Can human intuition be a Transfinite Turing Machine ? Will this machine solves 3-SAT$_{\aleph_0}$ in a Cantor Space ?

Abstract

Let 3-SAT$_{\aleph_0}$ be the problem obtained by adding to 3-SAT a countable infinite number of clauses and variables. Any solution of this problem can be expressed in a cantor space $\{0,1\}^{\mathbb{N}}$. Checking if $s \in \{0,1\}^{\mathbb{N}}$ is a solution can be done in $\aleph_0$ steps, but finding one by brute force takes $\aleph_1$ iterations in almost all cases. The question addressed by this paper is the following: is there an implication between $\mathcal{P} = \mathcal{NP}$ conjecture and the problem of solving 3-SAT$_{\aleph_0}$ in a countable time? First, a $\mathbf{T}_{\omega_1}$ transfinite extension of Turing machine will be introduced. When executed for a finite time, $\mathbf{T}_{\omega_1}$ and a classical Turing machine are indistinguishable. But when the number of steps reaches ω_0, complex discrepancies can appear. Unlike Turing machines with Oracle, $\mathbf{T}_{\omega_1}$ is provided with an explicit description. Moreover, the definition of $\mathbf{T}_{\omega_1}$ leads to a generalization of Rice's theorem that specifies what kind of implications exist between finite and infinite machines. It will be shown that some questions concerning their mutual relationships cannot be solved, including most $\mathcal{P} = \mathcal{NP}$ conjecture relevant questions. In a second part, a time countable algorithm for 3-SAT$_{\aleph_0}$ will be provided. In this purpose, a geometric representation of the problem will be put forward, where the domain of admissible solutions can be assimilated to the line segment [0, 1]. The clauses themselves will be expressed by means of periodic functions canceling out on certain key values of [0, 1]. In conclusion, the present paper intends to make five contributions: (1) introducing a new tool in hypercomputation theory, (2) showing that 3-SAT$_{\aleph_0}$ is soluble in a countable number of steps, (3) providing a geometric representation of 3-SAT, (4) discussing some new strategical ideas concerning $\mathcal{P} = \mathcal{NP}$ conjecture and (5) concluding with an informal and open-to-debate philosophical argument about the deductive power of a transfinite Turing Machine compared to human mind.