2017/19

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Intergenerational mobility, school inequality and social segregation*

Andreu Arenas† Jean Hindriks‡

September 29, 2017
First verion: June 29, 2017

Abstract

We study the role of school inequality and social segregation for human capital accumulation, inequality and intergenerational mobility. We augment the Becker-Tomes-Solon model of intergenerational mobility, introducing a regime switch model of social segregation at school. Depending on the social background of their parents, children have different probability of access to different school quality. We obtain that segregation and school inequality increase inequality in parental investment with ambiguous effect on the average level. However, we also find that segregation and school inequality increases average educational attainment and income levels. This is due to the complementarity between parental investment and school quality. We show that segregation and school inequality reduce intergenerational mobility if the variance of log parental income above the median is at least as high as the variance below the median. Lastly, we calibrate the model to the US income distribution and simulate the effects of changing segregation and school inequality on average human capital and intergenerational mobility. Our baseline simulation shows that de-segregation or school equalization policies would increase intergenerational mobility by 42%, while reducing average human capital by 0.13%, compared to current levels.

JEL Classification: I22; J62

Keywords: Intergenerational mobility; education, school system; equality of opportunity; segregation.

*We are grateful to Roland Bénabou, David de la Croix, Hubert Kempf, François Maniquet, Pierre Pestieau, and seminar participants at the 2nd Belgian-Japanese Public Finance Workshop at UCLouvain (March 2017, Belgium), LEER Workshop in Education Economics at KU Leuven (March 2017, Belgium), ECORES Summer School at UCLouvain (May-June 2017, Belgium), and Journées LAGV 2017 at Aix-en-Provence (June 2017, France) for comments and suggestions. Riccardo Fusari provided excellent research assistance.

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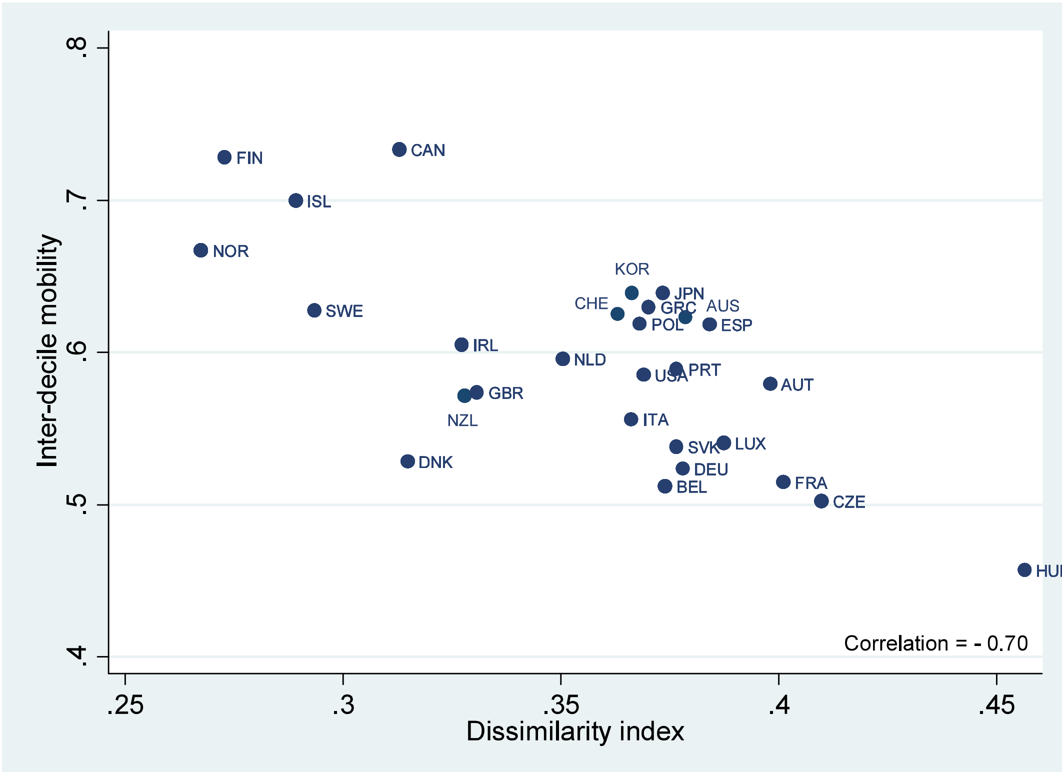
1 Introduction

Education is the central channel by which parents pass their economic status to their offsprings. From a policy perspective, the design of the school system is considered a key driver of equality of opportunity. The motto is that all children should have a fair shot at a good education. The persistence of inequality (the lack of intergenerational mobility) is often attributed to the large role of education for labor market outcomes, and the differences in school quality that are related to residential segregation and its inevitable fiscal and social spill-overs (Bénabou, 1996b). To give a snapshot of the relation between segregation in the school system and the intergenerational mobility in education we have used PISA data on student test scores and student socio-economic status (SES). For each country, we have ranked all students by their socio-economic decile and their test score decile. We have then computed the inter-decile mobility of each student by the ratio of the test score decile to the socio-economic decile. Averaging the individual mobility of all students within the same country we obtain the inter-decile mobility of the school system in that country. We have normalized this inter-decile mobility to take value between zero (no intergenerational mobility) and one (perfect intergenerational mobility).[1] On the other hand, we have for each country computed the social segregation of schools by using the social dissimilarity index (which measures the degree of dissimilarity between schools in terms of social composition). Pooling 5 waves of PISA data (between 2003 and 2015), figure 1 displays a strong cross-country negative correlation between school segregation and inter-generational mobility. The dispersion in the inter-decile mobility is remarkable with on the one hand Canada and Finland displaying school systems with inter-decile mobility close to 0.75 (i.e. 75% of perfect mobility), and on the other hand, Belgium, France, Hungary and Czech Republic close to 0.5 (50% of perfect mobility). The variation in segregation is also quite large across countries with the Nordic countries and Canada among the least segregated school systems and Hungary, France and Czech Republic among the most segregated school systems. The strong cross-country association between segregation and inter-decile mobility suggests that school segregation is a relevant issue for social mobility.[2]

[1] Perfect mobility implies that each social decile has equal probability to end up in each test score decile.

[2] We obtain a similarly strong correlation (of around 0.7) when using reading or science test scores.

Figure 1: School segregation and intergenerational mobility, OECD countries, 2003-2015



X-axis: school segregation - dissimilarity index, % of below median SES students that should change school to obtain equal social representation within each school s: $Dissimilarity = \sum_{s=1}^{s=S} \frac{1}{2S} |\%\text{ Low-income}_s - \%\text{ High-income}_s|$. Y-axis: inter-decile mobility- $Mobility = \frac{1}{N\alpha} \sum_{i=1}^{i=N} \left(\frac{\text{Math Test Score Decile}_i}{\text{Parental Socio-Economic Decile}_i} \right) - 1$

Within countries, Chetty *et al.* (2014) document large disparities in intergenerational mobility across neighborhoods in the US, with high mobility areas exhibiting less residential segregation, less income inequality, and better primary schools, among others. Crucially, children randomly growing up in less disadvantaged neighborhoods benefit from an increase in college attendance and earnings (Chetty *et al.*, 2016). Likewise, Goux and Maurin (2007) present similar evidence of neighborhood effects in France. This suggests that besides self-selection and inheritability, neighborhoods have a causal effect on outcomes and on intergenerational mobility. The aim of this paper is to present a theoretical framework that incorporates these insights to understand how school segregation and inequality contribute to the aforementioned lack of social mobility through its matching of different income groups into schools of different quality, which in turn feeds back into parental investment decisions and children's human capital.

We address this question by augmenting the benchmark Solon (1999) model of intergenerational mobility, that features a representative family having access to a uniform school system, mapping parental investment into human capital at a fixed rate, which in turn determines children's income. We augment the model by introducing school inequality and segregation. School inequality is modelled as difference in school quality and it captures policies that allow for heterogeneity in school management and curriculum. Social segregation is modelled as students from different social background having different probabilities of accessing high quality schools, and it captures differences in school choice mechanisms and policies that facilitate access to better schools from low-income neighborhoods.[3]

Formally, we adopt a Markov bivariate switching model where the transition probabilities for having access to a high quality school depend on parental income. The transition probabilities are a measure of the extent of social segregation across school types. By allowing for regime switches, the model can capture more complex, non-linear dynamic patterns in human capital accumulation. We study how the gap in school quality between good and bad schools as well as differences in transition probabilities affect optimal parental investment choices, human capital formation, and intergenerational mobility.

Our main finding is that school inequality and social segregation involve an equity-efficiency trade-off: more income and education on average but less equality, and in most cases, less mobility. Segregation and school inequality influence parental investment choices in different directions, depending on the parental income group. High-income parents, more likely to access high-quality schools, invest relatively more on their children's education, while poorer parents, less likely to access high-quality schools, invest relatively less. The decline in the share of income invested by low income families is larger than the increase in the share of income invested by high-income families, and as a result, there exists a tipping point level of segregation below which school inequality reduces average parental investment and above which school inequality increases average parental investment. Regardless of the effect on aggregate parental investment, however, school inequality and social segregation always increase aggregate human capital. This is because the parents reducing investment are those accessing lower quality school,

[3]The choice of combining segregation and school inequality has a long history that can be traced back to the civil rights movement: "The leading expert on desegregation did no believe that there was something magical about sitting next to whites in a classroom. It was however based on a belief that the dominant group would keep control of the most successful schools and that the only way to get full range of opportunities for a minority child was to get access to those schools" (www. civilrights.org/desegregation).

while those increasing investment are those accessing higher quality school, where parental investment is also more productive. Hence, the complementarity between parental investment and school quality generates efficiency gains from school inequality and segregation. The effect of school inequality and social segregation on intergenerational mobility, as measured by the intergenerational elasticity of human capital, depends on the distribution of parental income. It increases mobility among low income families (where low quality schools are a less effective channel for parents to pass their economic status to their offsprings), and it reduces mobility among high income families and across either group (where high quality school is more effective channel to transmit economic status). In most cases, when the income distribution is log normal for instance, segregation reduces intergenerational mobility.

We then calibrate the model to the US income distribution to run simulation on the effects of changing segregation or school inequality. The simulation results reveal that reducing segregation or school inequality would have significant impact on intergenerational mobility with very limited effect on efficiency. For instance some base line simulations show that either full de-segregation or school equalization would reduce the IGE by 42% while the average human capital would only decrease by 0.13%

1.1 Contribution to the literature

The main contribution of this paper is to include school inequality and social segregation within the classical model of intergenerational mobility (Solon (1999), Becker and Tomes (1986), Becker and Tomes (1979)). The Becker-Tomes-Solon model predicts that the intergenerational mobility is the same within and across income groups and that the inequality across groups falls over time even though aggregate inequality may even increase (Mulligan, 1999). Our model outlines the interaction between income inequality, school inequality and the intergenerational elasticity of income. In our model there is no transmission of abilities that could lead to an intergenerational persistence in educational attainments or income levels. Instead, we focus on the impact of segregation in school on the persistence of inequality, through its effect on parental choices. This approach makes persistent inequality less socially acceptable, and the efficiency gains of a segregated school system more surprising compared to existing literature (Hare and Ulph (1979), Cremer *et al.* (2010)), in which the efficiency gains are based on the heritability of child ability. In that literature, the efficiency argument is that children from richer families are assumed by heritability to be on average better students, and thus matching better students to better schools would maximize educational outcomes. In our model this mechanism is absent because we assume that ability is not heritable. Therefore, we concentrate on a different mechanism underlying the equity-efficiency tradeoff.[4]

Our paper relates to Bratsberg *et al.* (2007), that empirically show how intergenerational earnings mobility in Denmark, Finland and Norway, unlike those for the US and the UK, are highly nonlinear, and that the prospects for moving out of poverty in the US and the UK are poorer than in the Nordic countries. This could be related to Nordic educational policies that create a qualifications floor, to which most citizens can aspire, regardless of parental resources. The closest paper to ours is Becker *et al.* (2015), that departs from the standard Becker-Tomes-Solon framework by allowing for complementarities in the production of children's human capital, considering the possibility that highly educated parents are better equipped to transmit in turn their human capital to their children. This complementarity creates a non linearity in the production of human capital and possibly induces different persistence of economic status at the top and the middle of the income distribution. They also show that even without differences in innate ability and liquidity constraints, successive generations of the same family may stop regressing to the mean. They do not consider efficiency issues in their model but only intergenerational mobility. Our model highlights a different source of inequality persistence which is related to school inequality and segregation: children from disadvantaged families are less likely to attend high quality schools. This unequal opportunity creates also complementarity (reinforcing effect) between parental investment and school quality. We analyze the implications of this inequality of opportunity on efficiency (average earnings and educational attainment) and intergenerational mobility.

Our model can also be related to the cultural transmission model à la Bisin and Verdier (2001). In the cultural transmission model, children can become educated either because parents have been successful in educating them (socialization inside the family) or, if this is not the case, because the neighborhood where they live is of sufficiently high quality in terms of

[4] Checchi *et al.* (1999) propose another mechanism, based on self-confidence about talent, where selective schooling can act as a sorting mechanism that allows talented students from disadvantaged families to reveal themselves. As a result egalitarian education increases the risk that talented individuals from disadvantaged families remain stuck with low human capital leading to a less mobile but more equal system.

human capital (socialization outside the family). In different cases, the model can yield both cultural complementarity (higher neighborhood quality leads to higher effort) and substitutability (higher neighborhood quality leads to lower effort). One interpretation of our model is as a cultural transmission model with cultural complementarity between parental investment and the neighborhood (school) quality. This is because in our case, higher school quality leads to higher parental investment in the human capital of their children. Importantly, this is consistent with empirical evidence showing that the better the quality of the neighborhood, the higher is the parents' involvement in their children's education (Patacchini and Zenou, 2011)

Finally, our paper is related to a number of contributions considering segregation as an outcome (i.e., models with endogenous segregation). Loury *et al.* (1977), Durlauf (1996) or Bénabou (1996a), among others, show how optimal segregation relates to complementarity in family and social attributes and to the decreasing returns to community quality. A key result of this literature is that segregation/stratification makes inequality more persistent with the potential for concentration of poverty in ghettos consistently passed on from one generation to the next (i.e., the so-called underclass). Another lesson from this literature on endogenous segregation is that excessive segregation may become detrimental for performance. In De la Croix and Doepke (2009), segregation is endogenously determined as the equilibrium distribution of different income groups between public and private schools. This segregation depends on the interaction between income inequality and the political economy of public school funding. There is no dynamic of income transmission in their model, while in our model segregation is exogenously determined and we look at its impact on the dynamic of income transmission.

2 A regime switch model of segregation

We consider a simplified version of the Solon (1999) model. Each parent of generation $t-1$ has one child of generation t. Parents must allocate lifetime income y_{t-1} between own consumption C_{t-1} and investment I_{t-1} in the child's human capital. Parents cannot borrow against the child's future income and do not bequest income to the child. The resulting budget constraint of parents in generation $t-1$ is

$$y_{t-1} = C_{t-1} + I_{t-1} \tag{1}$$

Parental investment translates into the child's human capital according to the following human capital accumulation,

$$h_t = \theta log(I_{t-1}) + e_t, \qquad \theta > 0 \tag{2}$$

Where $\theta > 0$ represents a uniform school productivity parameter, and e_t represents the child's ability parameter independent of the parent investment choice (child's endowed attributes in the terminology of Becker and Tomes (1979)). We will assume that there is no intergenerational transmission of ability, and that

$$e_t = \mu + \epsilon_t \tag{3}$$

where $\mu > 0$ and ϵ_t is a mean zero iid random term with constant variance σ_ϵ^2. Human capital translates into income on the labor market as follows:

$$log(y_t) = h_t \tag{4}$$

The log-linear income function is useful to derive the intergenerational income elasticity. Parents seek to maximise a utility function over own consumption and child's income:

$$U_{t-1} = log(C_{t-1}) + log(y_t) \tag{5}$$

Note that this is a quasi-linear utility function in child's education attainment, since $log(y_t) = h_t$. Thus parents do care only about the expected value of h_t which they correctly anticipate. For that reason they do not need to know the child's ability while investing in her education.[5] Substituting the budget constraints, the human capital production function and the labor market transmission function into the utility function, parents maximize:

$$\begin{aligned} U_{t-1} &= log(y_{t-1} - I_{t-1}) + h_t \\ &= log(y_{t-1} - I_{t-1}) + \theta log(I_{t-1}) + e_t \end{aligned} \tag{6}$$

[5]If the utility function is not separable between consumption and child's income , and if the parents do know the child ability while investing, then they would condition investment on the child ability. Parents would invest less in a high-ability child to expand their own consumption such as in Becker and Tomes (1979).

Solving with respect to I_{t-1} gives the optimal investment in child's human capital

$$I_{t-1}^* = \frac{\theta}{1+\theta} y_{t-1} \tag{7}$$

Hence, the school productivity parameter θ determines the proportion of parental income that is devoted to investment $\frac{\theta}{1+\theta}$. Becker and Tomes (1979) call this parameter $\frac{\theta}{1+\theta}$ the propensity to invest in children. In a uniform school system, as in the Becker-Tomes-Solon model, the propensity to invest in children is uniform, and parents are willing to invest more whenever schools are of higher quality.

2.1 School inequality, segregation and regime switching

We introduce school inequality with social segregation, such that families above the median income have access, with probability $p \geq \frac{1}{2}$, to high-quality schools that translate parental investment into human capital at a higher rate $\theta^H = \theta + \kappa$. On the other hand, families below the median income have access, with the same probability p, to low-quality schools with a lower productivity parameter $\theta^L = \theta - \kappa$. The average school productivity is constant $E(\theta) = \theta$, and school inequality is given by the school productivity gap $k = \frac{\theta^H - \theta^L}{2}$. Thus κ measures the school inequality (fixing the average productivity at θ), and p measures the inequality of opportunity (as measured by the school social dissimilarity index). It is worth noting that our modelling of school quality is via a multiplicative parameter in the human capital production function, and not an additive parameter. More specifically, school quality is perceived as a technological progress that enhances the productivity of parental investment in human capital. It acts as a multiplier on parental investment, increasing its productivity and slowing down the diminishing returns. As we will see shortly the implication of this modelling approach is that for any given family income, families with access to better schooling will invest more in education.[6]

The human capital accumulation with school inequality and social segregation is described as the following regime switch model where y_{t-1}^M is the median income of generation $t-1$.

[6]By contrast additive productivity shock would not change marginal incentive of parental investment which is counterfactual. Indeed, Patacchini and Zenou (2011) find that families in better neighborhoods (including school quality) invest (relatively) more in the education of their children. Kornrich and Furstenberg (2013) and Reardon (2011) show that total parental spending on children increased faster than income and become more unequal.

For $y_{t-1} \leq y^M_{t-1}$ (i.e., below median income),

$$h_t = \begin{cases} \theta^L log(I_{t-1}) + e_t, & \text{with probability p.} \\ \theta^H log(I_{t-1}) + e_t, & \text{with probability 1-p.} \end{cases} \tag{8}$$

For $y_{t-1} > y^M_{t-1}$ (i.e., above median income),

$$h_t = \begin{cases} \theta^L log(I_{t-1}) + e_t, & \text{with probability 1-p.} \\ \theta^H log(I_{t-1}) + e_t, & \text{with probability p.} \end{cases} \tag{9}$$

The top half of the distribution has access to a better human capital technology than the bottom half of the population. Given this bivariate Markov regime switch model, each parent in generation $t-1$ will choose I_{t-1} given the school quality available so as to maximize their preference over own consumption and child's income. That gives the distribution of parental investment conditional on school quality.

For $y_{t-1} \leq y^M_{t-1}$,

$$I^*_{t-1} = \begin{cases} \frac{\theta_L}{1+\theta_L} y_{t-1}, & \text{with probability p.} \\ \frac{\theta_H}{1+\theta_H} y_{t-1}, & \text{with probability 1-p.} \end{cases} \tag{10}$$

For $y_{t-1} > y^M_{t-1}$,

$$I^*_{t-1} = \begin{cases} \frac{\theta_L}{1+\theta_L} y_{t-1}, & \text{with probability 1-p.} \\ \frac{\theta_H}{1+\theta_H} y_{t-1}, & \text{with probability p.} \end{cases} \tag{11}$$

Let $\omega_h(\theta) = p\frac{\theta^H}{1+\theta^H} + (1-p)\frac{\theta^L}{1+\theta^L}$ and $\omega_l(\theta) = p\frac{\theta^L}{1+\theta^L} + (1-p)\frac{\theta^H}{1+\theta^H}$ denote the expected propensities to invest in children for the above-median and below-median income group, respectively. Then, $\omega_h(\theta) \geq \omega_l(\theta)$ for $p \geq 1/2$: the high income families displays a higher propensity to invest in children in the presence of segregation, consistent with the empirical findings in Patacchini and Zenou (2011). Note also that

$$\frac{\partial \omega_h(\theta)}{\partial p} = -\frac{\partial \omega_l(\theta)}{\partial p} = \frac{\theta^H}{1+\theta^H} - \frac{\theta^L}{1+\theta^L} > 0$$

which means that segregation increases the inequality in the propensity to invest in children between high income and low income families.

We can now use this regime switch model to focus on the mean behavior of the non-linear dynamic variables. We begin by analyzing the mean value of the dynamic pattern of parental investment. In the regime switch model the average investment level with segregation is given by[7]:

$$E[I_s] = \frac{1}{2}\left(\omega_h(\theta)\right) E[y_{t-1}|y_{t-1} > y^M_{t-1}] + \frac{1}{2}\left(\omega_l(\theta)\right) E[y_{t-1}|y_{t-1} \leq y^M_{t-1}]$$

Let $\gamma = \frac{E[y_{t-1}|y_{t-1} > y^M]}{E[y_{t-1}]}$ represent the income inequality ratio between the two income groups. For γ close to 1, the income inequality is minimum: both income groups have equal shares of total income. For γ close to 2, the income inequality is maximum: total income is concentrated in the high parental income group. Hence $\gamma \in (1, 2)$. Using this (inter-group) income inequality ratio, we can rewrite the average investment under segregation as

$$E[I_s] = [\gamma\omega_h(\theta) + (2-\gamma)\omega_l(\theta)]\frac{E[y_{t-1}]}{2}$$

3 Impact of segregation and inequality on investment

Mean investment in a uniform school system with no segregation is given by $E[I^*_{t-1}] = \frac{\theta}{1+\theta}E[y_{t-1}]$. The question is how segregation and school inequality affect the average level of investment in our regime switch model of human capital accumulation. The question is complex a priori because segregation and inequality will change both parental investment and the entire distribution of income. However we can consider the impact of an isolated change in the segregation and inequality for generation t for any given initial condition of generation $t-1$ income distribution and average income $E[y_{t-1}]$.

In the segregated and unequal school system, students from different social backgrounds have different opportunities of attending high-quality schools and because such schools provide superior chances in educational attainments and the labor market income, parental investment will reflect this asymmetry in the school systems and the inequality of opportunity. In the

[7] We drop the time index of the investment variable to ease notation and when there is no risk of confusion.

following proposition we derive the (short term) comparative statics properties of the mean investment level with respect to a mean-preserving change in school inequality ($\kappa = \frac{\theta^H - \theta^L}{2}$), the segregation rate ($1/2 \leq p \leq 1$), and the level of income inequality ($1 \leq \gamma \leq 2$).

Proposition 1. *(i) For any $\kappa > 0$ and $\gamma > 1$, average investment is an increasing function of social segregation p; (ii) For any $\kappa > 0$ and $p > 1/2$, average investment is an increasing function of income inequality γ; (iii) For any $\gamma > 1$, there exists $p^\circ = \frac{1}{2} + \frac{\kappa(1+\theta)}{(\gamma-1)((1+\theta)^2+\kappa^2)}$ such that average investment is an increasing function of school inequality κ if and only if $p \geq p^\circ$.*

The proof is provided in the Appendix. This means that there exists a threshold value $p = p^\circ$ such that average investment is an increasing function of school inequality κ because high-quality schools are sufficiently concentrated among the high-income group which also invests more income in the education of their children. Note that for some parameter values (i.e., if income inequality is very low, γ close to 1), $p = p^\circ$ could be larger than one. This complementarity between income and school quality compensates for the decreasing returns of the school quality on the propensity to invest in children ($\frac{\theta+k}{1+\theta+k} > \frac{\theta-k}{1+\theta-k}$). Moreover, from the threshold condition we can see that p° is a decreasing function of income inequality γ. In that sense we have a reinforcing effect of the school inequality and the income inequality on the level of average investment. This reinforcement effect creates polarization dynamics. For any segregation rate $p > 1/2$, average investment is higher either when there is more income and school equality or when there is more income and school inequality.

We illustrate this result in figure 2, in the polar case of $p = 1$. To understand this result we can concentrate on the average investment change within each income group. In the polar case of full segregation ($p = 1$), those above the median income will invest more because the productivity parameter is increased by the amount κ, whereas those below the median will invest less because the productivity paremeter is reduced by the same amount κ. For those below the median income ($y_{t-1} \leq y^M$), the average investment loss (relative to school equality - $\kappa = 0$) is given by

$$\Delta I_L^* = \left(\frac{\theta^L}{1+\theta^L} - \frac{\theta}{1+\theta}\right) E[y_{t-1}|y_{t-1} \leq y^M] = \left(\frac{-\kappa}{1+\theta}\right)\left(\frac{E[y_{t-1}|y_{t-1} \leq y^M]}{1+\theta-\kappa}\right) < 0$$

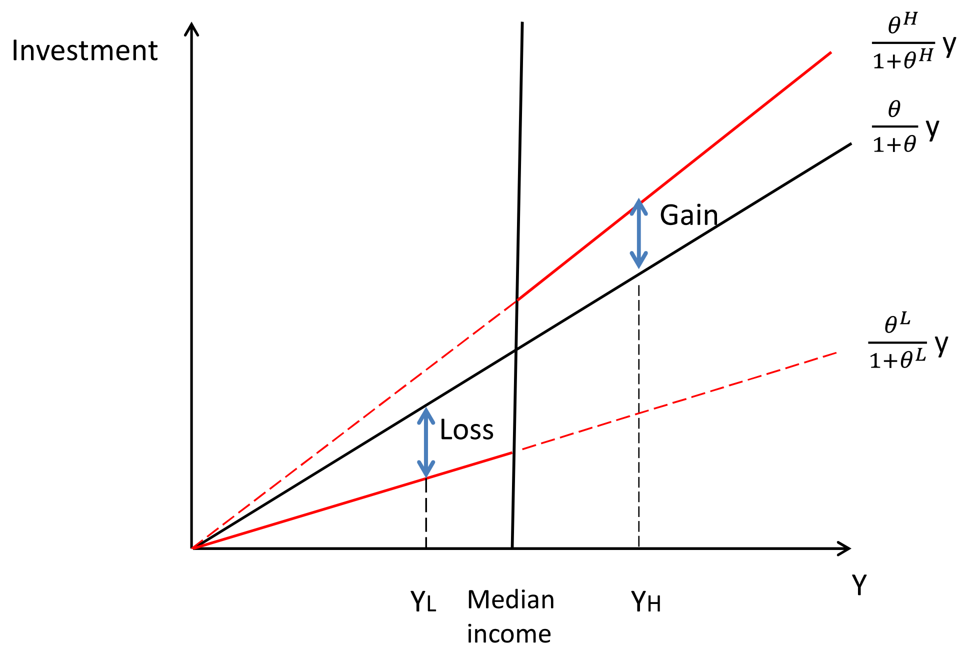
For those above the median income ($y_{t-1} > y^M$), the average investment gain (relative to school equality - $\kappa = 0$) is given by

$$\Delta I_H^* = \left(\frac{\theta^H}{1+\theta^H} - \frac{\theta}{1+\theta}\right) E[y_{t-1}|y_{t-1} > y^M] = \left(\frac{\kappa}{1+\theta}\right)\left(\frac{E[y_{t-1}|y_{t-1} > y^M]}{1+\theta+\kappa}\right) > 0$$

Therefore the net investment change is

$$\frac{1}{2}\left(\Delta I_H^* + \Delta I_L^*\right) = \frac{1}{2}\left(\frac{\kappa}{1+\theta}\right)\left(\frac{E[y_{t-1}|y_{t-1} > y^M]}{1+\theta+\kappa} - \frac{E[y_{t-1}|y_{t-1} \leq y^M]}{1+\theta-\kappa}\right) \lesseqgtr 0 \qquad (12)$$

Figure 2



The interpretation is simple. High income families attend high-quality schools and invest a higher proportion of income in education, whereas low-income families attending low-quality school will invest a smaller proportion of income. However there exist decreasing returns: the propensity to invest in children is a concave function of the school quality. Hence, the effect of a (mean preserving) school inequality is to decrease the average propensity to invest in children. Therefore, school inequality and segregation induce a smaller increase of investment just above the median income than the decrease just below the median income. This effect tends to lower average investment for a sufficiently small level of income inequality. However investment is proportional to income, so the investment change is proportional to the income gap. This complementarity effect between income and school quality tends to increase average investment, due to matching gains. Thus, with sufficiently high income group inequality, average investment increases with segregation. Note also that from the change in investment in each group, it is straightforward to see that school inequality and segregation increase inequality in investment across groups, since $(\Delta I_H^* - \Delta I_L^*) > 0$.

4 Impact of inequality and segregation on human capital

In a uniform school system without segregation, the average human capital level is

$$E[h_t] = \theta E[log(I_{t-1}^*)] + E[e_t] = g(\theta) + \theta E[log(y_{t-1})] + \mu \tag{13}$$

where $g(\theta) = \theta log(\frac{\theta}{1+\theta}) < 0$ with $g'(\theta) < 0 < g''(\theta) > 0$ for all $\theta \in (0, 1)$. Hence $g(\theta)$ is a negative, decreasing and convex function.

We now analyze the impact of school inequality and segregation on the average human capital level. We consider the short term impact of isolated changes in the segregation rate, school inequality and income inequality on the average human capital of generation t given the initial condition on the income distribution of generation $t-1$, the average income $E[y_{t-1}]$, and the median income $y_{t-1}^M = y^M$. Substituting for the optimal investment choices, the average human capital level is given by:

$$E[h_t, s] = \frac{1}{2}\left(pg(\theta^H) + (1-p)g(\theta^L) + (p\theta^H + (1-p)\theta^L)\phi E[log(y_{t-1})\right)$$

$$+\frac{1}{2}\left(pg(\theta^L) + (1-p)g(\theta^H)) + (p\theta^L + (1-p)\theta^H)(2-\phi)E[log(y_{t-1})\right) + \mu$$

$$= E[e_t]+\frac{1}{2}\left(g(\theta^H) + g(\theta^L)\right)+\frac{1}{2}(p\theta^H+(1-p)\theta^L)\phi E[log(y_{t-1})]+\frac{1}{2}(p\theta^L+(1-p)\theta^H)(2-\phi)E[log(y_{t-1})]$$

Where $\phi = \frac{E[\log(y_{t-1})| \log(y_{t-1})>\log(y^M)]}{E[\log(y_{t-1})]} \in (1, 2)$ is the inter-group log-income inequality. The comparative static properties of inequality and segregation on average human capital level are given in the following proposition:

Proposition 2. *(i) For any $\kappa > 0$ and $\phi > 1$, average human capital is an increasing function of social segregation p ; (ii) For any $\kappa > 0$ and $p > 1/2$, average human capital is an increasing function of log-income inequality ; (iii) For any $\phi > 1$ and $p > 1/2$, average human capital is an increasing function of school inequality κ.*

The proof is provided in the Appendix. Note the difference between the investment result in the previous proposition and the human capital result in this proposition: while average investment might either increase or decrease with school inequality, the average human capital is always increasing with (mean-preserving) school inequality under segregation. This is true although the impact of school inequality (with segregation) on parental investment is ambiguous. The low income group reduces their parental investment and the high income group increases it. The net effect depends on income inequality relative to school inequality. The reason for the increase of average human capital with school inequality is the complementarity between school productivity and parental investment in the formation of human capital. This complementarity makes the human capital transmission a convex function of parental income. This convexity comes from the interplay between optimal investment and the human capital technology. Indeed, segregation boosts parental investment in the high income group, where the productivity of investment is also higher, and reduces investment where the productivity is lower. This optimal response of parents to a change in the school system matters a lot for the intergenerational transmission of inequality. In fact, as put by Becker and Tomes (1979) "mechanical models of the intergenerational transmission of inequality that do not incorporate optimizing response of parents to their own or to their children's circumstances greatly understate the influence of family background on inequality" (p. 1165). Because students of different social backgrounds have different opportunities of attending high-quality schools and because such schools provide better educational attainments and labor market income, inequality in school quality contributes to educational inequality and as we will see shortly, in many cases to less social mobility as well.

This result is illustrated in figure 3 for the polar case $p = 1$. For those below the median income, whenever $p = 1$, the effect of school inequality and segregation on human capital, with respect to a case in which $\kappa = 0$, is given by:

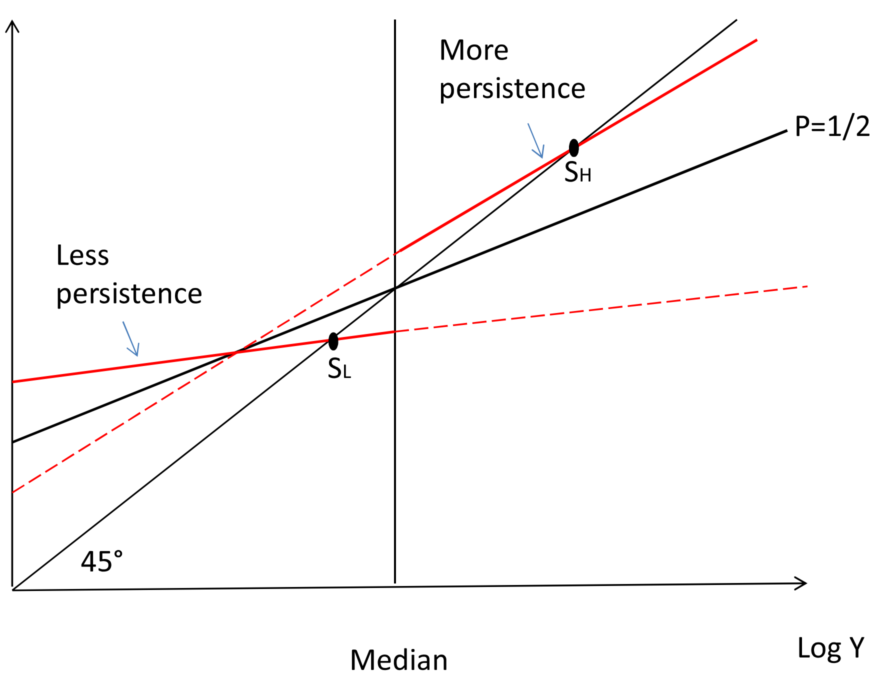
$$\Delta h_t = h_t^s - h_t = g(\theta^L) - g(\theta) + (\theta^L - \theta)\log(y_{t-1})$$

The first term is positive since $g(\theta)$ is decreasing and $\theta^L < \theta$; the second term is negative and proportional to parental income. Hence those who lose more from segregation are those closer to the median income. Those who lose less and could even win are those with lower income. The reason is that there is less influence of family income on inequality because the school is less productive. For those above the median income, whenever $p = 1$, the effect of school inequality and segregation on human capital, with respect to a case in which $\kappa = 0$, is given by:

$$\Delta h_t = h_t^s - h_t = g(\theta^H) - g(\theta) + (\theta^H - \theta) log(y_{t-1})$$

The first term is negative since $g(\theta)$ is decreasing and $\theta^H > \theta$; the second term is positive and increasing in parental income. This implies that those gaining more from segregation are those closer to the top, while those closer to the median gain less. On the other hand, regarding inequality, the effect is stronger than for investment, because the behavioral response is augmented with the differential productivity. Inequality decreases among those below the median, increases among those above the median income, and increases between groups (i.e., between those above and below the median income).

Figure 3



Given that empirical papers increasingly rely on quasi-experimental methods such as Regression Discontinuity Designs to estimate causal effects and that our model features a discontinuity based on a running variable (parental income), it is worth pointing out that the causal effect of the better schooling on investment and human capital at the threshold will not be, in general, representative of the average effect. For simplicity, consider the $p = 1$ case. The effect of a better schooling on investment at the threshold, (i.e., the effect that we would estimate via RDD) is given by $\left(\frac{\theta^H}{1+\theta^H} - \frac{\theta^L}{1+\theta^L}\right) y^M_{t-1}$. On the other hand, if we would run a Randomized Control Trial, the treatment effect that we would obtain, the Average Treatment Effect (ATE) is given by: $\left(\frac{\theta^H}{1+\theta^H} - \frac{\theta^L}{1+\theta^L}\right) E[y_{t-1}]$. Finally, if we would estimate the effect by differences-in-differences, exploiting variation in individuals' change of schooling over time, we would estimate an Average Treatment Effect on the Treated (ATT) given by $\left(\frac{\theta^H}{1+\theta^H} - \frac{\theta^L}{1+\theta^L}\right) E[y_{t-1}|y_{t-1} > y^M]$. The ATT is the largest of the possibly estimate treatment effects, and the difference between the ATE and the RDD estimate will depend on whether the distribution of income is very asymmetric. In general, with a log-normal distribution for parental income, the RDD estimate will be the most conservative. Notice that similar expressions hold for the effects on Human Capital. For instance, if $g(\theta^L) - g(\theta^H) = (\theta^H - \theta^L)log(y^M)$, a RDD would estimate a zero effect of segregation on human capital at the threshold, although the average effect is positive.

5 Intergenerational elasticity

To measure intergenerational mobility, we will compute the intergenerational elasticity of income. Substituting optimal investment into the earnings equation:

$$log(y_t) = \mu + g(\theta) + \theta log(y_{t-1}) + \epsilon_t \tag{14}$$

The inter-generational income elasticity IGE (i.e. the structural interpretation of the OLS estimate $\hat{\beta}$) in the uniform school system without segregation is given by $\beta = \theta$. Under segregation and school inequality, the regime switch model involves multiple auto-regressive equations that characterize dynamic of income transmission in different regimes. By permitting regime switches, this model is able to represent more complex (non linear) dynamic patterns of intergenerational income transmission.

Recall that $g(\theta) = \theta log(\frac{\theta}{1+\theta})$, and that ϕ is income inequality defined on log income. Since $log(y) = h$,it follows that ϕ is equivalent to the education inequality between the two income groups. Let also δ be defined as $\delta = \frac{E[(log(y_{t-1}))^2|y>y^M]}{E[(log(y_{t-1}))^2]}$ implying that $E[(log(y_{t-1}))^2|y < y^M] = (2-\delta)E[(log(y_{t-1}))^2]$.

Substituting optimal investment into the earnings equation for the regime switch model: for $y_{t-1} \leq y^M_{t-1}$

$$log(y_t) = \mu + pg(\theta^L) + (1-p)g(\theta^H) + (p\theta^L + (1-p)\theta^H)log(y_{t-1}))$$

and for $y_{t-1} > y^M_{t-1}$

$$log(y_t) = \mu + (1-p)g(\theta^L) + pg(\theta^H) + ((1-p)\theta^L + p\theta^H)log(y_{t-1}))$$

This can be rewritten in a single encompassing equation as:

$$log(y_t) = \mu + pg(\theta^L) + (1-p)g(\theta^H) + (2p-1)(g(\theta^H) - g(\theta^L))Z_{t-1}$$

$$+(p\theta^L + (1-p)\theta^H)log(y_{t-1})) + (2p-1)(\theta^H - \theta^L)log(y_{t-1})Z_{t-1} + \epsilon_t$$

where the random variable $Z_{t-1} = 1$ with probability p if $y_{t-1} > y^M$ and zero otherwise, and $Z_{t-1} = 1$ with probability $1-p$ if $y_{t-1} < y^M$ and zero otherwise . This bivariate regime switch model implies both a random intercept and a random coefficient in the auto-regressive process of income transmission. Notice that the auto-regressive process implies lower income persistence below the median income than above the median income. In the long run, persistence can also be reduced by possible switches across regimes, due to the error term ϵ_t that represents luck, and the reduction will be larger the larger the variance of ϵ. Persistence can also be reduced by a lower segregation rate p. When p is close to 1/2 we have a random switching model with equal probability of accessing high quality school for both income groups. We would like to compare the structural interpretation of the IGE arising from this non-linear regime switch model with the IGE arising from uniform school and no segregation, to understand the impact of segregation and school inequality on social mobility. To this aim, it is useful to use the classical omitted variable bias results. We estimate:

$$log(y_t) = \alpha_s + \beta_s log(y_{t-1}) + \eta_{t-1} \tag{15}$$

where the omitted term is $\eta_{t-1} = (2p-1)(g(\theta^H)-g(\theta^L))Z_{t-1}+(2p-1)(\theta^H-\theta^L)log(y_{t-1})Z_{t-1}+\epsilon_t$. Recall that $\beta = \theta$ in the uniform school with no segregation model. The omitted term is the sum of the omitted change in the intercept (which is negative since $g(\theta)$ is decreasing); and the omitted change in the slope (which is positive, since $\theta^H > \theta^L$), weighted by a measure of inequality in income. We have:

$$\begin{aligned}
\beta_s &= \frac{cov(log(y_t), log(y_{t-1}))}{Var(log(y_{t-1}))} \\
&= (2p-1)(g(\theta^H) - g(\theta^L))\frac{cov(Z_{t-1}, log(y_{t-1}))}{Var(log(y_{t-1}))} + (p\theta^L + (1-p)\theta^H) \\
&\quad +(2p-1)(\theta^H - \theta^L)\frac{cov(log(y_{t-1})Z_{t-1}, log(y_{t-1}))}{Var(log(y_{t-1}))}
\end{aligned}$$

First, note that with equal opportunity $p = \frac{1}{2}$, the regime switching model coincides to the random switching model and $\beta_s = \theta = \frac{\theta^H+\theta^L}{2}$. Hence *school inequality does not affect the social mobility if there is equal opportunity.*

Proposition 3. *Assume that $\theta + g(\theta)$ is increasing in θ. For any $\kappa > 0$, $p > 1/2$ and $\phi > 1$, the intergenerational elasticity of income β_s is an increasing function of social segregation p if $Var(log(y_{t-1})|y > y^M) \geq Var(log(y_{t-1})|y < y^M)$. This (sufficient) condition is verified for a log-normal income distribution.*

The proof is provided in the Appendix. Note, however, that in cases where the variance at the top is smaller, we could have that segregation increases intergenerational mobility as measured by the IGE. This is because the estimated IGE gives more weight to the groups with higher unexplained variation - like any regression coefficient- and segregation reduces persistence at the bottom while increasing it at the top. The intuition for this result is the following. Since education is the channel through which parents pass their economic status to their offspring, an increase in θ increases the room for the influence of parental background. This room will be amplified if there is a lot of variation in parental income. Hence, if there is more variation in parental income at the top than at the bottom, the increase in persistence at the top has stronger effects than the decrease in persistence at the bottom. The opposite happens if there is more variation in parental income at the bottom. Hence, the result on mobility depends on the overall distribution of parental (log-)income.

Proposition 4. *Assume that $\theta + g(\theta)$ is increasing in θ. For any $p > 1/2$ and $\phi > 1$, the intergenerational elasticity of income β_s is an increasing function of school inequality κ if $Var(log(y_{t-1})|y > y^M) \geq Var(log(y_{t-1})|y < y^M)$. This (sufficient) condition is verified for a log-normal income distribution. Under equal opportunity $p = \frac{1}{2}$, and we have $\frac{\partial \beta_s}{\partial \kappa} = 0$.*

The proof is provided in the Appendix. The intuition for this result is the same as the one for the previous proposition.

6 Steady state

The steady state is characterized by a stationary distribution so we drop the time index from now on. The intergenerational income transmission at the steady state with uniform school and no segregation is characterized by the following linear auto-regressive process

$$E[log(y)] = \mu + g(\theta) + \theta E[log(y)] + E[\epsilon]$$

$$E^{\circ}[\log(y)] = \frac{\mu + g(\theta)}{1 - \theta} \tag{16}$$

We shall assume in the following that $\mu + g(\theta^H) > 0$. Hence since $g'(\theta) < 0$, it follows that $\mu + g(\theta^L) > \mu + g(\theta^H) > 0$, and that $E[log(y)] > 0$ - the average level of human capital is positive. The intergenerational income transmission with school inequality and segregation is characterized by the following regime switch model with a random intercept and random coefficient:

$$\begin{aligned} log(y) = \mu + pg(\theta^L) + (1-p)g(\theta^H) + (2p-1)(g(\theta^H) - g(\theta^L))Z \\ + (p\theta^L + (1-p)\theta^H)log(y_{t-1})) + (2p-1)(\theta^H - \theta^L)log(y) + \epsilon \end{aligned}$$

Where the random variable Z is equal to 1 with probability 1/2 if $y > y^M$ and zero otherwise.

Proposition 5. *The steady state level of human capital is higher with school inequality ($\kappa > 0$) and segregation ($p > \frac{1}{2}$) than under $p = \frac{1}{2}$ and $\kappa = 0$.*

The proof is provided in the Appendix. The steady state level of human capital increases with school inequality and social segregation because of the complementarities between school quality and parental investment.

7 Calibration and Simulations

With the aim of providing quantitative estimation of the model predictions -most notably, the magnitude of the trade-off between intergenerational mobility and efficiency-, we calibrate the model to the US income distribution and simulate the effects on mobility and efficiency of changing segregation and school inequality, with the same starting conditions. In our calibration of the model, we assume that parental income is distributed as in the US, according to a Generalized Beta of the second kind, or Dagum I, with parameters $a = 3.008$ $b = 41865$, $p = 0.592$ and $q = 1$, which represent MLE coefficients for the US in 2013 (Clementi and Gallegati, 2016). For this particular distribution and calibration, the variance at the top is actually slightly smaller than the variance at the bottom. As in the proof of proposition 3, the sufficient condition for $\frac{\partial \beta_s}{\partial p} > 0$ does not require necessarily that the variance at the top is greater than the variance at the bottom. What is required is that $\pi \geq \frac{1}{2}\left(\frac{E[log(y_{t-1})]+1}{E[log(y_{t-1})]}\right)$. In our calibration, $\pi = 0.992$, and $\frac{1}{2}\left(\frac{E[log(y_{t-1})]+1}{E[log(y_{t-1})]}\right) = 0.548$, which means that the conditions for $\frac{\partial \beta_s}{\partial p} > 0$ and $\frac{\partial \beta_s}{\partial \kappa} > 0$ are satisfied for any $p > 1/2$ and $\phi > 1$.

As a benchmark, we set $\theta = \frac{1}{3}$, and $\mu = 7.9$, that give steady state values for the average log-income that are similar to those in (Clementi and Gallegati, 2016). We then explore how aggregate human capital and intergenerational mobility, measured by the IGE, are affected by different segregation and school inequality (i.e, different values of p and κ). Since the unit of measure of Human Capital is not obvious, we use as a benchmark (normalized to be one) the case in which there is equality of opportunity (i.e., $p = \frac{1}{2}$) and an intermediate value of school inequality ($\kappa = 0.03$). For every set of parameter values, we perform 10000 simulations; for each simulation, we take the moment of interest, and present the averages below.

Table 1 reports the results of increasing segregation. The simulations are qualitatively in line with the model predictions, but reveal that the decline in mobility is much sharper than the efficiency gain that comes along with larger values of p. In the US, using PISA data, we estimate the segregation parameter p to be around 0.87 (in figure 1 this translates into a dissimilarity index of 0.37). Then the simulation reveals that eliminating segregation (i.e. reducing p from 0.9 to 0.5) would reduce the IGE from 0.52 to 0.30 (that is a 42% reduction of IGE) while reducing average human capital level by only 0.13%.

In a similar vein, table 2 reports the results of increasing school inequality, for different levels of segregation. As predicted by the model, with no segregation ($p = 0.5$), school inequality does not affect intergenerational mobility (the small variations with k are not systematic and due to randomness). Likewise, as predicted by the model, school inequality increases aggregate human capital, although this effect is quantitatively very small. The second panel in table 2 reports the results of increasing school inequality, under $p = 0.7$. In this case, the effects of school inequality become larger due to the interaction with segregation. Moreover, we see again that the reduction of intergenerational mobility is relatively much larger than the efficiency gain from school inequality, that remains rather small. With segregation rates closer to the current level in the US ($p = 0.9$), the pattern is even sharper. Hence, these results suggest that policies targeted at reducing either segregation or school inequality would have large benefits on intergenerational mobility, with small efficiency costs. This is also the broad picture that emerges from these simulations: in the trade-off between intergenerational mobility and efficiency, different educational policies would trigger a much higher impact on intergenerational mobility than on efficiency.

Table 1: Simulation: $\theta = 0.3$, $\kappa = 0.03$

p	IGE	Aggregate HK
0.5	.3001	1
0.6	.3554	1.0003
0.7	.4108	1.0007
0.8	.4662	1.001
0.9	.5216	1.0013
1	.5771	1.0017

Table 2: Simulations for $\theta = 0.3$ and different values of κ

$p = 0.5$			$p = 0.7$			$p = 0.9$		
κ	IGE	Aggregate HK	κ	IGE	Aggregate HK	κ	IGE	Aggregate HK
0	.29998	.99991	0	.29998	.99991	0	.29998	.99991
.01	.30001	.99992	.01	.33695	1.00014	.01	.3739	1.00036
.02	.29999	.99995	.02	.37387	1.00039	.02	.44782	1.00083
.03	.29998	1	.03	.41082	1.00066	.03	.52168	1.00131
.04	.29993	1.00006	.04	.44774	1.00094	.04	.59556	1.00182
.05	.29991	1.00014	.05	.4848	1.00124	.05	.66934	1.00234
.06	.29977	1.00025	.06	.52168	1.00157	.06	.74325	1.00288
.07	.30003	1.00039	.07	.55853	1.00191	.07	.8169	1.00344
.08	.29989	1.00052	.08	.59536	1.00227	.08	.89061	1.00402

8 Conclusion

In a keynote speech, Alan Krueger said "There is a cost to the economy and society if children from low-income families do not have anything close to the opportunities to develop and use their talents as the more fortunate children from better off families who can attend better schools, receive college preparatory tutoring and draw on a network of family connections in the job market" (January 12, 2012, Council of Economics Advisers)". Following up on this statement, we study the role of school inequality and social segregation for human capital accumulation, inequality and intergenerational mobility. Motivated by cross-country associations between school segregation and intergenerational mobility, and by recent within country findings on the causal effects of neighborhoods, we extend the Becker-Tomes-Solon model of intergenerational mobility to allow for school segregation by using a regime switch model. A central channel for inequality persistence in Becker-Tomes-Solon is the parental investment in the human capital of their offspring, with richer parents investing more in their offspring than poorer ones. In our model, the effect of school segregation and school inequality on parental investment can go either way because of the diminishing returns in parental investment. For large income gap, segregation increases the average parental investment, but it is the other way around for low income gap. Segregation produces a shifting of parental investments towards the richer families. Given that high income families can attend better schools (on average), this shift of parental investment induces efficiency gains due to the complementarity between school quality and parental investment. Segregation increases the average level of human capital, reduces income persistence at the bottom of the distribution and increases it at the top of the distribution, the overall effect on intergenerational mobility is ambiguous and depends on the overall income distribution. The implication of the model, in contrast to the standard models, is that poorer and richer families need not regress to the same mean. We calibrate and simulate the model to provide further insights on the impact of de-segregation and school equalization policies on average income and intergenerational mobility. Our simulation results suggest that those school policies have a big impact on intergenerational mobility with small efficiency costs.

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Appendix

Proposition 1

Proof. Part (i).

$$\frac{\partial E[I_s]}{\partial p} = \left(\frac{\theta^H}{1+\theta^H} - \frac{\theta^L}{1+\theta^L} \right) (\gamma - 1) E[y_{t-1}] = \left(\frac{\theta^H - \theta^L}{(1+\theta^H)(1+\theta^L)} \right) (\gamma - 1) E[y_{t-1}] > 0$$

Which means that aggregate investment is an increasing function of social segregation p.

Part (ii). It is straightforward to see that average investment is increasing in γ since $\omega_h(\theta) > \omega_l(\theta)$ for $\kappa > 0$ and $p > 1/2$:

$$\frac{\partial E[I_s]}{\partial \gamma} = \frac{1}{2}\omega_h(\theta) E[y_{t-1}] - \frac{1}{2}\omega_l(\theta) E[y_{t-1}] > 0$$

Part (iii). The effect of school inequality κ on average investment is given by :

$$\frac{\partial E[I_s]}{\partial \kappa} = \frac{1}{2} \left(\frac{\partial \omega_h(\theta)}{\partial \kappa} \right) \gamma E[y_{t-1}] + \frac{1}{2} \left(\frac{\partial \omega_l(\theta)}{\partial \kappa} \right) (2-\gamma) E[y_{t-1}]$$

where the first term

$$\frac{\partial \omega_h(\theta)}{\partial \kappa} = p\frac{1}{(1+\theta^H)^2} - (1-p)\frac{1}{(1+\theta^L)^2}$$

is negative for $p = 1/2$ and positive for $p = 1$. So there exists intermediate value $1/2 < p < 1$ such that this expression is equal to zero. The second term is,

$$\frac{\partial \omega_l(\theta)}{\partial \kappa} = -p\frac{1}{(1+\theta^L)^2} + (1-p)\frac{1}{(1+\theta^H)^2} < 0$$

for all p and $k > 0$.

Hence, we can define a threshold for p solving

$$p\left(\gamma(1+\theta^L)^2 + \gamma(1+\theta^H)^2 - (2-\gamma)(1+\theta^L)^2 - (2-\gamma)(1+\theta^H)^2\right) - \gamma(1+\theta^H)^2 + (2-\gamma)(1+\theta^L)^2 = 0$$

such that $\frac{\partial E[I_s]}{\partial \kappa} > 0$ iff:

$$p > \frac{\gamma(1+\theta^H)^2 - (2-\gamma)(1+\theta^L)^2}{(\gamma(1+\theta^L)^2 + \gamma(1+\theta^H)^2 - (2-\gamma)(1+\theta^L)^2 - (2-\gamma)(1+\theta^H)^2)}$$

$$p > \frac{2(\gamma - 1)\left((1+\theta)^2 + \kappa^2\right) + 4\kappa(1+\theta)}{4(\gamma - 1)\left((1+\theta)^2 + \kappa^2\right)}$$

$$p > \frac{1}{2} + \frac{\kappa(1+\theta)}{(\gamma - 1)\left((1+\theta)^2 + \kappa^2\right)}$$

□

Proposition 2

Proof.

$$\frac{\partial E[h_t, s]}{\partial p} = \kappa\phi E[log\, y_{t-1}] - \kappa(2-\phi)E[log\, y_{t-1}] = \kappa E[log\, y_{t-1}]2(\phi - 1) > 0$$

$$\frac{\partial E[h_t, s]}{\partial \phi} = \frac{1}{2}E[log\, y_{t-1}]\left(p\theta^H + (1-p)\theta^L - p\theta^L - (1-p)\theta^H\right) = \kappa E[log\, y_{t-1}]\,(2p-1) > 0$$

$$\frac{\partial E[h_t, s]}{\partial \kappa} = \frac{1}{2}\left(g'(\theta^H) - g'(\theta^L)\right) + ((2p-1)(\phi - 1)E[log\, y_{t-1}]) > 0$$

where $g'(\theta^H) - g'(\theta^L) > 0$ by the convexity of $g(\theta)$. □

Proposition 3

Proof. Note that

$$\frac{\partial \beta_s}{\partial p} = 2(g(\theta^H) - g(\theta^L))\frac{cov(Z_{t-1}, log(y_{t-1}))}{Var(log(y_{t-1}))} + (\theta^L - \theta^H)$$

$$+2(\theta^H - \theta^L)\frac{cov(log(y_{t-1})Z_{t-1}, log(y_{t-1}))}{Var(log(y_{t-1}))}$$

Recall that $g(\theta^H) - g(\theta^L) < 0$. With $\theta + g(\theta)$ increasing in θ, $g(\theta^H) - g(\theta^L) < (\theta^L - \theta^H)$, and a sufficient condition for $\frac{\partial \beta_s}{\partial p} > 0$ is:

$$cov(log(y_{t-1})Z_{t-1}, log(y_{t-1})) > cov(Z_{t-1}, log(y_{t-1})) + \frac{Var(log(y_{t-1}))}{2}$$

First, note that as long as $E[log(y_t) \geq 1]$,

$$cov(log(y_{t-1})Z_{t-1}, log(y_{t-1})) - cov(Z_{t-1}, log(y_{t-1})) =$$

$$\frac{1}{2}\left(Var(log(y_{t-1})|y > y^M) + (\phi E[log(y_{t-1})] - 1)(\phi - 1)E[log(y_{t-1})]\right)\right)$$

For $\frac{\partial \beta_s}{\partial p} > 0$, the difference in the covariances above must be larger than $\frac{1}{2}Var(log(y_{t-1}))$:

$$\frac{1}{2}\left((Var(log(y_{t-1})|y > y^M) - Var(log(y_{t-1})) + (\phi E[log(y_{t-1})] - 1)(\phi - 1)E[log(y_{t-1})]\right)\right) > 0$$

To see why $Var(log(y_{t-1})|y > y^M) \geq Var(log(y_{t-1})|y < y^M)$ is sufficient, note that:

$$Var(log(y_{t-1})|y > y^M) - Var(log(y_{t-1})|y < y^M)$$

$$= \delta E[(log(y_{t-1})^2] - \phi^2 E[log(y_{t-1})]^2 - (2-\delta)E[(log(y_{t-1})^2] + (2-\phi)^2 E[log(y_{t-1})]^2$$

$$= 2(\delta - 1)E[(log(y_{t-1})^2] - 4(\phi - 1)E[log(y_{t-1})]^2$$

Now, define $\pi = \frac{2(\delta-1)E[(log(y_{t-1})^2]}{4(\phi-1)E[log(y_{t-1})]^2}$, such that $\pi \geq 1$ iff $Var(log(y_{t-1})|y > y^M) \geq Var(log(y_{t-1})|y < y^M)$. Using this, we can rewrite the sufficient condition for $\frac{\partial \beta_s}{\partial p} > 0$ as:

$$\frac{1}{2}(Var(log(y_{t-1})|y > y^M) - Var(log(y_{t-1})) + \frac{1}{2}(\phi E[log(y_{t-1})] - 1)(\phi - 1)E[log(y_{t-1})] > 0$$

$$\frac{1}{2}\left((\delta - 1)E[log(y_{t-1})^2] - (\phi - 1)E[log(y_{t-1})]^2 + (1-\phi)E[log(y_{t-1})]\right)\right) > 0$$

$$\frac{1}{2}\left(2\pi(\phi - 1)E[log(y_{t-1})]^2 - (\phi - 1)E[log(y_{t-1})]^2 + (1-\phi)E[log(y_{t-1})]\right) > 0$$

$$\frac{1}{2}\left((\phi - 1)E[log(y_{t-1})]\left((2\pi - 1)E[log(y_{t-1})] - 1\right)\right) > 0$$

Hence, the sufficient condition for $\frac{\partial \beta_s}{\partial p} > 0$ will be satisfied whenever:

$$\pi \geq \frac{1}{2}\left(\frac{E[log(y_{t-1})] + 1}{E[log(y_{t-1})]}\right)$$

Given that $E[log(y_{t-1})] \geq 1$, $\pi \geq 1$ is a sufficient condition. Note however that with more plausible values of $E[log(y_{t-1})]$, with π just a little larger than $\frac{1}{2}$ (i.e., for a wide range of values such that $Var(log(y_{t-1})|y > y^M) < Var(log(y_{t-1})|y < y^M)$), the condition will still be satisfied. Finally, note that whenever y is log-normally distributed, $log(y)$ follows a symmetric distribution, such that $\pi = 1$, and hence the condition will always be satisfied. □

Proposition 4

Proof.

$$\frac{\partial \beta_s}{\partial \kappa} = (2p-1)(g'(\theta^H) + g'(\theta^L))\frac{cov(Z_{t-1}, log(y_{t-1}))}{Var(log(y_{t-1}))} + (1-2p)$$

$$+(2p-1)2\frac{cov(log(y_{t-1})Z_{t-1}, log(y_{t-1}))}{Var(log(y_{t-1}))}$$

Where the first two terms are negative, and the last one is positive. First, note that $\theta + g(\theta)$ increasing in θ implies $(g'(\theta^H) + g'(\theta^L)) + 2 \geq 0$. Second, note that as long as $E[log(y_t) \geq 1]$, $cov(log(y_{t-1})Z_{t-1}, log(y_{t-1})) > cov(Z_{t-1}, log(y_{t-1}))$. Then it follows that a sufficient condition for $\frac{\partial \beta_s}{\partial \kappa} > 0$ is given by

$$cov(log(y_{t-1})Z_{t-1}, log(y_{t-1})) > cov(Z_{t-1}, log(y_{t-1})) + \frac{Var(log(y_{t-1}))}{2}$$

Which is exactly the same sufficient condition that we obtained for $\frac{\partial \beta_s}{\partial p} > 0$. □

Proposition 5

Proof. In the steady state:

$$E[log(y), s] = \frac{\frac{(\mu+g(\theta^L))+(\mu+g(\theta^H))}{2}}{(1 - \left(p\theta^L + (1-p)\theta^H + \frac{\tilde{\phi}}{2}(2p-1)(\theta^H - \theta^L)\right)} =$$

$$\frac{\frac{(\mu+g(\theta^L))+(\mu+g(\theta^H))}{2}}{(1 - \left(\theta^L(p - \tilde{\phi}(p-\frac{1}{2})) + \theta^H((1-p) + \tilde{\phi}(p - \frac{1}{2})\right)}$$

$$> \frac{\mu + g(\theta)}{1-\theta} = E^\circ[\log(y)]$$

Note that we previously assumed $\mu + g(\theta^H) > 0$, and that we denote the steady state inequality by $\tilde{\phi} \geq 1$. Since $g(\theta^L) > g(\theta^H)$, $E[log(y)] > 0$. Under segregation, the numerator is larger, since $g(\theta)$ is negative and convex, and the denominator is smaller. □