immaq/core/discussion-papers.html)

Long-term care insurance with family altruism: theory and empirics*

Justina Klimaviciute[†], Pierre Pestieau[‡] and Jérôme Schoenmaeckers[§]

April 4, 2018

Abstract

This paper studies long-term care (LTC) insurance in the presence of family altruism. In the first, theoretical, part of the paper, we explore whether and how family solidarity affects the application to LTC of Arrow's (1963) theorem of the deductible, which is shown to apply in models without family by a number of papers. We consider two types of family altruism, perfect and imperfect, and find that Arrow's theorem generally holds, even though some departures from the standard model and some differences between the types of altruism exist. Our analysis highlights a complex interplay between parents' insurance and their children's aid, which implies that a number of intuitive conjectures are not always verified. For instance, while one would expect the deductible to be increasing in the child's degree of altruism, this is unambiguously verified only under certain conditions. Given the ambiguity of some results, in the second part of the paper, we resort, more generally, to an empirical test of the relation between LTC insurance and children's altruism using the data from the *Health and Retirement Study* (HRS). Our findings suggest that children's altruism has a negative impact on parents' LTC insurance purchases, even though some results also point to this relationship being more complex than one might think.

Keywords: long-term care insurance, deductible theorem, altruism, family aid

JEL Classification: D64, I13, J14

*Financial support from the Belgian Science Policy Office (BELSPO) research project CRESUS and the "Chaire Marché des risques et création de valeur" of the FdR/SCOR is gratefully acknowledged.

[†]CREPP, University of Liege, e-mail: justina.klimaviciute@ulg.ac.be.

[‡]CREPP, University of Liege, CORE, Université Catholique de Louvain and PSE, e-mail: p.pestieau@ulg.ac.be.

[§]CREPP, University of Liege, e-mail: jerome.schoenmaeckers@ulg.ac.be.

1 Introduction

In a number of papers (Drèze et al., 2016, Klimaviciute and Pestieau, 2018a,b), Arrow's (1963) theorem of the deductible, i.e. the proposition that it is optimal to focus insurance coverage on the states with largest expenditures, was applied to the case of long-term care (LTC) insurance.¹ These papers show that if LTC insurance involves some loading costs, then the optimal contract should cover expenditures that are incurred above a certain amount (spending or years of dependency). That way, the consumption of elderly people would tend to be smoothed across states of nature. In this work the role of the family was assumed away except that joy of giving bequests are implicit. This restriction is questionable given that the bulk of LTC comes from the family most often informally.

In this paper we want to introduce family altruism, both descending and ascending, and analyze to what extent it modifies the results obtained without it. In particular, we want to see whether the promise of assistance from altruistic children does not invalidate the above results and if not, what is the incidence of altruism on the deductible.²

We adopt a simple static setting with ex ante identical individuals. Children and parents coexist and the parents face three states of nature. They can be either healthy, or slightly dependent, or severely dependent. Both parents and children have an exogenous level of resources. We implicitly expect that children have enough resources to assist their dependent parents.

We consider two models. In the first, we have perfect altruism between parents and children and all the decisions including that of buying a LTC insurance are taken in such a way that they maximize the overall utility of both parties. In the second, children's altruism depends on the level of dependence of their parents. Their decision to help their parents is made after the state of dependence of the latter is known.

Anticipating on the rest of the paper, we compare the results with and without family assistance. What prevails from our analysis is that the insurance with deductible is the most efficient way of reimbursing losses inflicted by dependence whatever the type of family solidarity. However, the exact implications on the deductible differ depending on the type of altruism. The main conclusion is that the deductible is the same at both severity

¹For a survey of the literature on the economics of LTC, see Cremer et al. (2012) and Norton (2000).

²There exist a number of papers dealing with the interaction between family altruism and LTC insurance schemes. However, these papers focus on public schemes and do not consider the possibility of a deductible. See, for instance, Cremer et al. (2016) and Pestieau and Sato (2008).

levels of dependence in the absence of family solidarity as well as in the case of perfect altruism. Turning to the case of imperfect altruism, we show that the deductible will be state-invariant only if the degree of altruism is the same in both states. If the child is more altruistic towards his parent in case of severe dependence, the deductible is then higher granted that the third derivative of the utility functions is non-positive. Otherwise, the result is ambiguous.

An important implication of altruism, differently from the standard model with no family help, is the presence of an interplay between the deductible and the child's aid. As we will show, a consequence of this interplay is that a number of intuitive conjectures are not always verified. Let us mention the most evident. First, we would expect that the child's aid decreases with the level of his parent's wealth and increases with his degree of altruism, in the case of imperfect altruism. Also we would find normal to see parents purchasing less insurance when their children are more altruistic. As we will see, answers to these questions are not straightforward.

Given the ambiguity of some results, we have tested empirically the role of altruism on the purchase of insurance using the data from the *Health and Retirement Study* (HRS), a panel of elderly Americans. We know that in the US more than in Europe there is a sizeable LTC insurance market, which motivates our choice of database. Our findings suggest that children's altruism has a negative impact on parents' LTC insurance purchases, even though some results also point to this relationship being more complex than one might think.

The rest of the paper is organized as follows. Section 2 provides a theoretical analysis of LTC insurance in the presence of family altruism focusing in particular on the implications for the applicability of Arrow's (1963) theorem of the deductible. The cases of perfect and imperfect altruism are first studied separately and then compared in the summary sub-section. Section 3 is devoted to the empirical study of the relationship between LTC insurance and children's altruism. Finally, Section 4 provides some concluding remarks, while some more technical material is presented in the Appendices.

2 Theoretical models: insurance, deductible and family altruism

As discussed in the Introduction, in this section we are interested in exploring the implications of family altruism to the application of Arrow's (1963) theorem of the deductible for LTC.

We consider a representative family consisting of an elderly parent and his adult child. The parent has

a wealth w and the child has an (exogenous) income y . The family faces the risk of the parent becoming dependent and needing LTC. In particular, with probability π_1 , the parent experiences dependence with a low severity level in which case he has LTC needs (expressed in terms of costs incurred) L_1 , with probability π_2 , he suffers from a heavy dependence with LTC needs $L_2 > L_1$, and with probability $1 - \pi_1 - \pi_2$, he remains healthy. To protect from this risk, private LTC insurance can be bought and is characterized by a premium P and the reimbursement of fractions α_1 and α_2 of LTC needs in, respectively, state 1 and state 2 (with $0 \leq \alpha_1 \leq 1$ and $0 \leq \alpha_2 \leq 1$). Consistently with empirical evidence, we assume that LTC insurance is not actuarially fair and denote by λ the insurance loading cost. The premium is therefore given by

$$P = \pi_1(1 + \lambda)\alpha_1 L_1 + \pi_2(1 + \lambda)\alpha_2 L_2 \quad (1)$$

We study two cases of family altruism. In the first case, we assume perfect altruism between the parent and the child who take all the decisions by maximizing the sum of their utilities. In the second case, the parent is self-concerned while the child's altruism depends on the level of the parent's dependence.

2.1 Perfect altruism

In the case of perfect altruism, the expected utility function of the family writes as

$$U_F = \pi_1 \left[u \left(\underbrace{w - P - (1 - \alpha_1) L_1 + a_1}_{m_1} \right) + v \left(\underbrace{y - a_1}_{c_1} \right) \right] + \pi_2 \left[u \left(\underbrace{w - P - (1 - \alpha_2) L_2 + a_2}_{m_2} \right) + v \left(\underbrace{y - a_2}_{c_2} \right) \right] + \\ + (1 - \pi_1 - \pi_2) \left[u \left(\underbrace{w - P + a_3}_d \right) + v \left(\underbrace{y - a_3}_{c_3} \right) \right]$$

where $u(\cdot)$ is the utility function of the parent and $v(\cdot)$ is the utility function of the child. Moreover, m_1 , m_2 and d are the parent's consumption levels in the states of light, heavy and no dependence, while c_1 , c_2 and c_3 are the consumption levels of the child in these states. Finally, a_1 , a_2 and a_3 are the amounts of (financial) aid provided by the child to the parent in the three states of nature.³

³ a can be seen as encompassing both (the value of) assistance in time and financial aid. We assume that the child's income is sufficiently large compared to the parent's wealth so that he makes a transfer to the parent.

The parent and the child jointly decide what should be the levels of the child's help in each state of nature and what insurance coverage the parent should buy. This is done by maximizing the expected utility of the family subject to the constraint (1) for the insurance premium, which gives the following Lagrangian:

$$\mathcal{L} = U_F + \mu [P - \pi_1(1 + \lambda)\alpha_1 L_1 - \pi_2(1 + \lambda)\alpha_2 L_2]$$

where μ is the Lagrange multiplier associated with the constraint for the premium. The FOCs write as follows:

$$\frac{\partial \mathcal{L}}{\partial a_1} = u'(m_1) - v'(c_1) = 0 \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial a_2} = u'(m_2) - v'(c_2) = 0 \quad (3)$$

$$\frac{\partial \mathcal{L}}{\partial a_3} = u'(d) - v'(c_3) = 0 \quad (4)$$

$$\frac{\partial \mathcal{L}}{\partial P} = -\pi_1 u'(m_1) - \pi_2 u'(m_2) - (1 - \pi_1 - \pi_2) u'(d) + \mu = 0 \quad (5)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_1} = u'(m_1) - \mu(1 + \lambda) \leq 0, \quad \alpha_1 \frac{\partial \mathcal{L}}{\partial \alpha_1} = 0 \quad (6)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_2} = u'(m_2) - \mu(1 + \lambda) \leq 0, \quad \alpha_2 \frac{\partial \mathcal{L}}{\partial \alpha_2} = 0 \quad (7)$$

We are now going to explore whether the equilibrium insurance policy is in line with Arrow's theorem of the deductible. For this, first note that from (6) and (7), we have that either $\alpha_j = 0$ or $u'(m_j) = \mu(1 + \lambda)$, with $j = 1, 2$. The second equality is equivalent to

$$(1 - \alpha_j)L_j = w - P + a_j - u'^{-1}(\mu(1 + \lambda))$$

Note that since $u'(m_1) = u'(m_2) = \mu(1 + \lambda)$, (2) and (3) imply $v'(c_1) = v'(c_2)$ and thus $a_1 = a_2 = a$.

Denoting

$$w - P + a - u'^{-1}(\mu(1 + \lambda)) \equiv D,$$

we can write

$$\alpha_j = \max \left[0; \frac{L_j - D}{L_j} \right]$$

From this we can see that if the needs L_j are lower than D , it is optimal for the individual to have zero insurance coverage and to bear all the costs himself, whereas if the needs are higher than D , the optimal insurance is such that the individual actually pays the amount D and the rest is covered by the insurer. This is precisely the message of Arrow's theorem of the deductible. Thus, like in the model without family aid,⁴ Arrow's theorem holds with perfect family altruism.

Further, focusing on interior solutions (i.e. assuming that the needs are higher than the deductible at both severity levels of dependence), we can combine (5) with (6) and (7) to get

$$\frac{u'(d)}{u'(m_j)} = \frac{1 - \pi_1(1 + \lambda) - \pi_2(1 + \lambda)}{(1 - \pi_1 - \pi_2)(1 + \lambda)} < 1 \quad (8)$$

This is the same condition as in the model without family aid. This means that $d > m_j$, and, more particularly in this context, that

$$w - P + a_3 > w - P - D + a$$

$$\Leftrightarrow$$

$$D > a - a_3 > 0$$

since it can be shown that $a > a_3$, where $a = a_1 = a_2$.

On the other hand, if there are no loading costs ($\lambda = 0$), then we have $u'(d) = u'(m_j)$, from which it follows that $v'(c_3) = v'(c_j)$ and so $a_3 = a$. It also follows that $D = a - a_3 = 0$. Therefore, just like in the model without family aid, we have that the deductible is strictly positive as long as loading costs are not zero.

Finally, we investigate how the deductible depends on the parent's initial wealth w . A common result in the model without family aid is that the deductible is increasing (resp. decreasing and constant) in initial wealth under DARA (resp. IARA and CARA) preferences.⁵ In the presence of the family, however, the relationship

⁴For the analysis of the model without family aid, see the *laissez-faire* section in Klimaviciute and Pestieau (2018a).

⁵DARA (resp. IARA and CARA): decreasing (resp. increasing and constant) absolute risk aversion. For more details, see Appendix A.

between the deductible and wealth becomes more complicated. We show in Appendix A that the derivative of D with respect to w has four terms. Two terms reflect the result obtained in the model without family aid, i.e. their sum is positive (resp. negative and equal to zero) when the parent's preferences are DARA (resp. IARA and CARA). However, there are two more terms which reflect the (direct)⁶ effect of w on the child's aid in the healthy state and in the states with dependence. This effect is negative in both cases but pushes the deductible into different directions. The reduction of the child's aid in the healthy state decreases the parent's wealth in that state and thus makes it less desirable to transfer resources to the states of dependence, i.e. pushes for buying less insurance (and so having a larger deductible). On the other hand, the decrease of the child's aid in the states of dependence calls for buying more insurance and thus pushes for a lower deductible. The comparison of the effect on the child's aid in different states of nature depends on the properties of the parent's and the child's utility functions and in particular, on their third derivatives. The effect on the deductible therefore depends on the third derivatives as well. More precisely, we show in Appendix A that under CARA, the deductible is decreasing in w if $v''' \leq 0$ and ambiguous otherwise⁷, under IARA, it is decreasing if $u''' \geq 0$ and $v''' \leq 0$ and ambiguous otherwise, while under DARA, the effect is always ambiguous. We therefore see that the presence of family altruism modifies the relationship between the deductible and the parent's wealth.

Summing up the analysis of this subsection, the main conclusion is that perfect family altruism does not alter the application of Arrow's theorem but makes the deductible interdependent with family aid, which creates some departures from the standard model.

2.2 Imperfect altruism

We now turn to the case of imperfect altruism. Altruism in our model is imperfect in several ways. First, we now assume that only the child can be altruistic towards his parent, whereas the parent is self-concerned. Second, the weight that the child gives to the utility of the parent is not necessarily equal to one but is rather reflected by his degree of altruism $\gamma \leq 1$. Moreover, this degree of altruism is not necessarily the same in all states of nature. For simplicity, we assume that the child is not altruistic when the parent is healthy,⁸ but his

⁶Since aid and insurance are interdependent, the full effect of w on aid depends also on the effect on D , as can be seen in equations (20) and (21) in Appendix A.

⁷Note that CARA always implies $u''' > 0$.

⁸This implies that in our theoretical model the child provides no aid in this state of nature. In our empirical analysis, however, we do study children's aid provided to non-dependent parents. The theoretical assumption of zero altruism can be seen as a

altruism is triggered by the parent's dependence. We denote by γ_1 and γ_2 the degree of the child's altruism respectively in the state of a light and in the state of a heavy dependence. We do not a priori impose how γ_1 and γ_2 compare with each other, but it might be reasonable to expect $\gamma_1 \leq \gamma_2$.⁹

Since the family is no longer making joint decisions, it is important to introduce the timing of choices. In particular, we assume that the parent moves first by choosing his insurance coverage before the realization of the dependence risk. Then, if the parent becomes dependent, the child, observing the severity level of dependence and the parent's insurance coverage, decides how much aid he is going to provide. If the parent remains healthy, no action is taken.

We solve the model reasoning backwards and thus starting with the choices of the child. In state j ($j = 1, 2$), the utility of the child writes as

$$U_k^j = v \left(\underbrace{y - a_j}_{c_j} \right) + \gamma_j u \left(\underbrace{w - P - (1 - \alpha_j) L_j + a_j}_{m_j} \right)$$

The FOC for a_j is

$$-v'(c_j) + \gamma_j u'(m_j) = 0$$

From this we can derive

$$\frac{\partial a_j}{\partial P} = \frac{\gamma_j u''(m_j)}{v''(c_j) + \gamma_j u''(m_j)} > 0 \quad \text{but} \quad < 1 \quad (9)$$

$$\frac{\partial a_j}{\partial \alpha_j} = -\frac{\gamma_j u''(m_j) L_j}{v''(c_j) + \gamma_j u''(m_j)} < 0 \quad (10)$$

Turning to the parent, his expected utility is given by

$$U_p = \pi_1 u \left(\underbrace{w - P - (1 - \alpha_1) L_1 + a_1}_{m_1} \right) + \pi_2 u \left(\underbrace{w - P - (1 - \alpha_2) L_2 + a_2}_{m_2} \right) + (1 - \pi_1 - \pi_2) u \left(\underbrace{w - P}_d \right)$$

normalization reflecting the expectation that altruism and aid in the healthy state are likely to be negligible compared to the states with dependence.

⁹Note that a is a financial transfer that encompasses both assistance in time and financial aid. In case of severe dependence, institutionalization can be necessary, which means that the children will move from assistance in time to financial aid.

and the Lagrangian for the choice of insurance can be written as

$$\mathcal{L} = U_p + \mu [P - \pi_1(1 + \lambda)\alpha_1 L_1 - \pi_2(1 + \lambda)\alpha_2 L_2]$$

When choosing P and α_j , the parent anticipates the effect that they will have on the child's aid, i.e. (9) and (10). The FOCs can then be written as follows:

$$\frac{\partial \mathcal{L}}{\partial P} = -\pi_1 u'(m_1) \frac{v''(c_1)}{v''(c_1) + \gamma_1 u''(m_1)} - \pi_2 u'(m_2) \frac{v''(c_2)}{v''(c_2) + \gamma_2 u''(m_2)} - (1 - \pi_1 - \pi_2) u'(d) + \mu = 0; \quad (11)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_1} = u'(m_1) \frac{v''(c_1)}{v''(c_1) + \gamma_1 u''(m_1)} - \mu(1 + \lambda) \leq 0, \quad \alpha_1 \frac{\partial \mathcal{L}}{\partial \alpha_1} = 0; \quad (12)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_2} = u'(m_2) \frac{v''(c_2)}{v''(c_2) + \gamma_2 u''(m_2)} - \mu(1 + \lambda) \leq 0, \quad \alpha_2 \frac{\partial \mathcal{L}}{\partial \alpha_2} = 0; \quad (13)$$

From (12) and (13), we have that either $\alpha_j = 0$ or $u'(m_j) = \mu(1 + \lambda) \frac{v''(c_j) + \gamma_j u''(m_j)}{v''(c_j)}$. The second equality is equivalent to

$$(1 - \alpha_j)L_j = w - P + a_j - u'^{-1} \left(\mu(1 + \lambda) \frac{v''(c_j) + \gamma_j u''(m_j)}{v''(c_j)} \right)$$

Denoting

$$w - P + a_j - u'^{-1} \left(\mu(1 + \lambda) \frac{v''(c_j) + \gamma_j u''(m_j)}{v''(c_j)} \right) \equiv D_j, \quad (14)$$

we can write

$$\alpha_j = \max \left[0; \frac{L_j - D_j}{L_j} \right]$$

These conditions are similar to the ones obtained in the perfect altruism case (and in the model with no family), except that here the level of the deductible D_j is generally state-specific. Indeed, we can ask ourselves whether D_1 and D_2 can be equal and if not, how they compare.

In fact, it can be verified that D_1 and D_2 can clearly be equal if $\gamma_1 = \gamma_2$ but not necessarily if $\gamma_1 \neq \gamma_2$. To see this, assume first that $\gamma_1 = \gamma_2 = \gamma$ and $D_1 = D_2 = D$. Then from the child's problem it follows that $a_1 = a_2$. This in turn implies that $c_1 = c_2$ and $m_1 = m_2$. Using all this in (14), we can see that indeed there

are no contradictions and $D_1 = D_2$ holds.

Now let us take the case $\gamma_1 < \gamma_2$ and assume again that $D_1 = D_2 = D$. In that case, from the child's problem we have $a_1 < a_2$. So, in (14) there is already one term which calls for $D_1 < D_2$. But we also need to compare $\frac{v''(c_1) + \gamma_1 u''(m_1)}{v''(c_1)}$ and $\frac{v''(c_2) + \gamma_2 u''(m_2)}{v''(c_2)}$, which gives

$$\frac{v''(c_2) + \gamma_2 u''(m_2)}{v''(c_2)} - \frac{v''(c_1) + \gamma_1 u''(m_1)}{v''(c_1)} = \frac{\gamma_2 u''(m_2) v''(c_1) - \gamma_1 u''(m_1) v''(c_2)}{v''(c_2) v''(c_1)}$$

Noting that $c_1 > c_2$ and $m_1 < m_2$, we will clearly have $\frac{\gamma_2 u''(m_2) v''(c_1) - \gamma_1 u''(m_1) v''(c_2)}{v''(c_2) v''(c_1)} > 0$ if $u''' \leq 0$ and $v''' \leq 0$.

Thus, when $u''' \leq 0$ and $v''' \leq 0$, we have $u'^{-1} \left(\mu(1 + \lambda) \frac{v''(c_2) + \gamma_2 u''(m_2)}{v''(c_2)} \right) < u'^{-1} \left(\mu(1 + \lambda) \frac{v''(c_1) + \gamma_1 u''(m_1)}{v''(c_1)} \right)$. This, together with $a_1 < a_2$, implies that $D_2 > D_1$, which is a contradiction to $D_1 = D_2 = D$. On the other hand, if $u''' \leq 0$ and $v''' \leq 0$ does not hold, the comparison of D_1 and D_2 becomes ambiguous.

The result of state-dependent deductibles is due to the fact that altruism is now imperfect and not invariant. First, since the degree of the child's altruism may vary according to the state of nature, the amount of aid anticipated by the parent varies as well, which has a direct consequence on the deductible chosen: the deductible tends to be higher in the state of nature where the child's altruism is stronger. Second, the parent knows that the child's aid is influenced by insurance coverage and takes the reaction of a_j into account. The reaction of aid to insurance coverage is also related to the child's degree of altruism and thus may also vary according to the state of nature, which also impacts the choice of deductible. However, since this reaction depends on the functional forms of the utility functions, we need some assumptions about the third derivatives, namely $u''' \leq 0$ and $v''' \leq 0$, to have an unambiguous comparison of deductibles.

Assuming interior solutions, we can combine (11) with (12) and (13) to get

$$\frac{u'(d)}{u'(m_j)} = \frac{[1 - \pi_1(1 + \lambda) - \pi_2(1 + \lambda)] v''(c_j)}{(1 - \pi_1 - \pi_2)(1 + \lambda) [v''(c_j) + \gamma_j u''(m_j)]} < 1 \quad (15)$$

This means that $d > m_j$, i.e.

$$w - P > w - P - D_j + a_j$$

\Leftrightarrow

$$D_j > a_j > 0$$

It is interesting to note that this result holds even if $\lambda = 0$, that is, even in the absence of loading costs, the deductible is strictly positive. Moreover, the deductible is always strictly higher than the child's aid. This comes from the fact that insurance discourages aid and, taking the reaction of aid into account, the parent prefers keeping a relatively low insurance coverage (i.e. a relatively high deductible) even when insurance is actuarially fair. To a certain extent, the parent uses the child's altruism to increase his resources even though he could get full insurance.

We have seen above that the comparison of deductibles in different states of nature depends on the comparison of the child's degrees of altruism in these states. We now explore how the size of the deductible in a given state varies when the degree of altruism changes.

We first look at the case when $\gamma_1 = \gamma_2 = \gamma$ and $D_1 = D_2 = D$. Assuming interior solutions, we then have $a_1 = a_2 = a$, $c_1 = c_2 = c$ and $m_1 = m_2 = m$. The parent's expected utility can then be rewritten as

$$U_p = \pi_1 u \left(\underbrace{w - P - D + a}_m \right) + \pi_2 u \left(\underbrace{w - P - D + a}_m \right) + (1 - \pi_1 - \pi_2) u \left(\underbrace{w - P}_d \right)$$

where $P = \pi_1(1 + \lambda)(L_1 - D) + \pi_2(1 + \lambda)(L_2 - D)$.

The parent's problem is now to choose D , which, noting that $\frac{\partial a}{\partial D} = -\frac{\gamma u''(m)[\pi_1(1+\lambda)+\pi_2(1+\lambda)-1]}{v''(c)+\gamma u''(m)} > 0$, gives the following FOC:

$$\begin{aligned} & (\pi_1 + \pi_2) u'(m) [\pi_1(1 + \lambda) + \pi_2(1 + \lambda) - 1] \frac{v''(c)}{v''(c) + \gamma u''(m)} + \\ & + (1 - \pi_1 - \pi_2) u'(d) [\pi_1(1 + \lambda) + \pi_2(1 + \lambda)] = 0 \end{aligned} \tag{16}$$

From this we can derive

$$\frac{\partial D}{\partial \gamma} = -\frac{[Z]}{SOC_D}$$

where $SOC_D < 0$ is the second-order condition for D and

$$[Z] \equiv (\pi_1 + \pi_2) u''(m) \frac{\partial a}{\partial \gamma} [\pi_1(1 + \lambda) + \pi_2(1 + \lambda) - 1] \frac{v''(c)}{v''(c) + \gamma u''(m)} +$$

$$+ (\pi_1 + \pi_2) u'(m) [\pi_1(1 + \lambda) + \pi_2(1 + \lambda) - 1] \frac{\left[-\frac{\partial a}{\partial \gamma} [v'''(c)\gamma u''(m) + v''(c)\gamma u'''(m)] - v''(c)u''(m) \right]}{[v''(c) + \gamma u''(m)]^2}$$

with $\frac{\partial a}{\partial \gamma} = -\frac{u'(m)}{v''(c) + \gamma u''(m)} > 0$ being the direct effect of γ on aid.¹⁰

The sign of $\frac{\partial D}{\partial \gamma}$ depends on the sign of $[Z]$. While the first term of $[Z]$ is always positive, the second term is clearly positive only if $v'''(c) \leq 0$ and $u'''(m) \leq 0$. Otherwise, the sign of the second term is ambiguous. This implies that $\frac{\partial D}{\partial \gamma} > 0$ if $v'''(c) \leq 0$ and $u'''(m) \leq 0$, but the sign is ambiguous otherwise.

To see the intuition for this result, it is instructive to study the FOC (16). This FOC has two terms: the first one is negative and reflects the cost of having a higher deductible, while the second one is positive and reflects the benefit of a higher deductible (which is a higher consumption in the healthy state). It should be noted that changes in γ only affect the first term, i.e. the cost.

The cost of having a higher deductible depends on the family aid. The fact that this aid goes up when D increases, reduces the cost of a higher deductible: the first term of the FOC is multiplied by $\frac{v''(c)}{v''(c) + \gamma u''(m)} < 1$ which appears due to the family aid. The smaller $\frac{v''(c)}{v''(c) + \gamma u''(m)}$, the larger is the reduction in the cost of a higher deductible.

The impact of γ can be decomposed into two parts. The first part comes from the fact that a rise in γ increases a and this in turn increases m . Thus, the marginal utility in case of dependence decreases, which also reduces the cost of a higher deductible and pushes for increasing D (this is reflected by the first term of $[Z]$).

The second part appears because an increase in γ also affects $\frac{v''(c)}{v''(c) + \gamma u''(m)}$ (reflected by the second term of $[Z]$). This second part consists itself of two effects: direct and indirect. The direct effect of γ is a decrease in $\frac{v''(c)}{v''(c) + \gamma u''(m)}$, which implies a decrease in the cost of a higher deductible and thus pushes for a higher D . The indirect effect comes through the levels of c and m . Since γ increases a , this means that c is reduced and m goes up. If $v'''(c) < 0$ and $u'''(m) < 0$, we then have that $v''(c)$ increases (i.e. becomes smaller in absolute value) and $u''(m)$ decreases (i.e. becomes larger in absolute value), which means that $\frac{v''(c)}{v''(c) + \gamma u''(m)}$ is reduced. On the

¹⁰Since a is a function of D , the full effect of γ on aid depends also on the effect on D , i.e. the full effect is equal to $\frac{\partial a}{\partial \gamma} + \frac{\partial a}{\partial D} \frac{\partial D}{\partial \gamma}$.

other hand, if $v'''(c) > 0$ and $u'''(m) > 0$, we then have that $v''(c)$ decreases (i.e. becomes larger in absolute value) and $u''(m)$ increases (i.e. becomes smaller in absolute value), which pushes for increasing $\frac{v''(c)}{v''(c)+\gamma u''(m)}$ and thus the cost of having a higher D . Therefore, if $v'''(c) \leq 0$ and $u'''(m) \leq 0$ does not hold, some ambiguity arises, which prevents from definitively signing $\frac{\partial D}{\partial \gamma}$.

In Appendix B, we study the case when $\gamma_1 \neq \gamma_2$. In that case, the deductibles in the two states of dependence are different and things become more complicated since we then have interactions between these deductibles. In general, the most clear-cut results can be obtained if we assume either $v''' = 0$ and $u''' = 0$ or $v''' = 0$ and $u''' < 0$. In these cases, we know that the deductible increases in the state of nature in which the degree of altruism has increased, while the effect on the deductible in the other state is ambiguous. In all other cases, however, the effect on both deductibles is ambiguous. The results become somewhat closer to the case of a single deductible if we assume that the degree of altruism in one state is zero. For instance, if we suppose that $\gamma_1 = 0$, then we will have $\frac{\partial D_2}{\partial \gamma_2} > 0$ and $\frac{\partial D_1}{\partial \gamma_2} \leq 0$ as long as $v''' \leq 0$ and $u''' \leq 0$, which is the same condition as in the case of a single deductible. Otherwise, both signs will be ambiguous.

Finally, as in the case of perfect altruism, we explore how the deductible depends on the parent's initial wealth w . For this analysis, which is derived in Appendix C, we concentrate on the case with $\gamma_1 = \gamma_2 = \gamma$ and $D_1 = D_2 = D$. The results we obtain have some similarities with those found in the case of perfect altruism. We show that the derivative of D with respect to w has four terms and that two of these terms, as with perfect altruism, reflect the result of the model without family aid: their sum is positive (resp. negative and equal to zero) when the parent's preferences are DARA (resp. IARA and CARA). The third term is always negative and reflects the fact that an increase in the parent's initial wealth discourages the child's aid in the states of dependence, which calls for buying more insurance on the market (lower deductible). Such a term is also present with perfect altruism. On the other hand, with imperfect altruism we do not have a term associated with the aid in the healthy state since we assume that the child's altruism is zero in that state. Instead, we now have a term related to the reaction of the child's aid to insurance coverage. The sign of this term depends on the assumptions about the third derivatives of the utility functions.

We analyze the sign of $\frac{\partial D}{\partial w}$ by looking separately at the cases of DARA, CARA and IARA. Interestingly, we obtain the same signs and conditions for these signs as in the case of perfect altruism, even though, as it can be seen from the derivative of D , the underlying considerations are not exactly the same in both cases.

Table 1: **Comparison of deductibles**

	Perfect altruism	Imperfect altruism	No family
D_1 vs D_2	$D_1 = D_2$	$D_1 = D_2$ if $\gamma_1 = \gamma_2$, $D_2 > D_1$ if $\gamma_1 < \gamma_2$ and $u''' \leq 0$ and $v''' \leq 0$, $D_2 \leq D_1$ if $u''' \leq 0$ and $v''' \leq 0$ does not hold.	$D_1 = D_2$
D_j vs 0	$D_j > a_j - a_3 > 0$ if $\lambda > 0$, $D_j = a_j - a_3 = 0$ if $\lambda = 0$.	$D_j > a_j > 0$	$D_j > 0$ if $\lambda > 0$, $D_j = 0$ if $\lambda = 0$.

Namely, we find that under CARA, the deductible is decreasing in w if $v''' \leq 0$ and ambiguous otherwise, under IARA, it is decreasing if $u''' \geq 0$ and $v''' \leq 0$ and ambiguous otherwise, whereas under DARA, the effect is always ambiguous. The ambiguity under DARA arises because already the sum of the first three terms has an ambiguous sign. When initial wealth increases, under DARA, individuals become less risk averse and require less insurance (larger deductible), which is reflected by the first two terms. However, the decrease in the child's aid, on the contrary, pushes for buying more insurance (the third term).

2.3 Summary

The analysis above has shown that, overall, Arrow's theorem holds in the presence of family altruism. Nevertheless, some departures exist with respect to the standard model without family aid. Tables 1, 2 and 3 summarize the main results obtained with perfect and imperfect altruism and contrasts them to the findings of the model with no family.

As it can be seen from Table 1, the implications of perfect altruism are somewhat closer to those of the model without family. First, both of them predict equal deductibles in the two dependence states of nature, while with imperfect altruism, this is not necessarily the case. State-dependent deductibles under imperfect altruism thus constitute the first important departure from a straightforward application of Arrow's theorem.

Another departure in the case of imperfect altruism is the fact that the deductible is strictly positive even in the absence of loading costs. This comes from two sources. The first one is the parent's anticipation of the

Table 2: Effect of parent's initial wealth

ARA (parent's utility)	Perfect altruism	Imperfect altruism ($\gamma_1 = \gamma_2$)	No family
DARA	$\frac{\partial D}{\partial w} \leq 0$	$\frac{\partial D}{\partial w} \leq 0$	$\frac{\partial D}{\partial w} > 0$
CARA	$\frac{\partial D}{\partial w} < 0$ if $v''' \leq 0$ $\frac{\partial D}{\partial w} \leq 0$ if $v''' > 0$	$\frac{\partial D}{\partial w} < 0$ if $v''' \leq 0$ $\frac{\partial D}{\partial w} \leq 0$ if $v''' > 0$	$\frac{\partial D}{\partial w} = 0$
IARA	$\frac{\partial D}{\partial w} < 0$ if $u''' \geq 0$ and $v''' \leq 0$ $\frac{\partial D}{\partial w} \leq 0$ otherwise	$\frac{\partial D}{\partial w} < 0$ if $u''' \geq 0$ and $v''' \leq 0$ $\frac{\partial D}{\partial w} \leq 0$ otherwise	$\frac{\partial D}{\partial w} < 0$

child's reaction to insurance coverage: since insurance discourages aid, the benefit from improving insurance coverage is smaller than in the absence of family, which makes the parent prefer less than full insurance even when it is actuarially fair. This does not arise under perfect altruism since in that case, aid is chosen jointly by the parent and the child and it is optimally adapted to insurance coverage so that there is no need for the parent to anticipate the reaction of the child. On the other hand, even if, under imperfect altruism, the parent wanted to have full insurance between the healthy and the dependence states, to equalize consumption in these states, his deductible would not be zero but rather equal to the level of aid in the dependence states (and so strictly positive).¹¹ Under perfect altruism, there is also aid in the healthy state and it appears to be equal to the aid in the states of dependence when loading costs are zero, which, as can be seen in Table 1, implies a zero deductible in that case.

Nevertheless, departures from the standard model exist with perfect altruism as well. For instance, as it can be seen from Table 2, the presence of family altruism (both perfect and imperfect) modifies the way in which the deductible depends on the parent's initial wealth. As discussed in the previous sections, this comes from the fact that, differently from the standard model, there is an interplay between the deductible and the child's aid and that both of them are influenced by the parent's wealth.

It is interesting to note that this interplay also creates some ambiguity in terms of the effect that the parent's

¹¹If we allowed for some (low) degree of altruism in the healthy state, D would need to be equal to the difference between aid in the dependent and in the healthy state, but this difference is likely to be strictly positive as well.

Table 3: **Effect of child's altruism (imperfect altruism case)**

Case $\gamma_1 = \gamma_2 = \gamma$	Case $\gamma_1 \neq \gamma_2$ and $\gamma_2 \uparrow$
$\frac{\partial D}{\partial \gamma} > 0$ if $v''' \leq 0$ and $u''' \leq 0$, $\frac{\partial D}{\partial \gamma} \leq 0$ otherwise.	$\frac{\partial D_2}{\partial \gamma_2} > 0$ and $\frac{\partial D_1}{\partial \gamma_2} \leq 0$ if $v''' = u''' = 0$ or $v''' = 0$ and $u''' < 0$, $\frac{\partial D_2}{\partial \gamma_2} > 0$ and $\frac{\partial D_1}{\partial \gamma_2} \leq 0$ if $v''' \leq 0$, $u''' \leq 0$ and $\gamma_1 = 0$, $\frac{\partial D_2}{\partial \gamma_2} \leq 0$ and $\frac{\partial D_1}{\partial \gamma_2} \leq 0$ otherwise.

wealth has on the child's aid. While, as it was seen in the analysis, the direct effect of the parent's wealth is, as intuitively expected, negative (i.e. the child tends to decrease his aid when the parent becomes wealthier), the total effect needs to also take into account the impact of wealth on the deductible, which makes things more complicated. We can be sure that the total effect on aid is negative only if $\frac{\partial D}{\partial w} \leq 0$, but since this is not necessarily guaranteed, the influence of wealth on aid is generally not clear.

Finally, Table 3 summarizes our findings regarding the way in which the deductible is affected by the child's degree of altruism in the case where altruism is imperfect. As discussed in the previous section, this effect is in general ambiguous, but, under certain conditions, the intuitive expectation that the deductible should increase with the child's altruism can be confirmed. Once again, the results are not straightforward due to the interplay between insurance and aid. Moreover, as it was the case with the parent's wealth, the impact of altruism on the child's aid is generally ambiguous as well. While the direct effect is, in line with intuition, positive, the total effect is clearly positive only if $\frac{\partial D}{\partial \gamma} \geq 0$, which is, however, not always guaranteed. On the other hand, it should be noted that we cannot find clear conditions under which D would be decreasing in γ , whereas clear conditions for $\frac{\partial D}{\partial \gamma} > 0$ are easily found, which somewhat suggests that the latter result is more likely to hold. Seen more generally, this latter result seems to imply that it is reasonable to expect less insurance purchased by parents who have altruistic children. In the next section, we explore if this can be confirmed by empirical evidence.

3 Empirical relationship between LTC insurance and children’s altruism

One of the key relationships explored in the theoretical part of our analysis is that between the size of the deductible and the degree of the child’s altruism (γ). We have seen above that the way in which the deductible is influenced by the child’s altruism is in general ambiguous, but, in some cases, it is clear that γ has a positive effect on D . More generally, our theoretical analysis suggests that the child’s altruism might discourage the parent’s insurance coverage. The aim of this section is to investigate this question empirically.

More precisely, in this section we attempt to explore the empirical relationship between parents’ LTC insurance purchases and the extent of altruism of their children. To this end, we use data from the *Health and Retirement Study* (HRS), which is a longitudinal project sponsored by the National Institute on Aging and the Social Security Administration in the United States. The survey is a public resource for data on aging in America since 1990. More than 37,000 people older than 50 have been interviewed since the start of the study.¹² Respondents are visited on a biannual basis and questioned about health, socio-economic status (income, assets, insurances), relationships with family (visits, care, financial transfers) and everyday activities. Data from wave 3 (1996) to wave 11 (2012) is used for the analysis.

Table 4: **Informal help from children and dependency**

	Year	1996	1998	2000	2002	2004	2006	2008	2010	2012
Informal Help from Children in t (%)										
Dependent Parent in t	<i>No</i>	3.5	3.5	3.4	3.7	3.6	4.0	4.0	4.2	4.0
	<i>Yes</i>	44.0	44.6	44.1	47.3	47.8	46.4	44.7	47.0	48.5

While HRS provides us with a direct information about parents’ LTC insurance ownership, the measurement of children’s altruism is a more complicated question. Ideally, we would like to have an exogenous measure of altruism (as reflected by the exogenous parameter γ in our theoretical model). However, such a measure being unavailable in our data, we proxy the children’s altruism by the informal help that they provide to their parents while the parents are not yet dependent. Indeed, as it can be seen below from Table 4 (the cross-relationship between children’s support and parent’s dependency), not only dependent but also healthy parents

¹²As for the longitudinal aspect, the panel surveys a representative sample of approximately 20,000 people who can be followed during their aging process.

receive help from their children, even though help to non-dependent parents is considerably lower than that to dependent ones.¹³ Non-dependent parents are slightly less than 4% on average to report receiving help from their children, while if the parent is dependent, children are on average 10 times more likely to take care of him. We argue that the informal help provided to non-dependent parents can be considered as an exogenous measure of children’s altruism since it is obviously different from LTC and is therefore not affected by parents’ LTC insurance coverage. On the contrary, it rather seems reasonable to believe that parents base their insurance decisions on the presence and the amount of help they receive from their children when they are still healthy hoping to infer from it the degree of their children’s altruism and thus the amount of help on which they can count if they become dependent.¹⁴

Table 5: LTC insurance and presence of children

Year		1996	1998	2000	2002	2004	2006	2008	2010	2012
LTC Insurance in t										
Non-dependent Respondents with children	%	10.5	9.7	9.9	11.5	11.7	12.2	12.7	12.0	11.5
	<i>N</i>	14,940	17,893	16,495	15,338	16,975	15,535	14,486	18,086	17,088
Non-dependent Respondents without children	%	10.6	9.6	10.1	12.6	13.9	15.0	16.0	13.6	13.8
	<i>N</i>	1,193	1,333	1,178	1,016	1,224	1,079	976	1,386	1,300

Therefore, we concentrate our analysis on non-dependent respondents¹⁵, and our two main variables of interest are the purchase of LTC insurance and the informal help received by the parent from children, as a proxy for altruism. Before investigating this relationship, with some descriptive results and regressions, it should be noted that the two variables are binary: does the respondent have insurance and has she/he received help from her/his children? No information on insurance coverage and aid intensity is included in Tables 5 and 6. Table 5 summarizes information about LTC insurance for the different waves of the survey. About 11% of non-dependent respondents with children declare owning a LTC insurance. The variation in the insurance rate among respondents with and without children is minimal in the first years of the survey but increases after

¹³Examples of help to non-dependent parents can be help to manage money, help with chores, errands and transportation or financial help.

¹⁴It is important to note that the fact of providing informal aid should not necessarily be equated to the presence of altruism since caregiving can also be driven by other motives such as exchange or family norms. However, it seems that altruism is the prevailing motivation in many countries (see Klimaviciute et al., 2017).

¹⁵Dependency is defined as having 2 limitations in activities of daily living (ADL) or more. It concerns bathing, eating, dressing, walking across a room, and getting in or out of bed.

2000. These raw numbers do not explain yet the link that may exist between the two variables of interest.

Table 6 summarizes the cross-relationships between children’s support and insurance. The sample is restricted to respondents with children: the parents. The table suggests a negative relationship between informal support from children and the probability of parents having LTC insurance. These descriptive results have to be confirmed by robust econometric analysis.

Table 6: LTC insurance and informal help from children

	Year	1996	1998	2000	2002	2004	2006	2008	2010	2012
LTC Insurance in t if Parent non-dependent in t (%)										
Informal Help from Children in t	<i>No</i>	10.8	9.8	10.1	11.6	12.0	12.5	13.0	12.2	11.5
	<i>Yes</i>	3.0	5.3	4.2	6.4	4.5	6.2	6.3	8.7	10.0

Table 7 summarizes our regression results. In the regressions, in addition to studying the impact of help on LTC insurance at the extensive margin (i.e. the fact of helping; 3 first columns), we also look at the intensive margin (i.e. the intensity of help; 3 last columns). Whereas the information is processed for each year separately in Tables 4, 5 and 6, the data is now considered as a longitudinal panel, not fully balanced since some respondents have not answered in all the waves (deaths, attrition). Pooled probit and pooled linear probability models are run to verify the negative correlation between support from children and purchase of LTC insurance. The sample still concerns non-dependent parents, and this negative link is observed, significant and robust once other explanatory variables are added. Assuming that the variation between individuals is random and uncorrelated with the predictor or independent variables included in the model, pooled linear probability models with random effects are used to deal with time issues.¹⁶ That allows for time-invariant variables to play a role as explanatory variables (gender and education in the models). We cluster the standard errors because we observe repeated observations of individuals in the data. Clustered standard errors allow for intragroup correlation, relaxing the usual requirement for the observations to be independent. In other words, the observations are independent across groups (clusters) but not necessarily within groups.

¹⁶Random effects specification is approved by the values of Wald Chi² tests, meaning that all the coefficients in the models are different from zero.

Table 7: Probit models of LTC insurance

Explanatory variables	Dependent variable: LTC Insurance in t					
	Non-dependent Parents					
	All			Conditional on help		
	(1)	(2)	(3)	(1)	(2)	(3)
Help	-0.023*** (0.008)	-0.013*** (0.005)	-0.013*** (0.004)	-	-	-
Hours of Help	-	-	-	0.003 (0.003)	0.003 (0.003)	0.005 (0.003)
In couple	0.038*** (0.004)	0.038*** (0.004)	0.024*** (0.003)	0.018* (0.010)	0.023** (0.010)	0.024** (0.010)
Health Index	0.061*** (0.008)	0.052*** (0.005)	0.030*** (0.005)	0.073*** (0.018)	0.069*** (0.017)	0.060*** (0.016)
Medicaid	-0.057*** (0.006)	-0.018*** (0.007)	-0.011*** (0.003)	-0.030** (0.013)	-0.016** (0.008)	-0.011 (0.008)
Female	0.031*** (0.004)	0.029*** (0.004)	0.018*** (0.003)	0.028** (0.011)	0.032* (0.011)	0.022* (0.011)
Age	0.003*** (0.000)	0.003*** (0.000)	0.003*** (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Years of Education	0.014*** (0.001)	0.012*** (0.001)	0.012*** (0.000)	0.006*** (0.002)	0.005*** (0.001)	0.005*** (0.001)
Income	0.017*** (0.001)	0.017*** (0.001)	0.009*** (0.001)	0.024*** (0.005)	0.031*** (0.006)	0.027*** (0.005)
N	131,550	131,550	131,550	3,822	3,822	3,822
(Pseudo)-R²	0.052	0.033	0.032	0.080	0.036	0.036
Wald Chi²	-	-	1,376.5	-	-	89.24

Notes: (1) Pooled Probit Model; (2) Pooled Linear Probability Model; (3) Linear Probability Random Effect Model

The negative impact of children's help (at the extensive margin) on the purchase of LTC insurance is significant.¹⁷ The non-dependency condition is the same as for the results of Tables 4, 5 and 6 (having less than 2 ADLs). Other explanatory variables are binary: the fact of having a partner (+), benefiting from Medicaid (-), being a woman (+). These different regressions are controlled for education (+), income¹⁸ (+) and health¹⁹ (+). The different signs of parameters associated to the control variables go in the expected direction. The positive sign of the health index implies the potential absence of adverse selection in the LTC insurance market.

¹⁷This result is also present and robust if regressions are run for each wave of the survey separately.

¹⁸Individuals have been ranked according to deciles of income created for each year of the survey.

¹⁹Index of health is predicted from Principal Component Analysis (PCA). This is a statistical procedure that uses orthogonal transformation to convert a set of 8 different objective and subjective health measures, possibly correlated into a set of linearly uncorrelated variables called principal components. Built at each period, the higher the index, the better the health.

Finally, looking at the link between the intensity of help and LTC insurance purchase (conditional on this help), it seems that there is no clear correlation. However, a slightly positive sign appears in the last column (6) of Table 7.²⁰ The estimate remains positive but becomes significant if we do not cluster the standard errors. It could be explained by the fact that parents realize that, although they are not dependent, they are already relying heavily on the help of their children. They then might find LTC insurance an essential option because the myopia/denial of dependence could have disappeared for them. On the other hand, this finding can also be seen as supporting the ambiguity of our theoretical results and confirming that the relationship between LTC insurance and children's altruism might not be as straightforward as one might think.

4 Conclusion

The purpose of this paper was to check whether or not Arrow's deductible theorem could readily apply in designing an optimal LTC insurance with family solidarity. We considered two types of altruism between parents and children: perfect altruism leading to a unitary model of the family and imperfect altruism implying that the parents play a Stackelberg game with their children, whose altruism is contingent to their state of dependence. Not surprisingly, in the case of perfect altruism, the solution is about the same as in the case without family solidarity except that, to the extent that children are richer than their parents, these benefit from a better protection in case of big losses. In the case of imperfect altruism, the deductible theorem applies but the way in which the deductible is affected by the child's degree of altruism is in general ambiguous. We need particular conditions to make sure that the deductible increases when the child becomes more altruistic. More generally, our theoretical analysis highlights a complex interplay between insurance and the child's aid, which results in a number of departures from the standard model without family solidarity.

Given the ambiguity of the theoretical findings, we resorted to a more general empirical test of the relation between LTC insurance and children's altruism, proxied by the informal help provided to non-dependent parents. Our analysis shows that the presence of this help has a negative impact on the parents' insurance purchases, which seems to support our theoretical results proving that, under certain conditions, children's altruism discourages LTC insurance. On the other hand, the empirical effect of the intensity of aid is less clear

²⁰The lack of significant results could be caused by the small size of this conditional sample.

suggesting that the relation between insurance and altruism might be more complex, as it is also well seen in the general case of the theoretical analysis.

One could object that in most countries the LTC insurance market is extremely thin and that therefore our analysis is of little relevance. But precisely, one of the reasons for such a thinness of the market is the unpalatable rule of reimbursement.²¹ Our paper therefore propagates the idea of using LTC insurance with deductible, which we show to be the most efficient reimbursement scheme.

In this paper, we have only considered two models of family solidarity. For future research, it would be interesting to look at the various bargaining models that prevail in the literature.²²

References

- [1] Arrow, K. (1963). "Uncertainty and the welfare economics of medical care". *American Economic Review*, 53, 941-973.
- [2] Brown, J. R. and A. Finkelstein (2007). "Why is the market for long-term care insurance so small?". *Journal of Public Economics*, 91, 1967-1991.
- [3] Cremer, H., P. Pestieau and G. Ponthière (2012). "The economics of long-term care: A survey". *Nordic Economic Policy Review*, 2, 108-148.
- [4] Cremer, H., P. Pestieau and K. Roeder (2016). "Social long-term care insurance with two-sided altruism". *Research in Economics*, 70, 101-109.
- [5] Cutler, D. (1993). "Why Doesn't the Market Fully Insure Long-Term Care?". NBER Working Paper No. 4301.
- [6] Drèze, J., P. Pestieau and E. Schokkaert (2016). "Arrow's theorem of the deductible and long-term care insurance". *Economics Letters*, 148, 103-105.
- [7] Engers, M. and S. Stern (2002). "Long-term care and family bargaining". *International Economic Review*, 43, 73-114.

²¹See on this Pestieau and Ponthière (2011), Brown and Finkelstein (2007), Cutler (1993).

²²See, for instance, Engers and Stern (2002).

- [8] Klimaviciute, J., S. Perelman, P. Pestieau and J. Schoenmaeckers (2017). “Caring for dependent parents: altruism, exchange or family norm?”. *Journal of Population Economics*, 30, 835-873.
- [9] Klimaviciute, J. and P. Pestieau (2018a). “Long-term care social insurance: How to avoid big losses?”. *International Tax and Public Finance*, 25, 99-139.
- [10] Klimaviciute, J. and P. Pestieau (2018b). “Social insurance for long-term care with deductible and linear contributions”. *FinanzArchiv/Public Finance Analysis*, forthcoming.
- [11] Norton, E. (2000). “Long term care”, in A. Cuyler & J. Newhouse (Eds.): *Handbook of Health Economics*, Volume 1b, chapter 17.
- [12] Pestieau, P. and G. Ponthière (2011). “The long term care insurance puzzle”, in J. Costa-Font & C. Courbage (Eds.): *Financing Long Term Care in Europe: Institutions, Markets and Models*. Palgrave Macmillan.
- [13] Pestieau, P. and M. Sato (2008). “Long term care: the State, the market and the family”. *Economica*, 75, 435-454.

Appendix A. Perfect altruism: comparative statics w.r.t. w

Assuming interior solutions, the problem of the family can be seen as maximizing

$$(\pi_1 + \pi_2) \left[u \left(\underbrace{w - P - D + a}_m \right) + v \left(\underbrace{y - a}_c \right) \right] + (1 - \pi_1 - \pi_2) \left[u \left(\underbrace{w - P + a_3}_d \right) + v \left(\underbrace{y - a_3}_{c_3} \right) \right]$$

subject to $P = \pi_1(1 + \lambda)(L_1 - D) + \pi_2(1 + \lambda)(L_2 - D)$.

The FOCs for D , a ($= a_1 = a_2$) and a_3 write as follows:

$$(\pi_1 + \pi_2) u'(m) [\pi_1(1 + \lambda) + \pi_2(1 + \lambda) - 1] + (1 - \pi_1 - \pi_2) u'(d) [\pi_1(1 + \lambda) + \pi_2(1 + \lambda)] = 0 \quad (17)$$

$$u'(m) - v'(c) = 0 \quad (18)$$

$$u'(d) - v'(c_3) = 0 \quad (19)$$

Fully differentiating these FOCs w.r.t. w and combining, we can get:

$$\frac{\partial a}{\partial w} = \frac{-\frac{\partial D}{\partial w} u''(m) [\pi_1(1+\lambda) + \pi_2(1+\lambda) - 1] - u''(m)}{[u''(m) + v''(c)]} \quad (20)$$

$$\frac{\partial a_3}{\partial w} = \frac{-\frac{\partial D}{\partial w} u''(d) [\pi_1(1+\lambda) + \pi_2(1+\lambda)] - u''(d)}{[u''(d) + v''(c_3)]} \quad (21)$$

and

$$\begin{aligned} \frac{\partial D}{\partial w} = & \frac{(1 - \pi_1 - \pi_2)u''(d) [\pi_1(1+\lambda) + \pi_2(1+\lambda)]}{- [M]} + \\ & + \frac{(1 - \pi_1 - \pi_2)u''(d) [\pi_1(1+\lambda) + \pi_2(1+\lambda)] \left[\frac{-u''(d)}{u''(d)+v''(c_3)} \right]}{- [M]} - \\ & - \frac{(\pi_1 + \pi_2) u''(m) [1 - \pi_1(1+\lambda) - \pi_2(1+\lambda)]}{- [M]} - \\ & - \frac{(\pi_1 + \pi_2) u''(m) [1 - \pi_1(1+\lambda) - \pi_2(1+\lambda)] \left[\frac{-u''(m)}{u''(m)+v''(c)} \right]}{- [M]} \end{aligned} \quad (22)$$

where

$$\begin{aligned} [M] \equiv & (1 - \pi_1 - \pi_2)u''(d) [\pi_1(1+\lambda) + \pi_2(1+\lambda)]^2 \frac{v''(c_3)}{[u''(d) + v''(c_3)]} + \\ & + (\pi_1 + \pi_2) u''(m) [1 - \pi_1(1+\lambda) - \pi_2(1+\lambda)]^2 \frac{v''(c)}{[u''(m) + v''(c)]} < 0 \end{aligned}$$

The sign of $\frac{\partial D}{\partial w}$ is generally ambiguous. First, it can be shown that the sum of the first and the third terms is positive (resp. negative and equal to zero) when the parent's preferences are DARA (resp. IARA and CARA). To see this, let us first note that DARA (resp. IARA and CARA) means that

$$ARA(c) = \frac{-u''(c)}{u'(c)} < (\text{resp. } > \text{ and } =) ARA(z) = \frac{-u''(z)}{u'(z)} \text{ for } c > z,$$

where $\frac{-u''(x)}{u'(x)}$ is the Arrow-Pratt measure of absolute risk aversion at wealth x .

Since we have $d > m$, under DARA (resp. IARA and CARA) preferences we can write

$$\frac{-u''(d)}{u'(d)} < (\text{resp. } > \text{ and } =) \frac{-u''(m)}{u'(m)}$$

$$\iff$$

$$u''(d) > (\text{resp. } < \text{ and } =) \frac{u''(m)}{u'(m)} u'(d)$$

We can then multiply both sides by $(1 - \pi_1 - \pi_2) [\pi_1(1 + \lambda) + \pi_2(1 + \lambda)]$ and subtract from both sides $(\pi_1 + \pi_2) u''(m) [1 - \pi_1(1 + \lambda) - \pi_2(1 + \lambda)]$, which gives

$$\begin{aligned} & (1 - \pi_1 - \pi_2) u''(d) [\pi_1(1 + \lambda) + \pi_2(1 + \lambda)] - (\pi_1 + \pi_2) u''(m) [1 - \pi_1(1 + \lambda) - \pi_2(1 + \lambda)] \\ & > (\text{resp. } < \text{ and } =) \frac{u''(m)}{u'(m)} \left[\begin{array}{l} u'(d) (1 - \pi_1 - \pi_2) [\pi_1(1 + \lambda) + \pi_2(1 + \lambda)] \\ - (\pi_1 + \pi_2) [1 - \pi_1(1 + \lambda) - \pi_2(1 + \lambda)] u'(m) \end{array} \right] = 0 \end{aligned} \quad (23)$$

noting that the expression in the last big bracket is zero from equation (17).

The left-hand side of inequality (23) is exactly the sum of the numerators of the first and the third terms in (22).

The second and the fourth terms in (22) contain, respectively, $\frac{-u''(d)}{[u''(d)+v''(c_3)]}$ and $\frac{-u''(m)}{[u''(m)+v''(c)]}$, which, as can be seen from (21) and (20), reflect the direct effect of w on the child's aid respectively in the healthy state and in the states with dependence. This effect is negative in both cases but pushes the deductible into different directions. To analyze the sign of $\frac{\partial D}{\partial w}$, it is instructive to rewrite the expression (22) as

$$\begin{aligned} \frac{\partial D}{\partial w} &= \frac{(1 - \pi_1 - \pi_2) u''(d) [\pi_1(1 + \lambda) + \pi_2(1 + \lambda)] \left[\frac{v''(c_3)}{u''(d)+v''(c_3)} \right]}{-[M]} \\ &\quad - \frac{(\pi_1 + \pi_2) u''(m) [1 - \pi_1(1 + \lambda) - \pi_2(1 + \lambda)] \left[\frac{v''(c)}{u''(m)+v''(c)} \right]}{-[M]} \end{aligned} \quad (24)$$

Let us study different cases.

DARA. We know that

$$(1 - \pi_1 - \pi_2)u''(d) [\pi_1(1 + \lambda) + \pi_2(1 + \lambda)] - (\pi_1 + \pi_2)u''(m) [1 - \pi_1(1 + \lambda) - \pi_2(1 + \lambda)] > 0$$

where the first term is negative and the second term is positive.

DARA also implies that $u''' > 0$. Then, since $d > m$, $u''(d)$ is smaller in absolute value than $u''(m)$. For v , we have 3 possible cases:

- $v''' = 0$. Then $v''(c_3) = v''(c)$. This implies $\left[\frac{v''(c_3)}{u''(d)+v''(c_3)}\right] > \left[\frac{v''(c)}{u''(m)+v''(c)}\right]$. So, the expression in the numerator of (24) is ambiguous.
- $v''' > 0$. Then, since $c_3 > c$, $v''(c_3)$ is smaller in absolute value than $v''(c)$. This implies $\left[\frac{v''(c_3)}{u''(d)+v''(c_3)}\right] \leq \left[\frac{v''(c)}{u''(m)+v''(c)}\right]$. So, the expression in the numerator is ambiguous.
- $v''' < 0$. Then, since $c_3 > c$, $v''(c_3)$ is larger in absolute value than $v''(c)$. This implies $\left[\frac{v''(c_3)}{u''(d)+v''(c_3)}\right] > \left[\frac{v''(c)}{u''(m)+v''(c)}\right]$. So, the expression in the numerator is ambiguous.

Thus, under DARA, we always have $\frac{\partial D}{\partial w} \leq 0$.

CARA. We know that

$$(1 - \pi_1 - \pi_2)u''(d) [\pi_1(1 + \lambda) + \pi_2(1 + \lambda)] - (\pi_1 + \pi_2)u''(m) [1 - \pi_1(1 + \lambda) - \pi_2(1 + \lambda)] = 0$$

where the first term is negative and the second term is positive.

CARA also implies that $u''' > 0$. We have the same 3 possible cases as with DARA, but now we find $\frac{\partial D}{\partial w} < 0$ if $v''' \leq 0$ and $\frac{\partial D}{\partial w} \leq 0$ if $v''' > 0$.

IARA. We know that

$$(1 - \pi_1 - \pi_2)u''(d) [\pi_1(1 + \lambda) + \pi_2(1 + \lambda)] - (\pi_1 + \pi_2)u''(m) [1 - \pi_1(1 + \lambda) - \pi_2(1 + \lambda)] < 0$$

where the first term is negative and the second term is positive.

IARA implies no restriction on u''' . Thus, we now have 9 possible cases. It can be verified that we have $\frac{\partial D}{\partial w} < 0$ if $u''' \geq 0$ and $v''' \leq 0$. Otherwise, the sign is ambiguous.

Appendix B. Imperfect altruism: comparative statics w.r.t. γ_1 or γ_2

Assuming interior solutions, the parent's utility can be rewritten as

$$U_p = \pi_1 u \left(\underbrace{w - P - D_1 + a_1}_{m_1} \right) + \pi_2 u \left(\underbrace{w - P - D_2 + a_2}_{m_2} \right) + (1 - \pi_1 - \pi_2) u \left(\underbrace{w - P}_d \right)$$

where $P = \pi_1(1 + \lambda)(L_1 - D_1) + \pi_2(1 + \lambda)(L_2 - D_2)$.

The FOCs for D_1 and D_2 are as follows:

$$\pi_1 u'(m_1) \left[\pi_1(1 + \lambda) - 1 + \frac{\partial a_1}{\partial D_1} \right] + \pi_2 u'(m_2) \left[\pi_1(1 + \lambda) + \frac{\partial a_2}{\partial D_1} \right] + (1 - \pi_1 - \pi_2) u'(d) \pi_1(1 + \lambda) = 0$$

$$\pi_1 u'(m_1) \left[\pi_2(1 + \lambda) + \frac{\partial a_1}{\partial D_2} \right] + \pi_2 u'(m_2) \left[\pi_2(1 + \lambda) - 1 + \frac{\partial a_2}{\partial D_2} \right] + (1 - \pi_1 - \pi_2) u'(d) \pi_2(1 + \lambda) = 0$$

From the child's problem, we have

$$\frac{\partial a_1}{\partial D_1} = -\frac{\gamma_1 u''(m_1) [\pi_1(1 + \lambda) - 1]}{v''(c_1) + \gamma_1 u''(m_1)} > 0, \quad \frac{\partial a_1}{\partial D_2} = -\frac{\gamma_1 u''(m_1) \pi_2(1 + \lambda)}{v''(c_1) + \gamma_1 u''(m_1)} < 0,$$

$$\frac{\partial a_2}{\partial D_1} = -\frac{\gamma_2 u''(m_2) \pi_1(1 + \lambda)}{v''(c_2) + \gamma_2 u''(m_2)} < 0, \quad \frac{\partial a_2}{\partial D_2} = -\frac{\gamma_2 u''(m_2) [\pi_2(1 + \lambda) - 1]}{v''(c_2) + \gamma_2 u''(m_2)} > 0$$

Using this, the FOCs can be rewritten as

$$\begin{aligned} & \pi_1 u'(m_1) [\pi_1(1 + \lambda) - 1] \frac{v''(c_1)}{[v''(c_1) + \gamma_1 u''(m_1)]} + \\ & + \pi_2 u'(m_2) \pi_1(1 + \lambda) \frac{v''(c_2)}{[v''(c_2) + \gamma_2 u''(m_2)]} + (1 - \pi_1 - \pi_2) u'(d) \pi_1(1 + \lambda) = 0 \end{aligned} \quad (25)$$

and

$$\begin{aligned}
& \pi_1 u'(m_1) \pi_2 (1 + \lambda) \frac{v''(c_1)}{[v''(c_1) + \gamma_1 u''(m_1)]} + \\
& + \pi_2 u'(m_2) [\pi_2 (1 + \lambda) - 1] \frac{v''(c_2)}{[v''(c_2) + \gamma_2 u''(m_2)]} + (1 - \pi_1 - \pi_2) u'(d) \pi_2 (1 + \lambda) = 0
\end{aligned} \tag{26}$$

Fully differentiating (25) and (26) with respect to γ_1 , we get:

$$\frac{\partial D_1}{\partial \gamma_1} [A] + \frac{\partial D_2}{\partial \gamma_1} [B] + [C] = 0 \tag{27}$$

and

$$\frac{\partial D_1}{\partial \gamma_1} [B] + \frac{\partial D_2}{\partial \gamma_1} [E] + [F] = 0 \tag{28}$$

where

$$\begin{aligned}
A \equiv & \frac{\pi_1 [\pi_1 (1 + \lambda) - 1]^2 u'(m_1) \left[[\gamma_1 u''(m_1)]^2 v'''(c_1) - [v''(c_1)]^2 u'''(m_1) \gamma_1 \right]}{[v''(c_1) + \gamma_1 u''(m_1)]^3} + \\
& + \frac{\pi_1 [\pi_1 (1 + \lambda) - 1]^2 \left[u''(m_1) [v''(c_1)]^3 + [v''(c_1)]^2 [u''(m_1)]^2 \gamma_1 \right]}{[v''(c_1) + \gamma_1 u''(m_1)]^3} + \\
& + \frac{\pi_2 [\pi_1 (1 + \lambda)]^2 u'(m_2) \left[[\gamma_2 u''(m_2)]^2 v'''(c_2) - [v''(c_2)]^2 u'''(m_2) \gamma_2 \right]}{[v''(c_2) + \gamma_2 u''(m_2)]^3} + \\
& + \frac{\pi_2 [\pi_1 (1 + \lambda)]^2 \left[u''(m_2) [v''(c_2)]^3 + [v''(c_2)]^2 [u''(m_2)]^2 \gamma_2 \right]}{[v''(c_2) + \gamma_2 u''(m_2)]^3} + (1 - \pi_1 - \pi_2) u''(d) [\pi_1 (1 + \lambda)]^2; \\
B \equiv & \frac{\pi_1 [\pi_1 (1 + \lambda) - 1] \pi_2 (1 + \lambda) u'(m_1) \left[[\gamma_1 u''(m_1)]^2 v'''(c_1) - [v''(c_1)]^2 u'''(m_1) \gamma_1 \right]}{[v''(c_1) + \gamma_1 u''(m_1)]^3} + \\
& + \frac{\pi_1 [\pi_1 (1 + \lambda) - 1] \pi_2 (1 + \lambda) \left[u''(m_1) [v''(c_1)]^3 + [v''(c_1)]^2 [u''(m_1)]^2 \gamma_1 \right]}{[v''(c_1) + \gamma_1 u''(m_1)]^3} +
\end{aligned}$$

$$\begin{aligned}
& + \frac{\pi_2 [\pi_2(1 + \lambda) - 1] \pi_1(1 + \lambda) u'(m_2) \left[[\gamma_2 u''(m_2)]^2 v'''(c_2) - [v''(c_2)]^2 u'''(m_2) \gamma_2 \right]}{[v''(c_2) + \gamma_2 u''(m_2)]^3} + \\
& + \frac{\pi_2 [\pi_2(1 + \lambda) - 1] \pi_1(1 + \lambda) \left[u''(m_2) [v''(c_2)]^3 + [v''(c_2)]^2 [u''(m_2)]^2 \gamma_2 \right]}{[v''(c_2) + \gamma_2 u''(m_2)]^3} + (1 - \pi_1 - \pi_2) u''(d) \pi_1 \pi_2 (1 + \lambda)^2;
\end{aligned}$$

$$C \equiv \frac{\pi_1 [\pi_1(1 + \lambda) - 1] \gamma_1 [u'(m_1)]^2 [v'''(c_1) u''(m_1) + v''(c_1) u'''(m_1)]}{[v''(c_1) + \gamma_1 u''(m_1)]^3} -$$

$$- \frac{2\pi_1 [\pi_1(1 + \lambda) - 1] u'(m_1) \left[u''(m_1) [v''(c_1)]^2 + v''(c_1) [u''(m_1)]^2 \gamma_1 \right]}{[v''(c_1) + \gamma_1 u''(m_1)]^3};$$

$$E \equiv \frac{\pi_1 [\pi_2(1 + \lambda)]^2 u'(m_1) \left[[\gamma_1 u''(m_1)]^2 v'''(c_1) - [v''(c_1)]^2 u'''(m_1) \gamma_1 \right]}{[v''(c_1) + \gamma_1 u''(m_1)]^3} +$$

$$+ \frac{\pi_1 [\pi_2(1 + \lambda)]^2 \left[u''(m_1) [v''(c_1)]^3 + [v''(c_1)]^2 [u''(m_1)]^2 \gamma_1 \right]}{[v''(c_1) + \gamma_1 u''(m_1)]^3} +$$

$$+ \frac{\pi_2 [\pi_2(1 + \lambda) - 1]^2 u'(m_2) \left[[\gamma_2 u''(m_2)]^2 v'''(c_2) - [v''(c_2)]^2 u'''(m_2) \gamma_2 \right]}{[v''(c_2) + \gamma_2 u''(m_2)]^3} +$$

$$+ \frac{\pi_2 [\pi_2(1 + \lambda) - 1]^2 \left[u''(m_2) [v''(c_2)]^3 + [v''(c_2)]^2 [u''(m_2)]^2 \gamma_2 \right]}{[v''(c_2) + \gamma_2 u''(m_2)]^3} + (1 - \pi_1 - \pi_2) u''(d) [\pi_2(1 + \lambda)]^2;$$

$$F \equiv \frac{\pi_1 \pi_2 (1 + \lambda) \gamma_1 [u'(m_1)]^2 [v'''(c_1) u''(m_1) + v''(c_1) u'''(m_1)]}{[v''(c_1) + \gamma_1 u''(m_1)]^3} -$$

$$- \frac{2\pi_1 \pi_2 (1 + \lambda) u'(m_1) \left[u''(m_1) [v''(c_1)]^2 + v''(c_1) [u''(m_1)]^2 \gamma_1 \right]}{[v''(c_1) + \gamma_1 u''(m_1)]^3}$$

Combining (27) and (28), we have

$$\frac{\partial D_1}{\partial \gamma_1} = \frac{[F][B] - [C][E]}{[E][A] - [B]^2} \quad (29)$$

and

$$\frac{\partial D_2}{\partial \gamma_1} = \frac{[C][B] - [F][A]}{[E][A] - [B]^2} \quad (30)$$

Noting that the SOC for a local maximum for a function of two variables implies $[E][A] - [B]^2 > 0$, we have that the denominators of the two expressions are positive. For the numerators, we can obtain the following:

$$\begin{aligned} [F][B] - [C][E] &= \frac{(1 - \pi_1 - \pi_2)u''(d)\pi_1[\pi_2(1 + \lambda)]^2[G]}{[v''(c_1) + \gamma_1 u''(m_1)]^3} + \\ &+ \frac{\pi_1\pi_2[\pi_2(1 + \lambda) - 1][\pi_1(1 + \lambda) + \pi_2(1 + \lambda) - 1][G][H]}{[v''(c_1) + \gamma_1 u''(m_1)]^3 [v''(c_2) + \gamma_2 u''(m_2)]^3} \end{aligned} \quad (31)$$

and

$$\begin{aligned} [C][B] - [F][A] &= \frac{-(1 - \pi_1 - \pi_2)u''(d)\pi_2[\pi_1(1 + \lambda)]^2[G]}{[v''(c_1) + \gamma_1 u''(m_1)]^3} + \\ &+ \frac{\pi_1^2\pi_2(1 + \lambda)[1 - \pi_1(1 + \lambda) - \pi_2(1 + \lambda)][G][H]}{[v''(c_1) + \gamma_1 u''(m_1)]^3 [v''(c_2) + \gamma_2 u''(m_2)]^3} \end{aligned} \quad (32)$$

where

$$\begin{aligned} [G] &\equiv \gamma_1 [u'(m_1)]^2 [v'''(c_1)u''(m_1) + v''(c_1)u'''(m_1)] - \\ &- 2u'(m_1) [u''(m_1) [v''(c_1)]^2 + v''(c_1) [u''(m_1)]^2 \gamma_1] \end{aligned}$$

and

$$\begin{aligned} [H] &\equiv u'(m_2) [\gamma_2 u''(m_2)]^2 v'''(c_2) - [v''(c_2)]^2 u'''(m_2) \gamma_2 + \\ &+ u''(m_2) [v''(c_2)]^3 + [v''(c_2)]^2 [u''(m_2)]^2 \gamma_2 \end{aligned}$$

The signs of the expressions (31) and (32) are generally ambiguous. Differently from the case where altruism, and thus the deductible, is the same in both states of nature, here we have interactions between the deductibles in the two states, which makes things more complicated. To recall, in the case of a single deductible, we had $\frac{\partial D}{\partial \gamma} > 0$ if $v'''(c) \leq 0$ and $u'''(m) \leq 0$, while the sign was ambiguous otherwise. Here, if we assume that all third derivatives of the utility functions are zero, we have $\frac{\partial D_1}{\partial \gamma_1} > 0$, while the sign of $\frac{\partial D_2}{\partial \gamma_1}$ is ambiguous (it has one negative and one positive term). The same result can also be obtained if $v''' = 0$ and $u''' < 0$. However, in all

other cases, both signs are ambiguous. For instance, if both third derivatives are negative, then the term $[H]$ is ambiguous, which prevents from signing the expressions, unlike in the case of a single deductible.

The results become closer to the case of a single deductible if we assume that $\gamma_2 = 0$. Then, in state 2, the child provides no aid, which somewhat simplifies the derivations. In that case, the second term of (31) becomes

$$\frac{\pi_1 \pi_2 [\pi_2(1 + \lambda) - 1] [\pi_1(1 + \lambda) + \pi_2(1 + \lambda) - 1] [G] u''(m_2)}{[v''(c_1) + \gamma_1 u''(m_1)]^3}$$

and the second term of (32) becomes

$$\frac{\pi_1^2 \pi_2(1 + \lambda) [1 - \pi_1(1 + \lambda) - \pi_2(1 + \lambda)] [G] u''(m_2)}{[v''(c_1) + \gamma_1 u''(m_1)]^3}$$

Then, if we assume that the third derivatives are negative or zero, we have $\frac{\partial D_1}{\partial \gamma_1} > 0$, while the sign of $\frac{\partial D_2}{\partial \gamma_1}$ is ambiguous. Otherwise, both signs are ambiguous.

We have analogous results if we consider an increase in γ_2 rather than in γ_1 . For instance, if we set $\gamma_1 = 0$ and assume negative or zero third derivatives, we will then have $\frac{\partial D_2}{\partial \gamma_2} > 0$, while the sign of $\frac{\partial D_1}{\partial \gamma_2}$ will be ambiguous.

Appendix C. Imperfect altruism: comparative statics w.r.t. w

We focus here on the case $\gamma_1 = \gamma_2 = \gamma$. Using equation (16), we can derive

$$\frac{\partial D}{\partial w} = - \frac{[N]}{SOC_D}$$

where $SOC_D < 0$ is the second-order condition for D and

$$\begin{aligned} [N] \equiv & (1 - \pi_1 - \pi_2) u''(d) [\pi_1(1 + \lambda) + \pi_2(1 + \lambda)] - \\ & - (\pi_1 + \pi_2) [1 - \pi_1(1 + \lambda) - \pi_2(1 + \lambda)] \frac{v''(c)}{v''(c) + \gamma u''(m)} u''(m) - \\ & - (\pi_1 + \pi_2) [1 - \pi_1(1 + \lambda) - \pi_2(1 + \lambda)] \frac{v''(c)}{v''(c) + \gamma u''(m)} u''(m) \frac{\partial a}{\partial w} + \end{aligned}$$

$$+ (\pi_1 + \pi_2) u'(m) [\pi_1(1 + \lambda) + \pi_2(1 + \lambda) - 1] \frac{[v'''(c) [-\frac{\partial a}{\partial w}] \gamma u''(m) - v''(c) \gamma u'''(m) [1 + \frac{\partial a}{\partial w}]]}{[v''(c) + \gamma u''(m)]^2} \quad (33)$$

with $\frac{\partial a}{\partial w} = -\frac{\gamma u''(m)}{v''(c) + \gamma u''(m)} < 0$ being the direct effect of w on aid.²³

The sign of $\frac{\partial D}{\partial w}$ depends on the sign of $[N]$, and the sign of $[N]$ is generally ambiguous. It can be shown that the sum of the first and the second terms is positive (resp. negative and equal to zero) when the parent's preferences are DARA (resp. IARA and CARA). The third term is always negative. The sign of the last term depends on the assumptions about the third derivatives of the utility functions. Let us analyze separately the cases of DARA, CARA and IARA.

Under DARA, the sign of $\frac{\partial D}{\partial w}$ is ambiguous since already the sum of the first three terms has an ambiguous sign.

Under CARA, the sign of $\frac{\partial D}{\partial w}$ depends on the sum of the two last terms. CARA implies that $u'''(m) > 0$. Then, if $v'''(c) \leq 0$, the last term is negative and so we clearly have $\frac{\partial D}{\partial w} < 0$. If $v'''(c) > 0$, the sign of the last term is ambiguous and so the total sign is ambiguous as well.

Under IARA, the sum of the first three terms is negative, so the total sign is clearly negative if the last term is also negative or zero. Otherwise, the total sign is ambiguous. Thus, it can be verified that $\frac{\partial D}{\partial w}$ is clearly negative if $u''' \geq 0$ and $v''' \leq 0$.

²³The full effect is equal to $\frac{\partial a}{\partial w} + \frac{\partial a}{\partial D} \frac{\partial D}{\partial w}$.