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The environmental Kuznets curve in a world of irreversibility

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February 2008

ENVIRONMENTAL ECONOMICS & MANAGEMENT MEMORANDUM

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Abstract   This paper develops an overlapping generations model where consumption is the source of polluting emissions. Pollution stock accumulates with emissions but is partially assimilated by nature at each period. The assimilation capacity of nature is limited and vanishes beyond a critical level of pollution. We first show that multiple equilibria exist. More importantly, some exhibit irreversible pollution levels although an abatement activity is operative. Thus, the simple engagement of maintenance does not necessarily suffice to protect an economy against convergence toward a steady state having the properties of an ecological and economic poverty trap. In contrast with earlier related studies, the emergence of the environmental Kuznets curve is no longer the rule. Instead, we detect a sort of degenerated environmental Kuznets curve that corresponds to the equilibrium trajectory leading to the irreversible solution.

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1 Introduction

In the early 1990s, empirical literature developed with the purpose of studying the relationship between economic growth and pollution (World Bank 1992; Grossman and Krueger 1993, 1995, among others). The main finding was the detection of the environmental Kuznets curve (EKC). The EKC represents the inverted U-shaped relation between income and the concentration of some air (SO_2, CO, NO_x) and water (nitrates, heavy metals, fecal coliform) pollutants. The thinking behind its emergence is the following: during the first stages of industrialization, pollution grows rapidly as priority is given to wealth accumulation. People are more concerned with their employment and their income than with the quality of air or water (Dasgupta et al. 2002). In more advanced development stages, people attach more value to the environment as soon as revenues increase. This "green" awareness then calls for environmental regulations that succeed in reducing pollution.

Over the past 15 years, literature on the EKC has increased exponentially. As mentioned by Copeland and Taylor (2003), recent empirical studies mainly have tried to confirm or invalidate the existence of the EKC by adding new explanatory variables (like an index of democracy) to the regression, verifying its validity for other types of pollutants, or testing the robustness of the results to the data used. To summarize, these contributions (see Stern 1998, for a survey) show that the EKC generally is observed for the aforementioned short-lived and local pollutants while stock pollutants, like CO_2, seem to be monotonically and positively related to income (exceptions are Carson et al. 1997 and Schmalensee et al. 1998 who also detect an EKC for CO_2).

On the theoretical side, this literature attempts to provide (often complementary) explanations of the emergence of the EKC (see Dinda 2004 for a detailed survey). Among the arguments advanced is the role of income elasticity of demand for environmental goods (Brock and Taylor 2004) or the interaction between the scale, composition and technical effects incorporated in the growth process (Copeland and Taylor 2003). The nature of institutions (Jones and Manuelli 2001, Yu 2005), by determining the efficiency of environmental regulation, and free trade (Suri and Chapman 1998) also play crucial roles. In a simple static model, Andreoni and Levinson (2001) propose another explanation based on the existence of increasing returns in abatement technology. In growth models, John and Pecchenino (1994), Selden and Song (1995) and Stokey (1998) show that the EKC results from a switch to abatement activities or, the adoption of less polluting technologies. For instance, John and Pecchenino (1994) emphasize that the regime switch to maintenance creates two distinct development stages. In the first phase (for low levels of income and pollution), agents favour consumption and capital accumulation. In other words, they do not maintain the environment and economic growth is associated with environmental degradation. Once the economy has reached a sufficient level of wealth and/or suffers from important environmental damage, agents are willing to clean up the environment. During the second period, growth is then accompanied by a continuous improvement in the environment. The EKC results from the combination of these two stages.

Two main controversies surround the EKC. The first is methodological. Indeed, Stern (2003) and Perman and Stern (2003), by pointing out the econometric weakness of most of the empirical studies (bias due to the omission of explanatory variables,

problem of causality...) express reserves about the robustness of the studies' results and, even more, about the existence of such a relation. Stern (2003) also criticizes theoretical works that prove the existence of the EKC and notes that: "*it seems fairly easy to develop models that generate EKCs under appropriate assumptions... Furthermore, if, in fact, the EKC for emissions is monotonic as more recent evidence suggests, the ability of a model to produce an inverted U-shaped curve is not a particularly desirable property*". The second controversy is more ethical and concerns a possible interpretation of the EKC that it is possible to pollute as much as one wants today because once the economy becomes rich it will be possible to reverse the trend by compensating for past damage to the environment. Dasgupta and Mäler (2002) oppose the idea that economic development is "mechanically" sustainable, basing their argument on the notion of irreversibility of environmental damage introduced in earlier works in biology and ecology (Holling 1973, Peterman 1980).

These studies showed that some ecosystems can possess more than one stable equilibrium. Multiplicity implies that these ecosystems, when submitted to strong perturbations, are unable to recover their original state. The most famous evidence of irreversibility is given by the eutrophication of shallow lakes. These lakes are subject to pollution resulting from the use of fertilizers (that contains nutritive substances like white phosphorus and nitrates). Nutrients flow through rivers into the lakes. Beyond a certain threshold, this leads to the proliferation of microscopic algae (like phytoplankton) which, by producing light filters and capturing oxygen, disrupt the development of flora and fauna and even cause the extinction of natural species. These ecosystems finally reach a new equilibrium with less biodiversity. More importantly, putting this example aside, Dasgupta and Mäler (2002) stress that the notion of irreversibility also can be extended to global pollution problems like the repercussions of greenhouse gases emissions on the climate.

The potential irreversibility of pollution challenges the assumption, used almost systematically in growth models (see Keeler et al. 1971, Van der Ploeg and Withagen 1991, Smulders and Gradus 1993, John and Pecchenino 1994 among others), that Nature is able to assimilate pollutants at a constant rate. Many authors (Forster 1975, Comolli 1977 and Dasgupta 1982) have proposed a new formulation of the decay function incorporating the idea that high pollution levels drastically alter the waste assimilation capacity of Nature. In fact, from their point of view, it is unreasonable to think that the higher the level of pollution, the greater will be Nature's ability to absorb pollution. Following this recommendation, several authors (Forster 1975, Cesar and de Zeeuw 1994, Tahvonen and Withagen 1996, Tahvonen and Salo 1996 or Toman and Withagen 2000) consider an inverted U-shaped decay function that notably expresses a limited natural capacity to assimilate pollution. The major result of this literature, which basically uses optimal control models, is the existence of multiple equilibria among which are some associated with irreversible pollution.

This study addresses the following question: why may irreversible pollution challenge the emergence of the EKC? More precisely, our aim is to measure the repercussions of irreversibility on the relationship between growth and the environment. Three related questions immediately arise: What is the impact of environmental degradation on prospects for growth? Is it innocuous in relation to economic activities? If not, how does it compromise the process of wealth accumulation? To answer these questions,

[John and Pecchenino](1994)'s overlapping generations model is extended with an inverted U-shaped decay function. The reason is that the paper written by John and Pecchenino (1994) is one of the most famous to generate the EKC in growth models. In addition, our intuition tells us that the relationship obtained in their model with a stock pollutant is widely conditioned by a controversial assumption of a constant rate of assimilation.

We first prove the existence of multiple equilibria with diametrically opposite properties. Some of them notably exhibit irreversible pollution although maintenance activity is operative. This result differs from John and Pecchenino (1994), who show that maintenance is sufficient to improve environmental quality.[1] More interestingly, these "irreversible" equilibria have the characteristics of poverty traps. The important point here is that economic poverty results from ecological poverty. In fact, when agents do not have enough incentive to stop polluting, economic growth is accompanied by the accumulation of ecological debt. But, due to the irreversible character of some pollution, the debt may be such that, once the economy engages in maintenance, this effort does not suffice to avoid the irrevocable degradation of the environment. This, in turn, creates an economic recession since the agents, who have no other choice but to devote a sizeable share of their resources to maintenance, sacrifice wealth accumulation. Thus, environmental degradation is not innocuous to growth. To the contrary, it affects private sector decisions and may lead to a poverty trap.

This result contributes to the growing literature, initiated by [Azariadis and Drazen](1990), on poverty traps. It provides a new explanation of their emergence that resides in the existence of a threshold effect in the regeneration capacity of Nature. Finally, the dynamic analysis echoes [Dasgupta and Mäler](2002)'s warning: the EKC no longer makes sense once one admits the potential irreversibility of environmental damages. Rather, our numerical simulations detect a degenerated EKC corresponding to the equilibrium trajectory that leads to the poverty trap.

The paper is organized as follows: Sect. 2 presents the model; Sect. 3 provides a detailed analysis of the equilibrium; Sect. 4 studies the dynamics with numerical simulations; and Sect. 5 concludes.

2 The model

We develop an overlapping generations model *à la* [Allais](1947), [Samuelson](1958) and [Diamond](1965). In a perfectly competitive world, firms produce a single homogeneous good used for both consumption and investment. In addition, consumption generates polluting emissions.

2.1 The dynamics of pollution and the environment

In the absence of human activity, pollution accumulation, for non-negative levels of the stock P_t, is described by the following equation:

$$P_{t+1} = P_t - \Gamma(P_t),$$ (1)

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[1] This property being the key element explaining the second stage of development in their model.
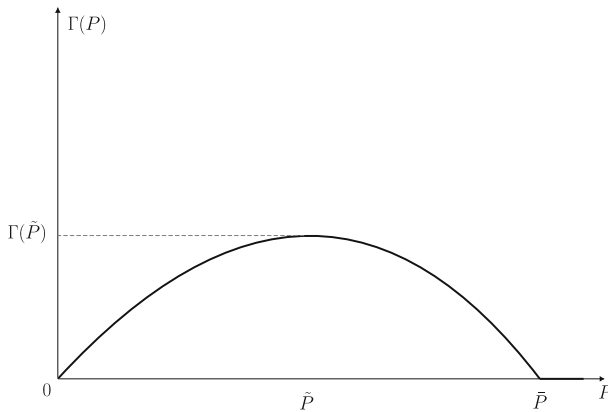
Fig. 1 The assimilation function

where $\Gamma(P_t)$ corresponds to the natural decay function that gives the amount of pollution assimilated by nature each period. Nature's ability to absorb pollution depends on the level of the pollutant concentration. Following Forster (1975), Cesar and de Zeeuw (1994) and Tahvonen and Withagen (1996), we assume an inverted U-shaped decay function (see Fig. 1). Its properties, summarized in the assumption below, convey the idea that after a certain point, high levels of pollution alter the natural regeneration capacity in an irreversible way.

Assumption 1 $\Gamma(P) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is continuous and satisfies $\Gamma(0) = 0$, $\exists! \bar{P} > 0$ such that $\Gamma(P) = 0 \ \forall P \geq \bar{P}$, $\Gamma(P) > 0 \ \forall P \in (0, \bar{P})$; $\Gamma''(P) \leq 0 \ \forall P \in [0, \bar{P})$.

For low pollution levels ($P_t \leq \tilde{P}$ where $\tilde{P}$ is such that $\Gamma'(P) = 0$), the volume absorbed by Nature at first increases with the stock. Beyond the turning point $\tilde{P}$, the assimilation of waste then decreases with the pollutant concentration. Finally, as soon as pollution reaches the threshold value $\bar{P}$, the regeneration capacity is permanently exhausted and pollution becomes irreversible. In other words, once the stock has exceeded the critical level $\bar{P}$, Nature reveals itself unable to assimilate pollution.

As mentioned in the introduction, this assumption is widely accepted to describe the assimilation of pollutants in ecosystems like shallow lakes. Is this formulation a good representation of global environmental problems like global warming?

As for the assimilation of carbon dioxide, there are two main natural stocks of carbon, that are called carbon sinks:

– Continental ecosystems (lands and forests). Assimilation is due to the photosynthesis and the storage of organic residues in soils.
– Oceans.

Let us focus on the storage capacity of oceans. The storage capacity can be interpreted as the assimilation capacity of Nature in our model. Interactions between oceans and the atmosphere are very complex. Simply note that the storage of CO_2 in Oceans is

related to the vertical oceanic circulation, namely, the thermohaline currents.[2] During the annual process of deep water formation, two billion of tons of CO_2 are diluted in the ocean and are captured in the depths for centuries.

Therefore, Oceans provide a huge reservoir for carbon sequestration. In fact, they hold 40,000 billion tons of carbon (fifty times the capacity of the atmosphere) and capture each year 25% of the anthropogenic CO_2 emissions.

Anthropogenic emissions disrupt the sink function of Oceans. The greenhouse effect means that emissions accelerate the rise in temperature.[3] Global warming, by reducing the density of seawater, compromises the thermohaline circulation and lower the deep water formation rates. With the slowing down in thermohaline circulation and lower deep waters formation, the ocean will capture less CO_2. All climate models (see notably Schmittner and Stocker 1997 or Wood et al. 1999) predict a slowing down in the process of carbon assimilation during the next decades. What is more worrying is that several studies (Corbière et al. 2007, Le Quéré et al. 2007), based on the observations of carbon concentration in Oceans, have detected such a deceleration in the North Atlantic and the austral Oceans in the last fifteen years. Therefore, predictions of climate models are confirmed: the weakening of the assimilation of carbon is already a reality.

In the worst scenario, oceans become a source rather than a sink, which means that they reject the gas, thus augmenting the atmospheric CO_2 stock. In turn, this will increase temperature in the northern hemisphere. This self-reinforcing process may lead to the collapse of the thermohaline circulation. In fact, many studies (Rahmstorf 1999, Wood et al. 1999) indicate that a complete shutdown of oceanic circulation and of the high latitude sinking motion may occur in the twenty second century. However, it is worth noting that uncertainty remains about the threshold value for CO_2 concentration that will engage this dramatic change. Indeed, as mentioned by Keller et al. (2004) (see also Schmittner and Stocker 1997), many models suggest that the thermohaline circulation may collapse when the CO_2 concentration rises above a critical value.[4]

Therefore, considering that the assimilation capacity depends on the stock and could collapse for a certain pollution threshold seems to be correct even if the exact shape of the assimilation function is unknown.

Let Q_t be an index of environmental quality such as the quality of air or water. Following John and Pecchenino (1994), pollution is assumed to impair Q_t according to the relation $Q_t = \bar{Q} - P_t$, where $\bar{Q}$ represents the highest stationary level of environmental quality reached when pollution is nil. Assuming non-negative pollution levels boils down to considering $\bar{Q}$ as the upper bound of the domain of definition of Q_t. The dynamics of environmental quality are given by:

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[2] The thermohaline currents take place in Norway and Labrador seas (near the north pole) where the surface waters descend and form deep waters. The formation of deep waters results from changes in water density.

[3] If we refer to the conclusions of the Intergovernmental Panel on Climate Change (IPCC, Fourth Report 2007), during the last century, mean temperatures have increased by 0.74°C. In addition, according to the previsions of the IPCC, this rise will be comprised in the range 1.1–6.4°C at the horizon of 2100.

[4] Note that the other sink, namely continental ecosystems, is also subject to the perturbations caused by CO_2 emissions. There is evidence to suggest that human activities also compromise continental natural storage processes (Fung et al. 2005). For instance, deforestation lowers the potential of assimilation by Nature. It may again involve a positive net transfer of CO_2 from land and forests toward the atmosphere.

$$Q_{t+1} = N(Q_t), \tag{2}$$

with $N(Q_t) : Q_t \in]-\infty, \bar{Q}] \to]-\infty, \bar{Q}]$ the function defined as follows:

$$N(Q_t) = \begin{cases} Q_t & \forall Q_t \leq \bar{Q} - \bar{P}, \\ Q_t + \Gamma(\bar{Q} - Q_t) & \forall Q_t \in]\bar{Q} - \bar{P}, \bar{Q}]. \end{cases} \tag{3}$$

Its properties derive immediately from those of the decay function. For the levels of Q_t corresponding to irreversible pollution ($Q_t \leq \bar{Q} - \bar{P}$), it is simply linear. Beyond the threshold level $\bar{Q} - \bar{P}$, this function is increasing in Q_t and concave (see Appendix A).

The next subsections set out the private agents' choices and trade-offs.

2.2 Production

Under perfect competition, firms produce the final good Y_t with a constant return to scale technology using labor L_t and capital K_t:

$$Y_t = F(K_t, L_t). \tag{4}$$

Since the production function is homogeneous of degree one, it can be expressed by its intensive form: $f(k_t)$ with $k_t = K_t/L_t$, the capital-labor ratio.

Assumption 2 $f(k) : \mathbb{R}^+ \to \mathbb{R}^+$ is $\mathcal{C}^2$. $\forall k > 0$, $f(k) > 0$, $f'(k) > 0$, $f''(k) < 0$. There is an upper bound to the attainable capital $\tilde{k} < \infty$ such that: $f(\tilde{k}) = \tilde{k}$.

Capital depreciates at a constant rate $\delta < 1$. Profit maximization yields:

$$w_t = f(k_t) - k_t f'(k_t), \tag{5}$$
$$r_t = f'(k_t) - \delta, \tag{6}$$

with w_t the wage rate and r_t the real rental rate of capital.

2.3 The households

We consider an infinite horizon economy composed of finite-lived agents. A new generation is born in each period $t = 1, 2, \ldots$, and lives for two periods: youth and old age. There is no population growth and the size of a generation is normalized to one. The young agent born in period t is endowed with one unit of labor, which is supplied inelastically to firms for a real wage w_t. She allocates this wage to savings s_t and maintenance m_t.[5,6] When retired, the agent supplies her savings to firms and earns

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[5] It is possible to reinterpret m_t as a tax levied by a one period-lived government in order to finance maintenance, for the benefit of agents living during its period of office (John et al. 1995).

[6] Note that there is no first period consumption. This simplifying assumption allows us to focus on the crucial trade-off between final good and environmental good consumptions (see, next page, the representative agent problem). In any event, adding a first period consumption would not change the qualitative results.

the return of savings $R_{t+1}s_t$ (with $R_{t+1} = 1 + r_{t+1}$ the interest factor). Her income is entirely devoted to the consumption c_{t+1}. The two budget constraints respectively write:

$$w_t = s_t + m_t, \tag{7}$$

$$c_{t+1} = R_{t+1}s_t. \tag{8}$$

The preferences of the agent born at date t are defined on old age consumption and environmental quality. They are described by the utility function $U(c_{t+1}, Q_{t+1})$.

Assumption 3 $U(c, Q) : \mathbb{R}^+ \times] - \infty, \bar{Q}] \rightarrow \mathbb{R}$ is $\mathcal{C}^2$ with: $U_1 \geq 0$, $U_2 \geq 0$, $U_{11}, U_{22} \leq 0$. The cross derivative is positive $U_{12} \geq 0$.[7] We further assume that $\lim_{c\to 0} U_1(c, Q) = +\infty$.

Following John and Pecchenino (1994) and John et al. (1995), polluting emissions E_t are imputed to the consumption c_t and contribute to the accumulation of pollutant stock. It is possible to control the level of emissions and to improve environmental quality through the maintenance m_t.

Remark Let us refer again to the issue of greenhouse gases emissions. The three mains sources of anthropogenic CO_2 emissions are:

– Transports,
– Residential heating,
– Industries and the energy sector.

So, emissions are due to consumption and production. Our model has a single production sector. It means that consumption and capital are formally the same good. Consequently, imputing emissions to production or to consumption does not really matter.

What should maintenance be in this context? there are many solutions intended to artificially restore the storage/assimilation capacity. Experts of the IPCC (2005) provide information on what they call the *Capture and Storage* of CO_2. This encompasses operations that consist in:

– separating CO_2 from industrial and energy-related sources,
– transporting CO_2 to a storage location and,
– isolating from the atmosphere for a long period.

The main technical storage methods are:

– geological storage in geological formations, such as oil and gas fields, unminable coal beds and deep saline formations. It is worth noting that the geological storage involves technologies that are close to the one developed by the oil and gas industry. So, it has been proved to be economically feasible.

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[7] This assumption expresses the existence of a complementarity between consumption and environmental quality in the agent's utility. An increase in environmental quality enhances the marginal utility of consumption and implies that the agent has a greater desire to consume. The alternative assumption $U_{12} < 0$ reflects, on the contrary, that the two goods are substitutes.

– ocean storage, which is still in the research phase.

Real emissions are represented by the following linear function: $E_t = \beta c_t - \gamma m_t$ with $0 \le \beta, \gamma < 1$. In the presence of human activity, the dynamics of environmental quality then becomes:[8]

$$Q_{t+1} = N(Q_t) - E_t. \tag{9}$$

In this framework, households typically face an intergenerational externality. When the agent consumes, she does not take into account the negative repercussions of her choice on the environmental quality bequeathed to future generations. In the same way, when the young agent decides her maintenance effort, she only cares about the environment she will enjoy in old age. But the agent ignores the future benefits of her "green" investment. The potential irreversibility of pollution strengthens the intergenerational dimension compared with John and Pecchenino (1994). Indeed, due to the present generations' decisions, future generations will likely suffer from an irrevocably degraded environment.

The representative agent born at date t divides her first period income between savings (which determines the consumption of the final good) and maintenance (which influences the "consumption" of the environmental good) in order to maximize her lifetime utility. Taking prices and environmental quality at the beginning of period t as given, the problem is written as:

$$\max_{s_t, m_t, c_{t+1}} U(c_{t+1}, Q_{t+1})$$

subject to,

$$\begin{cases} w_t = s_t + m_t \\ c_{t+1} = R_{t+1} s_t \\ Q_{t+1} = N(Q_t) - E(c_t, m_t) \\ m_t \ge 0. \end{cases}$$

The first order condition (FOC) reads:

$$-R_{t+1} U_1(c_{t+1}, Q_{t+1}) + \gamma U_2(c_{t+1}, Q_{t+1}) + \mu = 0, \tag{10}$$

with $\mu \ge 0$ the Lagrange multiplier that satisfies

$$\mu m_t = 0. \tag{11}$$

The next section is devoted to the study of the competitive equilibrium. The non-negativity constraint on m_t requires to distinguish the case where abatement is working i.e., $m_t \ge 0$, from the one where the agents do not maintain the environment i.e., $m_t = 0$. Moreover, each part of this study must also be divided in two subcases depending on whether or not, environmental quality has reached the irreversibility threshold $\bar{Q} - \bar{P}$.

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[8] It is worth noting that the main difference with John and Pecchenino (1994)'s setting precisely resides in those dynamics since they use the following specification: $Q_{t+1} = (1 - \Gamma)Q_t - \beta c_t + \gamma m_t$ where $0 < \Gamma < 1$ represents the constant rate of assimilation.

3 The competitive equilibrium

Prior to the equilibrium analysis, four important concepts have to be defined:

– the two frontiers delimiting the four possible cases enumerated above,
– the (intertemporal) competitive equilibrium,
– the poverty trap.

3.1 Definitions

3.1.1 The frontier case

We first provide a general definition of the frontiers. Next, a brief discussion is conducted on the issue of the admissibility of equilibria.

Imposing $m_t = \mu = 0$ in the FOC (10) yields:

$$R(w(k_t))U_1(c(w(k_t)), N(Q_t) - \beta c(k_t)) - \gamma U_2(c(w(k_t)), N(Q_t) - \beta c(k_t)) = 0. \tag{12}$$

This equation implicitly defines Q_t as a monotonically increasing function of k_t: $Q_t = Q^f(k_t)$ with $Q^{f\prime}(k_t) \geq 0$.[9]

Definition 1 In the $k - Q$ space, the first frontier, delimiting irreversible pollution levels from reversible ones, corresponds to the *irreversibility threshold: $Q_t = \bar{Q} - \bar{P}$*. The second frontier $Q^f(k_t)$, thereafter called *the indifference frontier*, represents the set of points (k_t, Q_t) where the agents are indifferent to whether or not they invest in depollution. When the system is located in the region above this manifold, environmental quality is sufficiently high and/or wealth is so low that maintenance is nil, while in the opposite situation, it is operative.

Analyzing the admissibility problem boils down to investigating the location of different equilibria with respect to the two frontiers separating, on the one hand, the interior space ($m_t \geq 0$) from the corner one ($m_t = 0$) and, on the other hand, the irreversible pollution space from the reversible zone.

This section addresses the existence of equilibria in the four regions of the $k - Q$ space. Now, assume that, for each dynamical system, there exists at least one stable steady state. Admissibility refers to the following reasoning: it is possible, during the convergence toward a stable solution of a determined zone, that the equilibrium path crosses one or the other frontier before reaching the steady state. But as soon as the trajectory goes through a frontier, the system is governed by new dynamics totally different from the ones valid in the previous region. In other words, the stable solution

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[9] Total differentiation of (12) gives

$$Q^{f\prime}(k) = \frac{R'w'U_1 + Rc'w'U_{11} - \beta c'U_{12} - \gamma c'(w'U_{12} - \beta U_{22})}{N'(\gamma U_{22} - RU_{12})},$$

and, under our assumptions, this ratio is positive.

in consideration is not admissible since, once the frontier is crossed, the economy will converge to another stable solution associated with the new significant dynamics.

3.1.2 The (intertemporal) competitive equilibrium

Let us call a *trajectory* of the economy a sequence of per capita variables $\{c_t, m_t, s_t\}$, of aggregate variables $\{L_t, K_t, Q_t\}$ and of prices $\{R_t, w_t\}$. Then, the equilibrium with perfect foresight is defined as follows:

Definition 2 An *intertemporal competitive equilibrium* is a trajectory of the economy such that:

i/ households and firms are at their optimum: the FOC (10) and the two conditions (5) and (6), for profit maximization, are satisfied,

ii/ all markets clear: $L_t = N = 1$ and $K_{t+1} = s_t (= k_{t+1})$,

iii/ budget constraints (7) and (8) are satisfied,

iv/ the dynamics of environmental quality are given by (9).

Equilibrium dynamics will be derived from this definition in both the zero and the positive maintenance regions.

3.1.3 The poverty trap

Having characterized equilibrium dynamics, we will next assess the existence of steady states for each region. Among the steady states, some may have the features of a poverty trap.

Definition 3 The *poverty trap* is the steady state that exhibits

– *environmental poverty*, which means that:

$$\lim_{t \to \infty} Q_t = Q_\infty,$$

with $-\infty < Q_\infty < \bar{Q} - \bar{P}$.

– *economic poverty*: assume multiple equilibria exist. Then, economic poverty refers to the steady state with the lowest level of capital (and consumption).

In other words, the poverty trap corresponds to the steady state with irreversible pollution and the lowest level of wealth.

The next sections are devoted to the equilibrium analysis.

3.2 Zero maintenance equilibrium

The first part of the analysis deals with the case where the non negativity constraint on maintenance is binding i.e., $m_t = 0$. A justification of this study is based on the observation that less developed economies may not be concerned with the protection

of the environment and favor wealth accumulation instead. In other words, these economies, in the first stages of development, may be too poor to have the incentive to abate polluting activities (Dasgupta et al. 2002).

In this context, the representative agent does not face any trade-off since she allocates her entire first period income to savings $w_t = s_t$. Equilibrium dynamics are derived from the combination of (5)–(8), (9) and the market clearing condition for capital:

$$\begin{cases} k_{t+1} = f(k_t) - k_t f'(k_t), \\ Q_{t+1} = N(Q_t) - E(k_t), \end{cases} \tag{13}$$

and, one may note that capital accumulation is independent of environmental quality. Emissions amount to a share β of the consumption: $E(k_t) = \beta c(k_t)$ with,

$$c(k_t) = (1 - \delta)k_t + k_t f'(k_t). \tag{14}$$

Before studying the existence of solutions to the system (13), let us define capital's share of output and the elasticity of substitution between capital and labor as follows:

$$s(k_t) = \frac{k_t f'(k_t)}{f(k_t)}, \tag{15}$$

$$\sigma(k_t) = -\frac{(1 - s(k_t)) f'(k_t)}{k_t f''(k_t)}. \tag{16}$$

In the remainder of the paper, the following conditions are supposed to hold:

Assumption 4 The technology satisfies:

$$\lim_{k_t \to 0} \frac{f(k_t) - k_t f'(k_t)}{k_t} > 1, \tag{17}$$

$$\sigma(k_t) \geq 1 - s(k_t). \tag{18}$$

These conditions are common in the literature that studies the equilibrium properties of the OLG one sector model (without pollution issues). Condition (17) is analogous to *the strengthened Inada condition* introduced by Galor and Ryder (1989) to avoid the trivial equilibrium with zero capital. It ensures that the first unit of capital is sufficiently efficient, in terms of labor productivity (recall that the numerator in (17) corresponds to the wage $w(k_t)$). It is also a necessary condition of existence of a non trivial steady state (see also De la Croix and Michel 2000). Condition (18) states that the elasticity of substitution between capital and labor is higher than the labor share of output.[10] It is similar to the one imposed by Wendner (2004) who also studies existence, uniqueness and stability of equilibrium for the OLG economy. This condition

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[10] These conditions are notably satisfied by the CES technologies, $F(K, L) = (\alpha K^{-\rho} + (1 - \alpha)L^{-\rho})^{-\frac{1}{\rho}}$, when $\rho \in (-1, 0]$. In addition, the second one seems quite reasonable since most of the estimations of labor's share of output and the elasticity of substitution give values respectively comprised in the range $[0.6, 0.7]$ and close to 1.

excludes technologies for which capital's share in output, $s(k)$, decreases too strongly in response to a rise in the capital-labor ratio. It implies $c'(k) > 0, \forall k > 0$.

The first proposition states the existence conditions of corner steady states by paying particular attention to the level of environmental quality compared with the irreversibility threshold.

Proposition 1 *For the dynamics given by* (13),

- *i/ there is no steady state associated with irreversible pollution,*
- *ii/ there exists a corner steady state that exhibits a level of environmental quality above $\bar{Q} - \bar{P}$ if and only if*

$$\max_{P \in [0, \bar{P}]} \Gamma(P) \geq E(k_{cr}^*), \qquad (19)$$

where k_{cr}^* is the steady state level of capital.

Proof See Appendix A.1.   □

The lack of steady state characterized by the irreversibility of pollution is explained by the environment's inability to stabilize to a constant long run level. In fact, when agents do not depollute, environmental quality perpetually deteriorates since the assimilation capacity of Nature is exceeded and there is no force able to compensate for the polluting emissions. The only means to stop the degradation is to cease consuming and, when capital is essential to production, to furthermore stop all productive activity. But, under the assumptions made on preferences, this limit case can be excluded. In terms of dynamics, it implies that if the economy is located in this subspace, the equilibrium path will necessarily cross the indifference frontier to reach the region where the agents have the incentive to maintain the environment.

Condition (19), for a solution with reversible pollution, corresponds to a rewriting, in our general equilibrium framework, of the condition used by Tahvonen and Withagen (1996). It conveys the idea that the maximum potential of natural assimilation is intrinsically higher than the stationary level of emissions. In contrast to the previous situation, this condition ensures that environmental quality will reach, in the long run, a constant level when stationary emissions will be entirely absorbed by Nature.

The stability analysis (of corner steady states) is performed in the Appendix B.1. It is worth noting that, if the inequality in (19) holds with ">" instead of "≥", then there exists a reversible asymptotically stable corner steady state. Finally, by construction, reversible steady states satisfy the admissibility condition concerning their location with respect to the irreversibility threshold.

Thereafter, the focus is put on the very distinct features of the positive maintenance equilibrium.

3.3 Positive maintenance equilibrium

When households are sufficiently wealthy and/or suffer from high environmental damages, they decide to engage in maintenance: $m_t \geq 0$. In this case, their optimization

problem admits an interior solution and the FOC (10) rewrites:

$$R_{t+1}U_1(c_{t+1}, Q_{t+1}) = \gamma U_2(c_{t+1}, Q_{t+1}). \tag{20}$$

This condition expresses the trade-off inherent in the maintenance decision at period t. An increase in the effort m_t is a means to improve environmental quality and thus to enhance welfare (RHS of (20)). However, it also implies a fall in the non-environmental component of welfare (LHS) since both savings and old age consumption decrease. Consequently, the agent chooses m_t to equate the marginal benefit of maintenance to its marginal cost.

The equilibrium analysis consists of considering the system of Eqs. (5)–(9), (20) and the market clearing conditions. Combining these equations yields the expression of the maintenance decision as a function of the capital stock

$$m_t = f(k_t) - k_t f'(k_t) - k_{t+1}, \tag{21}$$

and, since consumption is still given by (14), the emissions function E_t rewrites:

$$E(k_t, k_{t+1}) = \beta \left((1 - \delta)k_t + k_t f'(k_t)\right) - \gamma \left(f(k_t) - k_t f'(k_t) - k_{t+1}\right). \tag{22}$$

By substituting Eqs. (6) and (14) into the FOC (20), we get the following equation,

$$R(k_{t+1})U_1(c(k_{t+1}), Q_{t+1}) - \gamma U_2(c(k_{t+1}), Q_{t+1}) = 0, \tag{23}$$

which implicitly defines an equilibrium relation, valid for any t, between Q_t and k_t:

$$Q_t = Q^e(k_t). \tag{24}$$

It is worth noting that this increasing[11] relation governs the dynamics in the whole positive maintenance space.

Finally, the substitution of Eq. (22) into the law of motion for the environment (9), together with (23), completely characterizes the equilibrium dynamics:

$$\begin{cases} R(k_{t+1})U_1(c(k_{t+1}), Q_{t+1}) - \gamma U_2(c(k_{t+1}), Q_{t+1}) = 0, \\ Q_{t+1} = N(Q_t) - E(k_t, k_{t+1}). \end{cases} \tag{25}$$

The existence conditions of interior steady states are summarized in the following propositions. Two distinct cases are envisioned depending on whether or not environmental quality has exceeded the irreversibility threshold. The analysis is restricted to

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[11] Total differentiation of (23) gives:

$$Q^{e'}(k_t) = -\frac{R'U_1 + Rc'U_{11} - \gamma c'U_{12}}{RU_{12} - \gamma U_{22}},$$

and, under our hypothesis, this derivative is positive.

the interval $[0, \bar{k}]$, where $\bar{k}$ is defined as follows:

$$\bar{k} = \inf\{k > 0/w(k) = k\}.$$

This restriction is discussed in the Appendix A.2.1. It is shown that it does not affect the relevancy of our analysis.

Proposition 2 *When capital is not essential to production ($f(0) > 0$), there exists a steady state with irreversible pollution. Let us now consider the case where capital is essential to production ($f(0) = 0$). Then, there exists an irreversible steady state if*

$$\lim_{k \to 0} E'(k) < 0. \tag{26}$$

Proof See Appendix A.2.2. □

In both cases, stationary emissions $E(k)$ are negative in the neighbourhood of zero. Considering negative emissions boils down to assuming that there exists a man-made environment. This situation, reported by Forster (1975) among others, is all the more likely in our model since the impact of an increase in k on maintenance exceeds its repercussions on consumption.[12] When $\beta > \gamma$, it also supposes that the difference between the parameters of emissions diffusion β and maintenance efficiency γ is relatively tight.

Note that this property guarantees the stabilization of environmental quality to a constant level since emissions will cancel (each others) for a level of capital in $[0, \bar{k}]$. But it certainly does not mean that, during the transitional dynamics, emissions $E(k_t, k_{t+1})$ can be negative.

Let (k_{ii}^*, Q_{ii}^*) be an interior steady state with irreversible pollution. The admissibility of interior irreversible steady states (thereafter IISS) requires $Q_{ii}^* \leq \bar{Q} - \bar{P}$, which is equivalent to $k_{ii}^* \leq \underline{k}$ with $\underline{k} = (Q^e)^{-1}(\bar{Q} - \bar{P})$. In the next step of the analysis, we refer to this condition to prove the existence of reversible steady states.

Proposition 3 *i/ Assume first that all IISS are admissible ($\sup \{k_{ii}^*\} \leq \underline{k}$) or that $\underline{k}$ is comprised between two successive, odd then even, levels k_{ii}^* ($\underline{k} \in [k_{ii}^{*a}, k_{ii}^{*a+1}]$, with a an odd number). Then there exists a reversible steady state if:*

$$\max_{P \in [0, \bar{P}]} \{\Gamma(P)\} \geq \max_{k \in [0, \bar{k}]} \{E(k)\}, \tag{27}$$

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[12] If one refers to the Cobb-Douglas example (obtained by letting $\rho \to 0$ in the CES), it is easy to show that

$$\lim_{k \to 0} \frac{c'(k)}{m'(k)} < \frac{\alpha}{1 - \alpha},$$

with $\alpha \in (0, 1)$ the capital's share of output. Therefore, under the common assumption that $\alpha < 1/2$, an increase in k has a stronger impact on $m()$ than on $c()$ in the neighbourhood of zero.

and,

$$Q^e(\bar{k}) \geq \bar{Q} - \tilde{P}. \tag{28}$$

ii/ Suppose next that no IISS is admissible (inf $\{k_{ii}^*\} > \underline{k}$) *or that $\underline{k}$ belongs to an interval whose bounds correspond to two successive, even then odd, levels k_{ii}^* ($\underline{k} \in [k_{ii}^{*b}, k_{ii}^{*b+1}]$, with b an even number). Then there exists a reversible steady state if* (28) *holds and if:*

$$\max_{P \in [0,\bar{P}]} \{\Gamma(P)\} \leq \max_{k \in [0,\bar{k}]} \{E(k)\}. \tag{29}$$

Proof See Appendix A.2.3. □

In the first case *i/*, condition (27) states that the maximum potential of assimilation is higher than the maximum volume of emissions on the significant domains of variation of k and Q. It is similar to the necessary and sufficient condition (19), used to show the existence of corner steady states, since, in the positive maintenance space, emissions reach their maximum when the depollution effort vanishes, that is, at the upper bound $\bar{k}$. In addition, (28) is a technical condition that ensures some correspondence between these domains.

In the second configuration *ii/*, the sense of the inequality in (29) is reversed. This alternative is considered by Tahvonen and Salo (1996) in their optimal control model with irreversible pollution. In contrast to (27), it implies that the transgression of the irreversibility threshold is most likely to occur.

By construction, reversible steady states also satisfy the admissibility condition imposing $Q^* > \bar{Q} - \bar{P}$. And one may note that in both cases the three types of equilibrium thus far discussed can exist simultaneously (see Fig. 6, in the Appendix, for the representation of the phase diagram).

A synthesis of equilibrium properties[13] shows that a first implication of the inverted U-shape decay function is the existence of multiple steady states. These equilibria have very distinct features since some of them exhibit a safe environment while others are associated with irreversible pollution. The coexistence of these two types of interior solutions challenges John and Pecchenino (1994)'s result that maintenance is a "sufficient" condition for improving environmental quality. Indeed, it appears that maintaining the environment does not necessarily ensure its improvement and, furthermore, does not protect Nature against irrevocable degradation.

The existence of multiple equilibria with irreversible pollution already has been shown, notably by Tahvonen and Withagen (1996), in partial equilibrium models with infinitely-lived agents. The original result here resides in the consequence of the interaction between capital and environmental quality. The analysis of interior solutions reveals that a process of unregulated growth[14] can drive the economy toward a poverty trap. Poverty traps correspond precisely to irreversible steady states. In fact, such a long-run state is first an ecological trap since environmental quality is below the irreversibility threshold $\bar{Q} - \bar{P}$. But it is also an economic trap because the level

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[13] The analysis of local stability is performed in Appendix B.2.

[14] Or, if we consider John et al. (1995)'s approach instead, a process of growth where the regulation of emissions is a myopic government's responsibility.

of wealth, measured by capital stock, is lower than the level reached at any reversible (corner or interior) solution.

More precisely, the main feature of the model is that ecological poverty generates economic poverty. The general thinking that explains the emergence of economic poverty is the following. In the "irreversible" region, pollutant concentration is such that, on the one hand, nature no longer assimilates pollution and, on the other, households suffer from the damages caused by pollution. It is worth noting here that agents continue maintenance activities despite the irreversibility of pollution. Indeed, the maintenance effort is the only means to slow down the accumulation of pollution and to limit the disutility it causes. In order to control this damage, they have no other option than to devote a sizeable share of their resources to maintenance. This decision is detrimental to consumption (that always remains positive according to preferences) and savings. A new stage of economic recession occurs since the agents' reaction to environmental degradation causes a break in capital accumulation. As the opportunity cost of maintenance becomes more and more severe, the economy fails to artificially restore environmental quality. Finally, it manages to stabilize to a steady state with a constant, but irrevocably degraded, environment and a very low level of wealth.

This result contributes to the growing literature on poverty traps. Since the seminal paper of Azariadis and Drazen (1990), numerous studies have tried to determine the factors explaining the emergence of such states (see, among others, Xepapadeas 1997, Azariadis 2000). Among the arguments frequently invoked is an insufficient investment in human capital or the existence of threshold externalities affecting production or education technologies. These externalities imply that the production of education, for instance, first exhibits decreasing returns to scale for low levels of human capital. Then, above a critical value of the stock, there is a change in the technology and the returns to scale become constant and even increasing. In this context, the country that does not reach the critical level of human capital is doomed to remain in a poverty trap while the country that accumulates knowledge to a level above the threshold can experience sustained growth.

Here, the new mechanism proposed is also based on the notion of threshold effects but it comes from the environmental side of the system instead. To summarize, economic activity, through the polluting emissions it creates, can lead to the exceeding of the ecological threshold beyond which the natural assimilation capacity vanishes. This, in turn, causes a stage of economic recession that drives the economy into a poverty trap.

Whether the economy will be dragged down into the poverty trap or if, to the contrary, it will enjoy a stage of economic growth combined with an increase in environmental quality obviously depends on its initial location. Any economy with initially low levels of wealth and environmental quality would probably have the greatest difficulties in escaping from this impoverishment region. The final part of the paper deals with this issue. This section performs numerical simulations intended to illustrate the most striking equilibrium trajectories. Particularly, a discussion on the possible emergence of an environmental Kuznets curve, in our setting, will be conducted.

4 Dynamic analysis: the degenerated EKC

The aim of this section is to isolate the impact of an inverted U-shaped decay function on environmental and economic dynamics. The key question is whether or not the potential irreversibility of pollution challenges the main result of John and Pecchenino (1994), that is, the existence of the EKC. To answer this question, the following functional forms are used:

For $P_t \geq 0$, the decay function is given by a function defined piecewise:

$$\Gamma(P_t) = \begin{cases} \theta P_t(\bar{P} - P_t) & \forall P_t < \bar{P}, \\ 0 & P_t \geq \bar{P}, \end{cases}$$

the volume of pollution assimilated is first increasing in the stock until the level $\bar{P}/2$, it then is decreasing. Beyond the critical threshold $\bar{P}$, the natural capacity to absorb pollution vanishes.

Following John and Pecchenino (1994), we use a Cobb-Douglas technology:

$$Y_t = A K_t^{\alpha} L_t^{1-\alpha},$$

with $\alpha \in (0, 1)$ and $A > 0$ a scale parameter.

The household's preferences are characterized by a separable utility function.[15] This function is growing and concave in consumption and the environment (for $Q \leq \bar{Q}$):

$$U(c_{t+1}, Q_{t+1}) = \log c_{t+1} - \frac{1}{2}(\bar{Q} - Q_{t+1})^2.$$

Numerical simulations are performed for the following set of parameters:

$$\{A = 2.52, \theta = 0.09, \gamma = 0.2, \beta = 0.3, \alpha = 0.3, \delta = 0.6, \bar{P} = 5, \bar{Q} = 7\}.$$

It is possible to show only two admissible steady states exist.[16] Both solutions are located in the positive maintenance space. The first is associated with reversible pollution while the other exhibits irreversibility.

Figure 2 first represents the attraction basins of each solution. The partition of the $k - Q$ space is straightforward: starting from any point (K_0, Q_0) located in the upper and dark (resp. lower and bright) area, the economy reaches, in the long run, the reversible (resp. irreversible) steady state. Therefore, when Q_0 is lower than the critical level $\bar{Q} - \bar{P}$, the dynamics lead the economy with certainty into the environmentally poor steady state. On the contrary, an economy initially endowed with a sufficient amount of environmental quality will enjoy, in the long run, a safe environment.

More importantly, it is worth noting that the set of initial points characterized by reversible pollution and associated with a convergence toward the irreversible state

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[15] The results do not depend on the (simplifying) assumption of separability.

[16] Parameter restrictions and equilibrium properties, for the numerical example, are summarized in Appendix C.
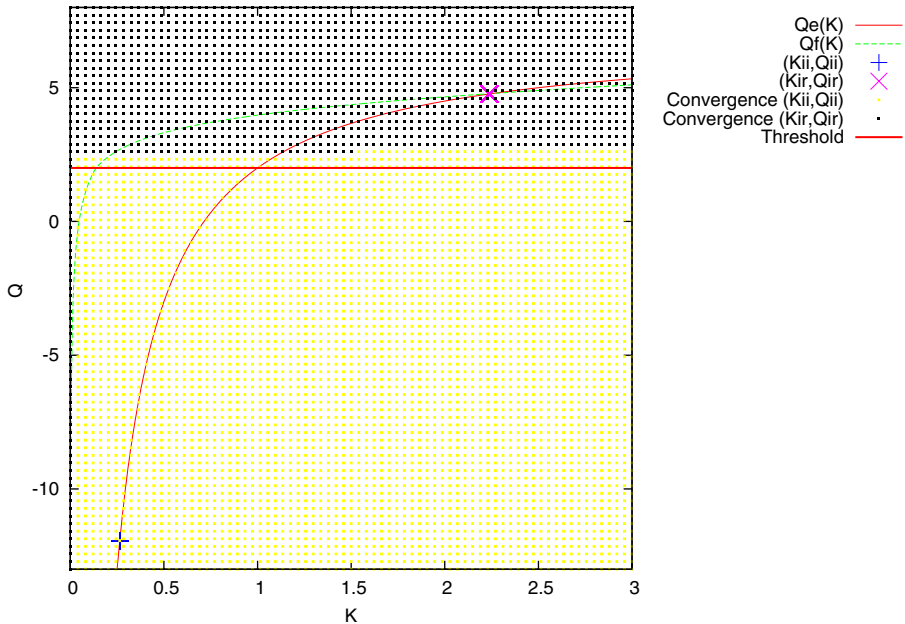
Fig. 2 The attraction basins. The figure contains the equilibrium relation $Q^e(k_t)$ the indifference frontier $Q^f(k_t)$ and the threshold $\bar{Q} - \bar{P} = 2$. The two steady states are located on $Q^f(k_t)$. Symbol "+" (*resp.* "x") represents the irreversible (resp. reversible) steady state

is not empty. These initial conditions belong to the lower basin, on the left of the indifference frontier (see Fig. 3). It means that an economy may be attracted to this kind of steady state despite its relatively high endowment in environmental quality.

We now focus on the equilibrium trajectories' properties. By analogy with John and Pecchenino (1994), the simulations start from initial conditions in the zero maintenance region. In addition, the only initial states (k_0, Q_0) considered are reversible. The dynamics exhibit very distinct features compared with John and Pecchenino (1994).[17, 18] Indeed, the analysis reveals that the EKC is no longer the rule and points instead to the following trajectories.

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[17] The authors detect a V-shaped relation between capital and environmental quality during the convergence toward the interior steady state. This EKC is the result of a break in maintenance. Starting from a point located in the zero maintenance region, agents first have no incentive to invest in maintenance and capital accumulation is accompanied by environmental degradation. When pollution emissions are sufficiently damaging (or equivalently, once the trajectory has crossed the frontier $Q^f(k_t)$), agents decide to engage in maintenance. In this second phase, environmental quality increases with capital until the positive maintenance steady state is reached. Finally, by combining these two different development stages, the authors obtain a sort of EKC.

[18] Among the related studies, we may also mention the work of Zhang (1999). Zhang (1999) focuses on the transitional dynamics in the John and Pecchenino (1994) model. Under specific conditions, notably involving preferences towards the environment, Zhang (1999) shows that this simple model can exhibit very complex dynamics with cycles or perpetual fluctuations. But, the author restricts the analysis to the positive maintenance region and does not take into account the potential irreversibility of pollution.
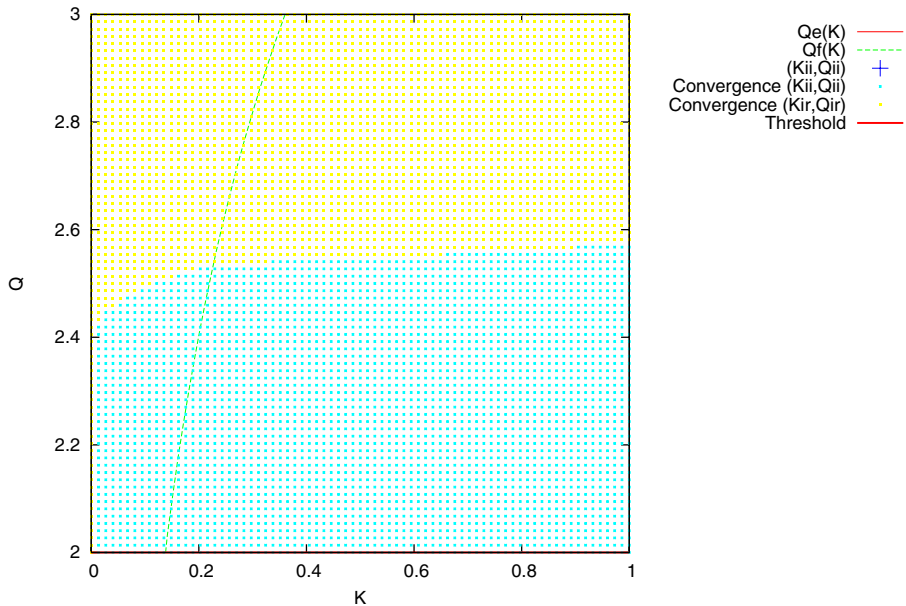
Fig. 3 Set of initial reversible conditions converging toward the irreversible state

The first trajectory (see Fig. 4), with Q_0 close to the upper bound $\bar{Q}$, corresponds to a monotonically decreasing convergence toward the reversible steady state. The initial endowment Q_0 is such that the economy remains in the zero maintenance area during almost all the transitional dynamics. Agents do not allocate any resources to maintenance and the economy enjoys a phase of sustained growth. In turn, the quality of the environment continuously declines. The degradation is first relatively slow but, as soon as capital approaches its stationary level, polluting emissions cause a severe fall in environmental quality (until its stabilization). When the indifference frontier $Q^f(k_t)$ is crossed, the economy starts to abate. However, this effort does not allow a stop to environmental deterioration.

The other striking trajectory, with diametrically opposite properties, is detected for an initial conditions with low, but still higher than the threshold, levels of environmental quality (see Fig. 5). This trajectory illustrates the case where the economy is dragged down into the ecological and economic poverty trap. The intuition behind the emergence of the degenerated EKC is the following. Starting from an initial state with relatively low levels of capital and environmental quality, there first is a stage of economic development accompanied by (slowly) worsening of environmental quality. During this phase, agents benefit from a sufficient environment and choose not to abate their activities. They favour consumption and wealth accumulation instead. The economy becomes rich but also accumulates an ecological debt that will be the responsibility of future generations. However, since agents do not take into account intergenerational externalities, this debt will exceed the gains from higher wealth. In fact, once the trajectory crosses the indifference frontier, maintenance becomes positive and the second development stage begins. During this second phase, agents are
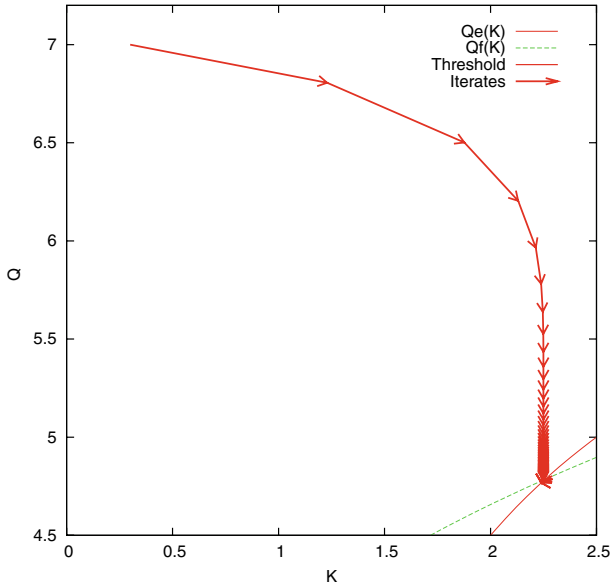
Fig. 4 Equilibrium trajectory leading to the reversible steady state
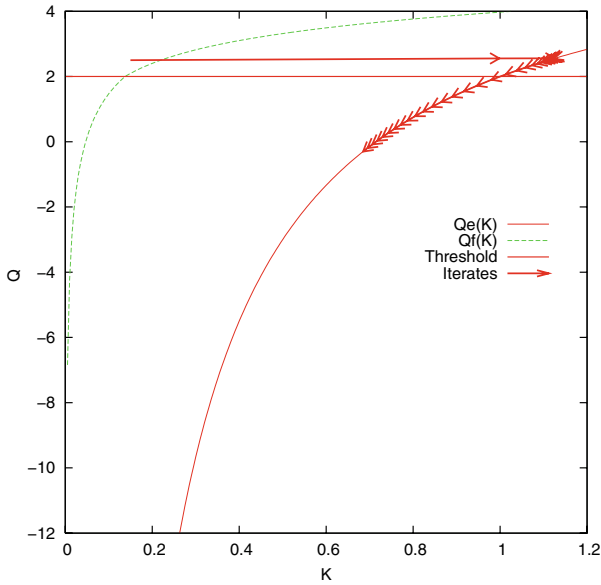
Fig. 5 Equilibrium trajectory: the degenerated EKC

willing to devote a share of their resources to maintenance to control the damages arising from pollution. However, due to their limited budget and the need to consume, they cannot allocate enough resources to maintenance and fail to compensate for the harmful effect of polluting emissions, which is exacerbated by the weakness of the

natural regeneration capacity.[19] Thus, environmental quality does not stop deteriorating. Moreover, this effort is made to the detriment of savings. The resulting break in capital accumulation is revealed in the fact that, at each date, the stock of capital will set up at a lower level.

This impoverishment mechanism finally re-occurs, from period to period. Its main implication resides in the exceeding of the irreversibility threshold. In this context, the economy is unable to stop the fall in environmental quality. This, in turn, causes a phase of economic recession since agents devote increasing amounts of resources to maintenance in order to limit the disutility of the unbroken rise in emissions. In the very long run, the economy reaches the steady state characterized by both a level of capital almost nil and a negative environmental quality.

The qualitative properties of this trajectory confirm Dasgupta and Mäler (2002)'s warning against any hasty interpretation of the EKC. Under the assumption of a constant rate of decay, the economy can pollute with complete impunity since it always will be able to reverse the trend by compensating for past environmental damage, with the support of an increasing natural assimilation of the pollution stock. If, on the contrary, the potential irreversibility of pollution is taken into account then the exceeding of critical ecological thresholds implies that the economy can not depend on the natural regeneration process anymore. Ultimately, it fails to absorb on its own the environmental debt accumulated from past polluting activities. The economy is then doomed to suffer irreparable degradation of the environment.

5 Conclusion

The purpose of this study is to confront the notion of irreversibility of pollution with the environmental Kuznets curve (EKC). Our analysis is intended to challenge the conclusions of John and Pecchenino (1994), who highlight an equilibrium relationship between growth and environmental quality that has the same characteristics as the EKC. The emergence of the EKC in their overlapping generations model is based on a break in abatement activity. A possible interpretation of this result is that it will always be possible to remedy the damage caused by pollution in the first stages of economic development, provided that the economy devotes, once the need arises, a sufficient amount of resources to maintenance. In this paper, we seek to discover the extent to which John and Pecchenino (1994)'s result is submitted to the assumption of a constant rate of pollution assimilation. Our approach echoes Dasgupta and Mäler (2002)'s statement that the concept of the EKC, and its interpretation given above, must be rejected once one admits the potential irreversibility of environmental damages.

To answer our question, their framework is extended by considering an inverted U-shaped assimilation function (similar to the one used by Forster 1975) rather than a constant rate of pollution assimilation. The equilibrium analysis reveals three important features of the model. First, there exist multiple equilibria with diametrically opposite properties. Some are associated with irreversible pollution although the abatement activity is operative. The major implication of this result lies in the fact that an

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[19] Here, the economy is located in the region where the assimilation function is decreasing in pollution.

economy, having vastly impaired the environment by placing greater importance on economic growth, may be unable to reverse the trend. In other words, the simple fact of engaging in maintenance may not suffice to avoid the convergence toward a long term state with the characteristics of an ecological poverty trap. Second, ecological poverty creates economic poverty in turn. Therefore, these steady states also correspond to poverty traps in terms of wealth. Finally, noteworthy equilibrium trajectories are illustrated with numerical simulations. Our intuition that the EKC, as it is depicted by John and Pecchenino (1994), is no longer the rule when the possibility of irreversibility is taken into account, is confirmed. The convergence toward a poverty trap makes appear a sort of degenerated EKC instead.

The existence of such a long-term state legitimizes the intervention of public authorities in the management of pollution problems, and will lead us, in future research, to study more deeply the means and consequences of such intervention. Further developments of this paper may consist notably of assuming that pollution proceeds from production activity, and that it is controlled thanks to a pollution permit market. In this context, the question is first to identify under what conditions this system of regulation allows an economy to avoid a drifting towards a poverty trap. In addition, it should be interesting to assess the effects of a reform of the pollution permit system on the growth perspectives of an economy.

Again, assuming emissions as a by-product of production, another interesting line of research will consist in introducing technological change in the present model. Ono (2003) extends John and Pecchenino (1994)'s model with a process of innovation and distinguishes two phases of growth: "the no-innovation growth regime" and "the innovation-led growth regime". Following his approach, we may consider technological improvements intended to promote a more efficient use of harmful emissions. One may expect that, with the opportunity to improve the production process, the economy will more likely be able to undo the environmental damage that accompanies growth and/or to remain above the irreversibility threshold.

Appendix
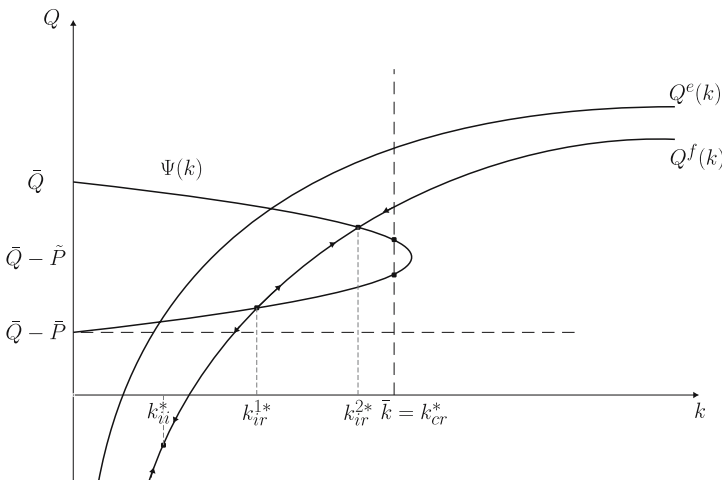
Fig. 6 The phase diagram

Note that the function $\Psi(k)$, implicitly defined by the following equation

$$\Gamma(\bar{Q} - Q) = E(k),$$

represents the second equilibrium relation between capital and environmental quality that holds at interior steady states (see Appendix A.2.3).

A Proofs of Propositions 1, 2 and 3

Let us first assess the properties of $N(Q)$:
$N(Q)$ is defined piecewise on the interval $]-\infty, \bar{Q}]$ (the upper bound comes from the fact that we consider only non negative pollution levels):

$$N(Q) = \begin{cases} Q & \forall Q \leq \bar{Q} - \bar{P} \\ Q + \Gamma(\bar{Q} - Q) & \forall \bar{Q} - \bar{P} < Q \leq \bar{Q} \end{cases}$$

For any value of Q lower than the critical threshold (that is $Q \leq \bar{Q} - \bar{P}$), $N(Q)$ is simply linear. Thus, we have $N(0) = 0$, $N(\bar{Q} - \bar{P}) = \bar{Q} - \bar{P}$ and $N'(Q) = 1$ $\forall Q \leq \bar{Q} - \bar{P}$.

Once the irreversibility frontier $\bar{Q} - \bar{P}$ is crossed, the natural regeneration process is operative and the law of motion becomes: $N(Q) = Q + \Gamma(\bar{Q} - Q)$. From the properties of the decay function $\Gamma(.)$, we know that $N(Q) > 0$ $\forall Q \in]\bar{Q} - \bar{P}, \bar{Q}]$, $\lim_{Q \to \bar{Q} - \bar{P}} N(Q) = \bar{Q} - \bar{P}$ and $N(\bar{Q}) = \bar{Q}$. The derivative $N'(Q) = 1 - \Gamma'(\bar{Q} - Q)$ is positive $\forall Q \leq \bar{Q}$. Il also appears that $N'(Q) \geq 1 \leftrightarrow Q \leq \bar{Q} - \tilde{P}$. Finally, this function is concave since $N''(Q) = \Gamma''(\bar{Q} - Q) \leq 0$.

A.1 Existence of corner SS (Proposition 1)

A steady state solves the following system of equations:

$$\begin{cases} k = f(k) - kf'(k), \\ \Gamma(\bar{Q} - Q) = E(k). \end{cases} \tag{30}$$

The first equation rewrites $w(k) - k = 0$. By definition, the wage is a non negative and increasing function of k: $w(k) \geq 0$ and $w'(k) = -kf''(k) \geq 0$ $\forall k \in \mathbb{R}^+$. Under Assumption 2, we have $\lim_{k \to 0} w(k) = f(0)$ and $\lim_{k \to 0} kf'(k) = 0$. Furthermore, it satisfies $\lim_{k \to +\infty} w(k)/k = 0$. Thus, the wage function is located below the 45° line when capital tends towards infinity. If capital is not essential to production ($f(0) > 0$), then there exists an intersection between $w(k)$ and k (where $m(k) = 0$). Otherwise, the previous property keeps holding if we impose: $\lim_{k \to 0} w(k)/k > 1$ (condition (17)).

It may exist an odd number, n, of intersections between $w(k)$ and the 45° line. Let us denote $J = \{k > 0/w(k) = k\}$ and $\bar{k}^j$, with $j = 0, \ldots, n - 1$, any element of J.

If pollution is irreversible, then the second equation in (30) becomes:

$$E(k) = 0.$$

At a corner steady state, emissions are simply given by $E(k) = \beta c(k)$ and this function has the same properties as $c(k)$. The consumption function writes:

$$c(k) = (1 - \delta)k + kf'(k).$$

Under Assumption 2, $c(k) \geq 0$, $\forall k > 0$. The first derivative is given by $c'(k) = 1 - \delta + f'(k) + kf''(k)$ and we impose the following condition (assumption (18)):

$$\sigma(k) \geq 1 - s(k) \leftrightarrow f'(k) + kf''(k) \geq 0,$$

with $\sigma(k)$, the elasticity of substitution, defined by (16) and $1 - s(k)$, the labor share of output (see (15)).

This assumption guarantees $c'(k) > 0$, $\forall k > 0$.

The solution to the second equation implies $c(k) = 0$. But this limit case can be excluded according to the assumption $\lim_{c \to 0} U_1(c, Q) = +\infty$. There is no corner SS that exhibits irreversible pollution.

In the reversible case, for $Q \in]\bar{Q} - \bar{P}, \bar{Q}]$, The set J corresponds to the set of steady state values for capital: $k_{cr}^{*j} = \bar{k}^j$ (such that $m(k) = 0$).[20] The corresponding level of environmental quality must solves: $\Gamma(\bar{Q} - Q_{cr}^{*j}) = \beta c(k_{cr}^{*j})$. Under the properties of $\Gamma()$, for each k_{cr}^{*j}, there exists a corresponding level Q_{cr}^{*j} iff the following condition (that corresponds to (19)) holds:

$$\max_{P \in [0, \bar{P}]} \Gamma(P) \geq E(k_{cr}^{*j}).$$

Note that there are exactly two Q_{cr}^{*j} if (19) holds with ">" instead of "$\geq$". If condition (19) holds with equality, then $Q_{cr}^{*j} = \bar{Q} - \tilde{P} > \bar{Q} - \bar{P}$. Otherwise, the ranking is such that $Q_{cr}^{*j+} > \bar{Q} - \tilde{P} > Q_{cr}^{*j-} > \bar{Q} - \bar{P}$.

A.2 Existence of interior SS

At steady state, the maintenance function writes as follows:

$$m(k) = w(k) - k.$$

Considering interior solutions boils down to imposing $m(k) \geq 0$. The set of points satisfying $m(k) = 0$ corresponds to J. Let us define $\bar{k}$ as follows:

$$\bar{k} = \inf\{k > 0 / w(k) = k\}$$

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[20] The subscript "c" (resp. "i") prevails for corner (resp. interior) solutions. The second subscript "i" (resp. "r") means that equilibrium pollution is irreversible (resp. reversible).

At this striking point, $w'(\bar{k}) < 1$ since the wage crosses the 45° line from above.

In the Appendix A.2.1, a discussion is conducted on the relevancy to restrict the analysis of interior SS (thereafter ISS) to the interval $[0, \bar{k}]$.

A.2.1 Restriction on the definition domain of k

For usual technologies such as the CES (when $\rho \in (-1, 0]$), $\bar{k}$ is unique. But, uniqueness of $\bar{k}$ is generally not guaranteed and one may wonder what happens when there are multiple intersections between the wage and the 45° line.

If there are multiple intersections, then the domain of definition of k can be separated into two classes of intervals.

- Let $I_a = [\bar{k}^a, \bar{k}^{a+1}]$, with a an even number ($a = 0, 2, 4, \ldots$) and $\bar{k}^0 = 0$, be the class of intervals on which $w(k) > k \; \forall k \in]\bar{k}^a, \bar{k}^{a+1}[$ and $w(k) = k$ for $k = \bar{k}^a, \bar{k}^{a+1}$ (exception is $\bar{k}^0$ that may satisfy $w(k) > k$ if capital is not essential to production),
- Let $I_b = [\bar{k}^b, \bar{k}^{b+1}]$, with b an odd number ($b = 1, 3, 5, \ldots$) and $\bar{k}^1 = \bar{k}$, be the class of intervals on which $w(k) < k \; \forall k \in]\bar{k}^b, \bar{k}^{b+1}[$ and $w(k) = k$ for $k = \bar{k}^b, \bar{k}^{b+1}$.

Bounds $\bar{k}^{a+1} = \bar{k}^b$ (resp. $\bar{k}^a = \bar{k}^{b+1}$) correspond to "attractors" (resp. "repulsive" states) for the dynamics in the zero maintenance region (ZMR), see the appendix B.1.

According to these definitions,

Lemma 1 *i/ an ISS cannot belong to intervals I_b since $w(k) < k \leftrightarrow m(k) < 0$. Thus, any ISS is located into the first class of intervals I_a.*

ii/ as for the dynamics, if the economy is endowed with a stock $k_t \in I_b$, then it will converge toward the interval I_a or toward the bound $\bar{k}^{a+1}$ with $a = b - 1$.

Proof The proof is as follows: assume first that (k_t, Q_t) belongs to the ZMR, then, according to (13), $k_t \in I_b \leftrightarrow k_{t+1} = w(k_t) < k_t$: capital decreases (environmental quality may increase or decrease) until the interval I_a is reached. Assume next that (k_t, Q_t) lies in the positive maintenance region (PMR), then according to (21), $k_t \in I_b \leftrightarrow k_{t+1} = w(k_t) - m_t < k_t$ since $m_t \geq 0$ and the same convergence toward I_a occurs. If we refer to the equilibrium relation 24, it means that environmental quality decreases too. □

Regions composed of the union of two successive intervals, $I_c = I_a U I_b$ with $c = 0, 2, 4$, are "independent" from each other. An economy starting from a point in I_c necessarily remains in this interval: convergence is conditional on the initial location. In a certain sense, the domain of variation of k is divided into independent intervals whose equilibria can be studied separately.

What are the implications of this feature on our analysis?

We know that it may exist an ISS in any interval I_a. The point is to check that our choice of restricting the analysis to the first interval $I_0 = [0, \bar{k}]$ does not weaken the results.

We are aiming at studying existence of two types of steady states with opposite properties:

– the "desirable" ones that encompass reversible (CSS and) ISS,
– the poverty trap as defined in Sect. 3.1.3.

As we will see in Appendix A.2.3, it does not make sense to set, for each interval I_a, existence conditions. Thus, we have to proceed in a different way in order to show that the followed approach is relevant.

Let me state some intermediary results regarding the existence of admissible (with respect to the irreversibility threshold $\bar{Q} - \bar{P}$) ISS in the successive intervals I_a:

Lemma 2 *i/ If there exists an admissible reversible ISS (ARISS) with a corresponding level of capital in I_a then there exists no admissible irreversible ISS (AIISS) in the following intervals $I_{a'}$ with $a' > a$.*

ii/ If there exists an AIISS with a corresponding level of capital in I_a then there exists no ARISS in the preceding intervals $I_{a''}$ with $a'' < a$.

iii/ If there is no AIISS in I_a then there is no AIISS in the following intervals $I_{a'}$ with $a' > a$.

Proof A proof by contradiction establishes the first two results. Assume that there exist an ARISS in I_a and an AIISS in $I_{a'}$. The corresponding levels of capital are denoted k_{ir}^* and k_{ii}^* and satisfy $k_{ir}^* < k_{ii}^*$. It is possible to deduce from the equilibrium relation that: $Q_{ir}^* = Q^e(k_{ir}^*) < Q_{ii}^* = Q^e(k_{ii}^*)$. The admissibility of the RISS implies $\bar{Q} - \bar{P} < Q_{ir}^*$. Thus, we have $\bar{Q} - \bar{P} < Q_{ir}^* < Q_{ii}^*$. But, this contradicts the admissibility of the IISS that imposes: $Q_{ii}^* \leq \bar{Q} - \bar{P}$. The same reasoning applies to prove the second result.

As for the third item, the proof is straightforward. Assume that there is no *AIISS* in I_a, it means that there is no $k \in I_a$ such that the corresponding level $Q = Q^e(k)$ is below $\bar{Q} - \bar{P}$. As the equilibrium relation is increasing, it implies that any $k \in I_{a'}$ is associated with a level $Q^e(k)$ that is necessarily greater than the threshold. □

These "results" provide the following information. First, situations where AIISS and ARISS simultaneously exist in different intervals, with a higher level of wealth at the AIISS, cannot occur. In other words, the definition of the poverty trap and its identification with the AIISS are valid. Next, if there no AIISS in $I_0 = [0, \bar{k}]$ then such steady states cannot exist in the following intervals. Finally, if it possible to prove the existence of both type of equilibrium in the same interval, say I_0, then in the following intervals, only the ARISS type may exist. Therefore, the complete study of existence and admissibility of IISS and RISS on the interval $[0, \bar{k}]$ seems to be a sufficient exercise in order to support our analysis.

A.2.2 Existence of interior irreversible SS (Proposition 2)

At the interior equilibrium, emissions are defined as follows (see (22)):

$$E(k) = \beta c(k) - \gamma m(k)$$

The levels of emission corresponding to each bound of the interval $[0, \bar{k}]$ equal to: $\lim_{k \to 0} E(k) = -\gamma f(0) \leq 0$ and $E(\bar{k}) = \beta c(\bar{k}) > 0$. The first derivative is given by:

$E'(k) = \beta c'(k) - \gamma m'(k)$. In addition, it is easy to prove that emissions reach their maximum at the upper bound $\bar{k}$: $E(\bar{k}) = \max\{E(k) \,/\, k \in [0, \bar{k}]\}$.

Evaluating (25) at an irreversible SS yields:

$$\begin{cases} R(k)U_1(c(k), Q) - \gamma U_2(c(k), Q) = 0, \\ E(k) = 0. \end{cases} \tag{31}$$

If k is not essential to production ($f(0) > 0$) then emissions are negative in the neighbourhood of 0. If not, imposing $\lim_{k \to 0} E'(k) < 0$ (see condition (26) in Proposition 2), is sufficient to prove that there exists a positive value of capital for which emissions are nil. From the properties of $E(k)$, the second equation admits an odd number of positive solutions k_{ii}^* such that $E(k) = 0$.

According to (24), the first Eq. in (31) corresponds to the equilibrium relation $Q_t = Q^e(k_t)$ valid for all t. Substituting a solution k_{ii}^* in this equation gives the corresponding equilibrium value of environmental quality $Q_{ii}^* = Q^e(k_{ii}^*)$.

There exists at least one SS associated with a positive maintenance and irreversible pollution. The admissibility of this solution, from the point of view of its location with respect to the critical level $\bar{Q} - \bar{P}$, requires $Q_{ii}^* \le \bar{Q} - \bar{P}$, which is equivalent to $k_{ii}^* \le \underline{k}$ with $\underline{k} = (Q^e)^{-1}(\bar{Q} - \bar{P})$ since the relation $Q^e(k_t)$ is invertible.

A.2.3 Existence of Interior reversible SS (Proposition 3)

The system of equations (25) now writes:

$$\begin{cases} R(k)U_1(c(k), Q) - \gamma U_2(c(k), Q) = 0 \\ \Gamma(\bar{Q} - Q) = E(k) \end{cases}$$

where the first equation still characterizes the equilibrium relation: $Q = Q^e(k)$.

Thus, the analysis of existence boils down to an examination of the solutions to the following equation

$$\Gamma(\bar{Q} - Q^e(k)) = E(k), \tag{32}$$

and, considering the reversible pollution region impose to restrict the study to the interval $]\underline{k}, \bar{k}]$.[21]

Let us define $G(k) = \Gamma(\bar{Q} - Q^e(k))$. $G(k)$ has the same qualitative behaviour as the assimilation function $\Gamma(\bar{Q} - Q)$ since $G'(k) = -Q^{e'}(k)\Gamma'(\bar{Q} - Q^e(k))$ with $Q^{e'}(k) \ge 0$. Thus, it is first increasing from $\underline{k}$ to $(Q^e)^{-1}(\bar{Q} - \tilde{P})$ and next, decreasing until $(Q^e)^{-1}(\bar{Q})$. We also have $G(k) \ge 0 \; \forall k \in]\underline{k}, (Q^e)^{-1}(\bar{Q})]$, $G(\underline{k}) = G((Q^e)^{-1}(\bar{Q})) = 0$ and $\max\{G(k) \,/k \in]\underline{k}, (Q^e)^{-1}(\bar{Q})]\} = G((Q^e)^{-1}(\bar{Q} - \tilde{P})) = \Gamma(\tilde{P})$.

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[21] It is worth noting that there is *a priori* a problem with the domains of definition of these two functions. In fact, $E(k)$ depends on conditions on technology, preferences and the law of pollution diffusion. But, the LHS in (32) is also and especially characterized by the properties of the assimilation function. Thus, nothing guarantees the non-emptiness of the studied interval $]\underline{k}, \bar{k}]$ and, we will have to set conditions that ensure its significance.
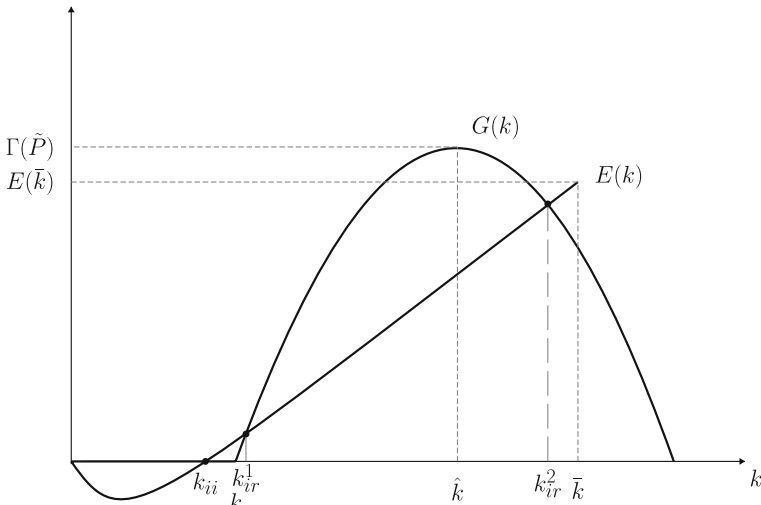
Fig. 7 Existence analysis: with $\hat{k} = (Q^e)^{-1}(\bar{Q} - \tilde{P})$

In order to determine the ranking between the functions $G(k)$ and $E(k)$ at the lower bound $\underline{k}$, it is possible to refer to the admissibility condition, for ISS, that imposes $k_{ii}^* \leq \underline{k}$. Two cases possibly occur:

- Suppose first that all ISS are admissible (sup $\{k_{ii}^*\} \leq \underline{k}$) or that $\underline{k}$ is comprised between two successive, odd then even, levels k_{ii}^* ($\underline{k} \in [k_{ii}^{*a}, k_i^{*a+1}]$, with a an odd number). In these cases, the ranking at $\underline{k}$ is: $E(\underline{k}) > G(\underline{k})$. If we further assume that[22]

$$\sup_{P \in [0, \bar{P}]} \{\Gamma(P)\} \geq \sup_{k \in [0, \bar{k}]} \{E(k)\},$$

and, $\bar{k} \geq (Q^e)^{-1}(\bar{Q} - \tilde{P})$ (see conditions (27) and (28) in Proposition 3), then $]\underline{k}, \bar{k}]$ is a non empty interval and the Eq. (32) admits a solution. In fact, under these conditions, the ranking is reversed at the striking point $(Q^e)^{-1}(\bar{Q} - \tilde{P})$: $E((Q^e)^{-1}(\bar{Q} - \tilde{P})) \leq G((Q^e)^{-1}(\bar{Q} - \tilde{P}))$. It means that the two curves $E(k)$ and $G(k)$ intersect at least once when k varies in the interval $]\underline{k}, \bar{k}]$.

- Consider now that there is no admissible ISS (inf $\{k_{ii}^*\} > \underline{k}$) or that $\underline{k}$ belongs to an interval whose bounds correspond to two successive, even then odd, levels k_{ii}^* ($\underline{k} \in [k_i^{*b}, k_i^{*b+1}]$, with b an even number). If (28) holds, proving the existence of a reversible SS requires, by symmetry, to reverse the inequality in (27).

Figure 7 illustrates how the interior SS are determined. The functional forms we consider are those used in Sect. 4 for the numerical simulations.

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[22] This condition is tantamount to (19) since $\max\{E(k)\} = E(\bar{k})$.

B Local dynamics

B.1 Stability of corner SS

The Jacobian associated with a corner SS writes:

$$J = \begin{pmatrix} w'(k^*) & 0 \\ -\beta c'(k^*) & N'(Q^*) \end{pmatrix}$$

Stability requires that the roots of the characteristic polynomial be located into the unit circle, all other configuration being unstable. We know that the trace corresponds to the sum of the (real parts) eigenvalues of J and, the determinant is the product, in modulus, of the eigenvalues. Here, the two eigenvalues are $\lambda_1 = w'(k^*)$ and $\lambda_2 = N'(Q^*)$. Thus, sufficient conditions for stability are:

$$w'(k^*) < 1, \tag{33}$$
$$N'(Q^*) < 1. \tag{34}$$

Condition (33) implies that the economy is able to absorb a capital shock whereas inequality (34) expresses the environment capacity to recover its original state when submitted to an exogenous perturbation.

By construction, intersections between the wage function and the 45° line successively satisfy $w'(k) < 1$ then $w'(k) > 1$. Thus, it is possible to exclude any k_{cr}^{*j} such that $w'(k_{cr}^{*j}) > 1$. If condition (19) holds with equality, the corresponding value Q_{cr}^{*j} is such that $N'(Q_{cr}^{*j}) = 1$, the steady state is unstable. Otherwise, each k_{cr}^{*j}, satisfying $w'(k) < 1$, is associated with two levels of environmental quality but only one meets the condition $N'(Q_{cr}^{*j+}) < 1$. Therefore, the steady state with the highest Q is locally stable.

B.2 Stability of interior SS

B.2.1 Reversible pollution

Linearizing the system (25) around a steady state (k^*, Q^*) yields:

$$\begin{cases} dQ_{t+1} = Q^{e'}(k^*)dk_{t+1}, \\ dQ_{t+1} = N'(Q^*)dQ_t - (E'(k^*) - \gamma)dk_t - \gamma dk_{t+1}. \end{cases} \tag{35}$$

From this system, we get the Jacobian matrix:

$$J = \begin{pmatrix} -\dfrac{E'(k)-\gamma}{Q^{e'}(k)+\gamma} & \dfrac{N'(Q)}{Q^{e'}(k)+\gamma} \\ -\dfrac{(E'(k)-\gamma)Q^{e'}(k)}{Q^{e'}(k)+\gamma} & \dfrac{N'(Q)Q^{e'}(k)}{Q^{e'}(k)+\gamma} \end{pmatrix}$$

Now, it is clear that $det(J) = 0$. In fact, the studied dynamics boil down to a one dimensional system because of the existence of the equilibrium relation (24). Therefore, the first eigenvalue is nil $\lambda_1 = 0$ while the second is equal to the trace $\lambda_2 = tra(J)$ with,

$$tra(J) = \frac{N'(Q)Q^{e'}(k) - (E'(k) - \gamma)}{Q^{e'}(k) + \gamma}.$$

We summarize all possible cases:

1/ if $(N'(Q) - 1)Q^{e'}(k) < E'(k) \leq \gamma + N'(Q)Q^{e'}(k)$ then $1 > tra(J) \geq 0$
   The conditions $E'(k) \geq 0$ and $N'(Q) \leq 1$ (with one of the two inequalities being strict) suffice to satisfy the necessary stability condition $E'(k) > (N'(Q) - 1)Q^{e'}(k)$.
2/ if $\gamma + N'(Q)Q^{e'}(k) < E'(k) < 2\gamma + (1 + N'(Q))Q^{e'}(k)$ then $tra(J) < 0$ and $|tra(J)| < 1$.

Thus, the double condition

$$(N'(Q) - 1)Q^{e'}(k) < E'(k) < 2\gamma + (1 + N'(Q))Q^{e'}(k), \tag{36}$$

defines an interval of variation, for the emissions function derivative, on which local stability of any interior reversible steady state is guaranteed. More precisely, the convergence is monotonic in the first case whereas it is oscillatory in the second one.

B.2.2 Irreversible pollution

This case is obtained by fixing $N'(Q) = 1$. Following the same process, the sufficient condition for stability writes:

$$0 < E'(k) < 2(\gamma + Q^{e'}(k)). \tag{37}$$

Note that condition (37) imposes that the derivative of the emission function with respect to capital is positive. It also supposes that the impact of a rise in capital on emissions is less than a bound particularly defined by its impact on environmental quality at equilibrium (measured by $Q^{e'}(k^*)$). This condition ensures that the economy is able to assimilate a capital shock and to recover in a few periods its original state. When the level of environmental quality is above the critical point $\bar{Q} - \bar{P}$, the condition (37) generalizes (36). We shall note that the inequality $N'(Q^*) \leq 1$ is sufficient to guarantee that the repercussions of a shock on environmental quality are absorbed from period to period.[23]

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[23] The linear formulation of environmental dynamics (see note 8) does not allow to catch this new effect. The specificity (and limits) of this approach is due to the fact that Nature is always able to assimilate a shock on the environment which returns to its steady state level in a finite time. To the contrary, in our framework, if $N'(Q) > 1$ then, this shock echoes in a more than proportional manner and the stability property is lost.

C Simulations

Parameter restrictions

We impose a restriction on the parameters related to the technology and the emissions function: $\gamma(1-\alpha)-\beta\alpha \geq 0$. We also set the domain of variation of the scale parameter: $A \in [\underline{A}, \bar{A}]$ with

$$\underline{A} = \left(\frac{2}{\gamma\bar{P}}\right)^{1-\alpha} \frac{1}{1-\alpha},$$

and,

$$\bar{A} = \left(\frac{\theta\bar{P}^2(1-\alpha)}{4\beta((1-\delta)(1-\alpha)+\alpha)}\right)^{1-\alpha} \frac{1}{1-\alpha}.$$

These conditions cover the assumptions made in the general setting.

Equilibrium properties

In this framework, the equilibrium analysis provides the following information. There exist five steady states:

– one interior irreversible locally stable solution,
– two interior reversible solutions. The stable steady state is the one that exhibits the highest level of capital,
– two corner reversible solutions, the stable steady state being the one associated with the highest level of environmental quality.

For the functional forms used, it is possible to compute analytically the global dynamics characterizing the four possible regions.

Moreover, by using the following set of parameters values,

$$\left\{A = 2.52, \theta = 0.09, \gamma = 0.2, \beta = 0.3, \alpha = 0.3, \delta = 0.6, \bar{P} = 5, \bar{Q} = 7\right\},$$

we show that there exist two steady states that satisfy the criterion of admissibility: the interior irreversible solution and the high interior reversible solution (the corner steady states are inadmissible in the sense that they are located in the positive maintenance region).

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