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DISCUSSION PAPER

Center for Operations Research
and Econometrics

Voie du Roman Pays, 34
B-1348 Louvain-la-Neuve
Belgium
http://www.uclouvain.be/core

CORE
Voie du Roman Pays 34, L1.03.01
B-1348 Louvain-la-Neuve, Belgium.
Tel (32 10) 47 43 04
Fax (32 10) 47 43 01
E-mail: immaq-library@uclouvain.be
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Multidimensional poverty measurement with individual preferences[1]

Koen Decancq[2], Marc Fleurbaey[3],
and François Maniquet[4]

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Abstract

We propose a new class of multidimensional poverty indices. To aggregate and weight the different dimensions of poverty, we rely on the preferences of the concerned agents rather than on an arbitrary weighting scheme selected by the analyst. The Pareto principle is, therefore, satisfied among the poor. The indices add up individual measures of poverty that are computed as a convex transform of the fraction of the poverty line vector to which the agent is indifferent. The axiomatic characterization of this class is grounded on new principles of interpersonal poverty comparisons and of inequality aversion among the poor. We illustrate our approach with Russian survey data between 1995 and 2005. We find that, compared to standard poverty indices, our preference sensitive indices lead to considerable differences in the identification of the poor and in subgroup poverty comparisons.

JEL Classification: D63, D71.
Keywords: multidimensional poverty measurement, preferences.

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[2] CSB, University of Antwerp, CORE, Université catholique de Louvain, and CES, Katholieke Universiteit Leuven. Email: koen.decancq@uantwerp.be.
[3] Princeton University. Email: mfleurba@princeton.edu.
[4] CORE, Université catholique de Louvain. Email: francois.maniquet@uclouvain.be.

1 Introduction

A growing consensus has emerged that well-being is multidimensional, and that income, even suitably deflated for differences in prices, does not qualify as a good proxy for it.[1] There are at least two reasons. First, private good markets, as well as labor markets, may fail to be competitive, so that individuals suffer from rationing. Second, some relevant goods may not be private and marketable (think of education, security or health).

As a consequence of this growing consensus, poverty is increasingly measured as a multidimensional phenomenon. The common practice consists of defining a threshold in each dimension of well-being, and claiming that an agent is deprived in a dimension if she experiences a lower level than the threshold. Measuring multidimensional poverty then requires a method to aggregate these deprivations across the different dimensions for all poor individuals.

Hence, measuring multidimensional poverty requires two central ethical choices. First, the so-called identification issue concerns the demarcation of the set of poor agents. Some researchers adopt the union definition of poverty in which deprivation in at least one dimension is sufficient to qualify as poor,[2] others follow the intersection definition and require agents to be below the threshold in all dimensions. Intermediate positions can be taken in which deprivation in a limited number of dimensions is sufficient to qualify as poor (see, for instance, Alkire and Foster [1, 2]). Second, ethical choices about the relative importance of the dimensions (i.e. the weight assigned to the deprivation in each dimension) and whether the dimensions are seen as complements or substitutes have to be made before multidimensional poverty can be measured.

Typically, the researcher measuring poverty makes both ethical choices. For obvious reasons, this practice can be criticized for being arbitrary. Ravallion [30] writes: "those with a stake in the outcomes will almost certainly be in a better position to determine what weights to apply than the analyst calibrating a measure of poverty."

Turning to the opinions of the poor themselves, a large-scale participatory consultation by the World Bank at the end of the 1990s has indeed endorsed the view that poverty is a multidimensional phenomenon, while at the same time documenting the diversity of views held by the individuals involved (see Narayan et al. [27]). The question now arises whether poverty can effectively be measured as a multidimensional phenomenon, without

[1] See Kolm [25], Atkinson and Bourguignon [5], Sen [33], Maasoumi [26], and Ravallion [29], among many others.

[2] See, for instance, Tsui [35], Atkinson [4], Bourguignon and Chakravarty [8], Bossert, Chakravarty and D'Ambrosio [7], and Bosmans, Ooghe, and Lauwers [6].

becoming overly arbitrary on the embedded ethical choices.

In this paper, we therefore propose to use the individuals' own preferences to identify the poor and to aggregate across dimensions. That is, we enrich the model by considering that individuals have possibly different preferences over the different poverty dimensions, and we propose a class of measures capturing the idea that these preferences should be respected when measuring poverty.[3]

Taking preferences into account has the immediate consequence of transforming multidimensional poverty measurement into an aggregation problem of one-dimensional individual well-being levels. That is, the ethical choice of how to weight the goods or assessing their complementarity or substitutability is left to the agents themselves. Respecting preferences also changes the outlook of the identification issue. The agents identify themselves whether they are poor, by comparing their multi-attribute situation with the poverty line vector by means of their own preferences. In other words, the relevant threshold becomes a well-being threshold. From the outset we do not assume agreement among the agents about the poverty line vector or the corresponding well-being threshold.

A difficulty with the preference-based approach, on the other hand, is that comparison between agents is lost. For instance, we can no longer assume that two agents consuming the same bundle of goods are equally poor. Likewise, we can no longer assume that an identical increase in consumption has the same impact on two agents if they have different preferences. In this paper, we propose a way to deal with both issues.

Our main theoretical result is a characterization of a class of poverty measures. For this class, there is a single poverty line vector but preferences still play a role in identification, because an agent qualifies as poor if and only if she prefers the poverty line vector to the bundle she is consuming. Moreover, the specific index that evaluates an individual situation is one minus the fraction of the poverty line vector to which the agent is indifferent. Figure 1 illustrates the resulting class of poverty measures.

This particular way of measuring individual poverty is similar to the ray index proposed by Samuelson [31], the distance function studied by Deaton [10], and the notion of egalitarian-equivalence due to Pazner and Schmeidler [28]. This index has been axiomatically analyzed in the literature on fair social orderings (see, e.g., Fleurbaey and Tadenuma [16], and for a synthetic presentation of various results, Fleurbaey and Maniquet [15]). One contribution of this paper is to provide a new justification for it, in the specific context of poverty measurement.

[3]Philosophers have raised the issue of "expensive tastes" to question the idea of relying on individual preferences. This issue is discussed in the next section.

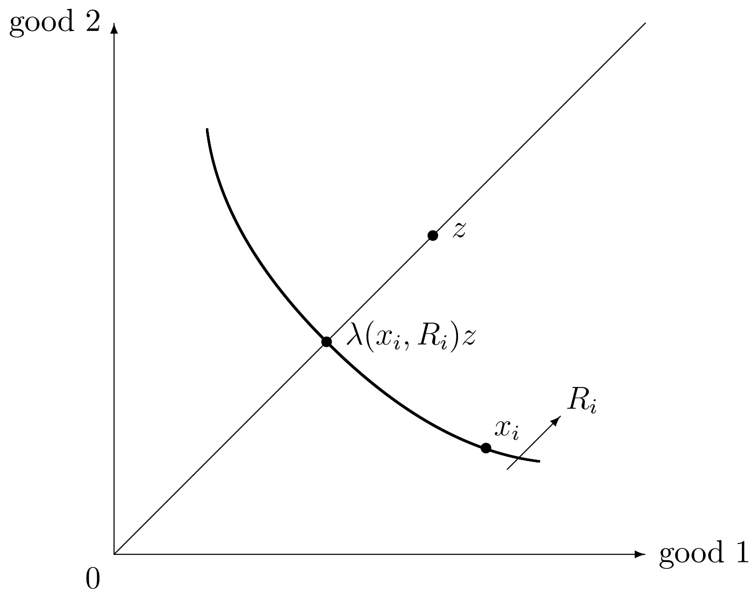

Figure 1: Illustration of the individual poverty index of an agent consuming x_i with preferences R_i: it is a decreasing and convex transform of $1 - \lambda(x_i, R_i)$.

Taking preferences into account in the measurement of poverty raises new empirical problems. In this paper we illustrate how one can tackle these problems. Using an existing Russian survey data set (RLMS-HSE) between 1995 and 2005, we estimate indifference maps based on a happiness regression. This allows us to compute the proposed poverty indices, to assess the evolution of poverty and to compare our results with other unidimensional and multidimensional measures. We find that taking preferences into account leads to important differences in the identification of the poor.

The remainder of the paper is organized as follows. The next section motivates our approach, by showing how preferences can be estimated and by illustrating that different individuals are identified as poor when preferences are taken into account. Four theoretical sections follow. In Section 3, we define the model and derive a representation theorem that extends the classical one-dimensional result of Foster and Shorrocks [18]. In Sections 4 and 5, we introduce the main requirements and characterize a class of poverty measures. Section 6 discusses a generalization towards discrete variables. Section 7 returns to the data and provides an empirical illustration of the proposed class of poverty measures. Section 8 concludes. Econometric details of the empirical application and proofs of the theorems are provided in two appendices.

2 Estimating preferences and identifying the poor

The central idea of our approach is to rely on the preferences of the concerned individuals to identify the poor and to measure poverty. In this section, we illustrate first how preferences can be estimated with a satisfaction regression. Then we show how the identification of the poor changes when these preferences are taken into account. We say that an individual is identified as poor if and only if she prefers a poverty line vector z to her actual multi-attribute situation.

We use data from the Russian Longitudinal Monitoring Survey (RLMS-HSE) between 1995 and 2005. This period was particularly turbulent due to the fast Russian transition towards a market economy and the severe financial crisis of August 1999. To capture some of the monetary and non-monetary effects of these changes, we include four "goods" in our poverty analysis: equivalized household expenditures, health, housing quality and the employment status.[4] The first three dimensions are (approximately) continuous in nature whereas the employment status is a binary variable. For this illustration, the poverty line vector z is set at 60% of the bundle that consists of the pooled median value of equivalized household expenditures, health and housing quality and being unemployed.

We estimate heterogenous preferences on the basis of self-reported life satisfaction information (see Decancq et al. [11], for a similar approach). Life satisfaction is measured in the RLMS-HSE by the following question: *"To what extent are you satisfied with your life in general at the present time?"*, with answers on a five point-scale ranging from "not at all satisfied" to "fully satisfied". The self-reported life satisfaction of individual i in period t is denoted (S_{it}). We start from a standard happiness regression with life satisfaction as the explained variable and a series of usual explanatory variables, including the vector of individual outcomes for the four dimensions of poverty (X_{it}) after a dimension-specific Box-Cox transformation, a time trend (γ_t) and some observable socio-demographic characteristics (Z_{it}) such as education, social status, marital status, average expenditures and employment level in a small geographical reference group, and the presence of wage arrears, which used to be a common phenomenon during the late nineties in Russia. As unobservable personality traits are likely to influence self-reported life-satisfaction, we control for these time-invariant factors by including individual fixed effects (α_i) in the regression. We allow for preference heterogeneity by including interaction effects between the outcome vector and a vector of five dummies (D_{it}) capturing whether the respondents are young (below the age of 33), male, living in a rural area, obtained higher education

[4] More details on the data set and on the estimation of the preferences are given in the Appendix 1.

and have a minority status. This leads to the following model:

$$S_{it}^{*} = \alpha_i + \gamma_t + (\beta + \Lambda D_{it})' X_{it} + \delta' Z_{it} + v_{it},$$

where S_{it}^{*} is a latent satisfaction variable, β and δ are vectors of direct effects and Λ a matrix with interaction effects to be estimated. The idiosyncratic error term v_{it} is assumed to follow a logistic distribution function.

The relevant estimation results are given in Table 1. To reach a parsimonious and tractable model, the least significant interaction effects in Λ have been dropped consequently until all the remaining interaction effects are significant at the 10% level. The coefficients of the remaining interaction terms are presented in the second part of the table. It can be seen that young people give relatively less importance to health and relatively less importance to their employment status, men care relatively more about health and about being employed than women, and so on.

Figure 2 illustrates the resulting indifference maps in the expenditures-health space for two population subgroups: the young, higher educated women (their outcomes are depicted with a diamond) and the old, lower educated men (the triangles in the figure). In line with expectations, we see that the elder individuals are on average in worse health. The illustrated indifference maps show that preferences of the latter group (depicted by the solid curves) are generally steeper, meaning that their willingness to pay for an increase in health is also higher.

Individuals are identified as poor whenever they consider themselves worse off than with the poverty line vector z (depicted by the black dot on Figure 2). Consider, for instance, the young, higher educated women (the diamonds) situated in the south-east of the poverty line vector between both indifference curves through z. These individuals consider themselves to be poor, i.e., worse off than the poverty line vector. However, if these individuals had the steeper (solid) indifference map, then they would not have considered themselves to be poor. This example illustrates how taking preferences into account matters empirically for the identification of the poor.

This is further illustrated by comparing the poor according to different approaches to poverty measurement. In Table 2 each column gives the average characteristics of the poor in 2000 according to one of three identification methods. The first column shows that the 18,6% poor individuals identified according to the preference-sensitive method proposed in this paper are relatively old, predominantly female and in bad health. These are the individuals who give a large relative weight to the dimension on which they score badly. In the second column, these findings are compared to another multidimensional poverty

Table 1: Happiness equation

	life satisfaction	
expenditures transformed (Box-Cox parameter: $0,055$)	0,440***	(0,0351)
health transformed (Box-Cox parameter: $0,485$)	0,557***	(0,124)
house transformed (Box-Cox parameter: $-0,356$)	0,259*	(0,128)
unemployed	-0,172	(0,123)
young × health	-0,315*	(0,159)
young × unemployed	0,190*	(0,0881)
male × health	0,401*	(0,179)
male × unemployed	-0,361***	(0.0882)
rural × house	0,397*	(0,169)
higher educated × expenditures	0,0441**	(0,0146)
higher educated × unemployed	-0,201+	(0,107)
higher educated × house	-0,240*	(0,109)
minority × health	0,655*	(0,263)
minority × expenditures	-0,382***	(0,0849)
N	53873	
pseudo R^2	0,073	

Clustered standard errors in parentheses. Coefficients are obtained after controling for education level, social status, marital status, reference group expenditures, reference group employment level, the presence of wage arrears and year dummies.

+ $p < 0,10$, * $p < 0,05$, ** $p < 0,01$, *** $p < 0,001$

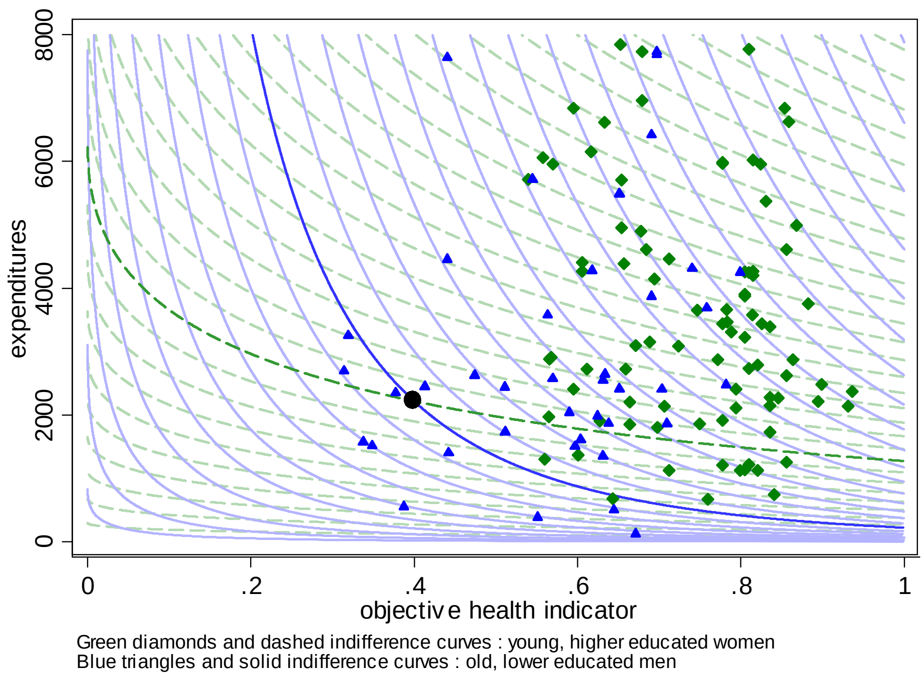

Figure 2: Indifference map of two subgroups in 2000

measure using the counting approach (see Atkinson [4] and Alkire and Foster [1, 2]). In the counting approach, the identification of the poor is based on the number of dimensions for which the individual falls below the threshold. In this illustration, we consider individuals as poor when they are below the threshold for at least two out of the four dimensions. The number of poor individuals is slightly lower (16,1%). The counting method identifies more people as poor who are unemployed, living in a relatively low quality house and who are lower educated. In the third column, we present the characteristics of the 18,6% poorest individuals according to the standard expenditure measure. They have low expenditures, are more often male, and are younger in general. As different people are indeed identified as poor according to the three methods, it is clear that the question whether and how to take their preferences into account may have strong implications for the design of targeted poverty alleviation programs.

In Figure 3, we consider the overlap between the poor according to each of the three methods in 2000. Slightly more than one third of the individuals (7,7%) are identified as belonging to the poor according to all measures. All other individuals are considered worst off by at least one method, but not by all methods. One finds remarkably little overlap between both multidimensional methods given that the same poverty line vector z is used. This finding stresses once more the empirical implications of taking the preferences of the

Table 2: Portrait of the poor in 2000

	preference sensitive	counting approach	expenditure poverty
number of poor (in %)	18,6	16,1	18,6
expenditure (in rubbles)	1655	1541	1055
health (on 0-1 scale)	0,52	0,56	0,62
house (in 100.000 rubbles)	2,40	2,04	2,37
unemployed (in %)	24,5	26,8	13,7
life satisfaction	1,97	1,96	2,02
male (in %)	32,1	38,6	40,0
young (in %)	25,7	29,2	32,2
higher educated (in %)	57,3	50,8	56,1
rural (in %)	35,2	46,1	36,9
minority (in %)	9,6	17,1	14,1

poor into account in the identification of the poor.

Before we move on to the theoretical derivation of a poverty index that takes individual preferences into account, we would like to add two comments on our general goal of taking preferences into account and on using a satisfaction regression to estimate them. The (McFadden) pseudo R-squared of our estimation is around 0,073, which is comparable to other studies using panel data (see, for instance Graham et al. [21]). This magnitude highlights that only a small part of the variation in life satisfaction can actually be explained.[5] One may wonder at this point how problematic this finding is for the idea of respecting preferences when identifying the poor. People have different opinions on how to trade-off dimensions of well-being. If the estimated preferences were only meant to capture this heterogeneity in opinions or in behavior, then the relatively small share of the explained variation in life satisfaction should be a source of concern, as we should aim at approaching the actual preferences of people as close as possible. One may argue,

[5] However, when considering for each respondent all pairwise comparisons between the ranking of available waves for the reported life satisfaction and the computed λ-values (the well-being index that is characterized in Theorem 3 below), we find inconsistencies in only 4,8% of the comparisons, i.e., when $S_{it} > S_{it'}$, but $\lambda_{it} < \lambda_{it'}$. This suggests that, in spite of assuming identical preferences within socio-demographic subgroups, the λ-values based on these estimations are broadly consistent with the ordinal information in the reported life satisfaction at the individual level.

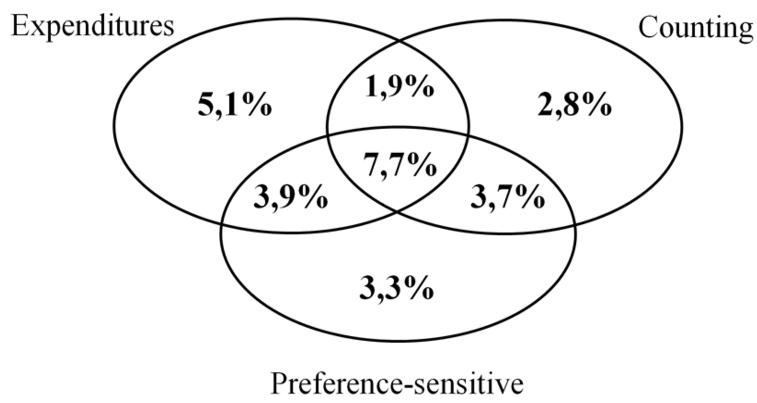

Figure 3: Overlap between the poor according to different approaches

however, that actual preferences are too idiosyncratic to be normatively compelling, for instance because people may make mistakes. Consequently, the actual preferences should be laundered before they are used in a normative judgement. This is precisely what the estimation carries out. We replace the actual individual preferences with the average preferences of the group to which the individual belongs, so that we end up only taking account of facts like the relative concern of elderly people for their health condition, the relative low worry of higher educated people on housing conditions, and so on. If the sample size would have allowed us to increase the number of these groups and to take account of more relevant characteristics, that would certainly have been desirable, but arguably not to the point that we should take the actual preferences of each agent individually.

This issue is connected to the problem of “expensive tastes”. Two sorts of expensive tastes should be distinguished. One consists of people being hard to satisfy because they have high aspirations. These expensive tastes are about their satisfaction level, not about their ordinal preferences. Because we only rely on ordinal preferences, our approach is immune to this form of expensive tastes. The second sort of expensive tastes appears when people prefer expensive goods for bad reasons (e.g., because they want to show off). Such people may end up appearing disadvantaged in our approach. The only way in which we partly eliminate this problem is by looking at average preferences, as explained in the previous paragraph. This obviously does not address the expensive tastes embodied in the average preferences of some groups. But we doubt that the group level preferences for basic aspects of life such as health and expenditures should be suspected of being questionable. We find it reasonable, to the contrary, to work on the assumption that such preferences are worthy of respect.

Finally, let us emphasize that our empirical application aims at illustrating the effect of adopting a preference-sensitive approach to the identification of the poor. It is not meant to be a definitive study on multidimensional poverty in Russia, which arguably requires a richer and more tailored data set. However, we have shown in this section that taking preferences into account makes a difference when identifying the poor, even with this crude data set. The next question is how the intensity of multidimensional poverty can be measured while taking preferences into account.

3 The model and a basic result

As discussed in the introduction, introducing preferences implies going from multi-dimensional attributes to a unidimensional notion of well-being. This section formalizes this simple observation. Let us first briefly introduce the framework.

An economic situation is a pair (x_N, R_N), where: N is the population; $x_N = (x_i)_{i\in N}$; each x_i is a ℓ-dimensional vector in a set $X \subseteq \mathbb{R}^\ell_+$, describing the multi-attribute situation (e.g., equivalized household expenditures, health, housing quality and employment status in the previous section) of individual i; $R_N = (R_i)_{i\in N}$; each R_i is i's preference relation over the set X of possible individual situations; the corresponding strict preference and indifference relations are denoted P_i and I_i, respectively.

We are looking for a *poverty index*, i.e., a function P such that for every (x_N, R_N) in the relevant domain $\mathcal{S}$, $P(x_N, R_N)$ is a real number measuring the extent of poverty in (x_N, R_N). It is assumed that $P(x_N, R_N)$ is continuous in x_N (which excludes simple headcount measures, but these can be considered a limit case of the measures we obtain).

Observe that by means of $P(x_N, R_N)$ economic situations can be compared in which preferences R_i, not just individual vectors x_i, are different. Here we follow Fleurbaey and Tadenuma [16], who advocate that social choice should extend beyond the comparison of options for a given population. As in their paper, the possibility to compare populations with different preferences is a direct side-product of the construction of the index, because even for a given population the index needs to compare individuals with different preferences.

We make the following assumptions on the domain $\mathcal{S}$ of economic situations (x_N, R_N). First, we assume that X is convex, compact and contains the ℓ-dimensional 0, the worst possible bundle of goods. The assumption of compactness is not standard.[6] It will give us that all continuous preferences have global maxima over all closed sets. Second, we assume that R_i may be any member of the set $\mathcal{R}$ of preferences which are continuous,

[6] A similar assumption is imposed in, for instance, Bourguignon and Chakravarty [8].

monotonic[7] (that is, for two bundles $x_i, x_i' \in \mathbb{R}_+^\ell$, if $x_i \leq x_i'$, then $x_i' \, R_i \, x_i$, and if $x_i \ll x_i'$, then $x_i' \, P_i \, x_i$), and convex. Third, the population N may have any positive finite size, and $\mathcal{N}$ denotes the set of possible populations. When the population has only one individual, we use the simpler notation $P(x_i, R_i)$ instead of $P(x_{\{i\}}, R_{\{i\}})$.

In the remainder of this section, we introduce basic requirements which constrain our search for P. We start by addressing the question "who is poor?". In our framework, it amounts to answering the question: which bundles make agent i with preferences R_i poor? We allow the answer to depend on individual preferences in a very general sense. Individuals are poor if they consider themselves worse off compared to a poverty line vector z which is allowed to depend on their preferences, i.e., to be a function $z(R_i)$. Note that in the previous section, we assumed the preference-sensitive functions $z(R_i)$ to coincide with a single poverty line vector. As it turns out, we will later obtain the result that indeed a single poverty line vector should be used, but we derive it as a result rather than imposing it here from the outset.

Our first axiom captures the idea that an individual is not poor, therefore irrelevant for P, if she prefers her situation x_i to $z(R_i)$. We call it *Focus*, by reference to Sen's Focus axiom (Sen [32]). The axiom requires that the poverty index, at the individual level, be independent of any change in the situation of a non-poor agent.

Axiom 1 Focus
There is a function $z : \mathcal{R} \to X$ such that for all $(x_N, R_N) \in \mathcal{S}$, $i \in N$, $x_i' \in X$, $R_i' \in \mathcal{R}$, if

$$x_i \, P_i \, z(R_i), \; x_i' \, P_i' \, z(R_i')$$

then

$$P((x_i', x_{-i}), (R_i', R_{-i})) = P(x_N, R_N).$$

We now adapt the classical requirement that an improvement in the situation of one agent cannot increase poverty. The core of our contribution in this paper is to use individual preferences to evaluate improvements in terms of individual well-being. We propose to apply a Pareto axiom, restricted to the poor: If the preference satisfaction of all poor agents weakly increases, then poverty weakly decreases. If, in addition, the preference satisfaction of at least one poor agent strictly increases, then poverty strictly decreases.

Axiom 2 Pareto among the Poor

[7] The three vector inequalities are denoted $\leq$, $<$ and $\ll$.

For all $(x_N, R_N), (x'_N, R_N) \in \mathcal{S},$ *if for all* $i \in N$ *such that* $z(R_i)\, P_i\, x_i,\ x'_i\, R_i\, x_i,$ *then*

$$P(x'_N, R_N) \leq P(x_N, R_N).$$

If, in addition, there is $j \in N$ *such that* $z(R_j)\ P_j\, x_j$ *and* $x'_j\, P_j\, x_j,$ *then*

$$P(x'_N, R_N) < P(x_N, R_N).$$

The next two axioms are standard axioms of the classical poverty measurement theory that do not need much adjustment to our framework. The next axiom is *Subgroup Consistency.* It requires that overall poverty decreases if it decreases in a subgroup of the population and the situation does not change for the other agents.

Axiom 3 Subgroup Consistency
For all $(x_N, R_N), (y_M, R_M), (y'_M, R'_M) \in \mathcal{S},\ P(y_M, R_M) \geq P(y'_M, R'_M)$ *if and only if*

$$P((x_N, y_M), (R_N, R_M)) \geq P((x_N, y'_M), (R_N, R'_M)).$$

This *Subgroup Consistency* axiom is a standard and powerful decomposability requirement. Observe that the decomposition it proposes does not allow the separation of the bundle from the preferences of an agent. That is the only difference with the standard axiom.

The above three axioms enable us to derive the representation theorem that we will use in the remaining of the paper. This result can even be simplified if the axiom of *Replication Invariance* is added. It requires that the poverty measure remains the same if the population is replicated and each replica of the current population exhibits the same characteristics as the current one. We need the following additional terminology. Let $r \in \mathbb{N}_{++}$ be a positive integer. The economic situation (x^r_N, R^r_N) is a replica of (x_N, R_N) if the set of agents is r times larger than N and is partitioned in r subgroups, one of which is N, and each subgroup has the same distribution of goods and preferences as N.

Axiom 4 Replication Invariance
For all $(x_N, R_N) \in \mathcal{S},$ *all* $r \in \mathbb{N}_{++},$

$$P(x_N, R_N) = P(x^r_N, R^r_N).$$

We are now equipped to state and prove the following representation result.[8] It is a straightforward generalization of a result obtained by Foster and Shorrocks [18] in

[8]Let us observe that, in the case $r = 1$, *Replication Invariance* boils down to the classical anonymity requirement: names of the agents do not matter, that is, only the list of bundles of goods and preference relations can influence poverty.

the standard one-dimensional framework. If we gather the above axioms, the resulting poverty index needs to be additively separable in individual characteristics (x_i, R_i).

Theorem 1 *A poverty index P satisfies* Focus, Pareto among the Poor, *and* Subgroup Consistency *if and only if there exist*

- *a continuous function $G : \mathbb{R}\times\mathcal{N} \to \mathbb{R}$, strictly increasing in its first argument,*
- *for all $N \in \mathcal{N}$, for all $i \in N$, a function $\phi_i^N : X \times \mathcal{R} \to \mathbb{R}$ such that ϕ_i^N is continuous in its first argument, $\phi_i^N(x_i, R_i) > \phi_i^N(x_i', R_i)$ whenever $z(R_i)\, R_i\, x_i'\, P_i\, x_i$, and $\phi_i^N(x_i, R_i) = 0$ whenever $x_i\, R_i\, z(R_i)$,*

so that for all $(x_N, R_N) \in \mathcal{S}$,

$$P(x_N, R_N) = G\left[\sum_{i\in N} \phi_i^N(x_i, R_i), N\right].$$

Moreover, if Replication Invariance *is added to the axioms, the poverty index can be simplified into*

$$P(x_N, R_N) = G\left[\frac{1}{|N|}\sum_{i\in N} \phi(x_i, R_i)\right],$$

for a continuous and strictly increasing function $G : \mathbb{R} \to \mathbb{R}$.

The proofs of all theorems are given in the second Appendix. Theorem 1 illustrates how taking preferences into account affects the definition of a multidimensional poverty index. The first consequence is that we return to a one-dimensional individual measure of poverty, i.e., the $\phi(x_i, R_i)$ measure, which is ordinally equivalent to $P(x_i, R_i)$. Preferences provide a powerful way of aggregating several dimensions into one complete order, and, therefore, into a one-dimensional individual measure. The second consequence is that we lose the comparability that is offered by restricting attention to quantities of goods alone. When preferences are ignored, two agents consuming the same bundle of goods can be assumed to experience the same poverty. This is no longer true once we aggregate dimensions with possibly heterogenous preferences. We cannot resort to classical one-dimensional poverty measurement either, as levels of preference satisfaction are not interpersonally comparable. What we do in the next sections, therefore, is to study a way of comparing poverty levels across individuals—more precisely, across preferences.

4 Inter-preference poverty comparisons

From the previous section we know that $P(x_i, R_i) \leq P(x_i', R_i)$ if and only if $x_i R_i x_i'$, and in this section we propose a way to extend this to comparisons between $P(x_i, R_i)$ and $P(x_i', R_i')$.

First, we introduce a natural constraint on inter-preference comparisons of poverty. It requires that an agent moving to an indifference curve below her initial curve (by a change of bundle and, possibly, preferences) should be considered poorer. This requirement can be justified in two ways. First, it involves a respect for preferences that extends the Pareto principle to inter-preference comparisons. When an individual has two (initial and final) indifference curves that do not cross, she has a firm judgment about where the best situation is, even if she changes her preferences when moving from one curve to the other. Second, it is also an informational simplicity requirement. By introducing preferences into the analysis, we allow a poverty index to depend on much more information than in the classical theory of poverty measurement. This axiom limits the information that can be taken from preferences to evaluate an economic situation, by implying that one can focus on the indifference curves containing the current bundles.

To state the condition, we need to introduce some additional terminology. For all $R_i \in \mathcal{R}$ and $x_i \in X$, we let $L(x_i, R_i) \subseteq X$ denote the (closed) lower contour set of R_i at x_i, that is, the set of bundles that agent i deems not better than x_i when she has preferences R_i. In a similar way, we define the upper contour set, $U(x_i, R_i)$, and the indifference surface $I(x_i, R_i) = L(x_i, R_i) \cap U(x_i, R_i)$.

Axiom 5 Nested Contours
For all $(x_N, R_N) \in \mathcal{S}$, $i \in N$, $x_i' \in X$, $R_i' \in \mathcal{R}$, if

$$U(x_i, R_i) \cap L(x_i', R_i') = \varnothing,$$

then

$$P(x_i, R_i) \leq P(x_i', R_i').$$

In combination with *Continuity*, *Nested Contours* implies that when $I(x_i, R_i) = I(x_i', R_i')$, then $P(x_i, R_i) = P(x_i', R_i')$. Therefore, adding *Nested Contours* to the axioms already present in Theorem 1 implies that the second argument of the ϕ_i functions can be simplified from R_i to $I(x_i, R_i)$. The results of this section build on this fact.

Obviously, *Nested Contours* does not help us for the harder comparisons between indifference curves that cross. We propose the following extension of this requirement. It is illustrated in Figure 4. *Nested Contours* implies that a situation (x_i, R_i) can have lower poverty than another situation (x'_i, R'_i) only if the lower contour set of the former is not included in the lower contour set of the latter. Now introduce a third situation (x''_i, R''_i), such that $L(x_i, R_i)$ is included in the union of the two other lower contour sets.

It is not obvious that (x_i, R_i) should be considered a worse situation than either of the other situations, because, as in the figure, the lower contour set at (x_i, R_i) may contain bundles which are not in one of the other lower contour sets. From *Nested Contours* we know that if $L(x_i, R_i)$ was included in the *intersection* of the other two lower contour sets, then (x_i, R_i) would be at least as poor as both of them. What the new axiom requires is that if $L(x_i, R_i)$ is included in the *union* of the two other lower contour sets, then (x_i, R_i) is at least as poor as the poorest of the other two.

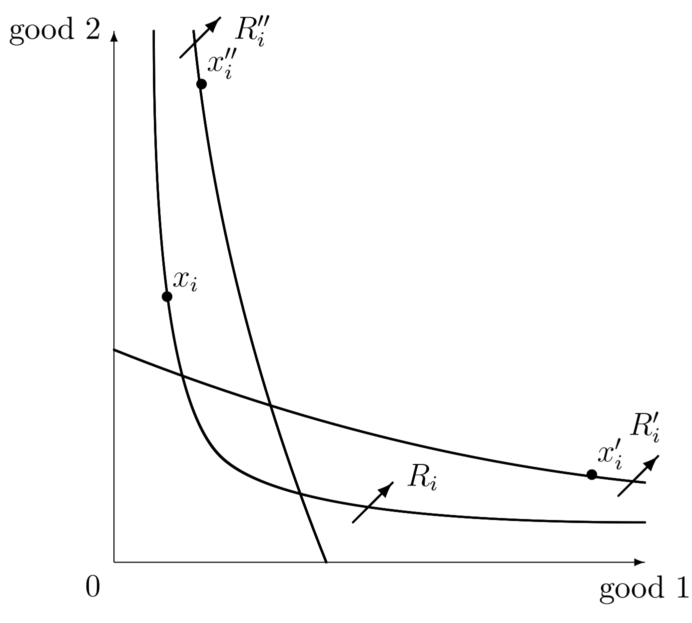

Figure 4: An illustration of *Nested Unions*: $P(x_i, R_i) \geq \min\{P(x'_i, R'_i), P(x''_i, R''_i)\}$

Axiom 6 Nested Unions

For all $(x_N, R_N) \in \mathcal{S}$, $i \in N$, *if for* $x'_i, x''_i \in X$ *and* $R'_i, R''_i \in \mathcal{R}$,

$$L(x_i, R_i) \subseteq L(x'_i, R'_i) \cup L(x''_i, R''_i),$$

then

$$P(x_i, R_i) \geq \min\{P(x'_i, R'_i), P(x''_i, R''_i)\}.$$

Observe from the figure that the union of two lower contour sets is the lower contour set of a convex preference, and that by *Nested Contours*, this corresponds to a situation that is at least as good as the best of the two. The new axiom therefore implies that it must be just as good as the best of the two.

Our second result is that adding this requirement is equivalent to imposing that there exist a unique preference relation R^w that is "the worst" in the sense that an agent with such preferences is at least as poor as with any other preferences, whatever his situation x. Formally, this means that there is $R^w \in \mathcal{R}$ such that for all $(x_i, R_i) \in \mathcal{S}$,

$$P(x_i, R^w) \geq P(x_i, R_i).$$

More importantly, $P(x_i, R_i)$ is then evaluated by the position of the indifference curve for R^w that is tangent from above to $I(x_i, R_i)$. This defines a class of poverty indexes that is already quite restricted.

Theorem 2 *Let P be a poverty index in the family characterized in Theorem 1, satisfying* Nested Contours*. The following two claims are equivalent.*

1. *P satisfies* Nested Unions*.*

2. *There exist*

   - *$R^w \in \mathcal{R}$,*
   - *a continuous and strictly increasing function $G : \mathbb{R} \to \mathbb{R}$,*
   - *a function $\phi : X \times \mathcal{R} \to \mathbb{R}$ such that*
     - *ϕ is continuous in its first argument,*
     - *$\phi(0, R^w) = 1$, $\phi(x_i, R^w) = 0$ for all $x_i \in X$ such that $x_i \, R^w \, z(R^w)$, and $\phi(x_i, R^w) > \phi(x_i', R^w)$ whenever $z(R^w) \, R^w \, x_i' \, P^w \, x_i$, and*
     - *$\phi(x_i, R_i) = \phi(x_i', R^w)$ if and only if $I(x_i, R_i)$ is tangent to $I(x_i', R^w)$ from below,*

   so that for all $(x_N, R_N) \in \mathcal{S}$,

$$P(x_N, R_N) = G\left[\frac{1}{|N|}\sum_{i=1}^{|N|} \phi(x_i, R_i)\right].$$

Let us provide an intuitive explanation for the tangency property (the impatient reader can jump to the next section). Figure 5 represents the indifference curve that partitions the consumption set of an agent with worst preferences into the bundles with which she qualifies as poor (below the curve through $z(R^w)$) and those with which she does not qualify as poor (above that curve). Let us look at agent j. Bundle z lies at the tangency between the indifference curve through $z(R^w)$ and one indifference curve for j.

Consider three bundles for agent j, x_j, x_j', x_j'', such that $x_j'' \, P_j \, x_j' \, I_j \, z \, P_j \, x_j$. We show that necessarily, $z(R_j) \, I_j \, z$.

First, we cannot have $z(R_j) \, I_j \, x_j$. Indeed, by the unrestricted domain assumption, there exists some $R_j' \in \mathcal{R}$ such that $I(x_j, R_j)$ and $I(z(R^w), R^w)$ are indifference curves for R_j' as well. By *Nested Contours* and *Continuity*, we should have $\phi(x_j, R_j') = \phi(x_j, R_j)$ and $\phi(z(R^w), R_j') = \phi(z(R^w), R^w)$. By *Focus* and *Continuity*, we should have $\phi(x_j, R_j) = \phi(z(R^w), R^w) = 0$. By transitivity, we should have $\phi(x_j, R_j') = \phi(z(R^w), R_j')$, violating *Pareto among the Poor*.

Second, we cannot have $z(R_j) \, I_j \, x_j''$ either. Indeed, there necessarily exists a bundle z' such that $z' \gg z$ and $x_j'' \, P_j \, z'$. By *Pareto among the Poor*, $\phi(z', R_j) > \phi(x_j'', R_j)$. By *Focus*, on the other hand, $\phi(z', R^w) = \phi(z, R^w) = 0$, violating the fact that R^w is the worst preference relation.

We are therefore left with $z(R_j) \, I_j \, z$. What this shows is that $z(R_j)$ must lie on the indifference curve of R_j that is tangent from below to the indifference curve of R^w through $z(R^w)$ (such a tangent indifference curve always exists because X is compact). This reasoning can be extended to any other indifference curve of R^w below the one containing $z(R^w)$.

5 Sensitivity to the degree of poverty

Theorem 2 above tells us how to construct functions $\phi(\cdot, R_i)$ for all $R_i \in \mathcal{R}$, once $\phi(\cdot, R^w)$ has been defined. How to define these ϕ functions —or equivalently, the relation R^w— more completely is the topic of this section.

Consider the idea that providing more resources to a poor agent decreases poverty more the poorer the agent is. A natural embodiment of this idea requires that if two agents consume bundles having the property that one bundle dominates the other one in each dimension, then poverty decreases more whenever the agent with the smallest bundle receives an additional amount Δx (that is proportional to the difference between the two bundles) than if Δx is assigned to the agent with the largest bundle.

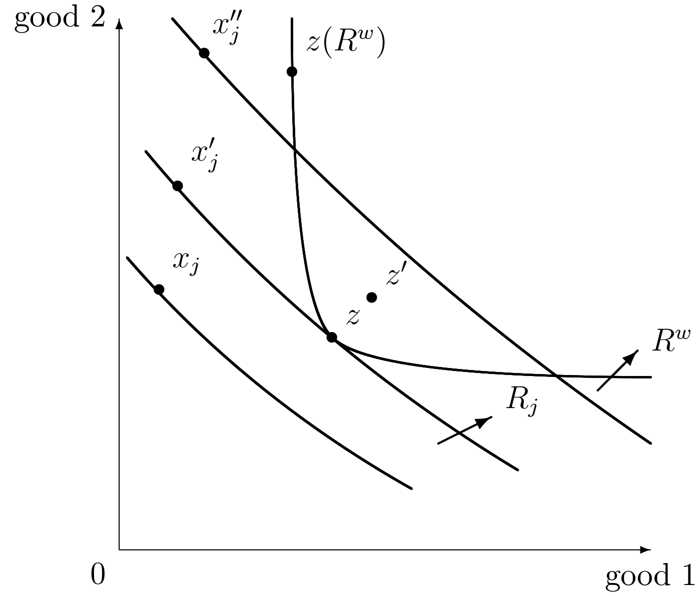

Figure 5: The poverty threshold $z(R_j)$ needs to be equivalent to z

Axiom 7 Poverty Sensitivity
For all $(x_N, R_N) \in \mathcal{S}$, $\mu \in (0,1)$, if there exist $j, k \in N$ such that $x_j \gg x_k$, then

$$P((x_j + \mu(x_j - x_k), x_{-j}), R_N) > P((x_k + \mu(x_j - x_k), x_{-k}), R_N).$$

We could equivalently write the axiom as a transfer principle, starting from the economic situation $((x_j + \mu(x_j - x_k), x_{-j}), R_N)$ and transferring $\mu(x_j - x_k)$ to individual k, thereby generating situation $((x_k + \mu(x_j - x_k), x_{-k}), R_N)$.[9]

Poverty Sensitivity defines who, between agents j and k, is poorer than the other only as a function of the bundles they consume. It turns out that this is incompatible with our general project of taking preferences into account. Indeed, this axiom is incompatible with *Pareto among the Poor*.[10]

Lemma 1 *No poverty index P satisfies* Pareto among the Poor *and* Poverty Sensitivity*.*

The following graphical sketch of the proof will help us understand where the difficulty

[9] Such a transfer principle is weaker than the standard multidimensional transfer principle considered in the literature on multidimensional inequality (see Weymark [37], among others). In that literature, typically rank reversals are allowed and, more importantly, there is no requirement of component-wise dominance imposed between bundles of donor and recipient so that transfers may go in opposite directions for different dimensions.

[10] This finding is similar to a result of Fleurbaey and Trannoy [17].

comes from and how to circumvent it. In Figure 6, two pairs of economic situations are represented. When agents j and k consume x_j and x_k, agent k is poorer, so that, by *Poverty Sensitivity*, there is less poverty in (x'_j, x'_k) than in (x_j, x_k). When agents j and k consume x''_j and x''_k, agent j is poorer, so that there is less poverty in (x'''_j, x'''_k) than in (x''_j, x''_k). The problem is that (x_j, x_k) and (x'''_j, x'''_k) are Pareto equivalent, and (x'_j, x'_k) and (x''_j, x''_k) are Pareto equivalent, too. By a successive application of *Pareto among the Poor* and *Poverty Sensitivity*, we should have $P(x_N, R_N) > P(x'_N, R_N) = P(x''_N, R_N) > P(x'''_N, R_N) = P(x_N, R_N)$, which entails a contradiction.

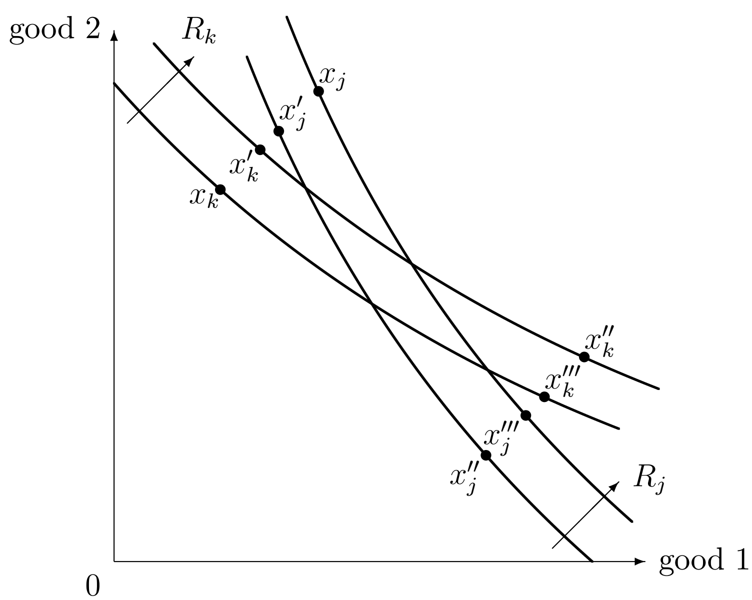

Figure 6: *Pareto Efficiency among the Poor* and *Poverty Sensitivity* are incompatible

The difficulty is related to the fact that the relationship "is poorer than" defined in terms of component-wise dominance between bundles (condition $x_j \gg x_k$ in the axiom) is too permissive. With this definition, agent k qualifies as poorer than agent j at (x_j, x_k) but richer at (x'''_j, x'''_k) whereas their respective well-being remains the same.

To avoid the difficulty revealed by Lemma 1, we propose to weaken *Poverty Sensitivity*. We weaken it in two directions. First, we allow the set of bundles among which the transfer among poor agents is required to decrease poverty to be a subset of X. This subset is required to be convex, so that the bundles obtained after the equalizing transfer are also in the set, guaranteeing that a further transfer reducing inequality is still considered desirable.

Second, we further require that the agent who is consuming more before the transfer is also considered strictly less poor than the other agent by the index P. This is consistent with our general objective of making our poverty judgments depend on preferences.

Either restriction would allow us to escape the difficulty revealed by Lemma 1. We choose to impose both restrictions, in order to obtain a reasonably weak axiom. As our last result will prove, combining this axiom with the axioms we already defined will allow us to single out a quite specific family of indices.

We need the following terminology. Let $\mathcal{T}$ be defined by: for all $T \in \mathcal{T}$, (i) $T \subset X$, (ii) T is convex, and (iii) for all $R_i \in \mathcal{R}$, all $x_i \in X$, $I(x_i, R_i) \cap T \neq \emptyset$.

Axiom 8 $\mathcal{T}$-Poverty Sensitivity
There exists $T \in \mathcal{T}$ such that for all $(x_N, R_N) \in \mathcal{S}$, all $\mu \in (0, \frac{1}{2})$, if there exists $j, k \in N$ such that

- $x_j, x_j', x_k, x_k' \in T$,
- $x_j \gg x_k$,
- $P(x_j, R_j) < P(x_k, R_k)$,

then

$$P(x_N, R_N) > P((x_j - \mu(x_j - x_k), x_k + \mu(x_j - x_k), x_{-jk}), R_N).$$

The following theorem proves that it is possible to add *$\mathcal{T}$-Poverty Sensitivity* to the list of axioms of Theorem 2. The theorem also characterizes the consequence of *$\mathcal{T}$-Poverty Sensitivity* on the choice of R^w. Only one preference relation can be chosen as the worst, i.e., the Leontief preferences with the cusps along a particular ray. This finding is equivalent to identifying a single poverty line vector z that is independent of preferences —although preferences remain important because an agent is poor if and only if she consumes a bundle to which she *prefers* z. Individual poverty, as measured by the ϕ function, can be any decreasing and convex (that is, inequality averse) function computed from the fraction λ of z to which an agent is indifferent.

Theorem 3 *Let P be a poverty index in the family characterized in Theorem 2. It satisfies* $\mathcal{T}$-Poverty Sensitivity *if and only if there exist*

- $z \in \mathbb{R}^\ell_{++}$ *such that* $x_i \, R^w \, x_i' \Leftrightarrow \min_{l \in \{1,\ldots,\ell\}} \frac{x_{il}}{z_l} \geq \min_{l \in \{1,\ldots,\ell\}} \frac{x_{il}'}{z_l}$,
- *a continuous and strictly increasing function* $G : [0, 1] \to \mathbb{R}$,

- *a continuous, decreasing and convex function* $\phi : [0,1] \to [0,1]$

so that for all $(x_N, R_N) \in \mathcal{S}$,

$$P(x_N, R_N) = G\left[\frac{1}{|N|}\sum_{i=1}^{|N|}\phi\left(1 - \min\{1, \lambda(x_i, R_i)\}\right)\right],$$

where $\lambda(x_i, R_i) = \lambda$ *if and only if* $x_i\, I_i\, \lambda z$.

Here is the intuition of why only Leontief preferences can be used as worst preferences. Let us consider Figure 7. Let us assume that R^w is not Leontief and $\mathcal{T}$-*Poverty Sensitivity* is satisfied with respect to some ray from the origin. By Theorem 2, we already know that $\phi(\tilde{x}_j, R^w) = \phi(x_j, R_j)$ and $\phi(x'_j, R_j) = \phi(x''_j, R^w)$. $\mathcal{T}$-*Poverty Sensitivity* implies that

$$\phi(x'_j, R_j) - \phi(x_j, R_j) \geq \phi(x'''_j, R^w) - \phi(x''_j, R^w),$$

which is equivalent to

$$\phi(x''_j, R^w) - \phi(\tilde{x}_j, R^w) \geq \phi(x'''_j, R^w) - \phi(x''_j, R^w). \tag{1}$$

The problem comes from the fact that the distance between the indifference curves of R^w through $\tilde{x}_j$ and x''_j can be made arbitrarily small, for a fixed $x'''_j - x''_j$. In other words, we can choose R_j so as to have an arbitrarily small $\phi(x''_j, R^w) - \phi(\tilde{x}_j, R^w)$ compared to a fixed $\phi(x'''_j, R^w) - \phi(x''_j, R^w)$. As a consequence, no continuous function ϕ can satisfy Eq. 1. This proves that $\mathcal{T}$-*Poverty Sensitivity* cannot be satisfied for T as represented in the figure. It turns out that whatever T, a construction like the one in the figure can be made, unless R^w is of the Leontief type and T is precisely the ray at which R^w has cusps.

The theorem is silent about two things: the shape of the decreasing and convex transformation function ϕ and the choice of the single poverty line vector z. These choices remain the degrees of freedom of the ethical observer. There are cases, however, in which the poverty judgement does not dependent on the specific choice of ϕ. This occurs in case of the so-called Three I's of Poverty" (TIP) dominance. Jenkins and Lambert [22] have shown that whenever there is TIP dominance of one distribution over another distribution, then there is unanimous agreement in the class of all additive poverty measures based on a decreasing and convex transformation function ϕ of the poverty gap that poverty is higher in the first distribution (see also Zheng [38]).

We need the following terminology to extend their result to our class of indices. Let $(\tilde{x}_N, \tilde{R}_N) \in \mathcal{S}$ be the permutation of (x_N, R_N) such that agents are ranked from poor to rich according to λ, that is, $\forall\, i, j \in N : i < j \Rightarrow \lambda(\tilde{x}_i, \tilde{R}_i) \leq \lambda(\tilde{x}_j, \tilde{R}_j)$.

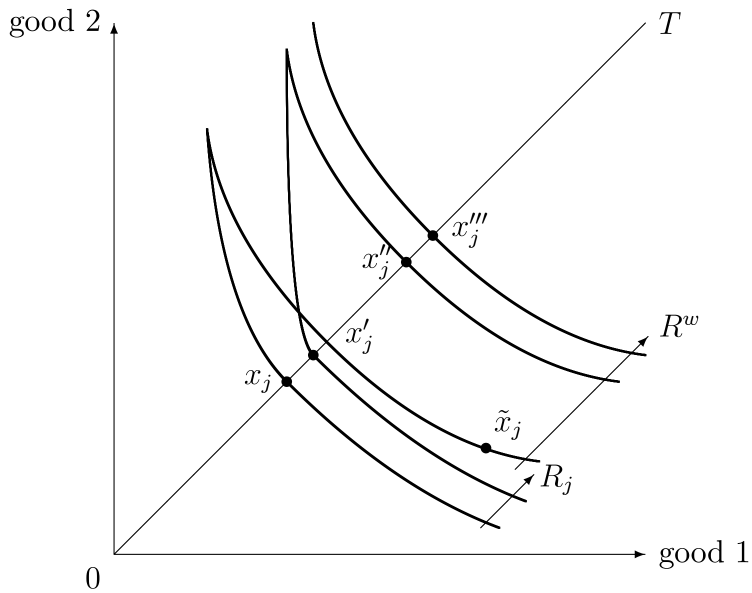

Figure 7: Why worst preferences need to be Leontief preferences

Corollary 1 *Let* $(x_N, R_N), (x'_N, R'_N) \in \mathcal{S}$. *Then, all poverty indices* P *satisfying the axioms of Theorem 3 conclude that*

$$P(x_N, R_N) \geq P(x'_N, R'_N)$$

if and only if for all $K \leq |N|$*:*

$$\sum_{k=1}^{K} \left(1 - \min(1, \lambda(\tilde{x}_k, \tilde{R}_k))\right) \geq \sum_{k=1}^{K} \left(1 - \min(1, \lambda(\tilde{x}'_k, \tilde{R}'_k))\right). \tag{2}$$

Testing for TIP dominance can be done easily by the comparing the corresponding TIP curves visually. A TIP curve plots the cumulative poverty gaps for the population ranked from poor to rich. In other words, it consists of the pairs $\left(K/|N|; \sum_{k=1}^{K} \left(1 - \min(1, \lambda(\tilde{x}_k, \tilde{R}_k))\right)\right)$ for all $K \leq |N|$. Whenever the TIP curve of one distribution is everywhere above the TIP curve of another distribution, the sequence of inequalities (2) hold. In Section 7, we provide an empirical test of poverty dominance based on these TIP curves. We will find that dominance relationships between TIP curves occurs (surprisingly) frequently.

We end this section with a discussion of the poverty line vector z. In the multidimensionality literature, this vector consists in the dimension-wise thresholds below which an individual is viewed as deprived in that dimension. In our approach, however, the choice of the poverty line vector is concomitant to the choice of the Leontief preferences that are considered the worst preferences. Choosing a poverty line vector, therefore, amounts to dividing the consumption set in as many subspaces as there are goods, where one good can be considered as least abundant or most deprived. The poverty line vector and corresponding division of the consumption space should then be chosen so that an agent who is unable to trade-off between the most deprived good and any other good (i.e., who has Leontief preferences that only depend on the consumption of the most deprived good in the respective subspace) is ceteris paribus believed to be the worst off.

6 Generalization to discrete variables

In practice, some variables that one may want to include in a multidimensional analysis of poverty are discrete in nature (see, amongst others, Alkire and Foster [1] and Bossert et al. [7]). This was the case for the unemployment variable that we studied in Section 2. In this section, we briefly and informally describe therefore how the results obtained in the preceding sections generalize to the case in which agents' consumption bundles are composed of goods that come in discrete quantities, in addition to the goods represented by the continuous variables of our previous model.

To simplify the exposition, let us only add one binary variable, for instance an indicator whether the individual is being unemployed or not). That is, the consumption set is now $X \subseteq \mathbb{R}_+^\ell \times \{0,1\}$, and we describe the consumption of an agent by a list (x_i, d_i) such that $x_i \in \mathbb{R}_+^\ell$ and $d_i \in \{0,1\}$. The discussion of this section generalizes immediately to the more general case with several additional variables, and with discrete rather than binary variables, so that the consumption set will be a finite product of Euclidean spaces.

To generalize our previous results we require that individual preferences are able to compare bundles in which $d_i = 0$ with bundles in which $d_i = 1$. A simple way of doing so is by assuming that the divisible goods (which include food, for instance) are necessary to survive, with the consequence that whether the binary good is consumed or not does not matter for someone whose consumption of the necessary goods is zero. Formally, this is the assumption that for all admissible preferences $R_i \in \mathcal{R}$, if $x_i = (0, \ldots, 0)$, then $(x_i, 0)\, I_i\, (x_i, 1)$. This simplifying assumption is not necessary for the generalization to hold true, but we find it sufficiently realistic to use it in this brief section.

None of axioms 1 to 8 requires any rewriting, and Theorems 1 and 2 are still valid. This can be checked by going throughout the proofs that are provided in the appendix and observing that all steps are still valid if the model is changed to accommodate an additional binary variable. Without entering into the details, let us simply point out that Theorem 1 continues to hold because our assumption that divisible goods are necessary guarantees that individual poverty measures range in an interval, and all the other steps follow. Theorem 2 is not affected by the change in the definition of the consumption set, provided the arguments involving convex upper contour sets are now rewritten to bear on upper contour sets that are the union of sets that are convex in the two Euclidean spaces (corresponding to the spaces in which $d_i = 0$ and that in which $d_1 = 1$).

This means that the consequence of imposing axioms 1 to 8 when agents have this new consumption set still amounts to choosing some worst preferences and evaluating poverty of any agent by looking at the lowest possible poverty level according to the worst preferences associated to a bundle in the lower contour sets of this agent's consumption bundle, as required by Theorem 2.

The generalization is slightly more complicated when one adds $\mathcal{T}$-*Poverty Sensitivity*. How should we define the set of bundles $\mathcal{T}$ in which transfers will be said to decrease poverty? The easiest answer consists probably in requiring again this set to be convex. As a consequence, set $\mathcal{T}$ is either in the space defined by $d_i = 0$ or in the space $d_i = 1$. Unsurprisingly, if we choose the space $d_i = 1$, then the worst preferences must be defined by the following two requirements: 1) they must be Leontief in that space along a ray from 0 to a bundle z, and 2) they must have the strongest possible preference for bundles in the space $d_i = 1$ over bundles in the space $d_i = 0$, which means that all other admissible preferences must require a lower increase in x_i to leave the agent indifferent between a bundle with $d_i = 1$ and one with $d_i = 0$.

Intuitively, this is reminiscent of what we found in the previous section. Indeed, the worst preferences need to be the ones associated to the lowest ability to trade-off between the dimension in which one agent is deprived and the other dimensions. Here, if transfers are evaluated in the space $d_i = 1$, it must be the case that the worst preferences are the ones that suffer most from having $d_i = 0$ compared to $d_i = 1$.

Theorem 3, above, tells us that individual situations have to be evaluated with the fraction of the poverty line vector to which an agent is indifferent. The theorem, on the other hand, does not tell us anything about how to choose the poverty line. Once we generalize Theorem 3 by adding a binary consumption good (and, more generally, by adding discrete goods), we obtain that individual situations have to be evaluated with the fraction of the poverty line vector, combined with a reference consumption of the binary good, to which an agent is indifferent. As a consequence, we still have to choose the poverty line, but, in addition to it, we now have to choose one reference value for the binary variable (or, more generally, one reference value of the discrete variables) so as to choose the space in which individual situations are evaluated.

7 Empirical poverty comparisons in Russia

At this point, we return to the data set described in Section 2 and illustrate how poverty comparisons can be made that are robust for the specific choice of the ϕ function in Theorem 3. We will make use of Corollary 1 and present TIP curves for two comparisons: one between the different waves of the data set and one across subgroups of the population in 2000.

First, in Figure 8 we have depicted the TIP curve of the computed λ-values for each wave of the RLMS-HSE data between 1995 and 2005.[11] As these curves become flat above the poverty line, we only show their leftmost part. The most striking observation is that only few of the nine TIP curves cross, and, hence, that we get an almost complete ordering of the different waves according to multidimensional poverty. That means that the precise choice of the ϕ function is not very decisive for the evaluation of the evolution of poverty in Russia over the considered period. Clearly, the TIP curve of 1998 is everywhere above the other curves. In other words, 1998 is unambiguously the year with most multidimensional poverty according to all poverty measures that satisfy the axioms listed in Theorem 3. Given the severe financial crisis of 1999 in Russia, this result stands to reason. On the other hand, we see that the curve of 2005 is everywhere below the curve of 1995, indicating that poverty decreased over the considered period.

Second, we compare eight different subgroups of the population in 2000. The groups are obtained by each of the eight combinations of the dummy variables that indicate whether respondents are male, young and higher educated respectively. Figure 9 shows the TIP curves for these eight groups for the three approaches that we introduced in Section 2, i.e., the preference-sensitive poverty index proposed in this paper (on the left), a multidimensional poverty measure based on counting (in the middle), and a standard measure of expenditure poverty (on the right).

The left panel of Figure 9 shows that many TIP curves cross each other, so that poverty dominance does not allow us to compare all pairs of groups. Which subgroup of the population is poorer depends on the choice of the ϕ function, that is, on the chosen degree of poverty sensitivity among the poor. Some general comparisons remain possible, however. The groups of old women and of old higher educated men are the poorest. The

[11] No data have been collected in 1997 and 1999.

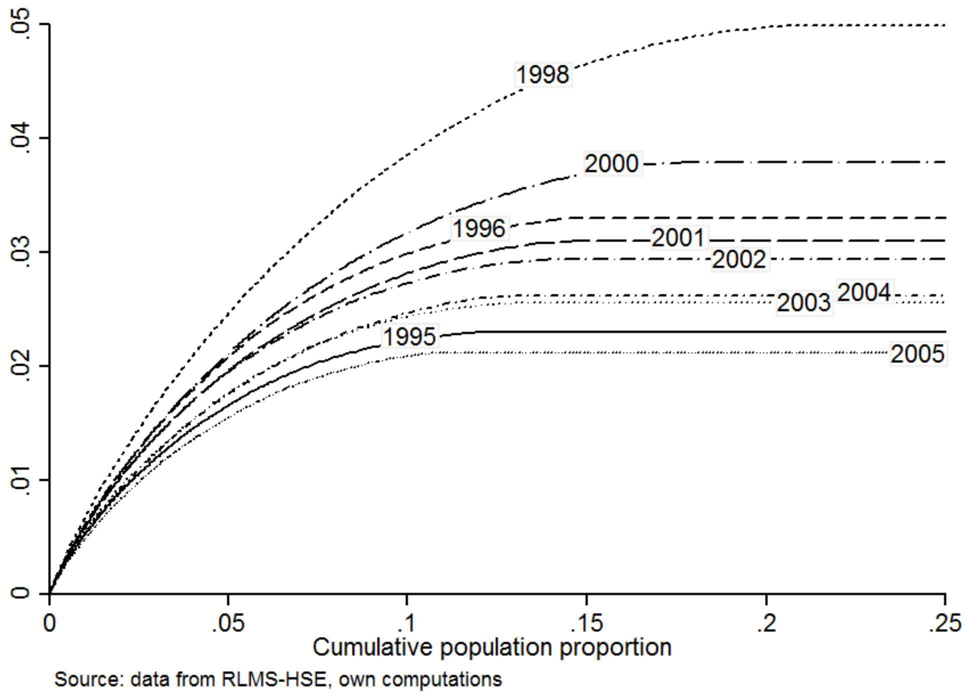

Figure 8: TIP curves for the preference sensitive approach for the different waves

young men cannot be compared with the old, lower educated men but they are less poor than all the other groups.

Comparing the TIP curves across the three panels of Figure 9 shows once again the effect of taking preferences into account. The figures reveal, indeed, that there are TIP dominance reversals among the different approaches. When we compare the left and middle panel of Figure 9, we see that the old, higher educated women are poorer than the young, lower educated women when preferences are taken into account, while the counting approach alters the ordering. These TIP dominance reversals between both multidimensional approaches are surprising, because exactly the same four dimensions of poverty are used in the analysis as well as the same poverty line vector z. It shows that the young, lower educated women are deprived in more dimensions, but that the old, higher educated women are deprived in precisely the dimensions they deem important.

Next, when we compare the left and right panel of Figure 9, we see, for instance, that the preference-sensitive approach concludes that old, higher educated women are unambiguously poorer than young, lower educated men, whereas the latter are unambiguously poorer according to the expenditure approach. This is because our approach takes account of additional dimensions of life which old women find important and in which they are deprived (mainly, health).

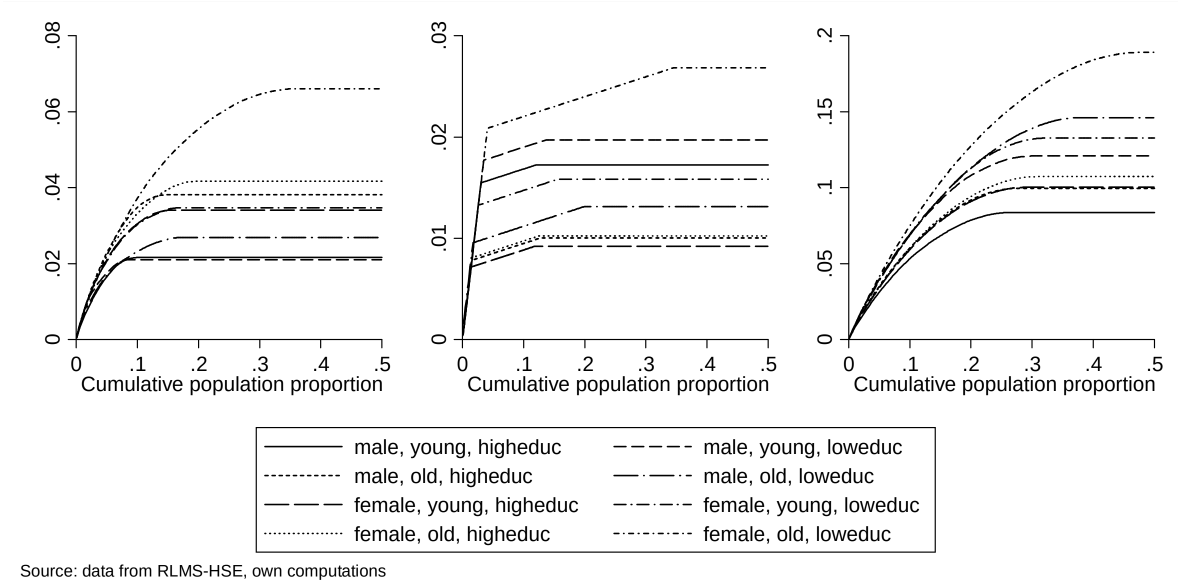

Figure 9: TIP curves for different groups with the preference-sensitive approach (left), the counting approach (middle), and the expenditure approach (right)

8 Conclusion

Measuring multidimensional poverty requires aggregating across dimensions and across agents. In this paper, we have studied the consequences of aggregating across dimensions at the level of each agent by taking the agent's preferences as the aggregation device. This approach forced us to find new ways of aggregating across agents, as individual levels of preference satisfaction cannot readily be compared. By introducing ways to build inter-preference poverty comparisons, we have been able to provide and characterize a family of poverty indices. These poverty indices aggregate individual measures of poverty that are a convex transformation of the fraction of the poverty line vector to which the individual herself is indifferent.

We have illustrated how the approach proposed in this paper can be implemented using existing Russian survey data from RLMS-HSE and we found some remarkable differences with standard (multidimensional) poverty measures. By taking preferences of the poor into account, different people are indeed identified as poor. The data that are needed to apply our approach are clearly more demanding than what is required to apply the other indices proposed in the literature. For instance, the counting approach is remarkably parsimonious in terms of the required data, whereas our approach requires a tailored data set that allows identification of the preferences in a wide set of dimensions.

We believe that the data requirement of our approach is the price to pay to develop an attractive way to measure multidimensional poverty without relying on arbitrary weights or arbitrary assumptions on the nature of the goods. The ray measure of individual poverty proposed in this paper is such that information about preferences is important primarily for the people who are not poor in all dimensions. In contrast, the situations of those who are below the poverty threshold and suffer similar deprivations in all dimensions can be quite accurately assessed without inquiring about their preferences. Although this seems a natural result, it is easy to conceive other measures that rely on preferences which do not have this property.

Finally, taking preferences into account when measuring poverty has clear policy implications, which we sketch here briefly. First, the different subgroups that are identified as poor with the preference-sensitive method call for a redirection of targeted poverty alleviation programs, e.g., those that are targeted to specific gender, age or education groups. In our empirical illustration with Russian data, for instance, a greater concern for the unemployed, the elderly, and women would be warranted. Second, the evaluation of poverty policies which have conflicting effects on different dimensions (e.g., improve health at some cost on income, or conversely) can be assessed in a more appealing way when population preferences are incorporated.

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Appendix: The satisfaction regression

We use the Russian Longitudinal Monitoring Survey (RLMS-HSE) data set between 1995 and 2005. The four "goods" we consider in the empirical sections are constructed as follows.[12] First, we use the square root of household size as equivalence scale for the real household *expenditures* (the reference year is 1992). Second, the measure of individual *health* is a composite index that consists of various objective health indicators such as indicators of diabetes, heart attack, aneamia, hospitalization and recent operations. The weights of the indicators in the health index are derived from the coefficients of an ordered logit regression using the self-assessed health as explained variable (for a similar approach, see van Doorslaer and Jones [36]). Third, the measure of *housing quality* is the predicted value of a logarithmic hedonic housing regression based on self-reported housing values and a series of housing characteristics after controlling for regional price differences, time trends and the size of the household (by using again the square root equivalence scale). Finally, the measure of *unemployment* is a binary indicator that takes 1 if the respondent is unemployed. The unemployment rate in the considered period fluctuates around 8%.

Recall that we estimate the following equation:

$$S_{it}^{*} = \alpha_i + \gamma_t + (\beta + \Lambda D_{it})' X_{it} + \delta' Z_{it} + v_{it}.$$

We observe the reported life satisfaction $S_{it} = k$ for k in $\{1, 2, \ldots, 5\}$ if the latent life satisfaction (S_{it}^{*}) lies within an interval between η_{k-1} and η_k :

$$S_{it} = k \text{ if } \eta_{k-1} < S_{it}^{*} \leq \eta_k.$$

The thresholds η_k are allowed to depend on the individual fixed effects, the observable socio-demographic characteristics and the time trend (for more details, see Jones and Schurer [23]). Finally, to allow for non-perfect substitutability between the three continuous dimensions of poverty, the outcomes of the individuals in these dimensions are transformed by a so-called Box-Cox transformation. For each dimension j in {expenditures, health, housing}, we have that:

$$X_{it}^{j} = \begin{cases} \left[\left(Y_{it}^{j}\right)^{\varepsilon_j} - 1\right] / \varepsilon_j & \text{when } \varepsilon_j \neq 0 \\ \log\left(Y_{it}^{j}\right) & \text{when } \varepsilon_j = 0, \end{cases}$$

where Y_{it}^{j} is the observed outcome of individual i in period t in dimension j. For the binary unemployment variable, $Y_{it} = X_{it}$. The three dimension-specific transformation

[12] Estimation results are available upon request.

parameters ε_j are chosen over a fine grid to maximize the overall fit of the model (Box and Cox [9]).

We follow the estimation method suggested by Jones and Schurer [23] that approximates the approach proposed by Ferrer-i-Carbonell and Frijters [14] and applied by Frijters et al. [20] to the RLMS-HSE. To reach a parsimonious and tractable model, the least significant interaction effects in Λ have been dropped consequently until all the remaining interaction effects are significant at the 10% level. All reported standard errors are corrected for clustering at the household level.

As can be seen from Table 1, the likelihood maximizing Box-Cox transformation parameters are respectively $-0,055$; $0,485$ and $-0,356$ for expenditures, health, and housing quality. The approximately logarithmic transformation of expenditures is a common finding in the happiness literature, suggesting important decreasing marginal returns of expenditures (see, amongst others, Layard et al. [24]). The marginal returns for health and housing are found to be respectively less and more decreasing.

(For on-line publication) Appendix: Proofs

Proof. (of Theorem 1) We focus on the "only if" part. Let P be a poverty index satisfying the axioms. Let $N \subset \mathbb{N}_{++}$. By *Subgroup Consistency* applied by decomposing N into $\{i\}$ and $N \setminus \{i\}$, there exist functions $g : X^{N \setminus \{i\}} \times \mathcal{R}^{N \setminus \{i\}} \to \mathbb{R}$ and $f : \mathbb{R}^2 \to \mathbb{R}$ such that for all $(x_N, R_N) \in \mathcal{S}$,

$$P(x_N, R_N) = f(P(x_i, R_i), g(x_{N \setminus \{i\}}, R_{N \setminus \{i\}})).$$

By applying the same argument to decompose g and the following functions, we reach the conclusion that there exists a function $f : \mathbb{R}^{|N|} \to \mathbb{R}$ such that for all $(x_N, R_N) \in \mathcal{S}$,

$$P(x_N, R_N) = f((P(x_i, R_i))_{i \in N}).$$

Fix R_N for the moment. The line segment from 0 to $z(R_i)$ is a compact set. By *Focus*, the range of $P(x_i, R_i)$ for fixed R_i is the image of this line segment. By continuity, the range of $P(x_i, R_i)$ is a compact interval, of strictly positive length by *Pareto among the Poor*.

We claim that f is continuous on this domain. Indeed, take a sequence $p_N^k \to p_N^*$ in this domain and suppose $f\left(p_N^k\right)$ does not tend to $f\left(p_N^*\right)$. It then has a subsequence p_N^t such that $f\left(p_N^t\right)$ stays away from $f\left(p_N^*\right)$. There is a corresponding sequence x_N^t on the line segments from 0 to $(z(R_i))_{i \in N}$ such that $p_N^t = (P(x_i^t, R_i))_{i \in N}$ for all t. As the sequence is in a compact set, it has a converging subsequence $x_N^s \to x_N^*$. By continuity, $P(x_N^s, R_N) \to P(x_N^*, R_N)$. Also by continuity, $p_N^s \to p_N^*$ implies that $p_N^* = (P(x_i^*, R_i))_{i \in N}$. Therefore $f\left(p_N^*\right) = P(x_N^*, R_N)$ is the limit of $f\left(p_N^s\right)$. This contradicts the assumption that $f\left(p_N^s\right)$, a subsequence of $f\left(p_N^t\right)$, stays away from $f\left(p_N^*\right)$.

Now let us drop the restriction on R_N. By *Focus*, one can normalize $P(x_i, R_i) = 0$ whenever $x_i R_i z\left(R_i\right)$. Therefore the range of $P(x_i, R_i)$ is the union of the intervals for every possible R_i, all these intervals having 0 as their lower bound, so that their union is an interval. The f function being continuous on any of these (Cartesian products of) intervals, it is continuous on (the Cartesian product of) their union.

So, f is a real-valued function defined over the Cartesian product of connected and separable spaces (intervals). By *Pareto among the Poor*, each dimension of the domain of f is essential (f cannot be constant in any $P(x_i, R_i)$). Therefore, by *Continuity* and *Subgroup Consistency*, we can apply Debreu's theorem on additive representation of separable preferences: there exists a continuous and strictly increasing function $G_N : \mathbb{R} \to \mathbb{R}$

and for all $i \in N$ a continuous function $f_i : \mathbb{R} \to \mathbb{R}$ such that

$$P(x_N, R_N) = G_N \left[\sum_{i=1}^{|N|} f_i(P(x_i, R_i)) \right].$$

By defining functions $\phi_i^N : X \times \mathcal{R} \to \mathbb{R}$ as follows: $\phi_i^N = f_i \circ P$, we get

$$P(x_N, R_N) = G_N \left[\sum_{i=1}^{|N|} \phi_i^N(x_i, R_i) \right].$$

By continuity, ϕ_i^N are continuous (in their first argument). By *Pareto among the Poor*, $\phi_i^N(x_i, R_i) > \phi_i^N(x_i', R_i)$ whenever $z(R_i)\, R_i\, x_i'\, P_i\, x_i$. By *Focus*, we can normalize $\phi_i^N(x_i, R_i) = 0$ whenever $x_i\, R_i\, z(R_i)$.

In order to prove the second part of the Theorem, assume it is wrong. For all G and ϕ^N such that $P(x_N, R_N) = G\left[\sum_{i=1}^{|N|} \phi(x_i, R_i), N\right]$, there exist $(x_N, R_N), (x'_{N'}, R'_{N'}) \in \mathcal{S}$ such that

$$\frac{1}{|N|} \sum_{i=1}^{|N|} \phi(x_i, R_i) = \frac{1}{|N'|} \sum_{i=1}^{|N'|} \phi(x_i', R_i') \tag{3}$$

and

$$P(x_N, R_N) \neq P(x'_{N'}, R'_{N'}). \tag{4}$$

Let $n = |N|$ and $n' = |N'|$. By *Replication Invariance*,

$$P(x_N, R_N) = P(x_N^{n'}, R_N^{n'}) = G \left[n' \sum_{i=1}^{|N|} \phi(x_i, R_i), nn' \right],$$

and

$$P(x'_{N'}, R'_{N'}) = P({x'_{N'}}^n, {R'_{N'}}^n) = G \left[n \sum_{i=1}^{|N'|} \phi(x_i', R_i'), nn' \right].$$

Eq. 3 implies that

$$n' \sum_{i=1}^{|N|} \phi(x_i, R_i) = n \sum_{i=1}^{|N'|} \phi(x_i', R_i'),$$

in contradiction to Eq. 4. ■

Proof. (of Theorem 2) We first prove that under *Nested Contours*, *Nested Unions* is equivalent to the following property P*:

For all $N \subset \mathbb{N}_{++}$, $i \in N$, $(x_{N\setminus\{i\}}, R_{N\setminus\{i\}}) \in \mathcal{S}^{N\setminus\{i\}}$, $\overline{\mathcal{R}}^i \subseteq \mathcal{R}$, $p \in \mathbb{R}$, if for all $R_i \in \overline{\mathcal{R}}^i$ there exists $x_i(R_i) \in X$ such that

$$P\left((x_{N\setminus\{i\}}, x_i(R_i)), (R_{N\setminus\{i\}}, R_i)\right) = p,$$

then for all $R_i' \in \mathcal{R}$, $x_i' \in X$, if

$$L(x_i', R_i') \subseteq \cup_{R_i \in \overline{\mathcal{R}}^i} L(x_i(R_i), R_i),$$

then

$$P\left((x_{N\backslash\{i\}}, x_i'), (R_{N\backslash\{i\}}, R_i')\right) \geq p.$$

Implication: *Nested Unions* implies P* for the case $\left|\overline{\mathcal{R}}^i\right| = 2$. It is a straightforward induction argument to extend the property to any finite number. Now, let $L(x_i', R_i') \subseteq \cup_{R_i \in \overline{\mathcal{R}}^i} L(x_i(R_i), R_i)$, for some possibly infinite $\overline{\mathcal{R}}^i$, and consider a sequence $y^k \to x_i'$ such that $x_i' P_i' y^k$ for all k. Necessarily for all k, given our assumption that X is compact, there is a finite subset $\overline{\overline{\mathcal{R}}}^i \subset \overline{\mathcal{R}}^i$ such that $L(y^k, R_i') \subseteq \cup_{R_i \in \overline{\overline{\mathcal{R}}}^i} L(x_i(R_i), R_i)$. By the property for finite numbers, this implies

$$P\left((x_{N\backslash\{i\}}, y^k), (R_{N\backslash\{i\}}, R_i')\right) \geq p.$$

By continuity,

$$P\left((x_{N\backslash\{i\}}, x_i'), (R_{N\backslash\{i\}}, R_i')\right) \geq p.$$

This proves the implication. The converse is omitted.

Part I: P* implies 2. By Theorem 1, we can restrict our attention to one agent situations. Let $p \in \mathbb{R}$. For all $R \in \mathcal{R}$, let $x_p(R) \in X$ be such that

$$P(x_p(R), R) = p.$$

Let $U^p \subseteq X$ be defined by

$$U^p = \bigcap_{R \in \mathcal{R}} U(x_p(R), R).$$

Because U^p is the (possibly infinite) intersection of compact and convex sets, it is compact and convex. Assume it is non-empty. Let $I^p \subseteq X$ be the lower frontier of U^p. By the unrestricted domain assumption, there exists $R^p \in \mathcal{R}, x^p \in I^p$ such that $I(x^p, R^p) = I^p$. We claim that $P(x^p, R^p) = p$. By construction of U^p, $P(x^p, R^p) > p$ is impossible, because the intersection is taken over all $R \in \mathcal{R}$, including R^p. Let $\overline{\mathcal{R}} \in \mathcal{R}$ be defined by

$$\overline{\mathcal{R}} = \{R \in \mathcal{R} | L(x_p(R), R) \cap I^p \neq \emptyset\}.$$

Observe that

$$L(x^p, R^p) = \bigcup_{R \in \overline{\mathcal{R}}} L(x_p(R), R).$$

By P*, $P(x^p, R^p) \geq p$, which proves the claim.

Now, let $\Pi \subset \mathbb{R}$ be the set of $p \in \mathbb{R}$ such that U^p is non-empty. By continuity of P, Π is a closed and convex interval of $\mathbb{R}$. By construction, for all $p, p' \in \Pi$

$$p < p' \Rightarrow intU^p \subset intU^{p'}.$$

Therefore, by the unrestricted domain assumption, there exists $R^{\Pi} \in \mathcal{R}$ such that for all $p \in \Pi$, $x^p \in I^p$, $I(x^p, R^{\Pi}) = I^p$. We now claim that P satisfies the property that there is a worst preference relation R^w (let us call this property *Worst Preferences*), for $R^w = R^{\Pi}$.

Let $x \in X, R \in \mathcal{R}$. We need to prove that $P(x, R) \leq P(x, R^{\Pi})$. By construction, $P(x, R) = p$ for some $p \in \Pi$. Therefore, $U(x, R) \supseteq U^p$, so that $x \in L(x_p(R^{\Pi}), R^{\Pi})$, which, by *Pareto Among the Poor*, implies $P(x, R^{\Pi}) \geq p$, the desired outcome.

Part II: 2 implies P*. Let $p \in \mathbb{R}$ and $\overline{\mathcal{R}} \in \mathcal{R}$ be such that for all $R \in \overline{\mathcal{R}}$ there exists $x(R) \in X$ such that $P(x(R), R) = p$. Let $x' \in X$ and $R' \in \mathcal{R}$ be such that

$$L(x', R') \subseteq \bigcup\nolimits_{R \in \mathcal{R}} L(x(R), R) \equiv L^p.$$

We need to prove that $P(x', R') \geq p$.

Let $x_p^w \in X$ be such that $x_p^w \in L^p$ and for all $x \in L^p : x_p^w \, R^w \, x$, that is, x_p^w is the best bundle for R^w in the lower contour of all R in $\overline{\mathcal{R}}$ through their $x(R)$. First, we claim that such a x_p^w exists. By the assumption that X is compact, L^p is compact. The claim follows from the assumption that all preferences are continuous. Second, we claim that $P(x_p^w, R^w) = p$. By construction, there is $R \in \overline{\mathcal{R}}$ such that $P(x_p^w, R) = p$. By *Worst Preferences*, $P(x_p^w, R^w) \geq p$. Assume $P(x_p^w, R^w) > p$. Let $x \in X$ be such that $x \, P^w \, x_p^w$ and $P(x, R^w) = p$. By construction of L^p and x_p^w, $U(x, R^w) \cap L^p = \emptyset$. By *Nested Contours*, $P(x, R^w) < P(x_p^w, R)$, a contradiction, which proves the claim.

Now, let $x'^w \in X$ be such that $x'^w \in L(x', R')$ and $x'^w \, R^w \, x$ for all $x \in L(x', R')$. By the same argument as to prove the claim above, $P(x', R') = P(x'^w, R^w)$. Finally, $L(x', R') \subseteq L^p$ implies that $P(x'^w, R^w) \geq P(x_p^w, R^w) = p$, which yields $P(x', R') \geq p$, the desired conclusion.

Part III: 2 implies 3. Let us construct function $\phi(\cdot, \cdot)$. By Theorem 1, we know that $\phi(z(R^w), R^w) = 0$. Without loss of generality, we can assume that $\phi(0^{\ell}, R^w) = 1$. By continuity, $\phi(\cdot, \cdot)$ is continuous in its first argument. Let $R_i \in \mathcal{R}$, $x_i, x_i' \in X$ be such that $L(x_i, R_i) \cap U(x_i', R^w) \neq \emptyset$ and $intL(x_i, R_i) \cap intU(x_i', R^w) = \emptyset$ (that is, $I(x_i, R_i)$ is tangent to $I(x_i', R^w)$ from below). We need to prove that $\phi(x_i, R_i) = \phi(x_i', R^w)$.

Assume $\phi(x_i, R_i) > \phi(x_i', R^w)$. Let $y \in L(x_i, R_i) \cap U(x_i', R^w)$. By continuity of ϕ, in a neighborhood of y there exists $x_i'' \in X$ such that $x_i'' \, P^w \, x_i'$ and $\phi(x_i'', R_i) > \phi(x_i', R^w)$. By *Pareto among the Poor*, $\phi(x_i', R^w) > \phi(x_i'', R^w)$ and therefore $\phi(x_i'', R_i) > \phi(x_i'', R^w)$, contradicting *Worst Preferences*.

Assume $\phi(x_i, R_i) < \phi(x_i', R^w)$. By continuity of ϕ, there exists $x_i'' \in X$ such that $x_i \, P_i \, x_i''$ and yet

$$\phi(x_i'', R_i) < \phi(x_i', R^w). \tag{5}$$

Let $R_i' \in \mathcal{R}$ satisfy: $I(x_i'', R_i) \in \mathcal{I}(R_i')$ and $I(x_i', R^w) \in \mathcal{I}(R^w)$. By *Nested Contours*, $\phi(x_i'', R_i') = \phi(x_i'', R_i)$ and $\phi(x_i', R_i') = \phi(x_i', R^w)$, which, in view of Eq. (5), violates *Pareto among the Poor*, because $x_i' P_i' x_i''$.

Part IV: It is easy to see that 3 implies 2. ■

Proof. (of Theorem 3) Let $T \in \mathcal{T}$ be the set of bundles for which *$\mathcal{T}$-Poverty Sensitivity* applies. Let $R^w \in \mathcal{R}$ be the preference relation for which *Worst Preferences* is satisfied. Let $z \in T$ be defined by: $z \, I^w \, z(R^w)$. Let $\lambda(\cdot,\cdot)$ be defined by $\lambda(x_i, R_i) = \lambda$ if and only if $x_i \, I_i \, \lambda z$.

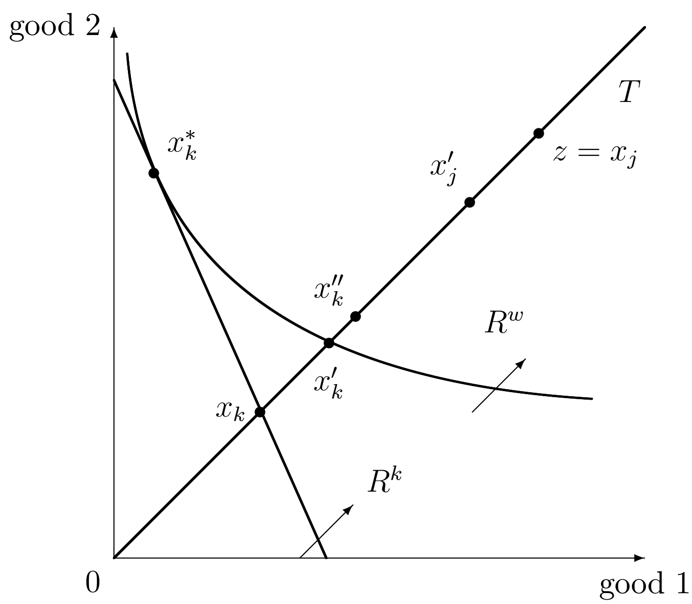

Figure 10: Illustration of the proof of Theorem 3

Assume R^w is not the Leontief preference ordering defined in the proposition (the construction of the key ingredients of the proof is illustrated in Figure 10, where the convexity of T is used by focusing on the line segment from 0 to z). Then, there exist $R_k \in \mathcal{R}$, $x_k^* \in X$ such that

$$x_k^* \in L(x_k^*, R_k) \quad \cap \quad U(x_k^*, R^w), \tag{6}$$

$$intL(x_k^*, R_k) \quad \cap \quad intU(x_k^*, R^w) = \emptyset, \tag{7}$$

$$\lambda(x_k^*, R_k) \quad < \quad \lambda(x_k^*, R^w). \tag{8}$$

Moreover, there exists a neighborhood $U_{x_k^*}$ of x_k^* such that for all $x_k \in U_{x_k^*}$, Eqs. (6)–(8) apply. Therefore, we can assume $\lambda(x_k^*, R^w) < 1$. By Theorem 2:

$$\phi(x_k^*, R_k) = \phi(x_k^*, R^w). \tag{9}$$

Let $x_k, x'_k, x_j, x'_j \in T$ be defined by

$$\begin{aligned} x_k &\in I(x_k^*, R_k), \\ x'_k &\in I(x_k^*, R^w), \\ x_j &= z, \\ x'_j &= x_j - (x'_k - x_k). \end{aligned}$$

By construction, $\phi(x_j, R^w) < \phi(x'_j, R^w)$, and $\phi(x_k, R_k) = \phi(x'_k, R^w)$.

By *Continuity* of P, Theorem 2 implies that there exists $x''_k \in T$ such that $x''_k \, P^w \, x'_k$ and

$$\phi(x_k, R_k) + \phi(x_j, R^w) < \phi(x''_k, R^w) + \phi(x'_j, R^w). \tag{10}$$

Let $N = \{j, k\}$. Let $R'_j = R'_k \in \mathcal{R}$ be such that

$$\begin{aligned} I(x_k, R'_k) &= I(x_k, R_k), \\ I(x''_k, R'_k) &= I(x''_k, R^w), \\ I(x'_j, R'_j) &= I(x'_j, R^w), \\ I(x_j, R'_j) &= I(x_j, R^w). \end{aligned}$$

By *Pareto among the Poor*, $P(x_j, R'_j) < P(x_k, R'_k)$. Therefore, by *$\mathcal{T}$-Poverty Sensitivity*, given that $x_j + x_k = x'_j + x'_k$,

$$P((x'_j, x'_k), (R'_j, R'_k)) < P((x_j, x_k), (R'_j, R'_k)).$$

By *Pareto among the Poor*,

$$P((x'_j, x''_k), (R'_j, R'_k)) < P((x'_j, x'_k), (R'_j, R'_k)).$$

By transitivity,

$$P((x'_j, x''_k), (R'_j, R'_k)) < P((x_j, x_k), (R'_j, R'_k)). \tag{11}$$

By Theorem 2 and Eq. (9),

$$\begin{aligned} P((x'_j, x''_k), (R'_j, R'_k)) &= G(\frac{1}{2}(\phi(x'_j, R^w) + \phi(x''_k, R^w))), \\ P((x_j, x_k), (R'_j, R'_k)) &= G(\frac{1}{2}(\phi(x_j, R^w) + \phi(x_k, R^w))). \end{aligned}$$

Consequently, by Eq. (10),

$$P((x'_j, x''_k), (R'_j, R'_k)) > P((x_j, x_k), (R'_j, R'_k)),$$

in contradiction to Eq. (11). ■

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