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Abstract

The Multiplicative MIDAS Realized DCC (MMReDCC) model of Bauwens et al. [5] decomposes the dynamics of the realized covariance matrix of returns into short-run transitory and long-run secular components where the latter reflects the effect of the continuously changing economic conditions. The model allows to obtain positive-definite forecasts of the realized covariance matrices but, due to the high number of parameters involved, estimation becomes unfeasible for large cross-sectional dimensions. Our contribution in this paper is twofold. First, in order to obtain a computationally feasible estimation procedure, we propose an algorithm that relies on the maximization of an iteratively re-computed moment-based profile likelihood function. We assess the finite sample properties of the proposed algorithm via a simulation study. Second, we propose a bootstrap procedure for generating multi-step ahead forecasts from the MMReDCC model. In an empirical application on realized covariance matrices for fifty equities, we find that the MMReDCC not only statistically outperforms the selected benchmarks in-sample, but also improves the out-of-sample ability to generate accurate multi-step ahead forecasts of the realized covariances.

Keywords: Realized covariance, dynamic component models, multi-step forecasting, MIDAS, targeting, model confidence set.

1. Introduction

Building models for predicting the volatility of high dimensional portfolios is important in risk management and asset allocation. Previous developments on time-varying covariances in large dimensions include the constant conditional correlation (CCC) model of Bollerslev [8], where the volatilities of each asset are allowed to vary through time but the correlations are time invariant, the RiskMetrics model by [27], and the DECO model by Engle and Kelly [18] who allow correlations to change over time and can be easily applied in vast dimensions. Recently, Andersen et al. [2], Barndorff-Nielsen and Shephard [4] and Barndorff-Nielsen et al. [3], among others, opened up a new channel for increasing the precision of covariance matrix estimates and forecasts by exploiting the information of high frequency asset returns. This development has motivated several researchers to investigate models directly fitted to series of realized covariance matrices (see Gouriéroux et al. [22], Jin and Maheu [25] and Chiriac and Voev [11], among others).

Despite the superiority of these models, illustrated for example by Hautsch et al. [24], there still remain technical and practical challenges one needs to deal with when constructing covariance matrix forecasts for high-dimensional systems. First and foremost, the well-known "curse of dimensionality" problem, implying that the number of parameters grows as a power function of the cross-sectional model dimension. In order to save parameters, a simple solution is represented by the so called covariance (or correlation) targeting approach of Engle [15], which consists in pre-estimating the constant intercept matrix in the model specification by linking it to the unconditional covariance matrix of returns. This method can be applied under the stationarity assumption of the model and is one of the most widely employed techniques to simplifying parameter estimation and reducing the computational burden when the numerical maximization of the likelihood function becomes difficult.

Recently, Bauwens et al. [5] investigated a wide class of multivariate models that simultaneously account for short and long-term dynamics in the conditional (co)volatilities and correlations of asset returns, in line with the empirical evidence suggesting that their level is changing over time as a function of the economic conditions (see, among others, Engle et al. [16]). Herein we focus on the Multiplicative MIDAS Realized DCC (MMReDCC) model, whose main ingredients are a multiplicative component structure, a Mixed Data Sampling (MIDAS) filter to modeling the secular dynamics and a DCC-type parameterization for the short term component, directly inspired by the multivariate GARCH literature.[1] The extensive out-of-sample forecasting comparison performed by Bauwens et al. [5], although not identifying a clear winner, shows that the MMReDCC model gives remarkably good performances in important financial applications such as Value-at-Risk forecasting and portfolio allocation. However, their results are limited to a relatively low dimensional setting (10 assets) and to a short-term forecasting horizon (1 day).

This paper extends the work by Bauwens et al. [5] along these directions: estimation for high-dimensional systems and multi-step forecasting. We contribute to the first line of research by developing of a computationally feasible procedure for the estimation of vast dimensional MMReDCC models. In this respect, it is important to remark that, although the introduction of a dynamic secular component in the structure of the model adds a major element of flexibility and enables to obtain more accurate forecasts than standard models reverting to constant mean levels (see Bauwens et al. [5]), it also dramatically increases the number of parameters to be estimated. Specifically, the long term component incorporates

[1] We refer to Engle [14] and Ghysels et al. [20] as leading references for detailed discussions of the DCC model and MIDAS regressions.

a scale intercept matrix with number of parameters equal to $n(n+1)/2$, where n denotes the number of assets. In a vast dimensional framework, this quickly translates into the impossibility of estimating the model since the intercept matrix cannot be directly targeted.

Therefore, we propose to overcome this estimation issue by proposing an iterative procedure inspired by the covariance targeting of Engle [15]. More precisely, based on a Method of Moments estimator, we profile out the parameters of the intercept matrix and iteratively maximize the likelihood in terms of the other parameters of interest. We refer to this as the Iterative Moment-Based Profiling (IMP) estimator, as opposed to the Quasi Maximum Likelihood (QML) estimator which directly maximizes the likelihood with respect to the full parameter vector.

It is worth noting that the proposed estimation procedure can be considered a *switching algorithm* in the sense discussed by Boswijk [9] and Cubadda and Scambelloni [13] since the maximization of the overall likelihood is obtained by switching between optimizations over different blocks of parameters. This idea has a long standing tradition in the econometric analysis of time series. A simple, well known example of switching algorithm is given by the Cochrane-Orcutt iterative estimation procedure. Compared to conventional switching algorithms, the procedure that is here implemented incorporates an additional targeting step. In particular, it reduces the dimension of the optimization problem to be solved by concentrating out some of the parameters, the elements of the intercept matrix, by means of an iteratively re-computed moment-based estimator. A comprehensive simulation study is performed to assess the finite-sample properties of the proposed estimator which is found to deliver unbiased estimates and to quickly converge, as no more than three iterations are required in general.

The second relevant contribution of the paper is the development of a resampling based procedure for the generation of multi-step ahead forecasts of the realized covariance matrices. The multiplicative component structure of the MMReDCC model makes the derivation of a closed-form expression for the h-step predictor impossible. Hence, to solve this issue we use a distribution-free procedure based on a residual bootstrap method. The bootstrap has been a standard tool for generating multi-step forecasts from non-linear and non-Gaussian time series models for more than two decades (see e.g. Clements and Smith [12]). Its use has been later extended to univariate volatility modeling (see e.g. Pascual and Ruiz [29]; Shephard and Sheppard [31]). More recently, Fresoli and Ruiz [19] have proposed a simple resampling algorithm that makes use of residual bootstrap to compute multi-step forecasts from DCC models. The bootstrap procedure which is implemented in this paper builds on the work of Fresoli and Ruiz [19] but the algorithm is adapted to the dynamic modeling of realized covariance matrices.

Finally, the results of two different applications to real data are presented and discussed. In the first one, we focus on a low dimensional setting (ten assets), in which both the IMP and one-step QML estimation procedures are feasible, and compare the estimates obtained by means of both algorithms. We find that the IMP-based estimates are sufficiently close to the QML ones, so that using the IMP method in large dimensions is a sensible approach. In the second application the MMReDCC model estimated for fifty assets by the IMP method is used to generate forecasts of the realized covariance matrix, up to twenty days ahead, and compared to existing benchmarks not accounting for short and long term (co)volatility dynamics. For the dataset considered, in correspondence of a forecasting horizon equal to one day, the MMReDCC model is outperformed by the benchmarks while it dominates for longer horizons up to ten days.

The remainder of the paper is organized as follows. Section 2 briefly recalls the structure of the MMReDCC model and explains the curse of dimensionality issue. Section 3 introduces the IMP algorithm and Section 4 presents the results of a Monte Carlo experiment aimed at assessing the finite sample statistical properties of the proposed estimation algorithm. The bootstrap procedure for computing multi-step ahead forecasts is explained in Section 5. Section 6 contains the empirical results for the in-sample estimation comparison and the out-of-sample forecasting exercise. Section 7 concludes with some final remarks.

2. The MMReDCC model

Let C_t be a $n \times n$ positive definite and symmetric (PDS) realized estimator of the latent integrated covariance (IC) matrix of daily returns. In the following, unless otherwise stated, we will refer to C_t as the realized covariance (RC), although any other consistent PDS estimator could be used. Conditionally on the set consisting of all relevant information up to and including day $t-1$, C_t is assumed to follow a n-dimensional central Wishart distribution:

$$C_t | I_{t-1} \sim W_n(\nu, S_t/\nu), \quad \forall t = 1, \ldots, T, \tag{1}$$

where ν ($> n-1$) is the degrees of freedom parameter and S_t is the PDS conditional mean matrix of order n. Under the assumption of absence of microstructure noise and other biases (see Barndorff-Nielsen and Shephard [4]), S_t represents the conditional covariance matrix of returns, which is our object of interest.

In the MMReDCC model, S_t is designed to directly capture the long run movements in the levels around which realized (co)variances (and by extension, correlations) fluctuate from day to day. To this extent, the model features a multiplicative decomposition of the conditional covariance matrix S_t into a smoothly varying or *secular* component $M_t = L_t L_t'$ and a short-lived component S_t^*, such that S_t can be rewritten as $S_t = L_t S_t^* L_t'$, where the matrix square root L_t can be obtained by a Cholesky factorization of M_t. These components can then be modeled separately.

First, the secular component is specified parametrically and extracted by means of a MIDAS filter assumed to be a weighted sum of K lagged realized covariance matrices over a long horizon, where the number of lags spanned in the MIDAS specification is usually chosen to minimize the trade-off between the highest in-sample likelihood value and the number of observations lost to initialize the filter. It is expressed as

$$M_t = \Lambda + \theta \sum_{k=1}^{K} \phi_k(\omega) C_{t-k}. \tag{2}$$

In the right hand side of Eq.(2), the first term Λ is a $n \times n$ symmetric and semi-positive definite matrix of constant parameters, θ is a positive scalar and $\phi_k(\cdot)$ is a weight function parametrized according to the restricted Beta polynomial

$$\phi_k(\omega) = \frac{\left(1 - \frac{k}{K}\right)^{\omega-1}}{\sum_{j=1}^{K}\left(1 - \frac{j}{K}\right)^{\omega-1}},$$

The scalar parameter ω dictates the shape of the function and in order to achieve a time-decaying pattern of the weights, it is constrained to be larger than 1. For identification, the constraint $\sum_{k=1}^{K}\phi_k(\omega) = 1$ is imposed.

Second, the dynamics of the short term component S_t^* is specified according to a scalar DCC parametrization that enables a separate treatment of conditional volatilities and correlations, thus allowing for a high degree of flexibility. Letting X be any square matrix of arbitrary size n, in the remainder the notation $diag(X)$ is used to denote a $n \times n$ diagonal matrix with non-zero elements equal to the diagonal elements of X. Therefore, assuming that $S_t^* = D_t^* R_t^* D_t^*$, where $D_t^* = diag\{S_t^*\}^{1/2}$, their scalar specifications correspond to the following equations:

$$S_{ii,t}^* = (1 - \gamma_i - \delta_i) + \gamma_i C_{ii,t-1}^* + \delta_i S_{ii,t-1}^*, \qquad \forall i = 1, \ldots, n \tag{3}$$

$$R_t^* = (1 - \alpha - \beta) I_n + \alpha P_{t-1}^* + \beta R_{t-1}^*, \tag{4}$$

where $\gamma_i > 0$, $\delta_i \geq 0$, $\gamma_i + \delta_i < 1$, $\alpha > 0$, $\beta \geq 0$, $\alpha + \beta < 1$, $C_t^* = L_t^{-1} C_t (L_t')^{-1}$ and $P_t^* = (diag\{C_t^*\})^{-1/2} C_t^* (diag\{C_t^*\})^{-1/2}$. The matrix C_t^* is the realized covariance matrix purged of its long term component and the matrix P_t^* is the corresponding short term realized correlation matrix. Mean reversion to unity in Eq.(3) and to an identity matrix in Eq.(4) is needed for identification of the different components. Let $\boldsymbol{\gamma} = \{\gamma_1, ..., \gamma_n\}$, $\boldsymbol{\delta} = \{\delta_1, ..., \delta_n\}$ for further use.

The parameters can be estimated by maximizing the following Wishart (quasi) log-likelihood function in one step:

$$\ell_T(\boldsymbol{\psi}) = -\frac{1}{2} \sum_{t=1}^{T} \left\{ \log |S_t(\boldsymbol{\psi})| + \mathrm{tr}\left[S_t(\boldsymbol{\psi})^{-1} C_t\right] \right\}. \tag{5}$$

The finite-dimensional parameter vector $\boldsymbol{\psi} = \{vech(\Lambda), \theta, \omega, \boldsymbol{\gamma}, \boldsymbol{\delta}, \alpha, \beta\}$[2], has length $\{n_\Lambda + 2n + 4\}$ where $n_\Lambda = n(n+1)/2 = O(n^2)$ denotes the number of unique parameters included in the intercept matrix Λ of Eq.(2). It is obvious that, as n increases, the curse of dimensionality problem quickly arises, leading to the number of parameters listed in the first two rows of Table 1. Observe that estimation becomes already cumbersome after $n = 20$ and almost impossible for $n \geq 50$.

[2]Note that $\boldsymbol{\psi}$ do not include the degree of freedom parameter ν, as the first order conditions for the estimation of the parameter vector $\boldsymbol{\psi}$ do not depend on ν by linearity in ν (see [5]).

Table 1: Number of parameters of MMReDCC models

	$n = 5$	$n = 10$	$n = 20$	$n = 50$	$n = 100$
n_Λ	15	55	210	1275	5050
ψ	29	79	254	1379	5254
$\tilde{\psi}$	14	24	44	104	204

Note: Entries report the number of parameters as a function of the dimension n; n_Λ denotes the number of unique parameters contained in the Λ matrix, ψ denotes the full vector of model parameters and $\tilde{\psi}$ the vector of parameters excluding n_Λ.

On the other hand, the last row of Table 1 shows that an intuitive way to keep the model tractable is to avoid estimating the parameters of the matrix Λ. This would be sufficient to reduce the order to $2n + 4 = O(n)$, thus making the model estimable also for large n.
In the following section we put forward a feasible estimation procedure that aims at overcoming the direct estimation of the long term component intercept matrix, thus crucially mitigating the computational complexity of the model.

3. An Iterative Moment based Profiling (IMP) algorithm

In this section we discuss an iterative procedure for fitting the MMReDCC model to large dimensional datasets. The basic idea underlying the proposed algorithm is to eliminate from the likelihood maximization the parameters of the intercept matrix Λ using a technique that builds upon the covariance targeting discussed in Pedersen and Rahbek [30] for BEKK and Engle et al. [17] for DCC models. First of all, notice that from Eq.(2) and the following relation

$$\Lambda = E(M_t) - \theta \sum_{k=1}^{K} \phi_k(\omega) E(C_{t-k}),$$

a moment based estimator of the Λ intercept matrix is

$$\hat{\Lambda} = \frac{1}{T} \sum_{t=1}^{T} \left[M_t - \theta \sum_{k=1}^{K} \phi_k(\omega) C_{t-k} \right]. \tag{6}$$

Obviously, given the latent nature of M_t, the estimator in Eq.(6) cannot be computed in practice and hence the covariance targeting approach cannot be applied in the usual way. It is worth noting that, if L_t and S_t^* were assumed to be independent, given $E(S_t^*) = I_n$, it would hold that $E(C_t) = E(M_t)$, implying that an asymptotically equivalent version of Eq.(6) could be explicitly computed replacing M_t by C_t. However, this is not the approach we pursue, since the assumption of independence of the short and long term sources is difficult to justify and would result in a rather counterintuitive and arbitrary constraint. Hence, we adopt a different method.

By noting from Eq.(6) that no estimate of Λ makes sense regardless of the value of (θ, ω), we make this dependence explicit and obtain an estimate of Λ as a function of (θ, ω), i.e. $\hat{\Lambda}(\theta, \omega)$. In this way, a different estimate of Λ is required for each different value of the other two parameters. Therefore, by substituting $\hat{\Lambda}(\theta, \omega)$ for Λ in the Wishart QML function stated in Eq.(5), the following moment based QML approximation is obtained:

$$\tilde{\ell}_T(\tilde{\boldsymbol{\psi}}) = -\frac{1}{2} \sum_{t=1}^{T} \left\{ \log |\tilde{L}_t(\theta, \omega) S_t^*(\tilde{\boldsymbol{\psi}}) \tilde{L}_t'(\theta, \omega)| + tr\left\{ \left[\tilde{L}_t(\theta, \omega) S_t^*(\tilde{\boldsymbol{\psi}}) \tilde{L}_t'(\theta, \omega) \right]^{-1} C_t \right\} \right\} \quad (7)$$

with $\tilde{\boldsymbol{\psi}} = (\omega, \theta, \boldsymbol{\psi}_{S^*})'$, $\boldsymbol{\psi}_{S^*}' = (\boldsymbol{\gamma}, \boldsymbol{\delta}, \alpha, \beta)$ and

$$\tilde{M}_t(\theta, \omega) = \tilde{L}_t(\theta, \omega) \tilde{L}_t'(\theta, \omega) = \hat{\Lambda}(\theta, \omega) + \theta \sum_{k=1}^{K} \phi_k(\omega) C_{t-k}. \quad (8)$$

The method we propose consists of estimating the parameters in $\tilde{\boldsymbol{\psi}}$ by a block-wise maximization of the moment-based QML function given in Eq.(7). First, conditional on some reasonable initial guess of (θ, ω), $\tilde{\ell}_T(\tilde{\boldsymbol{\psi}})$ is maximized with respect to the short term parameters ψ_{S^*} and then, conditional on $\hat{\boldsymbol{\psi}}_{S^*}$, the same function is maximized with respect to (θ, ω). The procedure is iterated for $j = 0, \ldots, J$ until some pre-specified convergence criterion is met.

To initialize the algorithm at $j = 0$, one can reasonably use as starting values the parameter estimates obtained by fitting the model to low dimensional subsets of data; also, an initial guess for the long term component $M_{t,0}$ could be either provided in a naive way, *i.e.* using the series of observed realized covariance matrices directly, or in a more sophisticated manner, by fitting to the data a nonparametric kernel smoother with an optimized bandwidth parameter. Note that in order to guarantee the positive definiteness of $\tilde{M}_t(\theta, \omega)$ in Eq.(8), it suffices to initialize $M_{t,0}$ from a PDS matrix and to impose $\theta > 0$. Given that the observed series of C_t, for every t, is PDS by definition, $\hat{\Lambda}(\theta, \omega)$ is assured to be at least semi-positive definite at each iteration $j > 0$.

Once $\Lambda_j(\theta_j, \omega_j)$ has been computed at the initial iteration $j = 0$, for every $j > 0$ the steps conducted in the algorithm are as follows:

Step 1 Plug $\Lambda_{j-1}(\theta_{j-1}, \omega_{j-1})$ into Eq.(2), then get $\tilde{M}_{t,j}$ and $\tilde{L}_{t,j} = \text{chol}(\tilde{M}_{t,j})$ for all t;

Step 2 For each asset $i = 1, \ldots n$, obtain the short term GARCH(1,1) parameters of Eq.(3) as follows

$$\{\gamma_{i,j}, \delta_{i,j}\} = \underset{\{\gamma_i, \delta_i\}}{\arg\max} \qquad \tilde{\ell}_T\left(\theta_{j-1}, \omega_{j-1}, \alpha_{j-1}, \beta_{j-1}\right);$$

Step 3 Conditional on the estimated vectors $\boldsymbol{\gamma}_j = (\gamma_{1,j}, \ldots, \gamma_{n,j})'$ and $\boldsymbol{\delta}_j = (\delta_{1,j}, \ldots, \delta_{n,j})'$, maximize the same log-likelihood function with respect to the short term DCC correlation parameters:

$$\{\alpha_j, \beta_j\} = \underset{\{\alpha, \beta\}}{\arg\max} \qquad \tilde{\ell}_T\left(\theta_{j-1}, \omega_{j-1}, \boldsymbol{\gamma}_j, \boldsymbol{\delta}_j\right);$$

Step 4 Finally, conditional on the vector of short term parameter estimates $\boldsymbol{\phi}_{S^*} = \{\boldsymbol{\gamma}_j, \boldsymbol{\delta}_j, \alpha_j, \beta_j\}$, maximize $\tilde{\ell}_T$ with respect to $\{\theta_j, \omega_j\}$; these estimates are used to compute an updated version of $\Lambda_j(\theta_j, \omega_j)$;

Step 5 Check for convergence otherwise update all parameter estimates and go back to Step 1.

It is worth to stress that although $\tilde{\ell}_T(\tilde{\boldsymbol{\psi}})$ looks like a profile likelihood, it is not since $\hat{\Lambda}(\theta, \omega)$ is not a QML estimator but a feasible moment estimator. This motivates our choice to refer to Steps $1-5$ as the Iterative Moment based Profiling algorithm, or IMP for short. This implies that $\tilde{\boldsymbol{\psi}}$ is typically less efficient than the standard QML estimator that maximizes Eq.(5) in one step. We come back to this issue in Section 6.1.

4. Simulation study

A Monte Carlo study is conducted to analyse the finite sample properties of the IMP estimator. We assume the MMReDCC to be the DGP and we generate 500 processes of length $T = 1000$ and 2000 for $n = 10, 20, 40$ and 50, with true parameter values inspired by the estimates given in Bauwens et al. [5], as summarized in Table 2.

It is important to stress that, in order to initialize the algorithm, parameter values have to be carefully chosen. This is a standard requirement in every optimization based procedure where the initial amount of information on the model parameters can be limited or even null. In our situation we are mainly concerned with the impact that different choices of $M_{t,0}$, more than the remaining set of parameters, may have on the convergence of the IMP algorithm. We evaluate this by performing a robustness check based on the two possible initializations of $M_{t,0}$ mentioned in Section 3.

In the first set of repetitions $M_{t,0}$ is computed by fitting to the series of simulated realized covariance matrices a Nadaraya-Watson kernel estimator with a single bandwidth parameter for the whole covariance matrix. As in Bauwens et al. [5] and Bauwens et al. [6], the optimal bandwidth is selected by a least squares cross-validation criterion, where the six-month rolling covariance is used as the reference for the computation of least squares. In the second (equivalent) simulation study, $M_{t,0}$ is obtained by substituting in Eq. (6) the observed C_t for the latent matrix M_t at each t. In both cases, the initial scalar model parameters are set equal to the values listed in Panel B of Table 2.

The estimation bias is evaluated by the relative bias (RB), computed as $\frac{1}{500}\sum_{i=1}^{500}\frac{\hat{\psi}_i-\psi}{\psi}$, along with the interquartile range (IQR), mean, minimum and maximum of the obtained parameter estimates. To save space, we report averaged bias results for the parameters of the MIDAS intercept matrix in a separate table.

Table 3 reports results from the first simulation exercise. The emerging picture looks encouraging. As expected, the relative biases decrease as n or T increases. The biases for the parameters of the short term volatility and correlation components are very small, being smaller than five per cent in most of the cases, with one exception recorded for $\bar{\gamma}$ at $T = 1000$ for $n = 10$. As for the scalar parameters in the MIDAS specification, the bias

Table 2: Simulation setting

Panel A: Parameters					
Long term component					
θ			0.5		
K			264		
ω			15		
Λ			$\Lambda_{i,i} = 0.02$, $\Lambda_{i,j} = 0.002$ for $i \neq j$		
Short term components					
γ_i			$\sim U(\gamma_0 - 0.02, \gamma_0 + 0.02)$, $\gamma_0 = 0.2$		
δ_i			$\sim U(2\delta_0 + \gamma_i - 1 + 0.01, 1 - \gamma_i - 0.01)$, $\delta_0 = 0.7$		
α			0.2		
β			0.7		
General					
ν			$2n$		
T			1000, 2000		
initial discarded observations			1000		
convergence tolerance			0.0001		
Panel B: Initial values					
θ_0	ω_0	$\gamma_{i,0}$	$\delta_{i,0}$	α_0	β_0
0.8	10	0.05	0.90	0.05	0.9

Note: In Panel A, for every $i = 1, ..., n$ it holds $\{\gamma_i + \delta_i\} < 1$. Entries of Panel B are scalar parameters chosen to initialize the algorithm in both sets of simulation exercises.

for θ is negative in seven out of eight cases (the exception occurs for $n = 10$ at $T = 2000$) and ranging from the maximum of 5.8% (in absolute value) for $n = 10$ and $T = 1000$ to the lowest value of 0.1% for $n = 50$ and $T = 2000$. The bias on the ω parameter, also generally negative, tends to decrease with n but is usually of higher order (from 1.1 to 12% in absolute value). A similar behavior is observed for the IQR measure, which decreases across n and T but remains on higher values for the parameter ω. However, this does not represent a major concern as the Beta weight function is not very sensitive to small variations of this parameter and therefore we do not expect the likelihood function to be either.

Table 4 gives an idea of the robustness of the results to the other initialization of the long term component. Entries can be directly compared to those in Table 3. As hoped for, the initial choice has a minor impact on the overall accuracy of the estimator, as the parameter biases are in the same range of magnitude and the comments made earlier are still valid under this alternative scenario. Figure C.2 contains plots of the Monte Carlo standard deviations of the estimated θ, ω, α and β parameters against the cross-section size.

Table 3: Simulation exercise I: summary statistics

	T= 1000					
	$\bar{\gamma}$	$\bar{\delta}$	α	β	θ	ω
	0.197	0.705	0.2	0.7	0.5	15
	n=10					
RB	0.098	-0.036	0.020	0.003	-0.058	-0.120
IQR	0.048	0.093	0.006	0.010	0.044	1.641
Mean	0.202	0.699	0.204	0.702	0.475	14.820
Min	0.176	0.660	0.191	0.679	0.393	7.460
Max	0.214	0.735	0.220	0.728	0.709	18.943
	n=20					
RB	0.049	-0.009	0.019	0.001	-0.056	-0.110
IQR	0.046	0.083	0.003	0.006	0.019	0.713
Mean	0.202	0.701	0.204	0.701	0.472	14.782
Min	0.171	0.633	0.197	0.687	0.430	12.802
Max	0.219	0.744	0.211	0.711	0.532	16.759
	n=40					
RB	0.028	0.023	0.015	0.002	-0.049	-0.080
IQR	0.042	0.077	0.002	0.002	0.011	0.372
Mean	0.208	0.715	0.203	0.701	0.476	14.810
Min	0.190	0.656	0.060	0.695	0.446	3.735
Max	0.217	0.762	0.222	0.799	0.705	16.500
	n=50					
RB	0.027	0.011	0.016	0.001	-0.045	0.012
IQR	0.042	0.076	0.001	0.002	0.008	0.293
Mean	0.208	0.716	0.203	0.701	0.473	15.182
Min	0.162	0.644	0.200	0.697	0.455	13.150
Max	0.220	0.814	0.207	0.705	0.525	15.830

	T= 2000					
	$\bar{\gamma}$	$\bar{\delta}$	α	β	θ	ω_2
	0.197	0.705	0.2	0.7	0.5	15
	n=10					
RB	-0.039	-0.002	0.033	0.003	0.053	-0.095
IQR	0.032	0.068	0.006	0.008	0.037	1.819
Mean	0.2	0.713	0.207	0.702	0.526	13.58
Min	0.153	0.669	0.183	0.523	0.072	1.949
Max	0.22	0.803	0.37	0.817	1.000	17.74
	n=20					
RB	0.036	0.011	0.024	0.002	-0.014	-0.080
IQR	0.031	0.079	0.002	0.004	0.015	0.622
Mean	0.209	0.708	0.205	0.702	0.496	13.801
Min	0.197	0.678	0.200	0.694	0.001	2.440
Max	0.221	0.739	0.220	0.748	0.598	15.270
	n=40					
RB	0.033	0.029	0.022	0.002	-0.014	-0.072
IQR	0.030	0.064	0.001	0.002	0.007	0.263
Mean	0.209	0.719	0.204	0.702	0.493	13.925
Min	0.181	0.671	0.182	0.674	0.172	1.000
Max	0.221	0.761	0.223	0.744	0.837	14.760
	n=50					
RB	0.029	0.027	0.017	0.001	-0.011	-0.037
IQR	0.030	0.056	0.001	0.002	0.007	0.220
Mean	0.208	0.720	0.203	0.701	0.494	14.442
Min	0.191	0.657	0.199	0.695	0.474	12.089
Max	0.222	0.763	0.207	0.707	0.529	16.238

Note: Summary statistics of the first set of simulations where $M_{t,0}$ is initialized using a nonparametric kernel estimator, see Section 3. To save on space, $\bar{\gamma}$ and $\bar{\delta}$ are reported as averaged values across series and replications. RB denotes the Relative Bias computed over 500 replications. True parameter values used to simulate the process at the top of the table.

In all cases, standard deviations tend to decline as the cross-section dimension grows, with a faster decline when $T = 2000$. The two approaches produce similar parameter standard deviations, with slightly bigger values recorded for θ and ω under the second simulation experiment in correspondence with the higher cross-section sizes.

If we move to analyzing the bias results for the scale MIDAS intercept matrix, Table 5 shows that under both sets of simulation exercises the estimator $\hat{\Lambda}(\theta, \omega)$ well approximates the true Λ matrix at all cross-section dimensions, with the parameter bias (averaged across diagonal and off-diagonal elements) clearly improving with increasing n and T. Again, the direct comparison of Panels A and B confirms that the algorithm initialized from the series

Table 4: Simulation exercise II: summary statistics

	T= 1000						**T= 2000**					
	$\bar{\gamma}$	$\bar{\bar{\delta}}$	α	β	θ	ω	$\bar{\gamma}$	$\bar{\bar{\delta}}$	α	β	θ	ω_2
	0.197	0.705	0.2	0.7	0.5	15	0.197	0.705	0.2	0.7	0.5	15
n=10												
RB	-0.077	0.013	0.019	0.002	-0.032	-0.007	0.043	0.006	0.025	0.002	-0.012	-0.069
IQR	0.070	0.113	0.007	0.010	0.043	1.664	0.028	0.046	0.004	0.007	0.024	0.870
Mean	0.191	0.708	0.204	0.701	0.484	14.892	0.211	0.707	0.205	0.701	0.494	13.959
Min	0.107	0.588	0.191	0.678	0.393	6.678	0.189	0.654	0.196	0.686	0.435	9.614
Max	0.231	0.833	0.218	0.723	0.927	19.237	0.232	0.748	0.213	0.715	0.623	15.535
n=20												
RB	0.048	-0.002	0.018	0.002	-0.045	-0.008	0.006	0.039	0.023	0.002	-0.011	-0.064
IQR	0.042	0.085	0.003	0.005	0.021	0.719	0.030	0.060	0.002	0.004	0.013	0.510
Mean	0.207	0.702	0.204	0.701	0.477	14.887	0.206	0.721	0.205	0.701	0.494	14.047
Min	0.196	0.649	0.197	0.689	0.429	6.751	0.164	0.680	0.199	0.692	0.467	11.108
Max	0.220	0.740	0.214	0.713	0.887	16.573	0.225	0.772	0.209	0.710	0.555	15.241
n=40												
RB	0.046	0.017	0.017	0.003	-0.045	0.005	0.050	0.016	0.021	0.002	-0.012	-0.057
IQR	0.042	0.080	0.002	0.003	0.010	0.421	0.029	0.053	0.001	0.002	0.007	0.245
Mean	0.209	0.713	0.203	0.702	0.477	15.078	0.210	0.718	0.204	0.701	0.494	14.148
Min	0.197	0.682	0.193	0.696	0.116	6.895	0.194	0.658	0.202	0.697	0.479	8.433
Max	0.219	0.756	0.216	0.807	0.955	49.985	0.223	0.768	0.212	0.708	0.726	14.638
n=50												
RB	0.029	0.028	0.016	0.002	-0.053	0.015	0.028	0.018	0.020	0.002	-0.017	-0.048
IQR	0.041	0.071	0.001	0.002	0.009	0.342	0.030	0.061	0.001	0.001	0.005	0.195
Mean	0.208	0.715	0.203	0.701	0.473	15.225	0.208	0.721	0.204	0.701	0.492	14.278
Min	0.190	0.632	0.200	0.698	0.457	14.190	0.184	0.674	0.202	0.698	0.437	11.178
Max	0.220	0.766	0.206	0.705	0.494	15.844	0.216	0.766	0.209	0.713	0.508	14.742

Note: Summary statistics of the second set of simulations where $M_{t,0}$ is initialized from the series of realized covariance matrices, see Section 3. To save on space, $\bar{\gamma}$ and $\bar{\delta}$ are reported as averaged values across series and replications. RB denotes the Relative Bias computed over 500 replications. True parameter values used to simulate the process at the top of the table.

of realized covariance matrices overall performs no worse than the one initialized from a nonparametric smoother.

To summarize, the simulation study carried out in this section suggests that the proposed algorithm works quite accurately in finite samples and converges irrespective of the initialization choice made. Overall, the moment-based estimator used for iteratively targeting the constant intercept matrix in the secular component does not create a severe bias problem in the estimation of the other parameters, thus representing a feasible solution to alleviate the curse of dimensionality issue that would otherwise prevent the use of the MMReDCC model in high dimensional applications. Both initialization methods for $M_{t,0}$

Table 5: Bias results for the scale MIDAS intercept matrix.

Panel A: Simulation exercise I				Panel B: Simulation exercise II			
T=1000		**T=2000**		**T=1000**		**T=2000**	
n=10		**n=10**		**n=10**		**n=10**	
$\mathbf{RB}_{\{i,i\}}$	0.080	$\mathbf{RB}_{\{i,i\}}$	0.033	$\mathbf{RB}_{\{i,i\}}$	0.075	$\mathbf{RB}_{\{i,i\}}$	0.060
$\mathbf{RB}_{\{i,j\}}$	0.079	$\mathbf{RB}_{\{i,j\}}$	0.000	$\mathbf{RB}_{\{i,j\}}$	0.066	$\mathbf{RB}_{\{i,j\}}$	0.040
n=20		**n=20**		**n=20**		**n=20**	
$\mathbf{RB}_{\{i,i\}}$	0.073	$\mathbf{RB}_{\{i,i\}}$	0.068	$\mathbf{RB}_{\{i,i\}}$	0.072	$\mathbf{RB}_{\{i,i\}}$	0.058
$\mathbf{RB}_{\{i,j\}}$	0.062	$\mathbf{RB}_{\{i,j\}}$	0.048	$\mathbf{RB}_{\{i,j\}}$	0.060	$\mathbf{RB}_{\{i,j\}}$	0.044
n=40		**n=40**		**n=40**		**n=40**	
$\mathbf{RB}_{\{i,i\}}$	0.073	$\mathbf{RB}_{\{i,i\}}$	0.062	$\mathbf{RB}_{\{i,i\}}$	0.073	$\mathbf{RB}_{\{i,i\}}$	0.058
$\mathbf{RB}_{\{i,j\}}$	0.061	$\mathbf{RB}_{\{i,j\}}$	0.046	$\mathbf{RB}_{\{i,j\}}$	0.159	$\mathbf{RB}_{\{i,j\}}$	0.043
n=50		**n=50**		**n=50**		**n=50**	
$\mathbf{RB}_{\{i,i\}}$	0.072	$\mathbf{RB}_{\{i,i\}}$	0.004	$\mathbf{RB}_{\{i,i\}}$	0.072	$\mathbf{RB}_{\{i,i\}}$	0.058
$\mathbf{RB}_{\{i,j\}}$	0.057	$\mathbf{RB}_{\{i,j\}}$	0.037	$\mathbf{RB}_{\{i,j\}}$	0.058	$\mathbf{RB}_{\{i,j\}}$	0.043

Note: $RB_{\{i,i\}}$ denotes averaged values over diagonal terms, while $RB_{\{i,j\}}$ denotes averages over off diagonal terms. Panel (a) reports summary statistics of the first simulation exercise where $M_{t,0}$ is initialized from a nonparametric smoother while Panel (b) reports results from the second simulation exercise where the series of observed realized covariance matrices are used.

can be used in practice. In the empirical section, we have opted for the nonparametric smoother.

5. Multi-step Forecasting

Models featuring short and long-run dynamics are particularly attractive for computing multi-step-ahead predictions, as their component dynamic structure is possibly expected to be beneficial for longer-term forecasts. The complex nonlinear structure of the MM-ReDCC model makes the analytical derivation of closed-form solutions impossible. In order to overcome this problem, we propose to compute multi-step predictions by means of a procedure based on bootstrap resampling.

At the outset, notice that Eq.(1) implies that $E(C_t|\Im_{t-1}) = S_t$, so that C_t can be represented as

$$C_t = S_t^{1/2} U_t (S_t^{1/2})' \tag{9}$$

where U_t is an element of a sequence of iid random matrices such that $E(U_t) = I_n$, and $S_t^{1/2}$ is any PDS matrix such that $S_t^{1/2}(S_t^{1/2})' = S_t$. The Wishart assumption of Eq.(1) is recovered if $U_t \sim W_n(\nu, I_n/\nu)$, but this assumption is not needed to justify the bootstrap procedure that we use for generating multi-step-ahead forecasts of the realized covariance matrix C_t. The procedure is described in the following six steps.

Step 1 Estimate the model on $\{C_t, t = 1, \ldots, T\}$ and obtain the estimated conditional covariances $\hat{S}_t$.

Step 2 Compute the estimated residuals

$$\hat{U}_t = \hat{S}_t^{-1/2} C_t (\hat{S}_t^{-1/2})', \qquad t = 1, \ldots, T$$

and rescale them to enforce their sample mean to be equal to I_n, namely:

$$\tilde{U}_t = (\hat{E}_u^{-1/2}) \hat{U}_t (\hat{E}_u^{-1/2})',$$

where $\hat{E}_u = (1/T) \sum_{t=1}^{T} \hat{U}_t$. The rescaled $\tilde{U}_t$ can then be used to generate bootstrap replicates of C_{T+j}, for $j = 1, \ldots, h$, where h denotes the chosen forecast horizon.

Step 3 Draw with replacement a bootstrap sample $\{\tilde{U}_{T+1|T}, \ldots, \tilde{U}_{T+h|T}\}$ of length h from the empirical CDF of $\{\tilde{U}_t, t = 1, \ldots, T\}$.

Step 4 For $j = 1 \ldots, h$, recursively generate a sequence of bootstrap replicates of C_{T+j} as follows

$$\begin{aligned}
M_{T+j|T} &= \hat{\Lambda}(\theta, \omega) + \hat{\theta} \sum_{k=1}^{K} \phi_k(\hat{\omega}) \tilde{C}_{T-k+j|T} \\
L_{T+j|T} &= M_{T+j|T}^{1/2} \\
C_{T+j|T}^{*} &= L_{T+j|T} C_{T+j} (L'_{T+j|T})^{-1} \\
P_{T+j|T}^{*} &= (diag\{C_{T+j|T}^{*}\})^{-1/2} C_{T+j|T}^{*} (diag\{C_{T+j|T}^{*}\})^{-1/2} \\
S_{ii,T+j|T}^{*} &= (1 - \hat{\gamma}_i - \hat{\delta}_i) + \hat{\gamma}_i C_{ii,T+j-1|T}^{*} + \hat{\delta}_i S_{ii,T+j-1|T}^{*} \\
R_{T+j|T}^{*} &= (1 - \hat{\alpha} - \hat{\beta}) I_n + \hat{\alpha} P_{T+j-1|T}^{*} + \hat{\beta} R_{T+j-1|T}^{*} \\
S_{T+j|T}^{*} &= (diag\{S_{T+j|T}^{*}\})^{1/2} R_{T+j|T}^{*} (diag\{S_{T+j|T}^{*}\})^{1/2} \\
S_{T+j|T} &= L_{T+j|T} S_{T+j|T}^{*} L'_{T+j|T} \\
C_{T+j|T} = &= (S_{T+j|T}^{1/2}) \tilde{U}_{T+j} (S_{T+j|T}^{1/2})'
\end{aligned}$$

Step 5 Repeat steps 3-4 B times, where B is set sufficiently large (e.g. B=10000). As a result the procedure generates an array of $h \times B$ bootstrap replicates $C_{T+j|T}^{(b)}$ $(b = 1, \ldots, B)$.

Step 6 Finally, the h-steps-ahead forecast can be computed as

$$\hat{S}_{T,j} = \frac{1}{B} \sum_{b=1}^{B} C_{T+j|T}^{(b)}.$$

Even if our primary interest is in forecasting from MMReDCC models, the proposed forecasting procedure is very general and can be readily adapted to any model that admits the representation in Eq.(9), where S_t is modeled as a function of past information $\mathcal{I}_{t-1}$. For example, in the empirical application which is being presented in Section 6.2, we also use it to generate multi-step ahead forecasts of C_t from the cRDCC model of Bauwens et al. [7]. To this purpose, the dynamic equations in step 4 must be replaced by those pertaining to the specific model of interest.

6. Empirical Applications

This section contains two empirical applications. The first application provides the estimation results for the IMP estimator in comparison with the standard QML estimator in the ideal case where both can be computed. The second one is performed in a large dimensional system and aims at evaluating both the full-sample fit of the model and its forecasting performance. Specifically, we evaluate the ability of the MMReDCC model to provide accurate multi-step-ahead covariance predictions against existing competitors not accounting for time-varying long term dynamics.

6.1. Small sample accuracy comparison

As regards the in-sample performance, we are interested in comparing the estimates provided by the IMP method to the QML ones that are obtained by maximizing the likelihood over the full parameter vector and can only be used in low dimensional cases (remember Table 1). For this purpose, we fix the cross-sectional dimension equal to ten assets and use three different datasets. An overview of the data being used is given in Appendix A. The first dataset comprises the assets used in Bauwens et al. [5] and includes series of daily realized covariance matrices estimated on five minute intraday returns over the period February 2001 to December 2009; the second and third sets comprise arbitrary selected subsamples of the dataset used in the work of Boudt et al. [10] which consists of series of daily realized covariance matrices obtained with the *CholCov* estimator over the period January 2007 to December 2012.[3] As already mentioned before, the choice of the realized estimator is not an issue here as the model can be fitted to any series of realized variance-covariance matrices as long as they are guaranteed to be PDS.

Estimation results for the MMReDCC model by both the IMP and the QML estimators are reported in Table 6.

In the three datasets considered, both methods appear to deliver similar estimates. Short term GARCH coefficients tend to be quite homogeneous across assets and generally significant; the same applies to the short term correlation estimates. As for the parameters driving the long term component, it can be noticed that the estimated θ and ω coefficients are regularly lower for the IMP than for the QML method. This is in line with the prevailing negative bias reported from the simulation study. The QML estimator, as expected, performs slightly better than the IMP but the small differences in the log-likelihood values indicate that the loss, in terms of goodness of fit, is negligible and that the proposed IMP algorithm represents an acceptable approach even when the model cannot be estimated by QML.

[3] Our analysis focuses on open-to-close covariance matrices, whereby noisy overnight returns have not been included in the construction of the estimators. We refer to the cited papers for further details.

Table 6: Application I. In-sample parameter estimates

Dataset 1				Dataset 2				Dataset 3			
QML		**IMPL**		**QML**		**IMP**		**QML**		**IMP**	
γ_i	δ_i	γ_i	δ_i	γ_i	δ_i	γ_i	δ_i	γ_i	δ_i	γ_i	δ_i
0.36 (0.04)	0.55 (0.05)	0.38 (0.06)	0.56 (0.04)	0.15 (0.03)	0.75 (0.06)	0.18 (0.05)	0.77 (0.05)	0.20 (0.05)	0.73 (0.12)	0.18 (0.07)	0.78 (0.08)
0.35 (0.63)	0.59 (0.03)	0.41 (0.07)	0.51 (0.04)	0.23 (0.05)	0.69 (0.07)	0.23 (0.05)	0.71 (0.06)	0.21 (0.06)	0.70 (0.23)	0.22 (0.04)	0.72 (0.05)
0.34 (0.04)	0.57 (0.06)	0.37 (0.05)	0.53 (0.06)	0.24 (0.06)	0.61 (0.09)	0.26 (0.06)	0.64 (0.07)	0.25 (0.05)	0.66 (0.07)	0.22 (0.06)	0.69 (0.06)
0.33 (0.04)	0.60 (0.06)	0.31 (0.04)	0.59 (0.05)	0.20 (0.28)	0.65 (0.41)	0.19 (0.08)	0.73 (0.09)	0.12 (0.07)	0.79 (0.20)	0.05 (0.01)	0.93 (0.02)
0.34 (0.03)	0.64 (0.05)	0.33 (0.04)	0.60 (0.04)	0.22 (0.1)	0.49 (0.06)	0.19 (0.08)	0.72 (0.10)	0.18 (0.14)	0.76 (0.41)	0.16 (0.03)	0.79 (0.04)
0.26 (0.03)	0.64 (0.05)	0.25 (0.03)	0.66 (0.04)	0.18 (0.09)	0.67 (0.12)	0.18 (0.06)	0.76 (0.08)	0.29 (0.10)	0.58 (0.17)	0.24 (0.07)	0.67 (0.09)
0.38 (0.03)	0.51 (0.05)	0.37 (0.03)	0.51 (0.03)	0.18 (0.2)	0.56 (0.09)	0.08 (0.04)	0.89 (0.05)	0.14 (0.05)	0.80 (0.50)	0.20 (0.13)	0.59 (0.15)
0.38 (0.04)	0.51 (0.05)	0.37 (0.03)	0.52 (0.04)	0.18 (0.09)	0.75 (0.09)	0.18 (0.04)	0.76 (0.04)	0.21 (0.10)	0.71 (0.11)	0.22 (0.06)	0.71 (0.07)
0.37 (0.04)	0.52 (0.05)	0.39 (0.04)	0.52 (0.04)	0.18 (0.06)	0.73 (0.08)	0.18 (0.05)	0.76 (0.05)	0.22 (0.06)	0.73 (0.21)	0.17 (0.10)	0.76 (0.09)
0.36 (0.04)	0.50 (0.04)	0.37 (0.04)	0.53 (0.05)	0.14 (0.04)	0.77 (0.08)	0.13 (0.04)	0.83 (0.05)	0.15 (0.06)	0.69 (0.10)	0.02 (0.08)	0.74 (0.03)
α	β	α	β	α	β	α	β	α	β	α	β
0.07 (0.00)	0.84 (0.01)	0.07 (0.00)	0.88 (0.00)	0.017 (0.00)	0.80 (0.1)	0.018 (0.00)	0.91 (0.05)	0.02 (0.00)	0.96 (0.01)	0.01 (0.00)	0.90 (0.04)
θ	ω	θ	ω	θ	ω	θ	ω	θ	ω	θ	ω
0.91 (0.05)	4.56 (0.06)	0.88 (0.05)	4.52 (0.05)	0.84 (0.18)	5.63 (2.4)	0.81 (0.18)	3.60 (0.65)	0.90 (0.14)	3.46 (0.71)	0.80 (0.22)	2.67 (0.51)
Frobenius Norm				**Frobenius Norm**				**Frobenius Norm**			
1.60				3.83				2.81			
Loglik		**Loglik**		**Loglik**		**Loglik**		**Loglik**		**Loglik**	
-18317		-18323		-28571		-28576		-29026		-29056	

Note: Each panel reports parameter estimates and corresponding standard errors in brackets. The Frobenius norm measures the difference between the two long term MIDAS intercepts and is computed as $\sqrt{\sum_{i,j=1}^{n} |\hat{\Lambda}_{i,j}^{QML} - \hat{\Lambda}_{i,j}^{IMP}|^2}$, where the superscripts QML, IMP denote respectively the one-step quasi-maximum likelihood and the iterative profile likelihood estimators. Number of in-sample observations is 2242 for Dataset 1 and 1499 for Dataset 2 and 3.

6.2. Forecasting performance

In this subsection we push the analysis to higher dimensions, with the aim of assessing the usefulness of the MMReDCC model in a forecasting framework. As benchmarks we consider the Consistent RDCC (cRDCC) model of Bauwens et al. [7] as the closest competitor and a simple Exponentially Weighted Moving Average (EWMA) model. The simple EWMA predictor appears a natural candidate due to its widespread diffusion among practitioners and in risk management systems like RiskMetrics. If applied to the realized covariance matrices, it is defined by

$$S_t = (1-\lambda)C_{t-1} + \lambda S_{t-1},$$

where the λ parameter is set equal to the value 0.94 (see also Golosnoy et al. [21]).

On the other hand, the choice of the cRDCC as a benchmark is supported by two main reasons. First, it assumes that conditional volatilities and correlations mean revert to constant quantities, thus it can be considered as a simplified version of the MMReDCC model despite not being formally nested in it. Second, the findings of Boudt et al. [10] show that the cRDCC model favorably compares with some widely used competitors, such as the HEAVY (Noureldin et al. [28]) and the cDCC (Aielli [1]) model, in forecasting Value-at-Risk. In order to estimate the cRDCC in high dimension, we apply a three stage QML estimation procedure as suggested by Bauwens et al. [7], where the constant long term covariance matrix is consistently targeted by the unconditional covariance. This drastically reduces the number of parameters to be estimated to $2n+2$.

The dataset comprises fifty of the most liquid equities of the S&P 500 traded over the period May 1997 – July 2008, for a total of 2524 observations. Tickers and descriptive statistics of the data are given in Appendix B.

Before turning to the out-of-sample analysis, it is worth first looking at the estimates obtained by fitting the MMReDCC and cRDCC models over the full sample period. As emerges from Panel A of Table 7, the MMReDCC outperforms the cRDCC in terms of the AIC and BIC criteria, which are both minimized for the MMReDCC. The univariate GARCH(1,1) parameters $\bar{\gamma}$ and $\bar{\delta}$, reported in averaged values across series, largely agree with each other, while the correlation estimates are markedly different across the two models.

To closely examine the difference between the MMReDCC and cRDCC models, consider the conditional correlations between two selected stocks, APOL and GCI, presented in Figure 1. The parameter estimates from the MMReDCC produce large and more persistent shifts in the conditional correlations, including a marked increase at the beginning of May, 2007, lasting until the end of the sample. The cRDCC model, on the contrary, delivers conditional correlations that are nearly constant and exhibit little variation even near the spread of the financial crisis events in 2008. Given the close similarity between the models, this can be reasonably explained by the fact that the parameters θ and ω driving the long term (co)volatilities dynamics allow much more flexibility of the MMReDCC model and thus for a better responsiveness of correlations in periods of higher market volatility.

To determine whether the MMReDCC model can lead to forecasting gains we compute forecasts of the conditional covariance matrix of daily returns at different horizons making use of the bootstrap procedure explained in Section 5. A similar approach is also applied to the cRDCC model, while predictions from the EWMA are obtained analytically, since this model implies that $E(C_{t+h}|\Im_t) = E(C_{t+h-1}|\Im_t)$.

To shorten the computational time, estimation is performed using a fixed-rolling window scheme with window length equal to 2024 observations, that shifts forward every twenty days, over which parameter estimates are kept fixed. Given the full sample size of 2524

Table 7: Full sample estimates and Implemented loss functions.

Panel A: Full sample estimates		
	MMReDCC	**cRDCC**
$\bar{\gamma}$	0.396	0.351
$\bar{\delta}$	0.601	0.592
α	0.013	0.027
β	0.951	0.875
θ	0.719	
ω	2.068	
Lik	800141	677064
AIC	-632	-599
BIC	-629	-598

Panel B: Implemented Loss functions		
Frob	Frobenius distance	$\mathrm{tr}\left[(C_t - H_t)'(C_t - H_t)\right]$
Sfrob	Squared Frobenius distance	$\sum_{i=1}^{n} \lambda_i$
Euclid	Euclidean distance	$\mathrm{vech}(C_t - H_t)'\mathrm{vech}(C_t - H_t)$
ST	Stein	$\mathrm{tr}(H_t^{-1}C_t) - \log\lvert H_t^{-1}C_t\rvert - n$
vND	von Neumann Divergence	$\mathrm{tr}\left(C_t \log C_t - C_t \log H_t - C_t + H_t\right)$
QLIK	Qlike	$\log\lvert H_t\rvert + \mathrm{tr}\left(H_t^{-1}C_t\right)$

Note: Panel A reports full sample estimates from the MMReDCC and cRDCC model, with AIC and BIC criteria rescaled by the number of observations. Panel B contains the loss functions chosen to evaluate the models forecasting ability. H_t denotes the predicted conditional covariance matrix while C_t is the realized measure; λ_i are the eigenvalues of $(C_t - H_t)^2$ and n denotes the number of assets.

observations, this leads to a number of re-estimations of each model equal to twenty-five. The out-of-sample period starts on July, 2006 and covers roughly the last 500 observations of the sample, ending just before the heat of the financial crisis (July 2008). The horizons considered for predictions are $h = 1, 5, 10$ and 20 days.

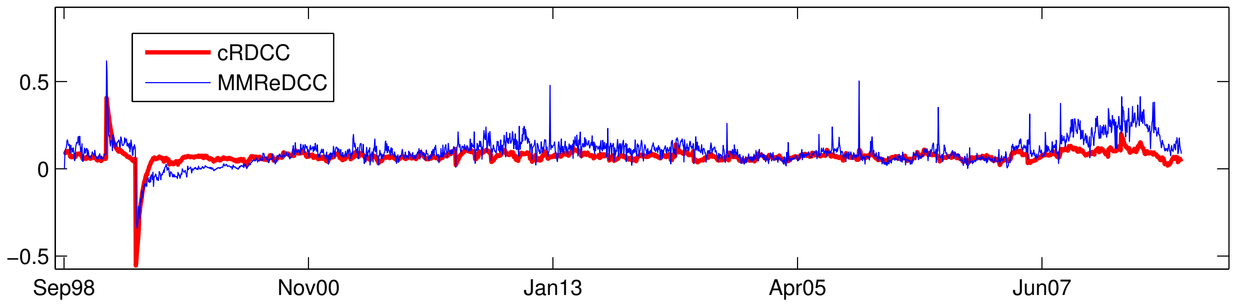
The comparison of the models forecasting ability is performed using the six consistent[4] loss functions defined in Panel B of Table 7, for which we report averaged values over the out-of-sample period. In order to evaluate the significance of the loss function differences we employ the model confidence set (MCS) approach of Hansen et al. [23], which identifies the single model or the set of models having the best forecasting performance at a given confidence level.[5]

Results are summarized by horizon in Table 8. According to Panel A, at the shortest horizon the MMReDCC model is outperformed by its competitors on all the selected criteria, and excluded by the MCS at both the 90 and 75% confidence levels. This suggests that

[4]The term *consistent* is used according to Laurent et al. [26].

[5]The MCS is computed at the 90 % and 75% confidence levels, with block-length bootstrap parameter and number of bootstrap samples used to obtain the distribution under the null respectively equal to 2 and 10000.

Figure 1: Estimated correlation of APOL-GCI.



for the data at hand, accurate one-step-ahead predictions can be obtained by employing simpler models that do not necessarily account for a time-varying long run level.[6] At this stage the choice between the EWMA and the cRDCC model appears almost indifferent, despite the latter being more often included in the MCS.

As we move further in time the situation is quickly reversed: at the 5-day horizon, the MMReDCC model minimizes five out of six loss functions while at the 10-day horizon it appears to deliver the optimal covariance forecasts according to the whole set of losses. This gain is confirmed by the inclusion of the model in the MCS resulting from all the selected criteria, differently from the competitors which are almost never included (EWMA and cRDCC are included three and two times, respectively, in the 75% MCS at $h = 5$, but never at $h = 10$). The predominance of the MMReDCC remains quite stable even at the longest horizon ($h = 20$), but the difference in the forecast accuracy between the MMReDCC and the benchmarks becomes smaller, with the cRDCC performing almost as good as the MMReDCC in terms of MCS inclusions (cRDCC excluded only in the von Neumann 75% MCS). These results appear to be in line with those of [21], where already at the 10-ahead horizon the differences in the forecast accuracy between their best component CAW model and the selected benchmarks were smaller than at shorter horizons.

Overall, the out-of-sample performance of the MMReDCC model in a moderately volatile time period appears to be good relative to the competing models especially at medium-term horizons, when it yields the most accurate forecasts. In light of these empirical results, it appears that the introduction of an additional component capturing the secular movements in the volatility and covolatility dynamics is well justified and useful to enhance a higher forecasting accuracy.

[6]This result is quite different from that of Bauwens et al. [5] where models with a time-varying long-run component dominate models with a constant long-run level at forecast horizon 1. The dataset of that paper covers the turbulent period of 2008 and 2009.

Table 8: Multi-step-ahead forecast evaluation

	Horizon 1			Horizon 5			Horizon 10			Horizon 20		
	MMReDCC	**EWMA**	**cRDCC**	**MMReDCC**	**EWMA**	**cRDCC**	**MMReDCC**	**EWMA**	**cRDCC**	**MMReDCC**	**EWMA**	**cRDCC**
Panel A: Loss functions												
Frob	0.133	**0.125**	0.126	0.138	**0.135**	0.139	**0.144**	0.148	0.148	**0.158**	0.167	0.160
Sfrob	0.040	**0.034**	0.037	**0.036**	**0.036**	**0.036**	**0.037**	0.038	0.038	**0.041**	0.042	0.043
Euclid	0.086	**0.080**	0.082	**0.087**	0.088	0.088	**0.091**	0.093	0.093	**0.099**	0.102	0.101
ST	65.159	59.018	**55.449**	**59.227**	61.684	60.245	**61.522**	63.611	63.289	67.748	**66.269**	66.667
vND	0.018	0.017	**0.017**	**0.018**	0.019	0.019	**0.019**	0.022	0.020	**0.022**	0.025	**0.022**
QLIK	-370.64	-376.78	**-380.35**	**-376.66**	-374.20	-375.64	**-374.43**	-372.34	-372.67	-368.07	**-369.55**	-369.15
Panel B: 90 % MCS												
Frob	0.014	**0.689**	**1.000**	**0.486**	**1.000**	**0.232**	**1.000**	0.001	0.015	**0.224**	**0.619**	**1.000**
Sfrob	0.044	**0.259**	**1.000**	**0.614**	**1.000**	**0.614**	**1.000**	0.052	0.052	**0.547**	**0.547**	**1.000**
Euclid	0.023	**0.464**	**1.000**	**1.000**	**0.311**	**0.420**	**1.000**	0.001	0.016	**0.506**	**0.506**	**1.000**
ST	0.000	0.000	**1.000**	**1.000**	0.000	0.018	**1.000**	0.000	0.003	**1.000**	0.000	**0.281**
vND	0.000	**0.263**	**1.000**	**1.000**	0.002	0.028	**1.000**	0.000	0.000	**1.000**	0.000	**0.159**
QLIK	0.000	0.000	**1.000**	**1.000**	0.000	0.017	**1.000**	0.000	0.004	**1.000**	0.000	**0.275**
Panel C: 75 % MCS												
Frob	0.013	**0.695**	**1.000**	**0.497**	**1.000**	0.227	**1.000**	0.003	0.016	**1.000**	0.218	**0.615**
Sfrob	0.047	**1.000**	**0.268**	**0.631**	**1.000**	**0.631**	**1.000**	0.051	0.051	**1.000**	**0.545**	**0.545**
Euclid	0.018	**0.461**	**1.000**	**1.000**	**0.315**	**0.429**	**1.000**	0.001	0.015	**1.000**	**0.511**	**0.511**
ST	0.000	0.000	**1.000**	**1.000**	0.000	0.015	**1.000**	0.000	0.003	**1.000**	0.000	**0.278**
vND	0.000	**0.265**	**1.000**	**1.000**	0.002	0.029	**1.000**	0.000	0.000	**1.000**	0.000	0.160
QLIK	0.000	0.000	**1.000**	**1.000**	0.000	0.014	**1.000**	0.000	0.003	**1.000**	0.000	**0.274**

Note: Panel A reports averaged values of the loss functions listed in Table 7 over the out-of-sample period, where the best performing model within each row is in bold. Entries in Panel B and C are p-values of the MCS with 10% and 25% size, respectively. Included models in bold.

7. Conclusions

The estimation procedure proposed in the paper allows to extend the range of applicability of the MMReDCC model to large dimensional portfolios such as those encountered in standard risk management practice. In order to reach this objective, we face two well-known challenges in multivariate time series modeling, namely high-dimensional estimation and multi-step ahead forecasting.

To face the former challenge, we implement a feasible estimation procedure, the Iterative Moment based Profiling (IMP) algorithm. It profiles out the parameters of the scale MIDAS intercept matrix and iteratively maximizes the likelihood in terms of the other parameters of interest. Whilst not providing an asymptotic inference theory for this method, we investigate the finite sample properties of the estimator via a simulation study, which demonstrates that the IMP estimator is comparable to the one-step QML ones in terms of bias and accuracy. We also compare the one-step QML estimator with the IMP estimator on real data sets of small dimension (ten) and find that not only the two estimators deliver very similar in-sample estimates, but also the loss of the IMP in terms of likelihood values can be considered as negligible. Another application illustrates the usefulness of the IMP algorithm when the model is applied to the realized covariances of fifty stocks. From the computational point of view, the IMP algorithm is found to be reliable and easy to apply despite the large number of parameters involved in the MMReDCC model. Given its flexibility, we fairly believe that it could be applied to datasets of larger dimensions.

As regards the second challenge, we develop a bootstrap approach to the generation of multi-step-ahead predictions. In an application to a portfolio of fifty stocks, we provide compelling evidence that the MMReDCC model is useful for out-of-sample forecasting purposes even when one has to work in such a dimension. If compared with existing multivariate competitors not accounting for time-varying long-term dynamics, the MMReDCC is found to deliver the most accurate predictions especially at medium-term horizons, thus indicating the importance of allowing for a long-run component.

Appendix A. Application I: Descriptive statistics of daily realized variances.

Symbol	Issue name	Mean	Max.	Min.	Std.dev.	Skewness	Kurtosis
	Dataset 1: February, 2001 – December, 2009						
AA	Alcoa	5.458	277.308	0.074	16.811	7.178	72.570
AXP	American Express	5.055	176.478	0.112	11.094	7.529	84.686
BAC	Bank of America	1.934	57.543	0.075	3.362	7.319	85.006
KO	Coca Cola	2.455	43.106	0.084	3.412	4.724	36.234
DD	Du Pont	2.073	115.378	0.126	4.155	13.296	288.066
GE	General Electric	4.944	160.241	0.294	8.935	7.635	92.124
IBM	International Business Machines	4.420	201.879	0.077	9.154	8.536	133.699
JPM	JP Morgan	2.529	63.874	0.163	3.728	6.442	68.505
MSFT	Microsoft	3.196	114.256	0.097	7.114	7.232	75.484
XOM	Exxon Mobil	1.414	56.505	0.039	2.254	9.715	180.206
	Dataset 2: January, 2007 – December, 2012						
ACAS	American Capital	8.576	331.786	0.060	20.844	7.226	78.667
AET	Aetna	8.163	771.525	0.109	26.593	17.882	467.969
AFL	Aflac Incorporated	9.113	675.348	0.133	27.345	13.811	284.791
AIG	American International Group	8.799	555.098	0.103	26.382	11.459	185.778
AIZ	Assurant	8.613	325.167	0.101	23.230	7.712	79.082
ALL	The Allstate Coprporation	8.213	543.714	0.186	24.593	11.277	189.052
AMP	Ameriprise Financial	7.679	264.761	0.129	17.790	6.098	54.262
AXP	American Express Company	8.076	945.750	0.095	30.571	21.795	618.891
BAC	Bank of America	8.450	332.586	0.130	22.830	8.458	96.824
BBT	BB&T Corporation	9.093	613.826	0.087	28.801	11.267	184.837
	Dataset 3: January, 2007 – December, 2012						
STI	SunTrust Banks	8.510	388.707	0.086	22.765	8.088	94.838
STT	State Street Corporation	8.985	315.656	0.039	23.192	7.265	74.391
TMK	Torchmark Corporation	8.748	537.273	0.022	24.479	10.543	176.515
TROW	T.Rowe Price Group	8.991	425.263	0.073	25.019	8.753	108.221
UNH	UnitedHealth Group	8.344	378.667	0.130	21.065	8.534	112.479
UNM	Unun Group	8.046	309.086	0.044	19.440	8.564	107.272
USB	U.S.Bancorp	10.176	2534.073	0.079	69.496	32.392	1164.053
WFC	Wells Fargo & Company	9.481	525.034	0.106	30.162	10.717	156.791
WU	The Western Union Company	8.775	484.124	0.056	25.204	9.954	145.509
ZION	Zions Bancorporation	15.302	6855.823	0.136	197.794	30.674	1005.621

Appendix B. Application II: descriptive statistics of daily realized variances.

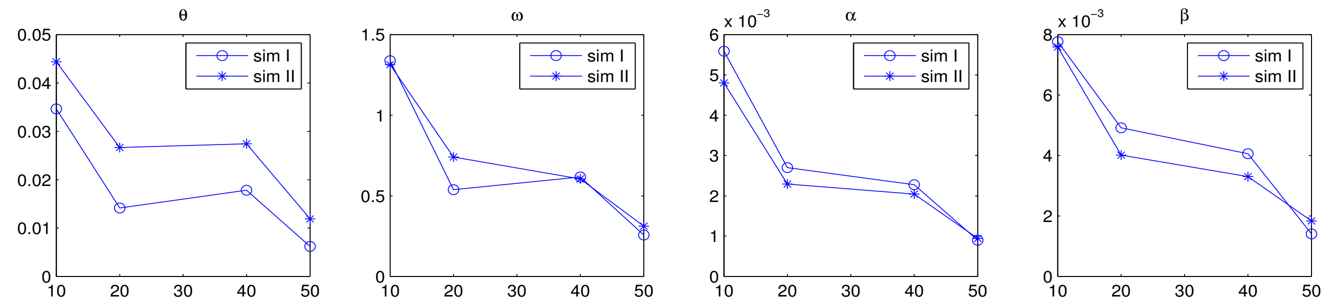
Estimation sample: May 12, 1997 to July 17, 2006 (2024 observations)						
Stock	Mean (e-03)	Max.	Min.(e-03)	Std.dev.(e-03)	Skewness	Kurtosis
ABT	0.434	0.025	0.026	0.948	15.958	333.606
AFL	0.491	0.027	0.020	1.208	13.097	225.997
APD	0.490	0.075	0.020	1.896	32.234	1221.466
AA	0.540	0.012	0.040	0.759	7.886	94.971
ALL	0.497	0.098	0.012	2.319	37.311	1554.460
AXP	0.549	0.041	0.014	1.553	16.921	374.949
AIG	0.391	0.073	0.027	1.724	38.011	1597.332
ADI	1.375	0.043	0.070	2.153	7.635	103.080
APOL	1.276	0.080	0.043	2.556	16.984	470.125
T	0.620	0.046	0.017	1.659	15.676	339.927
AZO	0.497	0.046	0.021	1.250	25.934	909.714
AVY	0.416	0.064	0.019	1.643	30.923	1136.941
BHI	1.000	0.098	0.060	3.284	23.106	603.142
BAC	0.477	0.054	0.015	1.681	20.789	578.406
BAX	0.447	0.059	0.022	1.884	22.785	609.476
BDX	0.481	0.036	0.021	1.109	19.322	541.972
BBBY	1.078	0.029	0.051	1.579	7.060	92.898
BMY	0.565	0.051	0.028	2.005	19.683	450.186
CPB	0.470	0.083	0.012	1.940	38.241	1616.776
COF	0.909	0.092	0.023	3.300	20.993	547.443
CAH	0.465	0.044	0.016	1.705	18.142	393.755
CTL	0.492	0.069	0.026	2.049	23.613	687.584
CTAS	1.104	0.113	0.018	2.839	31.100	1212.341
C	0.624	0.086	0.021	2.305	27.085	952.213
CLX	0.481	0.074	0.026	2.203	27.445	840.327
CMS	0.764	0.083	0.034	2.854	20.911	544.730
KO	0.314	0.009	0.015	0.464	7.565	101.565
CL	0.401	0.058	0.026	1.569	27.867	957.179
CMA	0.345	0.025	0.011	1.011	16.800	350.599
CSC	0.764	0.105	0.031	2.884	25.713	861.526
CAG	0.530	0.072	0.014	2.129	25.460	758.407
COST	0.773	0.135	0.042	3.358	33.314	1277.539
DOV	0.441	0.050	0.031	1.334	27.171	949.990
DOW	0.474	0.033	0.018	0.994	20.169	609.262
DTE	0.296	0.068	0.017	1.552	41.597	1818.052
EMN	0.432	0.089	0.027	2.141	35.535	1433.408
EIX	1.103	0.251	0.022	6.892	26.622	885.136
ETR	0.332	0.029	0.018	0.779	26.216	937.670
FDO	0.911	0.093	0.043	2.519	26.543	911.464
FISV	0.913	0.063	0.041	1.715	24.450	850.641
F	0.635	0.024	0.050	1.053	10.141	170.597
GCI	0.272	0.010	0.015	0.370	12.616	289.030
GPS	1.007	0.049	0.032	2.730	10.649	142.751
GE	0.445	0.037	0.013	1.233	19.948	515.188
GIS	0.231	0.008	0.013	0.334	9.989	181.369
GPC	0.427	0.089	0.018	2.039	40.541	1745.937
HNZ	1.240	0.075	0.030	2.547	19.600	511.929
HPQ	0.291	0.030	0.014	0.918	24.562	727.020
HD	0.902	0.047	0.022	1.965	11.388	195.042
HON	0.643	0.078	0.028	2.418	23.919	682.976

Forecasting sample: July 18, 2006 to July 18, 2008 (500 observations)						
Stock	Mean (e-03)	Max.	Min.(e-03)	Std.dev.(e-03)	Skewness	Kurtosis
ABT	0.191	0.004	0.020	0.263	8.887	122.525
AFL	0.268	0.004	0.015	0.406	4.815	35.550
APD	0.261	0.003	0.013	0.292	4.633	38.977
AA	0.559	0.007	0.051	0.682	4.078	25.551
ALL	0.232	0.002	0.012	0.304	3.423	18.970
AXP	0.498	0.015	0.010	0.919	8.966	127.909
AIG	0.523	0.010	0.017	1.020	4.375	27.892
ADI	0.432	0.014	0.046	0.816	11.614	168.440
APOL	0.939	0.079	0.035	4.173	14.929	261.548
T	0.278	0.004	0.023	0.399	6.116	50.180
AZO	0.303	0.010	0.016	0.517	13.313	242.173
AVY	0.243	0.005	0.019	0.446	7.402	70.878
BHI	0.506	0.009	0.079	0.563	8.511	108.889
BAC	0.495	0.016	0.012	1.223	7.682	79.279
BAX	0.190	0.004	0.020	0.306	7.569	80.297
BDX	0.150	0.003	0.017	0.210	8.356	104.636
BBBY	0.464	0.010	0.029	0.627	7.907	106.865
BMY	0.303	0.007	0.017	0.521	7.153	72.678
CPB	0.152	0.002	0.011	0.206	4.746	30.567
COF	0.923	0.013	0.028	1.481	3.995	25.175
CAH	0.186	0.005	0.011	0.294	9.695	135.019
CTL	0.279	0.021	0.022	1.008	17.453	350.144
CTAS	0.293	0.006	0.034	0.395	8.196	99.256
C	0.648	0.014	0.019	1.276	5.411	41.978
CLX	0.153	0.007	0.012	0.416	13.180	203.335
CMS	0.219	0.003	0.020	0.242	5.684	49.133
KO	0.112	0.002	0.005	0.181	7.972	78.506
CL	0.125	0.004	0.015	0.212	11.526	172.718
CMA	0.632	0.023	0.017	1.400	9.231	131.210
CSC	0.285	0.005	0.025	0.412	6.514	62.089
CAG	0.179	0.005	0.016	0.313	9.470	120.784
COST	0.338	0.010	0.030	0.564	11.158	166.572
DOV	0.245	0.003	0.025	0.327	5.352	39.200
DOW	0.347	0.007	0.028	0.616	6.593	55.276
DTE	0.177	0.002	0.015	0.202	4.481	34.565
EMN	0.292	0.005	0.024	0.429	6.244	57.701
EIX	0.223	0.005	0.026	0.320	7.899	96.229
ETR	0.203	0.003	0.017	0.281	5.482	44.736
FDO	0.770	0.017	0.033	1.332	7.121	70.492
FISV	0.276	0.006	0.031	0.366	7.898	96.806
F	0.802	0.009	0.091	1.101	4.107	23.288
GCI	0.305	0.012	0.018	0.658	12.646	218.866
GPS	0.564	0.007	0.028	0.681	4.354	30.343
GE	0.213	0.016	0.016	0.718	19.569	416.499
GIS	0.108	0.002	0.011	0.137	6.537	58.387
GPC	0.182	0.002	0.017	0.189	5.554	49.714
HNZ	0.309	0.003	0.043	0.280	4.217	28.816
HPQ	0.121	0.002	0.016	0.138	5.344	43.491
HD	0.289	0.005	0.024	0.387	6.301	64.372
HON	0.394	0.004	0.042	0.469	3.953	25.552

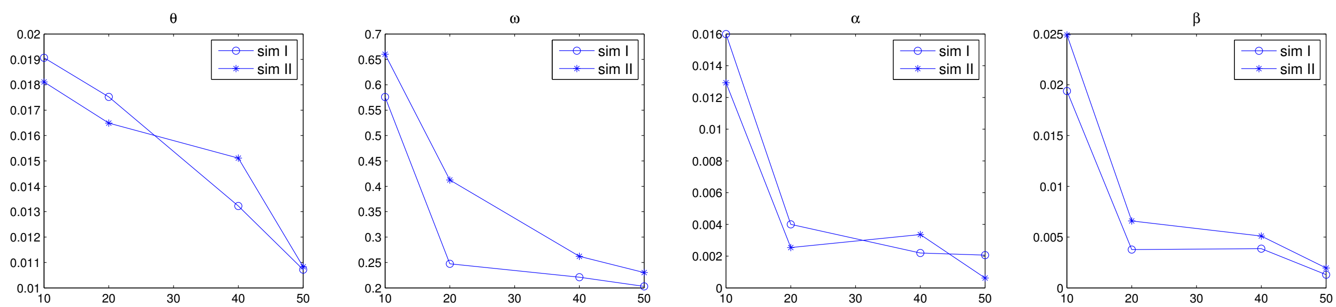
Appendix C. Figures

Figure C.2: Standard deviation of the IMP Monte Carlo estimated scalar parameters θ, ω, α and β against the cross-section dimension ranging from 10 to 50. Results from the two simulation studies for $T = 1000, 2000$ jointly reported respectively in Panel (a) and (b).

(a) $T = 1000$



(b) $T = 2000$



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