Alternative models of restructured electricity systems
Part 1: No market power

Jacqueline BOUCHER and Yves SMEERS

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Abstract

Different equilibrium concepts have been proposed by various authors (Schweppe et al, Hogan, Chao and Peck, Wu et al) to analyse competitive electricity systems. We establish correspondences between these different models though a single framework and provide additional interpretations of these equilibrium concepts. This unifying conceptual view also provides a computationally feasible approach to simulate the market. It also opens the way to the modelling of some imperfect markets.

1 Introduction

It is now more than a decade that the first major restructuring of the electricity industry took place in Chili. England and Wales, Scandinavia and New Zealand quickly followed suit. Order 888 in the US, the Council and European Parliament Directive in the European Union and movements in Australia later initiated a new wave of overhauling of the electricity sector. Electricity restructuring does not follow a single paradigm. A possible reason for this diversity is our imperfect understanding of the real impact of competition in the electricity industry. Significant progress has certainly been made in our knowledge but alternative structures remain difficult to select from. Microeconomics has been the main instrument in this ongoing analysis. It has also been recognized that computable models could shed additional insight. This paper reviews and analyses restructuring models through a single micro economic framework that leads itself to a unified computational framework.

Electricity restructuring can be achieved through quite different organizations. Descriptions of existing systems can now be found in many reports and books. The bare minimum is to have generators competing in the energy market . These may sell directly to the consumer or indirectly through power marketers. Generators together with other agents may compete for the provision of ancillary services. Infrastructure services remain a most controversial issue as the unbundling and decentralization of these services remains by nature limited. There is at least a common element in all proposals: whatever the final design, the system operator (SO) insures that the system is operated in a secure and reliable way. But its role may go beyond this minimum; depending on the degree of centralization, the system operator may also turn out to be a market maker in some services.

This explosion of goods and services and the introduction of new agents challenges the traditional modelling of the power system. The integrated character of the old regulated industry made it quite amenable to optimization. Problems could often be cast into cost minimization whether for looking at long term investment, machines dispatch or short term grid operation. An industry driven by several agents operating on multiple markets cannot be cast into a single optimization problem. Economic equilibrium emerges as a more suitable concept. Equilibrium is not very far from optimization. Roughly speaking one moves from optimization to equilibrium when one replaces a single optimization problem by a set of them and one further requires that quantities produced and consumed balance. In electricity restructuring terms, the old paradigm where a single agent optimizes the production and distribution of bundled electricity goods and services is replaced by one where different agents optimize the production, consumption and sales of different unbundled goods and services. The equilibrium is reached when all these production and consumption balance. We adopt the equilibrium paradigm in this paper and discuss various proposals of the literature in terms of interacting optimization problems. The mathematical structure underlying these different models is a quasi variational inequality problem hidden in some proofs of the appendix. In order to further simplify of the presentation the discussion is conducted by only referring to the now common standard three node example. A follow up paper will present the same material in full mathematical generality.

The aim of this paper is thus twofold. Its first objective is to cover a range of structures that go from the traditional integrated structure of the industry to a quite decentralized one. The second is to show that these different structures can be analysed independently from the standard optimization model of the regulated industry by directly resorting to the equilibrium concept.

The paper is organized as follows. The standard three node example is briefly recalled in section 1 as a general background for the rest of the paper. Discussions of electricity restructuring often refer to the so called optimal power flow problem and interpret it in terms of decentralized decisions. Section 2 reviews some of these interpretations. It indicates that they can all be obtained from reformulations of the dual of the optimal power flow model. Decentralization has been a main theme in discussions related to restructuring. Systems like the British Pool that appeared at the leading edge of competition some time ago are now seen as a paradigm of centralization. Several models have thus been proposed that allow for more and more decentralization in the markets. Section 3 accordingly surveys the models proposed by Schweppe et al [1988], Hogan [1992], Chao and Peck [1996], and finally Wu et al [1996]. It concludes with an equivalence theorem; the first three equilibria are equivalent and are competitive equilibria. The latter is more general. Difficulties for accommodating reliability in Chao and Peck's model and remedies thereof have been discussed in the literature. We take up this question in section 4 and show how Chao and Peck's model can be expanded by introducing new reliability services whose trade handles the reliability issue. The equivalence theorem of section 3 is then extended to this more general set up. The equivalence theorems of section 3 and 4 show a gap between competitive and Wu et al's equilibrium. This question is taken up in section 5. Specifically we refer to the old notion of social equilibrium to provide an new equilibrium that is less general than Wu et al but more general than the competitive equilibria. We also indicate how the social equilibrium can be interpreted in terms of decentralized energy and transmission markets. Section 5 concludes by showing that Wu et al equilibria encompass both competitive equilibria, social equilibria and other (regulated) equilibria. Conclusion terminates the paper.

2 Background

Concepts related to electricity restructuring are often analysed on the following three node example (Figure 1).

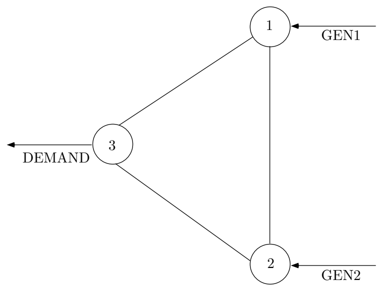

Figure 1: The standard three node example

Generators are respectively located at nodes 1 and 2 of the network; demand is concentrated at node 3. As most of the literature on restructuring, we focus on short term efficiency, that is to say on the economic operations of the electricity system during a given period when the long term characteristics (capacity and availability) of the generation, transmission and demand are given. The problem can then be fully specified by the following elements: generators are described by their short run marginal cost curves and the consumers are represented by their short run demand curves. The marginal cost curve of a plant is commonly modelled as a smooth function of the generation level. We suppose for the sake of simplification in this discussion that the following affine marginal cost curve apply to the plants

$$\begin{aligned} mc_1(g_1) &= \ell c_1 + qc_1 g_1, \qquad g_1, g_2 \geq 0 \\ mc_2(g_2) &= \ell c_2 + qc_2 g_2. \end{aligned} \tag{2.1}$$

(where ℓc and qc respectively stand for the linear cost and quadratic cost component of the total cost of the plant). We also suppose that the demand at node 3 is represented by an affine inverted demand curve that we write

$$p_3 = \alpha_3 - \beta_3 w_3 \tag{2.2}$$

where $w_3 \geq 0$ is the demand at node 3.

Access to the transmission system constitutes a major question in the restructuring of network industries. The peculiarities of the electric grid make it specially difficult to deal with access in the power industry. Kirchoff's laws, that rule electricity flows, are at the origin of these difficulties. A particularly simple presentation of these laws can be made on the three node example. Assume that all lines have identical characteristics except for some thermal limit on line 1-2. Kirchoff's laws imply the following. Because the indirect path 1–2–3 is twice as long as the direct path 1–3, a unitary injection of electricity in node 1 accompanied by a withdrawal at node 3 entails flows of 1/3 and 2/3 on the paths 1–2–3 and 1–3 respectively. The total flow on the line 1-2 is then equal to $f_{12} = (g_1 - g_2)/3$. The thermal limit on line 1–2 can then be expressed as

$$-uf_{12} \leq f_{12} \leq uf_{12}. \tag{2.3}$$

Reliability considerations, which are of utmost importance in the power industry are absent from this formulation. This simplifies the discussion but can lead one to overlook important aspects of the problem. In order to overcome this weakness we introduce here a simplified version of reliability considerations in the form of security constraints. These can take the form of constraints on generation levels such as

$$g_1 + g_2 \leq ug. \tag{2.4}$$

The construction of security constraints on the basis of in depth reliability analysis goes beyond the scope of this paper. It is relevant though to illustrate their role on the three node example. Suppose that the operator of the system wants to insure that its dispatch remains feasible in case of single line contingencies, that is if either one of the three lines goes down (this is commonly referred to as the $n-1$ security criterion). There are three single line contingencies in the three node example and Figure 2 shows the flow on the remaining operating lines in each of these contingencies.

It is easy to see that a flow equal to $g_1 + g_2$ goes through 1–3 when the line 2–3 is down; similarly a flow of the same value goes through 2–3 when

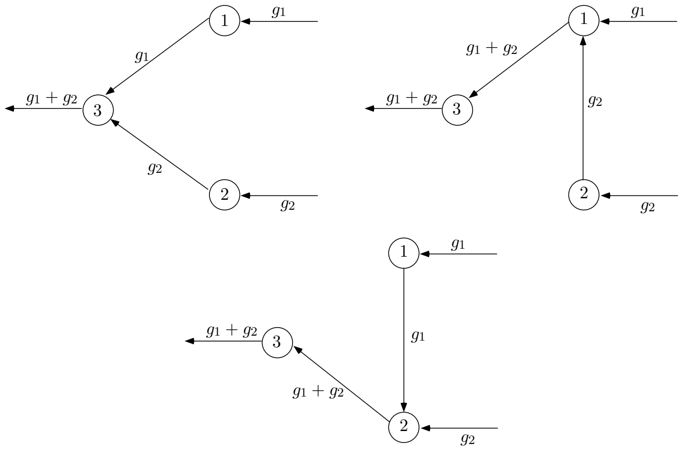

Figure 2: Flows on line in case of contingencies

the line 1–3 is down. The security constraint limits this flow on the remaining operating line in case of a contingency.

The three node problem is used throughout the course of the paper. It allows for a particularly intuitive presentation of the issues, albeit at the cost of sacrificing generality and practical realism. The presentation of this material in full mathematical generality is left to a follow up paper.

3 The vertically integrated industry, short run efficiency and the optimization paradigm

Define the variable cost functions $vc_i(g_i), i = 1,2$ and the willingness to pay function $u_3(w_3)$ such that

$$\begin{aligned} \frac{dvc_i}{dg_i} &= mc_i(g_i), \qquad i = 1,2 \\ \frac{du_3}{dw_3} &= mu_3(w_3) = \alpha_3 - \beta_3 w_3. \end{aligned}$$

Suppose in order to simplify the presentation that the economics of the system is such that the flow in line 1–2 goes always from node 1 to 2. The thermal limit constraint $-uf_{12} \leq \frac{g_1 - g_2}{3}$ is then always slack and can be dropped. We then introduce the following optimization problem hereafter referred to as the Optimal Power Flow model (OPF(3,3)).

$$\min vc_1(g_1) + vc_2(g_2) - u_3(w_3) \tag{3.1}$$

$$\text{s.t.} \qquad w_3 = g_1 + g_2 \tag{3.2}$$

$$(g_1 - g_2)/3 \leq uf_{12} \tag{3.3}$$

$$g_1, g_2, w_3 \geq 0 \tag{3.4}$$

Problem OPF(3,3), extended to an arbitrary number of nodes and lines, is a particular version of the Optimal Power Flow model commonly used in the power industry.

The analysis of the solution of OPF problem in terms of decentralization goes back to the fifties (Boiteux (1952)). Schweppe and his colleagues (1988) provided the most in depth analysis of these issues for the regulated industry and hinted at how it could be used as a paradigm for decentralization. More recently, restructuring led to a surge of short term efficiency analysis for decentralized systems. We shall mainly rely in the following on the work of Hogan (1992), Chao and Peck ((1996), (1997), (1998)), Wu et al (1996) and Oren et al (1995). Some of the main propositions of this literature can be directly derived from the application of straightforward duality theory to the OPF problem. We briefly review some of these propositions by first rewriting the OPF model in a way that allows for a richer set of dual variables . This is stated in problem OPF'(3,3) where perturbation variables are explicitly introduced in each constraint and dual variables are indicated at their right.

Problem OPF'(3,3)

$$W(v_1, v_2, v_3) \equiv \min vc_1(g_1) + vc_2(g_2) - u_3(w_3) \tag{3.5}$$

$$\text{s.t.} \qquad v_1 + g_1 = q_{13} \qquad p_1 \tag{3.6}$$

$$v_2 + g_2 = q_{23} \qquad p_2 \tag{3.7}$$

$$v_3 + w_3 = q_{13} + q_{23} \qquad p_3 \tag{3.8}$$

$$(q_{13} - q_{23})/3 \leq uf_{12} \qquad \eta \tag{3.9}$$

$$g_1, g_2, w_3 \geq 0 \tag{3.10}$$

OPF'(3,3) is a linearly constrained convex problem. Assuming a feasibly dispatch and some standard conditions on the demand curve, OPF'(3,3) has an optimal dispatch and optimal dual variables. Schweppe et al, emphasized the time varying character of these dual variables by interpreting them as spot prices. The term nodal price is now more commonly used in restructured systems and will therefore be adopted in the sequel. Nodal prices depend on location. We accordingly introduce the following definition.

Definition. *Nodal prices are the dual variables of the constraints (3.6), (3.7) and (3.8) of the problem OPF'(3,3) at optimality.*

The following proposition (see Schweppe et al (1988)) provides a first characterization of decentralized electricity systems.

Proposition 1 *Suppose that there exists an optimal dispatch. Then there exists a set of nodal prices (p_1^*, p_2^*, p_3^*) such that the optimal dispatch maximizes the surplus of the individual agents at these prices, that is (g_1^*, g_2^*, w_3^*) solves the following optimization problems:*

$$\begin{array}{ll} \max p_1^* g_1 - vc_1(g_1), & g_1 \geq 0 \\ \max p_2^* g_2 - vc_2(g_2), & g_2 \geq 0 \\ \max u_3(w_3) - p_3^* w_3, & w_3 \geq 0 \end{array} \tag{3.11}$$

Proof. See appendix.

The use of nodal prices to compute transmission tariffs goes back to the time of wheeling (see Schweppe et al (1986)). The underlying rational for this use of nodal prices is that transmission tariffs between two nodes should reflect the marginal cost of transmission, that is the marginal cost of a unitary injection and withdrawal at these two nodes. Assuming both the existence and uniqueness of the dual variables, this cost is expressed as a difference of dual variables. This is stated in Proposition 2.

Proposition 2 *Suppose that the function $W(v_1, v_2, v_3)$ is differentiable at the origin or equivalently that the dual variables (p_1^*, p_2^*, p_3^*) of OPF'(3,3) are unique at the optimal dispatch. Then, the marginal cost of transmitting electricity from one node to the other is well defined and is equal to the difference of the nodal prices at these nodes.*

Proof. See appendix.

Nodal prices are not necessarily unique. Specifically there may exist a set of nodal prices even when there is a single optimal dispatch. The nonuniqueness of the nodal prices may create practical difficulties. Indeed prices, and hence operations and profits of the agents may depend on the vagaries of an optimization algorithm and of its ending up at one or an other vector of nodal prices. In practice, nonuniqueness also increases the already quite high volatility of prices. Nonuniqueness of nodal prices occurs when the function $W(v)$ is not differentiable at the origin, which may be quite common. We retain for the rest of the presentation that non uniqueness may be both a reality and a difficulty in decentralized systems.

It should be noted that transmission prices between two nodes can always be defined as differences of nodal prices even when these are not unique. Simply this difference cannot be interpreted as a standard marginal cost for the very simple reason that the cost of transmission as a function of the injection and withdrawals is not differentiable. One should also note that transmission prices being the difference of possibly badly correlated nodal prices maybe more volatile than these prices.

It has been seen above that, whether unique or not, nodal electricity prices can be used to construct optimization problems that can be interpreted as behaviors of generators and consumers. Similarly, the organization of transmission can be described by optimization problems. Interpret q_{13} as a quantity of electricity injected in 1 and withdrawn in 3. Similarly q_{23} is the quantity of electricity injected in 2 and withdrawn in 3. One has $g_1 = q_{13}$ and $g_2 = q_{23}$ in the three node example. These injections/withdrawals require the use of the network. The following proposition characterizes the operation of transmission by an optimization problem. It was originally established by Hogan (1992):

Proposition 3 *Let q_{13}^*, q_{23}^* be the requests for using the network in the solution of the optimal dispatch. (q_{13}^*, q_{23}^*) solves the following optimization problem*

$$\max(p_3^* - p_1^*)q_{13} + (p_3^* - p_2^*)q_{23} \tag{3.12}$$

$$\begin{aligned} s.t. \quad & (q_{13} - q_{23})/3 \le uf_{12} \\ & q_{13}, q_{23} \ge 0 \end{aligned}$$

Moreover the objective function value of this optimization problem is always non negative.

Proof. See appendix.

An alternative statement of Proposition 3 is the following.

Proposition 4 *Let (g_1^*, g_2^*, w_3^*) be the optimal solution of the OPF. (g_1^*, g_2^*, w_3^*) also solves*

$$\max p_3^* w_3 - p_1^* g_1 - p_2^* g_2 \tag{3.13}$$

$$\begin{aligned} s.t. \quad & w_3 = g_1 + g_2 \\ & (g_1 - g_2)/3 \leq uf_{12} \\ & g_1, g_2, w_3 \geq 0. \end{aligned}$$

Moreover the objective function value of this optimization problem is always non negative.

This result, whether in the form of Proposition 3 or 4 plays a crucial role in many discussions of restructured electricity system (see Harvey et al (1997), Wu et al (1996)). Specifically it indicates that the value of the existing capacities of the network is maximized at the optimal dispatch. It also provides an objective function for an agent in charge of operating the network and it insures the solvency of this agent. This latter interpretation has been extensively debated.

An alternative interpretation of the organization of transmission has been proposed in Chao and Peck (1996). It is based on a reformulation of the load flow equations that can be motivated as follows. The load flow equations give the value of the flows on the lines as a function of the nodal generation and consumption. It is thus possible to express the use of the network in terms of line utilizations, these latter being expressed as functions of nodal generation and consumption. This approach is easily illustrated on the three node example by noting that a request to inject and withdraw q_{13} at nodes 1 and 3 implies a flow $q_{13}/3$ in line 1–2. Similarly a request to inject and withdraw q_{23} at nodes 2 and 3 induces a flow $-\frac{q_{23}}{3}$ in this line. The set of both q_{13} and q_{23} entails a flow $f_{12} = (q_{13} - q_{23})/3$ in line 1–2. We have the following proposition.

Proposition 5 *Let f_{12}^* be the flow in line 1–2 induced by the optimal dispatch and η^* be the dual variable associated with the thermal limit of line 1–2. Then f_{12}^* solves the following optimization problem*

$$\max \eta^* f_{12} \ s.t. f_{12} \leq uf_{12}. \tag{3.14}$$

Proof. See appendix.

This result also plays a crucial role in discussions of restructured electricity systems (see Chao and Peck (1996), (1997), (1998) and Wilson (1997)). Very much like propositions 3 and 4, it indicates that the value of the existing capacities of the network, but this time measured line by line, is maximized at the optimal dispatch. It also provides an objective function for the owner of each line. Again, this interpretation has been extensively debated.

4 The decentralized industry, short run efficiency and the equilibrium paradigm

The analysis of Section 3 can be summarized as follows. Suppose that the operations of the vertically integrated industry is represented by the optimal solution of the OPF model. Then, these operations can also be interpreted in terms of generators and consumers acting in a decentralized way under a given price system to maximize their profits. This suggests viewing the operations of the vertically integrated company in terms of economic equilibrium. These interpretations have generated alternative proposals for the restructuring of the industry. It has also been suggested by Wilson (1998) that additional insight could come from restructuring proposals that are not directly derived from the OPF model of the vertically integrated industry. In this section we briefly survey some equilibrium concepts proposed for restructured electricity systems and establish relations between them. Basically the idea is that many economic equilibrium concepts proposed for the restructured electricity industry are equivalent even if they were initially meant to correspond to different types of organizations.

4.1 Equilibrium A: Schweppe, Caramanis, Tabors and Bohn

A first interpretation of the solution of the OPF models in terms of a competitive equilibrium is due to Schweppe et al (1988). It can be stated as follows.

Let g_1^*, g_2^* and w_3^* be nodal injections and withdrawals of the generators and consumers. Let $p^* = (p_1^*, p_2^*, p_3^*)$ be a vector of nodal prices. The vectors (g_1^*, g_2^*, w_3^*) and p^* constitute an equilibrium of the power system iff

(i) g_i^* maximizes the profit of generator i, that is g_i^* is a solution of $\max p_i^* g_i - vc_i(g_i); g_i \geq 0$; $i = 1, 2$ (see relations 3.11);

(ii) w_3^* maximizes the surplus of the consumer, that is w_3^* is a solution of $\max u_3(w_3) - p_3^* w_3$, $w_3 \geq 0$ (see relations 3.11);

(iii) the nodal prices are determined by the SO by solving an OPF model on the basis of the available information on marginal cost and using the characteristics of the network.

This equilibrium truly makes the SO a Market Coordinator (Section 5.2 in Scheppe et al. (1988)). It also assumes that this market coordinator asks plant owners information on their marginal cost and solves an optimal dispatch problem.

4.2 Equilibria B and C: Hogan

Hogan (1992) relaxes this definition by not requiring that nodal prices be determined by solving an OPF problem. Instead, he assumes that the SO sells transmission services so as to maximize the profits accruing from these sales at the given nodal prices. This requires a definition of these services. They are conveniently seen as nodal Transmission Capacity Reservation (TCRs), that is reservation capacities to inject/withdraw some amounts of electricity at the different nodes of the grid (Harvey et al (1997)). In the physical market considered in this paper the reserved capacities to inject and withdraw at some node must, at equilibrium, be equal to the electricity injected and withdrawn at that node. Abusing notation, we shall use the same symbol for the quantities of electricity and network services.

At least two systems can be constructed on this basis, namely a mandatory energy pool and a fully bilateral energy market. The equivalence between these two systems when there is no market power has been argued at length in the literature (see for instance Parkinson (1996) and references therein, and Hunt and Schuttleworth (1996)). It is shown more formally here. This equivalence immediately points to a new class of equilibria obtained by mixing the mandatory pool and bilateral systems. This leads to the now widely accepted notion of non mandatory pool. The mandatory pool and bilateral equilibria can be defined as follows.

4.2.1 Equilibrium B: the Mandatory Pool

Let g_1^*, g_2^* and w_3^* respectively designate the injections and withdrawals of the generators and consumer at the different nodes of the network. Let p^* be a system of nodal prices. $(g_1^*, g_2^*, w_3^*), p^*$ is an equilibrium of the mandatory pool iff

(i) g_i^* maximizes the profit of generation i, that is g_i^* is a solution of $\max p_i^* g_i - vc_i(g_i); g_i \geq 0$; $i = 1, 2$ (see relations 3.11);

(ii) w_3^* maximizes the surplus of the consumer, that is w_3^* is a solution of $\max u_3(w_3) - p_3^* w_3$, $w_3 \geq 0$ (see relations 3.11);

(iii) the SO maximizes the profits accuring from the sales of TCRs by solving the problem (3.13), that is

$$\max p_3^* w_3 - p_1^* g_1 - p_2^* g_2$$

$$\begin{aligned} \text{s.t.} \quad & w_3 = g_1 + g_2 \\ & (g_1 - g_2)/3 \leq uf_{12} \\ & g_1, g_2, w_3 \geq 0. \end{aligned}$$

This interpretation of transmission services does not require that nodal TCRs be sold by pair, that is that a same amount of injection and withdrawal capacities be sold to each agent. It is the SO that insures an overall balance of these injection and withdrawal TCRs. This complies with the organization of a mandatory pool system where generators inject at some node without any consideration of physical bilateral contracts and hence without any concern for pairing injection at a node with withdrawal at some other node.

4.2.2 Equilibrium C: the bilateral model

An alternative version of the preceding equilibrium assumes that electricity is traded through bilateral transactions. It will be referred to as the bilateral model. Bilateral transactions require balanced amounts of nodal TCRs at the injection and withdrawal nodes covered by the transaction. Physical injection and withdrawal TCRs associated to bilateral energy transactions go by pairs. The equilibrium is then defined as follows Let g_1^*, g_2^* and w_3^* be injections and withdrawals at the nodes of the network. Because there is a single consumer, these injections and withdrawal define bilateral transactions respectively noted $q_{13}^*(= g_1^*)$ and $q_{23}^*(= g_2^*)$. Let p^* be the vector of nodal prices and t_{13}^*, t_{23}^* (where $t_{13}^* = p_3^* - p_1^*$ and $t_{23}^* = p_3^* - p_1^*$) be the transmission prices between nodes 1 and 3 and 2 and 3 respectively. The vectors q^*, p^*, t^* constitute an equilibrium of the bilateral electricity market iff

(i) q_{i3}^* maximizes the profit of the generator i selling at node 3 and paying the transmission price t_{i3}^*, that is q_{i3}^* is the solution of the following optimization problem.

$$\max(p_3^* - t_{i3}^*)q_{i3} - vc_1(q_{i3}), \quad \text{s.t. } q_{i3} \geq 0, i = 1, 2$$

(ii) w_3^* maximizes the surplus of the consumer, that is w_3^* is a solution of $\max u_3(w_3) - p_3^* w_3$, $w_3 \geq 0$ (see relations 3.11);

(iii) q_{13}^* and q_{23}^* maximize the profits accruing to the SO from the sales of TCRs, that is q_{13}^* and q_{23}^* are solutions of

$$\begin{aligned} &\max (p_3^* - p_1^*) q_{13} + (p_3^* - p_2^*) q_{23} \\ &\text{s.t.} \quad (q_{13} - q_{23})/3 \leq u f_{12} \\ &\qquad\quad q_{13}, q_{23} \geq 0 \end{aligned}$$

(see relation (3.12)).

4.3 Equilibrium D: Chao and Peck

The bilateral equilibrium paves the way to Chao and Peck's proposal (Chao and Peck (1996), (1997), (1998)). These authors introduce a decentralized market for transmission services and restrict the economic role of the SO to an information one. Specifically, generators engage in bilateral transactions and request line services from the owner of these lines in order to complete these energy transactions. The SO informs the agents about the amount of line services required to complete their transactions and the owners of the lines sell these services to maximize the profit accruing from these sales. These services are referred to as link based TCRs that is Transmission Capacity Reservation to send power on lines. The functioning of link based TCRs can be illustrated as follows. A unitary bilateral transaction between 1 and 3 requires 2/3 link based TCRs on line 1–3 and 1/3 link based TCR on both the lines 1–2 and 2–3. Similarly a unitary bilateral transaction between node 2 and 3 requires 2/3 link based TCRs on the line 2–3 and -1/3 and 1/3 link based TCRs on the line 1–2 and 1–3.

The equilibrium is then defined as follows. Let (q_{13}^*, q_{23}^*) be a vector of bilateral transactions. Suppose that these transactions saturate the thermal capacity of the line 1–2 in that direction. Let $f_{12}^* = \frac{q_{13}^* - q_{23}^*}{3}$ be this saturating flow in line 1–2. Let p^* be a system of nodal prices and η^* be the TCR price for using the line 1–2 in this direction (the direction in which the line is saturated). (q_{13}^*, q_{23}^*), f_{12}^* and (p^*, η^*) constitute an equilibrium of the power system iff

(i) q_{13}^* maximizes the profit of the generator 1 selling at node 3 and paying the link based TCRs on line 1–2, that is

$$\max \left(p_3^* - \frac{\eta^*}{3} \right) q_{13} - vc_1(q_{13}), \quad \text{s.t. } q_{13} \geq 0.$$

Similarly, q_{23}^* maximizes

$$\max\left(p_3^* + \frac{\eta^*}{3}\right) q_{23} - vc_2(q_{13}), \quad \text{s.t. } q_{23} \geq 0.$$

(Recall that the generator 2 consumes $-\frac{1}{3}$ TCR on line 1–2.)

(ii) w_3^* maximizes the surplus of the consumer, that is w_3^* is a solution of $\max u_3(w_3) - p_3^* w_3$, $w_3 \geq 0$ (see relation 3.11);

(iii) f_{12}^* maximizes the profit of the owner of line 1–2. That is f_{12}^* solves $\max \eta^* f_{12}$ s.t. $f_{12} \leq uf_{12}$ (see relation (3.12)).

4.4 Equilibrium E: Oren, Spiller, Varaiya and Wu

Oren et al (1995) and Wu et al (1996) take an altogether different view and define equilibria that do not involve network services. Specifically they consider a nodal system of the mandatory pool type and define the equilibrium as follows.

Let g_1^*, g_2^* and w_3^* be respectively the injections and withdrawals of the different generators and consumer at the nodes of the network. Let p^* be a system of nodal prices. $(g_1^*, g_2^*, w_3^*), p^*$ constitute an equilibrium for the power system iff

(i) g_i^* maximizes the profit of generator i, that is g_i^* is a solution of $\max p_i^* g_i - vc_i(g_i); g_i \geq 0; i = 1, 2$ (see relation 3.11);

(ii) w_3^* maximizes the surplus of the consumer, that is w_3^* is a solution of $\max u_3(w_3) - p_3^* w_3$, $w_3 \geq 0$ (see relations 3.11).

(iii) (g_1^*, g_2^*, w_3^*) is feasible for the network, that is it satisfies $w_3^* = g_1^* + g_2^*$, $\frac{g_1^* - g_2^*}{3} \leq uf_{12}$.

In contrast with the other equilibria, Wu et al do not introduce any behavioral assumption on the SO. The only obligation of this latter is to insure that the energy transaction are feasible for the network.

4.5 Equivalence theorems

The introduction of different economic equilibria obviously raises the question of their possible relation. This section concludes with an equivalence theorem: equilibria A, B, C and D, that is all equilibria except Wu et al's are equivalent when reliability considerations are absent. A, B, C and D equilibria are also equilibria in the sense of Wu et al but these latter are more general. A, B, C and D equilibria are also equivalent to the solution of an OPF. In this latter case they insure the same short term efficiency result as the OPF. Last but not least, there are competitive equilibria in an economy composed of generators, a SO and consumers where generators sell electricity to the SO and consumers buy electricity from the SO. These propositions are stated more formally as follows.

Theorem 1 *Consider a power system without reliability constraints. If there exists an optimal dispatch, then there exists at least an equilibrium of the type A, B, C and D. Moreover there is a one to one correspondence between equilibria of the type A, B, C and D. In other words any equilibrium of type A, B, C and D can be rewritten as an equilibrium of an other type. Any of these equilibria is also an equilibrium in the sense of Wu et al, but the converse is not true. Equilibria A, B, C and D are equivalent to the solution of the associated Optimal Power Flow model. They are also competitive equilibria.*

Proof. See appendix.

5 A stronger equivalence result with reliability

Theorem 1 only proves the equivalence of A, B, C and D equilibria when there is no reliability constraint. A discrepancy between link and node based TCRs may arise from the treatment of reliability. This possible divergence has been pointed out by Harvey et al (1997) and Hogan (1998). The key argument of their criticism is that the market for link based TCRs crucially depends on the availability of the lines. Specifically this market is continuously changing with line outages. It is contingent on line availability. Therefore a firm transmission service can only be acquired if the adequate amount of link based TCRs has been secured in each contingent market. In short, implementing Chao and Peck's proposal requires that all contingent (link based) TCR markets clear, an impossible task to achieve in practice. This shortcoming does not appear with nodal TCRs as the SO is in a position to allocate TCRs that remain valid in all contingencies. Indeed the possibility to inject and withdraw or the proper financial compensation at a node holds whatever the (assumed) contingencies on the line. On the contrary, the right to use a line disappears when this line fails.

Doing away with reliability is a significant drawback in power systems. Chao and Peck responded to Hogan's criticism by expanding their initial proposals to account for reliability. In Chao and Peck (1997) they endow the SO with the economic role of managing reliability. They do so by making the SO financially responsible of an insurance system dealing with contingencies in the transmission grid. In this system the SO sells insurance contracts against grid contingencies and pays for related damages according to these contracts. These authors further refined their system (Chao and Peck (1998)) and proposed that the SO buys TCRs as well as other services in order to insure reliability. Various remarks can be made to this latter part of the proposal. First it requires that the SO buys TCRs and hence intervenes on the market, an assumption that other authors (e.g. Oren et al (1995)) prefer to avoid in order to prevent the SO from exerting market power. Second, in contrast with the initial proposal of link based TRCs, this extension is not presented in a full equilibrium context. We hereby propose a reformulation of Chao and Peck's proposal that tries to avoid making the SO an active buyer of TCRs. This is achieved by unbundling transmission services into line utilization and some synthetic reliability services that need not be traded by the SO. We also cast this proposal in an equilibrium framework and show that the introduction of these latter services makes Schweppe, Hogan and Chao and Peck's equilibria fully equivalent.

Suppose that an adequate reliability level has been identified, for instance by one of the organizations discussed in Wilson (1997). Suppose also that reliability is expressed through a certain number of security constraints. It remains to construct a market system that achieves this level of reliability. We first indicate how the definition of Schweppe and Hogan's equilibria can be trivially adapted to account for the security constraints. We then show how Chao and Peck's equilibrium model can also be modified by introducing additional markets that we call markets for Network Reliability Reservation (NRR). The reader will easily recognise the similarity, in terms of economic concept, between NNRs and the SO_2 emission allowances introduced by Title IV of the 1990 Clean Air Act Amendments for dealing with Acid Rain (Joskow et al (1998)). As emission permits markets, NNR markets can be structured in different ways. Whatever the adopted organization the market power of the SO is reduced compared to Chao and Peck 's latest proposal (Chao and Peck (1998)).

The following sketches the small modifications necessary to deal with security constraints in equilibria A, B and C.

5.1 Equilibrium A: Schweppe, Caramanis, Tabors and Bohn

The new statement of the equilibrium is obtained by replacing condition (iii) by (iii)'(modifications are underlined) in the defintion of Section 3.1.

(iii)' The nodal prices are determined by the SO by solving an OPF model on the basis of the available information on marginal cost and using the characteristics of the network <u>and taking into account the security constraints</u>.

5.2 Equilibrium B and C: Hogan

These equilibria, whether of the mandatory pool or bilateral type, are easily modified by introducing the security constraints in the optimization problem of the SO. We indicate this modification on the bilateral model. The adaptations to the mandatory pool are similar.

Let g_1^*, g_2^* and w_3^* be the injections and withdrawals at the nodes of the network and $q_{13}^*(= g_1^*)$ and $q_{23}^*(= g_2^*)$ be the associated bilateral transactions. Let p^* be the vector of nodal prices and t_{13}^*, t_{23}^* (where $t_{13}^* = p_3^* - p_1^*$ and $t_{23}^* = p_3^* - p_1^*$) be the transmission prices between nodes 1 and 3 and 2 and 3 respectively. q^*, p^*, t^* constitutes an equilibrium of the bilateral model with security constraints iff

(i) q_{i3}^* maximizes the profit of generator i by solving the problem

$$\max(p_3^* - t_{i3}^*)q_{i3} - vc_i(q_{i3}), \quad \text{s.t. } q_{i3} \geq 0.$$

(ii) w_3^* maximizes the surplus of the consumer, that is w_3^* is a solution of $\max u_3(w_3) - p_3^* w_3$, $w_3 \geq 0$ (see relation 3.11).

(iii) (q_{13}^*, q_{23}^*) maximizes the profits accruing to the SO from the sale of TCRs, that is (q_{13}^*, q_{23}^*) is a solution of

$$\max(p_3^* - p_1^*)q_{13} + (p_3^* - p_2^*)q_{23}$$

$$\begin{aligned} \text{s.t.} \quad & (q_{13} - q_{23})/3 \leq uf_{12} \\ & q_{13} + q_{23} \leq ug \\ & q_{13}, q_{23} \geq 0 \end{aligned}$$

5.3 Equilibrium D: Chao and Peck

We now show how Chao and Peck's model can be expanded by introducing markets for reliability services. The principle is as follows. Transactions between supply and demand nodes are subject to two types of constraints, namely the thermal and security constraints. Link based TCRs take care of the limitations on injections and withdrawals arising from thermal constraints. It is submitted that one can similarly deal with security constraints through new markets of network services hereafter called Network Reliability Reservations (NNR). The approach is illustrated on the three node example. All TCRs in the rest of this section are link based TCRs.

A unitary transaction between nodes 1 and 3 requires 2/3 TCRs on line 1–3 and 1/3 TCR on both lines 1–2 and 2–3. Similarly a unitary bilateral transaction between node 2 and 3 requires 2/3 TCRs on the line 2–3 and -1/3 TCRs on the line 1–2. Consider now the NNR market. There is a single security constraint in the example. The constraint defines a single synthetic network service (NNR) for which one introduces a market. Each bilateral transaction appears with a unitary coefficient in the security constraint. This implies that the transaction requires a single NNR. To sum up a transaction between 1 and 3 requires 2/3 TCRs on line 1–3, 1/3 TCR on both line 1–2 and 2–3 and 1 NNR. Similarly a unitary transaction between 2 and 3 requires 2/3 TCR on 2–3, -1/3 and 1/3 TCR on 1–2 and 1–3 respectively and 1 NNR. In general as many NNR markets need to be created as the number of security constraints. A bilateral transaction on the energy market requires a certain amount of NNRs associated to a security constraint whenever the injection and withdrawal involved in this transaction appear in the security constraint. The required amount of NNRs is specified by the injection and withdrawal coefficient in the security constraint. It is negative when the coefficient of this transaction in the security constraint is negative.

The SO is obviously involved in the organization of the NNR markets. Specifically it needs to specify how many NNRs of different types are available and how many of them are necessary for a given transaction. This is quite similar to the informational role initially assumed in Chao and Peck (1996) for dealing with TCRs. In fact, most of this information is a by-product of the reliability analysis that the SO must conduct anyway. There is a difference between TCR and NNR though. While it is natural to assign property rights on TCRs on the basis of line ownership, this seems much more difficult with NNRs. One way to organize the system is to introduce an auction for each

NNR market. An other organization is to suppose an initial endowment of NNR and to let the market develop on a bilateral basis. Last NNRs may also be sold by the SO for a profit. We come back to these different possibilities in the next section. The discussion of the use of the proceeds accruing from the NNR markets and their possible insertion in Chao-Peck and Wilson's insurance process is beyond the scope of this paper.

To sum up, NRRs are directly linked to the security constraints and hence are distinct from TCRs. They are ancillary services that complement TCRs and deal with contingencies. They can also be seen as resulting from the unbundling of firm transmission services into TCRs and other reliability related services. A trader needs thus procure both TCRs and NRRs in order to perform its energy operations.

One can now introduce the definition of an equilibrium of this expanded market.

Let $q^*_{13}(= g^*_1)$ and $q^*_{23}(= g^*_2)$ be bilateral transactions concluded on the energy market between generators 1 and 2 respectively and the consumer located at node 3. Let p^* be a system of nodal prices, η^* the price of a TCR on line 1–2 in the direction 1–2 and λ^* the price of an NNR on the NNR market. By definition (q^*_{13}, q^*_{23}) and (p^*, η^*, λ^*) constitute an equilibrium of the power system iff

(i) q^*_{13} and q^*_{23} respectively maximize the profit of generators 1 and 2 after these latter pay for TCRs and NNRs. This is, q^*_{13} maximizes

$$\max \left(p^*_3 - \frac{\eta^*}{3} - \nu^*\right) q_{13} - vc_1(q_{13}), \quad \text{s.t. } q_{13} \geq 0$$

and q^*_{23} maximizes

$$\max \left(p^*_3 + \frac{\eta^*}{3} - \nu^*\right) q_{23} - vc_2(q_{23}), \quad \text{s.t. } q_{23} \geq 0.$$

(Recall that generator 2 consumes $-\frac{1}{3}$ TCR on line 1–2.)

(ii) w^*_3 maximizes the surplus of the consumer, that is w^*_3 is a solution of $\max u_3(w_3) - p^*_3 w_3$, $w_3 \geq 0$ (see relation 3.11).

(iii) The owner of line 1-2 maximizes the revenue accruing from its selling TCRs on this line, that is

$$f^*_{12} = \frac{q^*_{13} - q^*_{23}}{3} \text{ solves } \max \eta^* f_{12} \text{ s.t. } f_{12} \leq uf_{12}.$$

(iv) $q_{13}^* + q_{23}^*$ maximizes the value of available NNRs at price ν^*. That is q_{13}^*, q_{23}^* solve

$$\max \nu^*(q_{13} + q_{23})$$

$$\begin{aligned} \text{s.t.} \quad & q_{13} + q_{23} \leq ug \\ & q_{13}, q_{23} \geq 0. \end{aligned}$$

At this stage we shall suppose that condition (iv) is made achievable by assuming an owner of all NNR that sells them to generators. Consider again an economy composed of generators, consumers, the SO and the newly introduced owner of NNR. We can then state the following equivalence theorem.

Theorem 2 *Consider a power system with security constraints. If there exists an optimal dispatch, then there exists at least an equilibrium of the type A, B, C and D. Moreover there is a one to one correspondence between equilibria of the type A, B, C and D. In other words any equilibrium of type A, B, C and D can be rewritten as an equilibrium of an other type. Any of these equilibria is also an equilibrium in the sense of Wu et al, but the converse is not true. Equilibria A, B, C and D are equivalent to the solution of the associated Optimal Power Flow model. This equilibrium is also a competitive equilibrium in the above defined economy.*

Proof. See appendix.

6 Competitive industry and generalized equilibria

Theorem 2 shows that the equilibria considered by Wu et al (type E) are not identical to the others (types A, B, C and D). Specifically they are more of the former than of the latter. This discrepancy has been recognized by Wu et al and is further analysed in this section. Harvey et al (1997) attribute the multiplicity of Wu et al's equilibria to the absence of behavioral assumption on the SO. The remark is certainly to the point as Wu et al purposely avoid making assumptions on the SO. As they claim, this would give it an economic role incompatible with its market power. In order words Wu et al do not want to deal with restructured electricity systems where the SO is in a position to manipulate TCR prices. As argued in this section, the missing assumption on the SO in Wu et al equilibria can be substituted by alternative hypotheses on the organization of the transmission system that bypass the market power of the SO. Variations with respect these alternative assumptions in turn lead one to consider imperfect or regulated equilibria that depart from the competitive or OPF like equilibria examined in the previous section.

In order to proceed with this analysis we first introduce a more general description of restructured electricity systems that allows for different organizations of these latter. As in the rest of this paper we suppose that there is no market power. We first specify the goods and services traded in the electricity system and discuss the roles of the agents in a second stage. The following description is inspired by the unbundling between energy and services that characterised the restructuring in California. Needless to say unbundling in California went much beyond the sole separation of energy and transmission services used in this paper.

6.1 Goods and services

There are two markets namely energy and transmission services. The union of the two markets will be referred to as the global market. Each of these markets can be described as follows.

6.1.1 The energy market

Energy is traded in the energy market separately from transmission services.

6.1.2 The transmission services market

Two alternative organizations of the market may be considered. In the first one, the market for transmission services consists of nodal TCRs. The second organization is constructed around the Chao and Peck paradigm with the above reliability add up. This means that transmission services are unbundled into link based TCRs and NNRs. Different arrangements can in turn be conceived in each of these two cases depending on how property rights for these services are assigned to the different agents operating on the global market. These will be discussed after presenting the agents and their role.

6.2 The agents

Various agents operate in restructured electricity systems. In order to fix ideas we suppose that generators, consumers and power marketers are active in the energy market. Power marketers also operate in the transmission services market together with those agents which initially own TCRs and NNRs or organise these markets. Various roles of different economic importance can be envisaged for the SO. The following elaborates on these points.

6.2.1 Generators

They own and operate power plants at different nodes of the grid. They sell their output to power marketers. Because we assume there is no market power, these sales take place at marginal generation cost. In order to fix ideas we suppose that generators do not intervene on the transmission services market. Alternative assumptions can easily be introduced within the same conceptual and computational framework. Their discussion would unduly increase the length of the paper.

6.2.2 Consumers

They buy energy from power marketers. The quantities bought and the prices paid to the power marketers are related by their demand system. Again we suppose that consumers do not intervene on the market of transmission services.

6.2.3 The System Operator

One can think of many different roles for the SO (Oren (1998)). At minimum it should provide network information. Specifically, the SO should verify that the consolidated requested transmission services, whether nodal or link based TCRs and NNRs are feasible for the network, that is to say whether they satisfy the thermal limits and reliability constraints. A more active role is the one where the SO auctions TCRs and NNRs. It then seems natural to assume that the auction is designed to maximize the value of these transmission services. Last the SO can be made the sole seller of transmission services. It is then supposed that its aim is to maximize its profit. This implies that it must be sufficiently regulated to be prevented from manipulating the prices of these transmission services.

6.2.4 Power Marketers

They trade electricity between generators and consumers. We suppose that power marketers are the sole users of transmission services but alternative assumptions could be made. Their procurement of these services differs depending on the organization of their market. Specifically power marketers may buy link based TCRs from line owners. They may bilaterally trade nodal TCRs and NNRs between themselves and with those initially holding the property rights on these services. Last, they may buy these services through an auction or from the SO if this is the assumed organization of the market.

6.2.5 Owners of TCRs and NNRs

They sell these services to maximize their profits.

Quite many different organizations of the market can be conceived using these notions. We proceed from structures with minimal to maximal institutions.

6.3 Case 1: Minimal Institution - no market of transmission services

Consider first the case where the SO is deprived from any economic role. In this organization the SO is only required to indicate whether the consolidated transmission services requested by power marketers are feasible for the network, that is whether they satisfy the security constraints and the thermal limits on the lines. This is the idea underlying Wu et al's equilibrium (see condition (iii) in section 4.4). This minimal institutional organization can be applied whether transmission services are nodal or link based. It is then up to the traders to negotiate transmission services between them until they reach an equilibrium on the energy market that is feasible for the network. This organization may be quite close to the system of negotiated access to the network allowed in the EU directive.

The analysis of the equilibrium of this system is conducted in terms of link based TCRs and NNRs. The discussion can easily be adapted to nodal TCRs in bilateral markets.

Let (g_1, g_2, w_3) be the vector of energy generated and consumed and (q_{13}, q_{23}) be the vector of energy traded on the energy market. $f_{12} = \frac{q_{13}-q_{23}}{3}$, $r_1 = q_{13}$ and $r_2 = q_{23}$ are then the network services (TCRs and NNRs) requested to complete the energy transactions. Let (p_1^*, p_2^*, p_3^*) be the vector of nodal prices of energy. Because there is no market for TCRs nor NNRs, there is no price for these services.

An equilibrium $(g_1^*, g_2^*, w_3^*), (q_{13}^*, q_{23}^*), f_{12}^*, (r_1^*, r_2^*), (p_1^*, p_2^*, p_3^*)$ satisfies the following properties

(i) g_i^* maximizes the profit of generator i, that is g_i^* is a solution of $\max p_i^* g_i - vc_i(g_i); g_i \geq 0;\ i = 1, 2$ (see relation 3.11).

(ii) w_3^* maximizes the surplus of the consumer, that is w_3^* is a solution of $\max u_3(w_3) - p_3^* w_3,\ w_3 \geq 0$ (see relation 3.11).

(iii) Power marketers maximize their profit by buying and selling electricity at the given nodal prices. Power marketers trade within the residual possibilities of the network, that is those left by the other power marketers. These are determined on the basis of the amount of TCRs and NNRs already used by the other power marketers at equilibrium. The following states this maximization problem. The trade q_{13}^* of power marketer 1 maximizes its profit, given the residual capacities of the network. That is, it is a solution to the following optimization problem.

$$\begin{array}{lll} \max & (p_3^* - p_1^*) q_{13} & \\ & & \\ \text{s.t.} & (q_{13} - q_{23}^*)/3 \leq u f_{12}, & \eta^1 \\ & q_{13} + q_{23}^* \leq ug, & \nu^1 \\ & q_{13} \geq 0 & \end{array}$$

In this expression $q_{23}^*/3$ and r_2^* are respectively the amounts of TCRs and NNRs used by the other power marketer at the equilibrium. They specify the residual network capacities available to the first power marketer. The dual variables are written at the right of each constraint. They are upper indexed with a 1 to indicate that they refer to the first power marketer. The behavior of the second power marketer is described by a similar maximization problem. We let η^2 and ν^2 be the dual variables associated to the constraints of the second power marketer's problem.

(iv) Energy quantity and transmission services balance. That is

$$q_{13}^* = g_1^*, q_{23}^* = g_2^*, q_{13}^* + q_{23}^* = w_3^*, q_{13}^* = r_1^*, q_{23}^* = r_2^*, f_{12}^* = \frac{q_{13}^* - q_{23}^*}{3}.$$

This equilibrium requires more than Wu et al's. Specifically it assumes that the power marketer maximizes its profit within the residual possibilities of the network. The network is then a common infrastructure which is shared by all power marketers, but without any economic mechanism specified for the allocation of its services. This additional behavioral assumption on the use of the network is not due to our introduction of power marketers though. A similar behavior could have been assumed in the bilateral market. In fact, the assumption is rooted in the decentralization of market services combined with the absence of a market for these services. In contrast, the same behavioral assumption on the use of the network would have been impossible in the mandatory pool type of system assumed by Wu et al as generators only see nodal prices without any consideration of network services. It is easy to see that this new equilibrium is a social equilibrium in the sense of Debreu (1952). Indeed consider the economy where the generators, the consumers and the power marketers are the agents (the SO is not a trading agent in this economy). The balance constraints (iv) and the network constraints on the power marketers define a feasibility set for each agent, that is the set of feasible actions of each agent given the actions of the others. It is easy to see from the definition that an equilibrium point maximizes the profit or surplus of each agent within its feasibility set at this equilibrium point. This is the definition of Debreu's social equilibrium. This characterization is stated more formally in Theorem 3.

Theorem 3 *$(g_1^*, g_2^*, w_3^*), (q_{13}^*, q_{23}^*)$ and (p_1^*, p_2^*, p_3^*) is an equilibrium in the above sense iff (g_1^*, g_2^*, w_3^*) and (p_1^*, p_2^*, p_3^*) is a social equilibrium in the sense of Debreu (1952). A social equilibrium is an equilibrium in the sense of Wu et al but the converse is not true.*

Proof. See appendix.

There may be many social equilibria. This multiplicity arises because the set of feasible actions of one agent depends on the actions of the others. This can be used by public authorities to tilt the market to select one or the other of these equilibria to best fit their policy objectives (Rosen (1965)). This property suggests that social equilibria may in some case be interpreted as regulated equilibria. At this stage of the discussion, in the absence of regulation and referring to restructured electricity systems, multiple equilibria can be expected to arise when access to the network is obtained by a non market procedure such as a negotiated access. Specifically, the set of transmission possibilities accessible to a power marketer depends on the actions of the others. This is a true externality which is not internalized by a price process nor controlled by public action. These equilibria will be referred to in the following as Equilibrium F: Social Equilibrium.

The proof given in appendix gives the mathematical reason of this multiplicity. Each social equilibrium is a solution of a quasi variational problem and conversely. Mathematically, quasi variational inequality problems have multiple solutions. Specifically the usual strict monotonicity assumptions that guarantee solution uniqueness in variational inequality problems (Harker and Pang (1990)) are essentially useless in quasi variational inequalities (Harker (1991)).

6.4 Case 2: Bilateral markets of transmission services

Social equilibria appear on the energy market when there is no market for transmission services or when the rules of this market are not fully specified. This statement is not sufficient to characterize a relation between social equilibria and the competitive equilibria A, B, C and D. Recall that Wu et al showed that the mandatory pool equilibrium B (and hence also the other equilibria of type A, C and D) is an equilibrium of type F that satisfies an additional condition. Specifically they derive this condition from the OPF problem. We follow a similar objective here and seek additional conditions that relate social equilibria to A, B, C and D equilibria and to Wu's equilibria. As in Wu et al, an OPF like condition can also be introduced to restrict social equilibria to be a competitive equilibrium. We follow an alternative path and provide a condition in terms of bilateral markets for transmission services. This condition is based on an interpretation of the dual variables η and ν appearing in the definition of the Power Marketer's problem ((iii) in section 5.3) in terms of opportunity cost of TRCs and NNRs. In order to explore this interpretation suppose that the first power marketer buys a positive quantity at node 1 and sells at node 3, that is $q^*_{13} > 0$. Then the optimality condition for this trader states that $(p^*_3 - p^*_1) - \frac{\eta^*_1}{3} - \nu^*_1 = 0$. This means that the price paid for a TRCs is equal to the marginal profit made on the energy market after paying for NNRs. Similarly the price paid by the first power marketer for a NNR is equal to its marginal profit, given the price of energy and TCRs. This leads one to introduce the following definition.

Definition. *The optimal dual variables $\eta^{i,*}, \nu^{i,*}$ of power marketers' problem i are respectively the opportunity cost of a TCR and a NNR when the trade of this power marketer is optimal with respect to the residual network possibilities, that is when the power marketer behaves according to condition (iii) of section 6.3.*

It is now possible to give an interpretation of the competitive equilibria A, B, C and D in terms of social equilibria.

Theorem 4 *Equilibria of type A, B, C and D are social equilibria where*

$$\eta^{1,*} = \eta^{2,*}, \nu^{1,*} = \nu^{2,*}.$$

Proof. See appendix.

This condition imposed on opportunity costs can be interpreted as expressing equilibrium on the bilateral transmission services markets. Theorem 4 and the definition of the opportunity costs accordingly suggest the following definition of an equilibrium on the transmission market.

Definition. *The TCR market is in equilibrium if the opportunity cost of an additional link based TCR on some line is the same for all power marketers. Similarly a NNR market is in equilibrium if the opportunity cost of an additional NNR on this market is the same for all power marketers. The link based transmission market is in equilibrium if all link TCR and all NNRs markets are in equilibrium.*

Theorem 4 can then be restated in equivalent form as follows.

Theorem 5 *An equilibrium of type A, B, C or D is a social equilibrium such that there is also equilibrium on the transmission market.*

In contrast with the OPF condition imposed by Wu et al to single out competitive equilibria, the preceding condition can be entirely interpreted in terms of bilateral transmission services markets. Indeed one can assume that power marketers trade TCRs or NNRs (with corresponding trading activity on the energy market) as long as the opportunity costs of additional TCRs and/or NNR are different. This interpretation only requires a minimal informational role of the SO, namely that it indicates whether a set of transmission services is feasible. Needless to say, this requires that TCR and NNR markets be reasonably efficient. This bilateral trade of transmission services between power marketers can develop after an initial assignment of property rights on these services has been made. No further supervision of the SO is necessary as long as this assignment is compatible with the capacity of the network.

6.5 Case 3: Kicking off the bilateral trade with an auction

Efficient bilateral energy and transmission services markets single out a particular social equilibrium. This latter is also an equilibrium in the sense of Schweppe, Hogan and Chao and Peck. One can obviously think of many reasons why bilateral transmission markets may not be efficient. Specifically, efficiency may be affected by the small number of agents operating on the market. This argument is sometimes heard against nodal TCRs because of the large number of nodal markets that this system would imply and hence the possibly small number of participants in each of these markets. Chao an Peck's organization is claimed to be more advantageous from that point of view as all traders indeed negotiate with the owners of link based TCRs. A similar argument of efficiency can be invoked for NNR markets as many traders will need to request NNRs on each NNR market. In any case market efficiency is not guaranteed which implies that power marketer's opportunity costs may eventually not equalise. This would leave the end result undefined and may require some ad hoc procedure to make the consolidated set of network services feasible.

Analogous arguments of inefficient bilateral markets have been heard before. Specifically the reasoning was put forward when emission allowances were introduced by Title IV of the Clean Air Act Amendments. The introduction of auction of emission permits had accordingly been advocated at the time to mitigate the consequences of inefficient bilateral markets (Hausker (1992)). It appears that auctions may have indeed helped the development of efficient bilateral markets (Schmalensee et al (1998), see also Joskow et al (1998) for an a posteriori analysis of the argument of efficiency). The discussion of auctions goes much beyond the scope of this paper. It suffices to say for our purpose in this paper that a common objective of auctions is to maximize the value of the goods of services that are auctioned. The following proposition suggests that auctions might substitute efficient bilateral markets in the definition of the equilibrium.

Theorem 6 *An equilibrium of type A, B, C or D is a social equilibrium such that the value of all link based TCRs and NNRs is maximized at the prevailing prices.*

Proof. See appendix.

6.6 Case 4: Making the SO the sole seller of transmission services

It is easy to see that Theorem 6 can be rephrased in terms of an organization of the market where the SO is also the sole seller of link based TCRs and NNRs and maximizes the profit accruing from these sales.

6.7 Case 5: Towards imperfect markets: Equilibrium G

Relying on perfect transmission markets whether through bilateral transaction, auctions or a well regulated SO maybe overly optimistic. Differences between opportunity costs of TCRs and NNRs among the different power marketers is what characterises social equilibria that are not OPF like or competitive equilibria. This suggests structuring these alternative equilibria in terms of what drives differences between opportunity costs of transmission services; transaction costs could be a case in point. One can go beyond this first step and consider equilibria that, in contrast with condition (iii) in section 6.3, do not induce power marketers to make the most extensive use of the transmission system. This will happen in certain regulation of transmission services. For example, the prices of TCRs and NNRs can be chosen to satisfy some revenue requirement of the SO. This may lead to high access prices that would leave some of the transmission capacity idle. In order to explore this possibility, consider the equilibria obtained by replacing (iii) and (iv) in equilibria F by the following

(iii) Power marketers maximize their profit by buying and selling electricity at the given nodal prices and paying regulated transmission prices. The following states the resulting maximization problem.
The trade q^*_{13} of power marketer 1 maximizes its profit, given the prices of TCRs and NNRs. That is, it is a solution to the following optimization problem.

$$\begin{aligned} \max \quad & (p^*_3 - \tfrac{\eta^1}{3} - \nu^1 - p^*_1) q_{13} \\ \text{s.t.} \quad & q_{13} \geq 0. \end{aligned}$$

In this expression η^1 and ν^1 are respectively the TCR and NNR prices charged to power marketer 1 as a result of the regulation of these services. The behavior of the second power marketer is described by a similar maximization problem, η^2 and ν^2 being the TCR and NNR prices charged to this marketer.

(iv) Energy quantity and transmission services balance. That is

$$q^*_{13} = g^*_1, q^*_{23} = g^*_2, q^*_{13} + q^*_{23} = w^*_3, q^*_{13} = r^*_1, q^*_{23} = r^*_2, f^*_{12} = \frac{q^*_{13} - q^*_{23}}{3}.$$

Because η^i and ν^i are regulated TCR and NNR prices in this power marketer's problem, they may be different from zero even if available TCRs and NNRs are not used up. Specifically this phenomenon may occur when fully distributed cost allocation rules are used for pricing transmission services. Still generation and consumption in this new equilibria remain based on nodal prices, and hence satisfy the condition of Wu et al's equilibria. This is expressed in the following theorem.

Theorem 7 *An equilibrium of type G is an equilibrium in the sense of Wu et al. An equilibrium in the sense of Wu is an equilibrium of type G with regulated nodal transmission prices and conversely. An equilibrium of type G is a social equilibrium if the price of not fully used transmission services is zero.*

Proof. See appendix.

The difference between G and social equilibrium thus lies in the fact that the price of transmission services in the former may be regulated. A particular case of this regulation namely regulated nodal transmission prices lead to Wu et al's equilibria. Wu et al and G equilibria are equivalent when the regulated nodal prices can be unbundled into regulated prices of the unbundled transmission services. G equilibria do not insure the full use of the transmission capacities. G equilibria and hence also Wu et al's equilibria are thus quite general as they also include regulated transmission systems. They are treated in a follow up paper together with market power in the energy market.

7 Conclusion

This paper focuses on the wholesale energy market and on markets for infrastructure services and reliability requirements. As in most of the electricity restructuring literature, the paper concentrates on short term efficiency and leaves aside important issues related to time variability and stochasticity of demand and generation. More specifically, its aim is to review models of the restructured electricity industry in terms of the degree of decentralization that they allow. Different restructuring paradigms are thus analysed going from what may be considered as relatively centralized models to what we see at this stage as the most decentralized organization namely bilateral markets on both energy, transmission and ancillary services.

It is shown that models involving quite different levels of decentralization lead to the same equilibrium. These models certainly reflect different organizations of the industry but are equivalent when markets are perfect. Specifically the models proposed by Schweppe at al, Hogan and Chao and Peck arrive at the same equilibrium which is a true competitive equilibrium. Some of the shortcomings claimed against Chao and Peck's treatment of reliability and the resulting discrepancy that may have arisen between this equilibrium and others can be overcome by introducing markets for new unbundled reliability services. Interestingly it is possible to interpret these equilibria as resulting from fully bilateral markets in energy and transmission and reliability services. The relevant practical question is thus the extent to which market imperfections will jam this equivalence. In short market imperfections may differentiate organizational structures that are otherwise equivalent. Wu et al's equilibrium provides a concept for looking at this issue. It is indeed different from the other equilibrium to the extent that it includes the competitive equilibrium but is much broader. It is shown that it is even broader than social equilibria which themselves encompass the competitive equilibrium. In fact Wu et al's equilibria include competitive equilibria, social equilibria and some regulated equilibria. This may prove to be of practical interest. Social equilibrium different from competitive equilibria can indeed be interpreted in terms of market imperfection on the transmission and ancillary services markets. Regulated equilibrium can also be seen as reflecting the situation where markets of transmission and ancillary services have been replaced by a regulation of some sort. Wu et al's equilibrium and their slight generalization introduced here therefore provides a framework where departures from the competitive equilibrium due to market imperfection and regulation on the transmission and ancillary service market can be explored.

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Appendix

Proof of Proposition 1.
Take $v_1 = v_2 = v_3 = 0$ and write the dual of OPF'(3,3) as

$$\max_{p_1,p_2,p_3} \min_{\substack{g_1,g_2,w_3\\ q_{13},q_{23}\geq 0}} \{vc_1(g_1) + vc_2(g_2) - u_3(w_3) - p_1(g_1 - q_{13})$$
$$-p_2(g_2 - q_{23}) + p_3(w_3 - q_{13} - q_{23}) \text{ s.t. } (q_{13} - q_{23})/3 \leq uf_{12}\}$$

or by grouping terms

$$\max_{p_1,p_2,p_3} \min_{\substack{g_1,g_2,w_3\\ q_{13},q_{23}\geq 0}} \{[vc_1(g_1) - p_1g_1] + [vc_2(g_2) - p_2g_2]+$$
$$[p_3w_3 - u_3(w_3)] + p_1q_{13} + p_2q_{23} - p_3(q_{13} + q_{23}) \text{ s.t. } (q_{13} - q_{23}/3 \leq uf_{12}\}$$

For the optimal p_1^*, p_2^*, p_3^* of this dual problem the optimal dispatch maximizes

$$p_1^*g_1 - vc_1(g_1) \text{ s.t. } g_1 \geq 0$$
$$p_2^*g_2 - vc_2(g_2) \text{ s.t. } g_2 \geq 0$$
$$u_3(w_3) - p_3^*w_3 \text{ s.t. } w_3 \geq 0$$

Proof of Proposition 2.
The proof follows immediately from the fact that $p_i = \frac{\partial W}{\partial v_i}$ when the dual variables are unique.

Proof of Proposition 3.
The maximization problem 3.12 appears in the dual stated in the proof of Proposition 1. This proves the first statement of the proposition. To see that the objective function value of problem 3.12 is positive, consider the dual of this problem. Its objective function is equal to $(uf_{12})\eta$ where η is the nonnegative dual variable of the upper limit on the flow on line 1–2. By construction this expression is nonnegative.

Proof of Proposition 5.
The proof is similar to the one of Proposition 3. It is based on a slightly different formulation of the primal and dual problems. Specifically, consider the primal problem

$$\min vc_1(g_1) + vc_2(g_2) - u_3(w_3)$$

$$
\begin{array}{ll}
g_1 = q_{13} & p_1 \\
g_2 = q_{23} & p_2 \\
w_3 = q_{13} + q_{23} & p_3 \\
f_{12} = \frac{q_{13} - q_{23}}{3} & \xi \\
f_{12} \leq uf_{12} & \eta
\end{array}
$$

$$
g_1 \geq 0, g_2 \geq 0, q_{13} \geq 0, q_{23} \geq 0, w_3 \geq 0.
$$

The dual is written as

$$
\begin{aligned}
\max_{p_1,p_2,p_3,\xi,\eta} \min\{&[vc_1(g_1) - p_1 g_1] + [vc_2(g_2) - p_2 q_2] + [p_3 w_3 - u_3(w_3)] \\
&+(p_1 - p_3 - \tfrac{\xi}{3})q_{13} + (p_2 - p_3 + \tfrac{\xi}{3})q_{23} - \eta f_{12} \text{ s.t. } f_{12} \leq uf_{12}\}
\end{aligned}
$$

Again the optimization problem (3.14) directly appears in the expression of the dual.

Proof of Theorem 1

As can be seen from the proofs of Propositions 1, 3 and 4, the conditions describing equilibria A,B,C and D are all obtained from dual of equivalent reformulations of the OPF problem. All equilibria are thus equivalent to the solution of the OPF. To see that they are also competitive equilibria consider the definition of the mandatory pool and verify that it satisfies the usual condition of a competitive equilibrium in an economy where generators and the SO maximize their profits and the consumer maximizes its surplus.

Proof of Theorem 2

The proof is analogous to the one of Theorem 1: Equilibria are equivalent because the conditions that define them are obtained from duals of equivalent reformulations of an OPF problem. The only difference is that the OPF problem includes security constraints. As to the proof of the competitive equilibrium it suffices to note that the owner of NNRs must maximize the value of these NNRs in a competitive equilibrium.

Proof of Theorem 3

The proof that the above equilibrium is a social equilibrium in the sense of Debreu follows from the definition of this latter. In order to explore the relation between these equilibria and Wu et al's consider the following Quasi Variational Inequality problem (QVI) that will also be used in the proof of Theorem 6.

Problem QVI.
Find $(g_1^*, g_2^*, w_3^*), (q_{13}^*, q_{23}^*)$ satisfying

$$\begin{array}{l} w_3^* = g_1^* + g_2^* \\ g_1^* = q_{13}^* \\ g_2^* = q_{23}^* \\ \frac{q_{13}^* - q_{23}^*}{3} \le uf_{12} \\ q_{13}^* + q_{23}^* \le ug \\ g_1^*, g_2^* \ge 0 \end{array}$$

such that

$$mc_1(g_1^*)(g_1 - g_1^*) + mc_2(g_2^*)(g_2 - g_2^*) - mv_3(w_3^*)(w_3 - w_3^*) \ge 0$$

for all $(g_1, g_2, w_3, q_{13}, q_{23})$ satisfying

$$\left.\begin{array}{l} g_1 = q_{13} \\ g_2 = q_{23} \\ w_3 = q_{13} + q_{23} \\ \frac{q_{13} - q_{23}}{3} \le uf_{12} \\ q_{13} + q_{23} \le ug \\ g_1 \ge 0, q_2 \ge 0, w_3 \ge 0, q_{13} \ge 0, q_{23} \ge 0 \end{array}\right\} \text{(set } K\text{)}$$

$$\begin{array}{ll} \eta^1 & \frac{q_{13} - q_{23}^*}{3} \le uf_{12} \\ \nu^1 & q_{13} + q_{23}^* \le ug \end{array}\Bigg\} \text{(set } (K_1(q_{23}^*))$$

$$\begin{array}{ll} \eta^2 & \frac{q_{13}^* - q_{23}}{3} \le uf_{12} \\ \nu^2 & q_{13}^* + q_{23} \le ug \end{array}\Bigg\} \text{(set } K_2(q_{13}^*))$$

where the dual variables of the constraints defining the sets $K_1(q_{23})$ and $K_2(q_{13})$ are written on the left of the corresponding inequalities.

Applying Theorem 4 of Harker (1991), one can state that $(g_1^*, g_2^*, w_3^*, q_{13}^*, q_{23}^*)$ is a solution of QVI iff there exists $\eta^{1*} \ge 0, \eta^{2*} \ge 0, \nu^{1*} \ge 0, \nu^{2*} \ge 0$ satisfying

$$\left.\begin{array}{l} \eta^{1*}\left(uf_{12} - \frac{q_{13}^* - q_{23}^*}{3}\right) = 0 \\ \eta^{2*}\left(uf_{12} - \frac{q_{13}^* - q_{23}^*}{3}\right) = 0 \\ \nu^{1*}[ug - (q_{13}^* + q_{23}^*)] = 0 \\ \nu^{2*}[ug - (q_{13}^* + q_{23}^*)] = 0 \end{array}\right\} \quad (CS)$$

and such that $(g_1^*, g_2^*, w_3^*, q_{13}^*, q_{23}^*) \in K$ is a solution of the following Variational Inequality problem (VI)

$$\begin{array}{r}mc_1(g_1^*)(g_1 - g_1^*) + mc_2(g_2^*)(g_2 - g_2^*) - mu_3(w_3^*)(w_3 - w_3^*) \\ + \left(\frac{\eta^{1*}}{3} + \nu^{1*}\right)(q_{13} - q_{13}^*) + \left(-\frac{\eta^{2*}}{2} + \nu^{*2}\right)(q_{23} - q_{23}^*) \geq 0\end{array}$$

for all $g_1, g_2, w_3, q_{13}, q_{23}$ in K.
Using a standard argument, $(g_1^*, g_2^*, w_3^*, q_{13}^*, q_{23}^*)$ is a solution of VI iff there exists p_1^*, p_2^*, p_3^* such that $p_1^*(g_1^* - q_{13}^*) = 0$, $p_2^*(g_2^* - q_{23}^*) = 0$, $p_3^*(w_3^* - q_{13}^* - q_{23}^*) = 0$ and $(g_1^*, g_2^*, w_3^*, q_{13}^*, q_{23}^*)$ is also a solution of the following variational inequality problem VI'.

$$\begin{array}{ll}[mc_1(g_1^*) - p_1^*](g_1 - g_1^*) & +[mc_2(g_2^*) - p_2^*](g_2 - q_2^*) \\ & +[p_3^* - mu_3(w_3^*)](w_3 - w_3^*) \\ & +(p_1^* + \frac{\eta^{1*}}{3} + \nu^{1*} - p_3^*)(q_{13} - q_{13}^*) \\ & +(p_2^* - \frac{\eta^{2*}}{3} + \nu^{2*} - p_3^*)(q_{23} - q_{23}^*) \geq 0\end{array}$$

for all $g_1, g_2, w_3, q_{13}, q_{23}$ non negative such that

$$\begin{array}{rcl}\frac{q_{13} - q_{23}}{3} & \leq & uf_{12} \\ q_{13} + q_{23} & \leq & ug.\end{array}$$

Using the complementarity condition (CS) on $\eta^{1*}; \eta^{2*}, \nu^{1*}, \nu^{2*}$ and Theorem 4 of Harker (1991) VI' is equivalent to the following set of conditions

(a) $[mc_1(g_1^*) - p_1^*](g_1 - g_1^*) \geq 0,$ for all $g_1 \geq 0$ and
$[mc_2(g_2^*) - p_2^*](g_2 - g_2^*) \geq 0,$ for all $g_2 \geq 0$

(b) $[p_3^* - mu_3(w_3^*)](w_3 - w_3^*) \geq 0,$ for all $w_3 \geq 0$

(c) $(p_1^* - p_3^*)(q_{13} - q_{13}^*) \geq 0,$ for all q_{13} such that
$\frac{q_{13} - q_{23}^*}{3} \leq uf_{12}$
$q_{13} + q_{23}^* \leq ug$
$q_{13} \geq 0$
and
$(p_2^* - p_3^*)(q_{23} - q_{23}^*) \geq 0,$ for all q_{23} such that
$\frac{q_{13}^* - q_{23}}{3} \leq uf_{12}$
$q_{13}^* + q_{23} \leq ug$
$q_{23} \geq 0$

(d) $w_3^* = q_{13}^* + q_{23}^*, g_1^* = q_{13}^*, g_2^* = q_{23}^*$

It is easy to see that (a), (b), (c) and (d) are respectively equivalent to (i), (ii), (iii) and (iv) in the definition of the equilibrium of section 6.3. It is also easy to see that (g_1^*, g_2^*, w_3^*) satisfies the condition (a), (b) and (d) and hence is an equilibrium in the sense of Wu et al. The converse is not true because condition (c) need not be satisfied by Wu et al's equilibria.

Proof of Theorem 4

Apply Theorem 6 of Harker (1991) to the solution of QVI.

Proof of Theorem 6

The proposition follows from Theorem 2. In equilibrium D the value of TCRs and NNRs is maximized at the prevailing prices.

Proof of Theorem 7

The proof that an equilibrium of type G is an equilibrium in the sense of Wu et al results from the consideration in the text. Consider now an equilibrium g_1^*, g_2^*, w_3^* in the sense of Wu et al and define regulated nodal TCR prices as follows

$$\xi_i^* = p_3^* - p_i^*, \qquad i = 1, 2.$$

The power marketers problem in this system can be stated as

$$\begin{array}{l} \max(p_3^* - \xi_i^* - p_i^*)q_{i3} \\ \text{s.t. } q_{i3} \geq 0 \end{array}$$

q_{i3}^* trivially solves this problem. This completes the proof of the second statement. The proof of the third statement is directly obtained by noting that the complementary slackness conditions CS in problem QVI (proof of Theorem 3) are satisfied under the assumption of the theorem. This implies that the equilibrium of type G is a solution to QVI and hence a social equilibrium.