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Mathieu PARENTI, Philip USHCHEV and Jacques-François THISSE

DISCUSSION PAPER

Center for Operations Research
and Econometrics

Voie du Roman Pays, 34
B-1348 Louvain-la-Neuve
Belgium
http://www.uclouvain.be/core

CORE
Voie du Roman Pays 34, L1.03.01
B-1348 Louvain-la-Neuve, Belgium.
Tel (32 10) 47 43 04
Fax (32 10) 47 43 01
E-mail: immaq-library@uclouvain.be
http://www.uclouvain.be/en-44508.html

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Toward a theory of monopolistic competition[1]

Mathieu Parenti[2], Philip Ushchev[3],
and Jacque-François Thisse[4]

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Abstract

We propose a general model of monopolistic competition with unspecified preferences. Our analysis applies to the case of symmetric/asymmetric preferences and costs. Our basic tool is the elasticity of substitution function, which is shown to depend on the actions taken by firms. We impose intuitive conditions on this function to guarantee the existence of a free-entry equilibrium. Comparative statics with respect to population size, GDP per capita and productivity shocks (the pass-through rate) is conducted by means of necessary and sufficient conditions in the case of symmetric firms.

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JEL classification: D43, L11, L13.

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[2] CORE-UCLouvain (Belgium) and NRU-Higher School of Economics (Russia)
[3] NRU-Higher School of Economics (Russia)
[4] CORE-UCLouvain (Belgium), NRU-Higher School of Economics (Russia) and CEPR
E-mail addresses: mathieu.parenti@uclouvain.be, ph.ushchev@gmail.com, jacques.thisse@uclouvain.be.

1 Introduction

The theory of general equilibrium with imperfectly competitive markets is still in infancy. In his survey of the various attempts made in the 1970s and 1980s to integrate oligopolistic competition within the general equilibrium framework, Hart (1985) has convincingly argued that these contributions have failed to produce a consistent and workable model. Unintentionally, the absence of a general equilibrium model of oligopolistic competition has paved the way to the success of the CES model of monopolistic competition developed by Dixit and Stiglitz (1977). And indeed, the CES model has been used in so many economic fields that a large number of scholars believe that this is *the* model of monopolistic competition. For example, Head and Mayer (2014) observe that this model is "nearly ubiquitous" in the trade literature. However, owing to its extreme simplicity, the CES model dismisses several important effects that belong to the core of economic theory. To mention a few, unlike what the CES predicts, prices and firm sizes are affected by entry, market size and consumer income, while markups vary with costs. Moreover, there is a growing empirical evidence that casts doubt on the theoretical relevance of this model (see, e.g. De Loecker et al., 2012; Simonovska, 2015).

In addition, tweaking the CES or using other specific models in the hope of obviating these difficulties does not permit to check the robustness of the results. Needless to say, we acknowledge that such a research strategy buys tractability, but at a cost that may be high because those models are not flexible enough to capture several important facets of market competition. For example, under the CES market prices do not depend on market size and individual income. By nesting quadratic preferences into a quasi-linear utility, Melitz and Ottaviano (2008) show that prices depend on market size, but suppress the per capita income effect. Prices depend on per capita income under the linear expenditure system in an open economy (Simonovska, 2015), but this effect disappears in a closed economy under additive preferences (Zhelobodko et al., 2012). Prices are independent of the number of competitors in the CES model of monopolistic competition, but not under oligopolistic competition. In sum, it seems fair to say that the state of the art looks like a scattered field of incomplete and insufficiently related contributions in search of a more general approach. The literature focuses on particular preferences. But, and this is a big but, it is not clear how the properties obtained under such specifications can be compared across models, neither what their imply for the market outcome.

Our purpose is to build a *general* model of monopolistic competition that has the following two desirable features. First, it encompasses all existing models of monopolistic competition, such as the CES, quadratic, CARA, additive, and homothetic preferences. Second, it displays enough flexibility to take into account demand and competition attributes in a way that will allow us to determine under which conditions many findings are valid. To this end, we consider a setting in which preferences are unspecified and characterize preferences through necessary and sufficient conditions for each comparative static effect to hold. This should be useful to the applied economists in discriminating between the different specifications used in their settings.

By modeling monopolistic competition as a noncooperative game with a continuum of players, we are able to obviate at least two major problems. First, we capture Chamberlin's central idea according to which the cross elasticities of demands are equal to zero, the reason being that each firm is negligible to the market. Second, because individual firms are unable to manipulate profits, firms do not have to make full general equilibrium calculations before choosing their profit-maximizing strategy.

Our main findings may be summarized as follows. First, at any symmetric market outcome, the individual demand for a variety depends only upon its consumption when preferences are additive. By contrast, when preferences are homothetic, the demand for a variety depends upon its relative consumption level and the mass of available varieties. Therefore, when preferences are neither additive nor homothetic, the demand for a variety must depend on its consumption level and the total mass of available varieties.

Even though the case of heterogeneous firms is studied in this paper, it should be clear that starting with symmetric firms allows us to insulate the impact of various types of preferences on the market outcome. In the words of Chamberlin: "We therefore proceed under the heroic assumption that both demand and cost curves for all the "products" are uniform throughout the group." We view this as a necessary first step toward the construction of a general model of monopolistic competition. That said, rather than making new assumptions on preferences and demands, we tackle the problem from the viewpoint of the theory of product differentiation. To be precise, the key concept of our model is the elasticity of substitution across varieties. We then exploit the symmetry of preferences over a continuum of goods to show that, under the most general specification of preferences, at any symmetric outcome the elasticity of substitution between any two varieties is a function of two variables only: the per variety consumption and the total mass of firms. Combining this with the absence of the business-stealing effect of oligopoly theory reveals that, at the market equilibrium, *firms' markup is equal to the inverse of* *the equilibrium value of the elasticity of substitution.* This result agrees with one of the main messages of industrial organization: the higher is the elasticity of substitution, the less differentiated are varieties, and thus the lower are firms' markup.

It should now be clear that the properties of the symmetric free-entry equilibrium depend on how the elasticity of substitution function behaves when the per variety consumption and the mass of firms vary. To be precise, by imposing plausible conditions to the elasticity of substitution function and using simple analytical arguments, we are able to disentangle the various determinants of firms' strategies. This will lead us to determine our preferred set of assumptions by building on the theory of product differentiation, as well as on empirical evidence.

Second, we provide necessary and sufficient conditions on the elasticity of substitution for the existence and uniqueness of a free-entry equilibrium. Our setting is especially well suited to conduct detailed comparative static analyses in that we can determine the necessary and sufficient conditions for all the thought experiments undertaken in the literature. The most typical experiment is to study the impact of market size. What market size signifies is not always clear because it compounds two variables, i.e., the number of consumers and their willingness-to-pay for the product under consideration. The impact of population size and income level on prices, output and the number of firms need not be the same because these two parameters affect firms' demand in different ways. An increase in population or in income raises demand, thereby fostering entry and lower prices. But an income hike also raises consumers' willingness-to-pay, which tends to push prices upward. The final impact is thus a priori ambiguous.

We show that a larger market results in a lower market price and bigger firms if and only if the elasticity of substitution responds more to a change in the mass of varieties than to a change in the per variety consumption. This is so in the likely case where the entry of new firms does not render varieties much more differentiated. Regarding the mass of varieties, it increases with the number of consumers if varieties do not become too similar when their number rises. Thus, like most oligopoly models but unlike the CES, *our model of monopolistic competition exhibits the standard pro-competitive effects associated with market size and entry.* However, anti-competitive effects cannot be ruled out a priori. Furthermore, an increase in individual income generates similar, but not identical, effects if and only if varieties become closer substitutes when their range widens. The CES is the only utility for which price and output are independent of both income and market size.

Our setting also allows us to study the impact of a cost change on markups. When all firms face the same productivity hike, we show that *the nature of preferences determines the extent of the pass-through*. Specifically, a decrease in marginal cost leads to a lower market price, but a higher markup, if and only if the elasticity of substitution decreases with the per capita consumption. In this event, there is incomplete pass-through. However, the pass-through rate need not be smaller than one.

Second, we consider Melitz-like heterogeneous firms. In this case, when preferences are non-additive, the profit-maximizing strategy of a firm depends directly on the range of strategies chosen by all the other types' firms, which vastly increases the complexity of the problem. Nevertheless, we are able to prove that, regardless of the distribution of marginal costs, firms are sorted out by decreasing productivity order, while a bigger market sustains a larger number of active firms. For each type of firm, the elasticity of substitution now depends on the number of active firms through the number of entrants and the cutoff cost. Interestingly enough, this approach paves the way to the study of asymmetric preferences such as those used in spatial models of product differentiation: *heterogeneous firms supplying symmetric varieties behave like homogeneous firms selling varieties whose elasticity of substitution is variety-specific*. In other words, a model with heterogeneous firms supplying symmetric varieties is isomorphic to a model with homogeneous firms selling asymmetric varieties.

Last, we show that the continuum assumption has a far-fetched implication: even though firms do not compete strategically, *our model is able to mimic oligopolistic markets with free entry* and to generate within a general equilibrium framework findings akin to those obtained in industrial organization.

Related literature. Different alternatives have been proposed to avoid the main pitfalls of the CES model. Behrens and Murata (2007) propose the CARA utility that captures price competition effects, while Zhelobodko et al. (2012) and Dhingra and Morrow (2015) use general additive preferences to work with a variable elasticity of substitution, and thus variable markups. Vives (1999) and Ottaviano et al. (2002) show how the quadratic utility model obviates some of the difficulties associated with the CES model, while delivering a full analytical solution. Bilbiie et al. (2012) use general symmetric homothetic preferences in a real business cycle model. Last, pursuing a different, but related, objective, Mrazova and Neary (2013) study a class of demands which implies an invariant relationship between the demand elasticity and the curvature of demand schedule. Subsection 2.3 explains how this class fits in our general model.

In the next section, we describe the demand and supply sides of our setting. In particular, the primitive of the model being the elasticity of substitution function, we discuss how this function may vary with the per variety consumption and the mass of varieties. In Section 3, we prove the existence and uniqueness of a free-entry equilibrium and characterize its various properties when firms are symmetric. We study the case of heterogeneous firms in Section 4, while Section 5 highlights unsuspected relationships between monopolistic and oligopolistic competition with free-entry. Section 6 concludes.

2 The model and preliminary results

2.1 Consumers and demand systems

Consider an economy with a mass L of identical consumers, one sector and one production factor – labor, which is used as the numéraire. Let $\mathcal{N}$ be the mass of "potential" varieties (e.g., the varieties for which a patent exists). Individual preferences are described by a *utility functional* $\mathcal{U}(\mathbf{x})$, where $\mathbf{x} \geq 0$ is a *consumption profile* that maps the space of potential varieties $[0, \mathcal{N}]$ into $\mathbb{R}_+$. For analytical simplicity, we focus on consumption profiles that have a finite mean and variance.

We now make two assumptions about $\mathcal{U}$, which are close to the "minimal" set of requirements for our model to be nonspecific while displaying the desirable features of existing models of monopolistic competition. First, the functional $\mathcal{U}$ is *symmetric* in the sense that any mapping from $[0, \mathcal{N}]$ into itself does not change the value of $\mathcal{U}$. Intuitively, this means that renumbering varieties has no impact on the utility level. Second, the utility function exhibits *love for variety* if, for any $N \leq \mathcal{N}$, a consumer strictly prefers to consume the whole range of varieties $[0, N]$ than any subinterval $[0, k]$ of $[0, N]$:

$$\mathcal{U}\left(\frac{X}{k} I_{[0,k]}\right) < \mathcal{U}\left(\frac{X}{N} I_{[0,N]}\right), \tag{1}$$

where $X > 0$ is the consumer's total consumption of the differentiated good and I_A is the indicator of $A \sqsubseteq [0, N]$. It can be shown that *consumers exhibit love for variety if* $\mathcal{U}(\mathbf{x})$ *is continuous and strictly quasi-concave.*

To determine the inverse demand for a variety, we assume that the utility functional $\mathcal{U}$ is differentiable in $\mathbf{x}$ in the following sense: there exists a unique function $D(x_i, \mathbf{x})$ such that, for any given N and for all $\mathbf{h}$ with finite mean and variance, the equality

$$\mathcal{U}(\mathbf{x} + \mathbf{h}) = \mathcal{U}(\mathbf{x}) + \int_0^N D(x_i, \mathbf{x})\, h_i \,\mathrm{d}i + \mathcal{O}\left(\int_0^N h_i^2 \mathrm{d}i\right) \tag{2}$$

holds.[1] The function $D(x_i, \mathbf{x})$ is the *marginal utility of variety* i. In what follows, we focus on utility functionals that satisfy (2) for all $\mathbf{x} \geq 0$ and such that the marginal utility $D(x_i, \mathbf{x})$ is decreasing and differentiable with respect to the consumption x_i of variety i. That $D(x_i, \mathbf{x})$ does not depend directly on $i \in [0, N]$ follows from the symmetry of preferences. Evidently, $D(x_i, \mathbf{x})$ strictly decreases with x_i if $\mathcal{U}$ is strictly concave.

Each consumer is endowed with y efficiency units of labor, so that the per capita labor income y is given and the same across consumers because the unit wage is 1. Maximizing the utility functional $\mathcal{U}(\mathbf{x})$ subject to (i) the budget constraint

$$\int_0^N p_i x_i \mathrm{d}i = Y, \tag{3}$$

where $Y \geq y$ is the consumer's income and (ii) the availability constraint

$$x_i \geq 0 \text{ for all } i \in [0, N] \quad \text{and} \quad x_i = 0 \text{ for all } i \in]N, \mathcal{N}]$$

yields the following inverse demand function for variety i:

$$p_i = \frac{D(x_i, \, \mathbf{x})}{\lambda} \quad \text{for all } i \in [0, N], \tag{4}$$

where λ is the Lagrange multiplier of the consumer's optimization problem. Expressing λ as a function of Y and $\mathbf{x}$, we obtain

$$\lambda(Y, \mathbf{x}) = \frac{\int_0^N x_i D(x_i, \mathbf{x}) \, \mathrm{d}i}{Y}, \tag{5}$$

which is the marginal utility of income at the consumption profile $\mathbf{x}$ under income Y.

Examples. 1. **Additive preferences**. Assume that preferences are additive over the set of available varieties (Spence, 1976; Dixit and Stiglitz, 1977):

$$\mathcal{U}(\mathbf{x}) \equiv \int_0^{\mathcal{N}} u(x_i) \mathrm{d}i, \tag{6}$$

where u is differentiable, strictly increasing, strictly concave and such that $u(0) = 0$. The CES, the CARA (Behrens and Murata, 2007) and the linear expenditure system

[1] For symmetric utility functionals, (2) is equivalent to assuming that $\mathcal{U}$ is Frechet-differentiable in L_2 (Dunford and Schwartz, 1988). The concept of Frechet-differentiability extends the standard concept of differentiability in a fairly natural way.

(Simonovska, 2015) are special cases of (6). The marginal utility of variety i depends only upon its own consumption:

$$D(x_i, \mathbf{x}) = u'(x_i).$$

2. Translog preferences. A tractable example of non-CES homothetic preferences is the translog (Feenstra, 2003). There is no closed-form expression for the marginal utility $D(x_i, \mathbf{x})$. Nevertheless, it can be shown that $D(x_i, \mathbf{x})$ may be written as follows:

$$D(x_i, \mathbf{x}) = \psi(x_i, A(\mathbf{x})),$$

where $\varphi(x, A)$ is implicitly defined by

$$\psi x + \beta \ln \psi = \frac{1 + \beta A}{\mathcal{N}} \qquad A(\mathbf{x}) = \int_0^{\mathcal{N}} \ln \psi(x_i, A) \mathrm{d}i.$$

3. **Quadratic preferences.** As an example of a preference which is neither additive nor homothetic, consider the quadratic utility:

$$\mathcal{U}(\mathbf{x}) \equiv \alpha \int_0^{\mathcal{N}} x_i \mathrm{d}i - \frac{\beta}{2} \int_0^{\mathcal{N}} x_i^2 \, \mathrm{d}i - \frac{\gamma}{2} \int_0^{\mathcal{N}} \left(\int_0^{\mathcal{N}} x_i \, \mathrm{d}i \right) x_j \mathrm{d}j, \tag{7}$$

where α, β,and γ are positive constants (Ottaviano et al., 2002). The marginal utility of variety i is given by

$$D(x_i, \, \mathbf{x}) = \alpha - \beta \, x_i - \gamma \int_0^{\mathcal{N}} x_j \mathrm{d}j,$$

which is linear decreasing in x_i.

Since the bulk of the applied literature focuses on additive or homothetic preferences, we find it important to provide a full characterization of the corresponding demands (see Appendix 1).

Proposition 1. *The marginal utility* $D(x_i, \mathbf{x})$ *of variety* i *depends only upon* (i) *the consumption* x_i *if and only if preferences are additive and* (ii) *the consumption ratio* $\mathbf{x}/x_i$ *if and only if preferences are homothetic.*

2.2 Firms: first- and second-order conditions

Let $\mathbf{\Omega}$ be the set of active firms. There are increasing returns at the firm level, but no scope economies that would induce a firm to produce several varieties. Each firm supplies a single variety and each variety is produced by a single firm. Consequently, a variety may be identified by its producer $i \in \mathbf{\Omega}$. The continuum assumption distinguishes monopolistic competition from other market structures in that it is the formal counterpart of the basic idea that a firm's action has no impact on the others. As a result, by being negligible to the market, each firm may choose its output (or price) while accurately treating market aggregates as given. However, for the market to be in equilibrium, firms must accurately guess what these market aggregates will be.

Firms share the same fixed cost F; marginal costs are constant but may be firm-specific. In other words, to produce q units of its variety, firm $i \in \mathbf{\Omega}$ needs $F + c_i q$ efficiency units of labor. Hence, firm i's profit is given by

$$\pi(q_i) = (p_i - c_i)q_i - F. \tag{8}$$

Since consumers share the same preferences, the consumption of each variety is the same across consumers. Therefore, product market clearing implies $q_i = Lx_i$. Firm i maximizes (8) with respect to its output q_i subject to the inverse market demand function $p_i = LD/\lambda$, while the market outcome is given by a Nash equilibrium. The Nash equilibrium distribution of firms' actions is encapsulated in $\mathbf{x}$ and λ. In the CES case, this comes down to treating the price-index parametrically, while under additive preferences the only payoff-relevant statistic is λ.

Plugging D into (8), the program of firm i is given by

$$\max_{x_i} \pi_i(x_i, \mathbf{x}) \equiv \left[\frac{D\left(x_i, \mathbf{x}\right)}{\lambda} - c_i\right] L\, x_i - F.$$

Setting

$$D_i' \equiv \frac{\partial D(x_i, \mathbf{x})}{\partial x_i} \qquad D_i'' \equiv \frac{\partial D^2(x_i, \mathbf{x})}{\partial x_i^2},$$

the first-order condition for profit-maximization are given by

$$D(x_i, \mathbf{x}) + x_i D_i' = \left[1 - \bar{\eta}(x_i, \mathbf{x})\right] D(x_i, \mathbf{x}) = \lambda c_i. \tag{9}$$

Since λ is endogenous, we seek necessary and sufficient conditions for a unique (interior or corner) profit-maximizer to exist regardless of the value of $\lambda c_i > 0$. The argument involves two steps.

Step 1. For (9) to have at least one positive solution regardless of $\lambda c_i > 0$, it is sufficient to assume that, for any $\mathbf{x}$, the following Inada conditions hold:

$$\lim_{x_i \to 0} D = \infty \qquad \lim_{x_i \to \infty} D = 0. \tag{10}$$

Indeed, since $\bar{\eta}(0, \mathbf{x}) < 1$, (10) implies that $\lim_{x_i \to 0}(1 - \bar{\eta})D = \infty$. Similarly, since $0 < (1 - \bar{\eta})D < D$, it ensues from (10) that $\lim_{x_i \to \infty}(1 - \bar{\eta})D = 0$. Because $(1 - \bar{\eta})D$ is continuous, it follows from the intermediate value theorem that (9) has at least one positive solution. Note that (10) is sufficient, but not necessary. For example, if D displays a finite choke price exceeding the marginal cost, it is readily verified that (9) has at least one positive solution.

Step 2. The first-order condition (9) is sufficient if the profit function π is strictly quasi-concave in x_i. If the maximizer of π is positive and finite, the profit function is strictly quasi-concave in x_i for any positive value of λc_i if and only if the second derivative of π is negative at any solution to the first-order condition. Since firm i treats λ parametrically, the second-order condition is given by

$$x_i D_i^{''} + 2D_i^{'} < 0. \tag{11}$$

This condition means that firm i's marginal revenue $(x_i D_i' + D)L/\lambda$ is strictly decreasing in x_i. It is satisfied when D is concave, linear or not "too" convex in x_i. Furthermore, (11) is also a necessary and sufficient condition for the function π to be strictly quasi-concave for all $\lambda c_i > 0$, for otherwise there would exist a value λc_i such that the marginal revenue curve intersects the horizontal line λc_i more than once.

Observe also that (11) means that the revenue function is strictly concave. Since the marginal cost is independent of x_i, this in turn implies that π_i is strictly concave in x_i. In other words, when firms are quantity-setters, the profit function π_i is quasi-concave in x_i if and only if π_i is concave in x_i.[2]

In sum, the profit function π_i is strictly quasi-concave in x_i for all values of λc_i if and only if

(**A**) *firm i's marginal revenue decreases in* x_i.

Another sufficient condition commonly used in the literature is as follows (Krugman, 1979; Vives, 1999):

(Abis) *the elasticity of the inverse demand* $\bar{\eta}(x_i, x)$ *increases in* x_i.

[2]It can be shown that (**A**) is equivalent to the well-known condition obtained by Caplin and Nalebuff (1991) for a firm's profits to be quasi-concave in its own price, i.e., the Marshallian demand $\mathcal{D}(p_i, \mathbf{p}, Y)$ for every variety i is such that $1/\mathcal{D}$ is convex in price. Since the Caplin-Nalebuff condition is the least stringent one for a firm's profit to be quasi-concave under price-setting firms, (**A**) is therefore the least demanding condition when firms compete in quantities.

It is readily verified that **(Abis)** is equivalent to

$$-x_i \frac{D_i''}{D_i'} < 1 + \bar{\eta}.$$

whereas (11) is equivalent to

$$-x_i \frac{D_i''}{D_i'} < 2.$$

Since $\bar{\eta} < 1$, **(Abis)** implies **(A)**.

Outputs and prices. Consider two different firms such that $c_i > c_j$, hence

$$\left[\frac{D\left(x_i, \mathbf{x}\right)}{\lambda} - c_i \right] L x_i < \left[\frac{D\left(x_i, \mathbf{x}\right)}{\lambda} - c_j \right] L x_i.$$

This implies that, given $\mathbf{x}$, firm j earns strictly higher profits than firm i. In other words, there is *perfect sorting across firms' types* at any equilibrium, while *firms with a higher productivity make higher profits.*

In addition, more efficient firms produce more than less efficient firms. Indeed, dividing (9) for a c_i-type firm by the same expression for a c_j-type firm, we obtain

$$\frac{D(x_i, \mathbf{x}) \left[1 - \bar{\eta}(x_i, \mathbf{x})\right]}{D(x_j, \mathbf{x}) \left[1 - \bar{\eta}(x_j, \mathbf{x})\right]} = \frac{c_i}{c_j}. \tag{12}$$

The condition **(A)** of Section 2 means that, for any given $\mathbf{x}$, a firm's marginal revenue $D(x, \mathbf{x}) \left[1 - \bar{\eta}(x, \mathbf{x})\right]$ decreases with x. Therefore, it ensues that $x_i > x_j$ if and only if $c_i < c_j$.

Furthermore, since $p_i = D(x_i, \mathbf{x})/\lambda$ and D decreases in x_i for any given $\mathbf{x}$, more efficient firms sell at lower prices than less efficient firms. As for the markups, (12) implies

$$\frac{p_i/c_i}{p_j/c_j} = \frac{1 - \bar{\eta}(x_j, \mathbf{x})}{1 - \bar{\eta}(x_i, \mathbf{x})}.$$

Consequently, more efficient firms enjoy higher markups – a regularity observed in the data (De Loecker and Warzynski, 2012) – if and only if $\bar{\eta}(x, \mathbf{x})$ increases with x, i.e., (**Abis**) holds.

The following proposition is a summary.

Proposition 2. *Assume that the active firms are heterogeneous. If* (**A**) *holds, then* *at the Nash equilibrium more efficient firms produce larger outputs, charge lower prices and earn higher profits than less efficient firms. Furthermore, more efficient firms have higher markups if* (**Abis**) *holds, but lower markups otherwise.*

Note that this proposition implies that the firm-level pass-through is *incomplete* if and only if **(Abis)** holds. A firm being negligible to the market, a change in its marginal cost does not affect $\mathbf{x}$ but only its own output. However, when all firms are hit by the same shock (e.g. a change in the exchange rate), the equilibrium consumption profile also changes, so that the equilibrium pass-through rate captures the changes in $\mathbf{x}$. We return to this issue in Section 3.

2.3 The elasticity of substitution

Definition. The elasticity of substitution plays a central role in the CES model of monopolistic competition. In what follows, we build on this idea and use the concept of *elasticity of substitution function* applied to unspecified preferences (Nadiri, 1982).

Consider any two varieties i and j such that $x_i = x_j = x$. In Appendix 2, we show that the elasticity of substitution between i and j, conditional on $\mathbf{x}$, satisfies

$$\bar{\sigma}(x, \mathbf{x}) = \frac{1}{\bar{\eta}(x, \mathbf{x})}, \tag{13}$$

where

$$\bar{\eta}(x_i, \mathbf{x}) \equiv -\frac{x_i}{D(x_i, \mathbf{x})} \frac{\partial D(x_i, \mathbf{x})}{\partial x_i}$$

is the elasticity of the inverse demand for variety i.

Evaluating $\bar{\sigma}$ at a symmetric consumption pattern, where $\mathbf{x} = xI_{[0,N]}$, yields

$$\sigma(x, N) \equiv \bar{\sigma}(x, xI_{[0,N]}).$$

Hence, regardless of the structure of preferences, at any symmetric consumption pattern *the elasticity of substitution depends only upon the individual consumption and the mass of varieties.* This implies that we can reduce the dimensionality of the problem. When firms are symmetric, that is, they share the same marginal cost, the consumption vector $\mathbf{x}$ can be summarized with the number of varieties N and the consumption per variety x. When firms are heterogeneous, the consumption pattern is no longer symmetric. Yet, it is important to notice that σ keeps its relevance. Indeed, assumption (**A**) implies that all firms of a given type c will supply the same output. Making σ type-specific (see

Section 4 for details) allows us to use it for studying heterogeneous firms at the cost of one additional dimension, i.e. the firm's type.

Given the above considerations, we may thus consider the function $\sigma(x, N)$ as the *primitive* of the model. There are two more reasons for making this choice. First, we will see that what matters for the properties of the symmetric equilibrium is how $\sigma(x, N)$ varies with x and N. More precisely, we will show that the behavior of the market outcome can be characterized by necessary and sufficient conditions stated in terms of the elasticity of substitution σ with respect to x and N, which are denoted $\mathcal{E}_x(\sigma)$ and $\mathcal{E}_N(\sigma)$. To be precise, the signs of these two expressions ($\mathcal{E}_x(\sigma) \gtreqless 0$ and $\mathcal{E}_N(\sigma) \lesseqgtr 0$) and their relationship ($\mathcal{E}_x(\sigma) \gtreqless \mathcal{E}_N(\sigma)$) will allow us to characterize *completely* the market outcome.

Second, since the elasticity of substitution is an inverse measure of the degree of product differentiation across varieties, we are able to appeal to the industrial organization theory of product differentiation to choose the most plausible assumptions regarding the behavior of $\sigma(x, N)$ with respect to x and N.

Our approach could be equivalently reformulated by considering the manifold (σ, $\mathcal{E}_x(\sigma)$, $\mathcal{E}_N(\sigma)$), which is parameterized by the variables x and N. Being generically a two-dimensional surface in $\mathbb{R}^3$, this manifold boils down to a one-dimensional locus in Mrazova and Neary (2013). The one-dimensional case encompasses a wide variety of demand systems, including those generated by additive preferences ($\mathcal{E}_N(\sigma) = 0$). We want to stress that the approach developed by Mrazova and Neary could equally be useful to cope with homothetic preferences ($\mathcal{E}_x(\sigma) = 0$).

Examples. To develop more insights about the behavior of σ as a function of x and N, we give below the elasticity of substitution for the different types of preferences discussed in the previous subsection.

(i) When the utility is additive, we have:

$$\frac{1}{\sigma(x, N)} = r(x) \equiv -\frac{xu''(x)}{u'(x)}, \tag{14}$$

which means that σ depends only upon the per variety consumption when preferences are additive.

(ii) When preferences are homothetic, Proposition 1 implies that the elasticity of substitution at a symmetric outcome depends solely on the mass N of available varieties.

Indeed, setting

$$\varphi(N) \equiv \eta(1, N)$$

and using (13) yields

$$\frac{1}{\sigma(x, N)} = \varphi(N). \tag{15}$$

For example, under translog preferences, we have $\varphi(N) = 1/(1 + \beta N)$.

Since the CES is additive, the elasticity of substitution is independent of N. Furthermore, since the CES is also homothetic, it must be that

$$r(x) = \varphi(N) = \frac{1}{\sigma}.$$

It is, therefore, no surprise that the constant σ is the only demand parameter that drives the market outcome under CES preferences.

Using (14) and (15), it is readily verified that $\mathcal{E}_N(\sigma) = 0$ if and only if preferences are additive, while $\mathcal{E}_x(\sigma) = 0$ if and only if preferences are homothetic. The CES is given by $\mathcal{E}_N(\sigma) = \mathcal{E}_x(\sigma) = 0$.

How does $\sigma(x, N)$ vary with x and N? The answer to this question a priori unclear. Therefore, we find it useful to discuss the various assumptions made in the literature, as well as to state the assumptions that we consider as our most preferred ones. Nevertheless, relaxing these assumptions does not generate ambiguity, but leads to less intuitive results that are briefly discussed.

Marshall (1920, Book 3, Chapter IV) has argued on intuitive grounds that the elasticity of the inverse demand $\bar{\eta}(x_i, \mathbf{x})$ increases in sales.[3] In our setting, (13) shows that this assumption amounts to $\partial\bar{\sigma}(x, \mathbf{x})/\partial x < 0$. However, this inequality does not tell us anything about the sign of $\mathcal{E}_x(\sigma)$ because x refers here to the consumption of *all* varieties. When preferences are additive, Marshall's argument can be applied because the elasticity of substitution is independent of $\mathbf{x}$. But this ceases to be true when preferences are non-additive. Nevertheless, as will be seen in 3.2.3, the pass-through rate is smaller than or equal to 100% if and only if $\mathcal{E}_x(\sigma) \leq 0$ holds. Although the literature on spatial pricing backs up this assumption, it also recognizes the possibility of a pass-through exceeding 100% (Greenhut et al., 1987). For this reason, our most preferred assumptions does not require $\mathcal{E}_x(\sigma) \leq 0$ to hold.

We now come to the relationship between σ and N. The spatial models of product

[3] We thank Peter Neary for having pointed out this reference to us.

differentiation featuring perfectly inelastic demands suggest that varieties become closer substitutes when N increases, the reason being that adding new varieties crowds out the product space (Salop, 1979; Anderson et al., 1992). Therefore, assuming $\mathcal{E}_N(\sigma) \geq 0$ seems natural.

To sum up, the folk wisdom would be described by the following two conditions:

$$\mathcal{E}_x(\sigma) \leq 0 \leq \mathcal{E}_N(\sigma). \tag{16}$$

However, these two inequalities turn out to be more restrictive than what they might seem at first glance. Indeed, they do not allow us to capture some interesting market effects and fail to encompass some standard models of monopolistic competition. For example, when preferences are quadratic, the elasticity of substitution decreases with N:

$$\sigma(x, N) = \frac{\alpha - \beta x}{\beta x} - \frac{\gamma}{\beta} N,$$

where $\mathcal{E}_N(\sigma) < 0$. This should not come as a surprise. Indeed, although spatial models of product differentiation and models of monopolistic competition are not orthogonal to each other, they differ in several respects (Anderson et al., 1992).

In order to compare spatial models of product differentiation, where individual demands are perfectly inelastic, and love-for-variety models of monopolistic competition, which focus on the trade-off between the per variety consumption and the mass of varieties, we consider the case in which a consumer's total consumption Nx is constant. Under this condition, we determine when an increase in N makes varieties closer substitutes.

When Nx is exogenous, it is readily verified that the following two relationships must hold:

$$\begin{aligned} \frac{\mathrm{d}x}{x} &= -\frac{\mathrm{d}N}{N} \\ \frac{\mathrm{d}\sigma}{\sigma} &= \frac{\partial \sigma}{\partial N}\frac{N}{\sigma}\frac{\mathrm{d}N}{N} + \frac{\partial \sigma}{\partial x}\frac{x}{\sigma}\frac{\mathrm{d}x}{x}. \end{aligned}$$

Plugging the first expression into the second, we obtain

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}N}\right|_{Nx=\mathrm{const}} = \frac{\sigma}{N}\left[\mathcal{E}_N(\sigma) - \mathcal{E}_x(\sigma)\right].$$

Therefore, the elasticity of substitution weakly increases with N if and only if

$$\mathcal{E}_x(\sigma) \leq \mathcal{E}_N(\sigma) \tag{17}$$

holds for all $x > 0$ and $N > 0$.

The condition (17) is less stringent than (16) because it allows the elasticity of substitution to decrease with N at a given level of x. In other words, entry may trigger more differentiation, perhaps because the incumbents react by adding new attributes to their products (Anderson et al., 1992). In addition, the evidence supporting the assumption $\mathcal{E}_x(\sigma) < 0$ being mixed, we find it relevant to investigate the implications of $\mathcal{E}_x(\sigma) > 0$ as well.

Finally, (17) seems to be the appropriate generalization of Krugman's (1979) idea that introducing new varieties triggers a pro-competitive effect, a result that is empirically confirmed by Feenstra and Weinstein (2015). Based on these considerations, we find it reasonable to consider (17) as one of our most preferred assumptions.

3 Symmetric monopolistic competition

As mentioned in the introduction, starting with the case of symmetric firms allows one to highlight in a simple way how the properties of $\sigma(x, N)$ affect the market outcome.

3.1 Existence and uniqueness of a symmetric free-entry equilibrium

We first determine prices, outputs and profits when the mass of firms is fixed, and then find N by using the zero-profit condition. When N is exogenously given, the market equilibrium is given by the functions $\bar{\mathbf{q}}(N)$, $\bar{\mathbf{p}}(N)$ and $\bar{\mathbf{x}}(N)$ defined on $[0, N]$, which satisfy the following four conditions: (i) no firm i can increase its profit by changing its output, (ii) each consumer maximizes her utility subject to her budget constraint, (iii) the product market clearing condition

$$\bar{q}_i = L\bar{x}_i \qquad \text{for all } i \in [0, N]$$

and (iv) the labor market balance

$$c\int_0^N q_i \mathrm{d}i + NF = yL$$

hold. The study of market equilibria where the number of firms is exogenous is to be viewed as an intermediate step toward monopolistic competition, where the number of firms is endogenized by free entry and exit.

Since we focus here on symmetric free-entry equilibria, we find it reasonable to study symmetric market equilibria, which means that the functions $\bar{\mathbf{q}}(N)$, $\bar{\mathbf{p}}(N)$ and $\bar{\mathbf{x}}(N)$ become scalars, i.e., $\bar{q}(N)$, $\bar{p}(N)$ and $\bar{x}(N)$. For this, consumers must have the same income, which holds when profits are uniformly distributed across consumers. Therefore, the budget constraint (3) must be replaced by the following expression:

$$\int_0^N p_i x_i \mathrm{d}i = Y \equiv y + \frac{1}{L}\int_0^N \pi_i \mathrm{d}i. \tag{18}$$

Being negligible to the market, each firm accurately treats Y as a given.

Each firm facing the same demand and being negligible, the function $\pi(x_i, \mathbf{x})$ is the same for all i. In addition, (**A**) implies that $\pi(x_i, \mathbf{x})$ has a unique maximizer for any $\mathbf{x}$. As a result, the market equilibrium must be symmetric.

Using $\pi_i \equiv (p_i - c)Lx_i - F$, (18) boils down to labor market balance:

$$cL\int_0^N x_i \mathrm{d}i + FN = yL,$$

which yields the only candidate symmetric equilibrium for the per variety consumption:

$$\bar{x}(N) = \frac{y}{cN} - \frac{F}{cL}. \tag{19}$$

Therefore, $\bar{x}(N)$ is unique and positive if and only if $N \leq Ly/F$. The product market clearing condition implies that the candidate equilibrium output is

$$\bar{q}(N) = \frac{yL}{cN} - \frac{F}{c}. \tag{20}$$

Plugging (20) into the profit maximization condition (22) shows that there is a unique candidate equilibrium price given by

$$\bar{p}(N) = c\frac{\sigma\left(\bar{x}(N), N\right)}{\sigma\left(\bar{x}(N), N\right) - 1}. \tag{21}$$

Clearly, if $N > Ly/F$, there exists no interior equilibrium. Accordingly, we have the following result: *If* (**A**) *holds and* $N \leq Ly/F$*, then there exists a unique market equilibrium. Furthermore, this equilibrium is symmetric.*

Rewriting the equilibrium conditions (9) along the diagonal yields

$$\bar{m}(N) \equiv \frac{\bar{p}(N) - c}{\bar{p}(N)} = \frac{1}{\sigma(\bar{x}(N), N)}, \tag{22}$$

while

$$\bar{\pi}(N) \equiv (\bar{p}(N) - c)\bar{q}(N)$$

denotes the equilibrium operating profits made by a firm when there is a mass N of firms.

Importantly, (22) shows that, for any given N, *the equilibrium markup* $\bar{m}(N)$ *varies inversely with the elasticity of substitution.* The intuition is easy to grasp. It is well know from industrial organization that product differentiation relaxes competition. When the elasticity of substitution is lower, varieties are worse substitutes, thereby endowing firms with more market power. It is, therefore, no surprise that firms have a higher markup when σ is lower. It also follows from (22) that the way σ varies with x and N shapes the market outcome. In particular, this demonstrates that assuming a constant elasticity of substitution amounts to adding very strong restraints on the way the market works.

Combining (19) and (21), we find that the operating profits are given by

$$\bar{\pi}(N) = \frac{cL\bar{x}(N)}{\sigma\left(\bar{x}(N), N\right) - 1}. \tag{23}$$

It is legitimate to ask how $\bar{p}(N)$ and $\bar{\pi}(N)$ vary with the mass of firms There is no simple answer to this question. Indeed, the expression (23) suffices to show that the way the market outcome reacts to the entry of new firms depends on how the elasticity of substitution varies with x and N. This confirms why static comparative statics under oligopoly yields ambiguous results.

We now pin down the equilibrium value of N by using the zero-profit condition. Therefore, a consumer's income is equal to her sole labor income: $Y = y$. A *symmetric free-entry equilibrium* (SFE) is described by the vector (q^*, p^*, x^*, N^*), where N^* solves the zero-profit condition

$$\bar{\pi}(N) = F, \tag{24}$$

while $q^* = \bar{q}(N^*)$, $p^* = \bar{p}(N^*)$ and $x^* = \bar{x}(N^*)$. The Walras Law implies that the budget constraint $N^* p^* x^* = y$ is satisfied. Without loss of generality, we restrict ourselves to the domain of parameters for which $N^* < \mathcal{N}$.

Combining (22) and (24), we obtain a single equilibrium condition given by

$$\bar{m}(N) = \frac{NF}{Ly}, \tag{25}$$

which means that, at the SFE, the equilibrium markup is equal to the share of the labor supply spent on overhead costs. When preferences are non-homothetic, (19) and (21) show that L/F and y enter the function $\bar{m}(N)$ as two distinct parameters. This implies that L and y have a different impact on the equilibrium markup, while a hike in L is equivalent to a drop in F.

For the condition (24) to have a unique solution N^* for *all* values of $F > 0$, it is necessary and sufficient that $\bar{\pi}(N)$ strictly decreases with N. Differentiating (23) with respect to N, we obtain

$$\begin{aligned}
\bar{\pi}'(N) &= \bar{x}'(N)\,\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{cLx}{\sigma\left(x, yL/(cLx+F)\right)-1}\right]\Bigg|_{x=\bar{x}(N)} \\
&= -\frac{y}{cN^2}\left(\sigma - 1 - x\frac{\partial\sigma}{\partial x} + \frac{cLx}{cLx+F}\frac{yL}{cLx+F}\frac{\partial\sigma}{\partial N}\right)\Bigg|_{x=\bar{x}(N)}.
\end{aligned}$$

Using (19) and (24), we find that the second term in the right-hand side of this expression is positive if and only if

$$\mathcal{E}_x(\sigma(x,N)) < \frac{\sigma(x,N)-1}{\sigma(x,N)}\left[1 + \mathcal{E}_N(\sigma(x,N))\right]. \tag{26}$$

Therefore, $\bar{\pi}'(N) < 0$ for all N if and only if (26) holds. This implies the following proposition.

Proposition 3. *Assume* (**A**). *There exists a free-entry equilibrium for all* $c > 0$ *and* $F > 0$ *if and only if* (26) *holds for all* $x > 0$ *and* $N > 0$. *Furthermore, this equilibrium is unique, stable and symmetric.*

Because the above proposition provides a necessary and sufficient condition for the existence of a SFE, we may safely conclude that the set of assumptions required to bring into play monopolistic competition must include (26). Therefore, throughout the remaining of the paper, we assume that (26) holds. This condition allows one to work with preferences that display a great deal of flexibility. Indeed, σ may decrease or increase with x and/or N. To be precise, varieties may become better or worse substitutes when the per variety consumption and/or the number of varieties rises, thus generating either price-decreasing or price-increasing competition. Evidently, (26) is satisfied when the folk wisdom conditions (16) hold.

Under additive preferences, (26) amounts to assuming that $\mathcal{E}_x(\sigma) < (\sigma - 1)/\sigma$, which means that σ cannot increase "too fast" with x. In this case, as shown by (25), there exists a unique SFE and the markup function $m(N)$ increases with N provided that the slope of m is smaller than F/Ly. In other words, *a market mimicking anti-competitive effects need not preclude the existence and uniqueness of a SFE* (Zhelobodko et al., 2012). When preferences are homothetic, (26) holds if and only if $\mathcal{E}_N(\sigma)$ exceeds -1, which means that varieties cannot become too differentiated when their number increases, which seems reasonable.

In what follows, (17) *and* (26) *are our most preferred assumptions*. The former states that the impact of a change in the number of varieties on σ dominates the impact of a change in the per variety consumption, and thus points to the importance of the variety range for consumers. The latter is necessary and sufficient for the existence and uniqueness of a SFE. Taken together, (17) and (26) define a range of possibilities which is broader than the one defined by (16). We will refrain from following an encyclopedic approach in which all cases are systematically explored. However, since (17) need not hold for a SFE to exist, we will also explore what the properties of the equilibrium become when this condition is not met. In so doing, we are able to highlight the role played by (17) for some particular results to hold.

3.2 Comparative statics

In this subsection, we study the impact of a shock in GDP on the SFE. A higher total income may stem from a larger population L, a higher per capita income y, or both. Next, we will discuss the impact of firm's productivity. To achieve our goal, it proves to be convenient to work with the markup as the endogenous variable. Setting $m \equiv FN/(Ly)$, we may rewrite the equilibrium condition (25) in terms of m only:

$$m\sigma\left(\frac{F}{cL}\frac{1-m}{m}, \frac{Ly}{F}m\right) = F. \tag{27}$$

Note that (27) involves the four structural parameters of the economy: L, y, c and F. Furthermore, it is readily verified that the left-hand side of (27) increases with m if and only if (26) holds. Therefore, to study the impact of a specific parameter, we only have to find out how the corresponding curve is shifted. Before proceeding, we want to stress that we provide below a complete description of the comparative static effects through a series of necessary and sufficient conditions.

3.2.1 The impact of population size

Let us first consider the impact on the market price p^*. Differentiating (27) with respect to L, we find that the right-hand side of (27) is shifted upwards under an increase in L if and only if (17) holds. As a consequence, the equilibrium markup m^*, whence the equilibrium price p^*, decreases with L. This is in accordance with Handbury and Weinstein (2015) who observe that the price level for food products falls with city size. In this case, (27) implies that the equilibrium value of σ increases, which amounts to saying that varieties get less differentiated in a larger market, very much like in spatial models of product differentiation.

Second, the zero-profit condition implies that L always shifts p^* and q^* in opposite directions. Therefore, firm sizes are larger in bigger markets, as suggested by the empirical evidence provided by Manning (2010).

How does N^* change with L? Differentiating (23) with respect to L, we have

$$\left.\frac{\partial \bar{\pi}}{\partial L}\right|_{N=N^*} = \frac{cx}{\sigma(x,N)-1} + \frac{\partial \bar{x}(N)}{\partial L}\frac{\partial}{\partial x}\left.\left(\frac{cLx}{\sigma(x,N)-1}\right)\right|_{x=x^*,N=N^*}. \tag{28}$$

Substituting F for $\bar{\pi}(N^*)$ and simplifying, we obtain

$$\left.\frac{\partial \bar{\pi}}{\partial L}\right|_{N=N^*} = \left.\left[\frac{cx\sigma}{(\sigma-1)^3}\left(\sigma - 1 - \mathcal{E}_x(\sigma)\right)\right]\right|_{x=x^*,N=N^*}.$$

Since the first term in the right-hand side of this expression is positive, (28) is positive if and only if the following condition holds:

$$\mathcal{E}_x(\sigma) < \sigma - 1. \tag{29}$$

In this case, a population growth triggers the entry of new firms. Otherwise, the mass of varieties falls with the population size. Indeed, when $\mathcal{E}_x(\sigma)$ exceeds $\sigma - 1$, increasing the individual consumption makes varieties much closer substitutes, which intensifies competition. Under such circumstances, the benefits associated with diversity are low, thus implying that consumers value more the volumes they consume. This in turn leads a fraction of incumbents to get out of business. Furthermore, when (29) holds, the labor market balance condition implies that N^* rises less than proportionally with L because q^* increases with L. Observe also that (26) implies (29) when preferences are additive, while (29) holds true under homothetic preferences because $\mathcal{E}_x(\sigma) = 0$.

The following proposition comprises a summary.

Proposition 4. *If $\mathcal{E}_x(\sigma)$ is smaller than $\mathcal{E}_N(\sigma)$ at the SFE, then a higher population size results in a lower markup and larger firms. Furthermore, the mass of varieties increases with L if and only if* (29) *holds in equilibrium.*

What happens when $\mathcal{E}_x(\sigma) > \mathcal{E}_N(\sigma)$ at the SFE? In this event, a higher population size results in a higher markup, smaller firms, a more than proportional rise in the mass of varieties, and a lower per variety consumption. In other words, a larger market would generate anti-competitive effects. Last, when $\mathcal{E}_x(\sigma) = \mathcal{E}_N(\sigma)$ at the SFE, as in the CES case, the population size has no impact on the market outcome.

3.2.2 The impact of individual income

We now come to the impact of the per capita income on the SFE. One expects a positive shock on y to trigger the entry of new firms because more labor is available for production. However, consumers have a higher willingness-to-pay for the incumbent varieties and can afford to buy each of them in a larger volume. Therefore, the impact of y on the SFE is a priori ambiguous.

Differentiating (27) with respect to y, we see that the left-hand side of (27) is shifted downwards by an increase in y if and only if $\mathcal{E}_N(\sigma) > 0$. In this event, the equilibrium markup decreases with y. To check the impact of y on N^*, we differentiate (23) with respect to y and get

$$\left.\frac{\partial \bar{\pi}(N)}{\partial y}\right|_{N=N^*} = \left.\left[\frac{\partial \bar{x}(N)}{\partial y}\frac{\partial}{\partial x}\left(\frac{cLx}{\sigma\left(x,N\right)-1}\right)\right]\right|_{x=x^*,N=N^*}.$$

After simplification, this yields

$$\left.\frac{\partial \bar{\pi}(N)}{\partial y}\right|_{N=N^*} = \left.\frac{L}{N}\frac{\sigma-1-\sigma\mathcal{E}_x(\sigma)}{(\sigma-1)^2}\right|_{x=x^*,N=N^*}.$$

Hence, $\partial\bar{\pi}(N^*)/\partial y > 0$ if and only if the following condition holds:

$$\mathcal{E}_x(\sigma) < \frac{\sigma-1}{\sigma}. \tag{30}$$

Note that this condition is more stringent than (29). Thus, if $\mathcal{E}_N(\sigma) > 0$, then (30) implies (26). As a consequence, we have:

Proposition 5. *If $\mathcal{E}_N(\sigma) > 0$ at the SFE, then a higher per capita income results in a lower markup and bigger firms. Furthermore, the mass of varieties increases with y if and only if* (30) *holds in equilibrium.*

Thus, when entry renders varieties less differentiated, the mass of varieties need not rise with income. Indeed, the increase in per variety consumption may be too high for all the incumbents to stay in business. The reason for this is that the decline in prices is sufficiently strong for fewer firms to operate at a larger scale. As a consequence, a richer economy need not exhibit a wider array of varieties.

Evidently, if $\mathcal{E}_N(\sigma) < 0$, the markup is higher and firms are smaller when the income y rises. Furthermore, (26) implies (30) so that N^* increases with y. Indeed, since varieties get more differentiated when entry arises, firms exploit consumers' higher willingness-to-pay to sell less at a higher price, which goes together with a larger mass of varieties.

Propositions 4 and 5 show that an increase in L is not a substitute for an increase in y and vice versa, except, as shown below, in the case of homothetic preferences. This should not come as a surprise because an increase in income affects the shape of individual demands when preferences are non-homothetic, whereas an increase in L shifts upward the market demand without changing its shape.

Finally, if preferences are homothetic, it is well known that the effects of L and y on the market variables p^*, q^* and N^* are exactly the same. Indeed, m does not involve y as a parameter because σ depends solely on N. Therefore, it ensues from (25) that the equilibrium price, firm size, and number of firms depend only upon the total income yL.

3.2.3 The impact of firm productivity

Firms' productivity is typically measured by their marginal costs. To uncover the impact on the market outcome of a productivity shock common to all firms, we conduct a comparative static analysis of the SFE with respect to c and show that *the nature of preferences determines the extent of the pass-through.* In particular, we establish that the pass-through rate is lower than 100% if and only if σ decreases with x, i.e.

$$\mathcal{E}_x(\sigma) < 0 \tag{31}$$

holds. Evidently, the pass-through rate exceeds 100% when $0 < \mathcal{E}_x(\sigma)$.

The left-hand side of (27) is shifted downwards under a decrease in c if and only if $\mathcal{E}_x(\sigma) < 0$. In this case, both the equilibrium markup m^* and the equilibrium mass of firms $N^* = (yL/F) \cdot m^*$ increases with c. In other words, when $\mathcal{E}_x(\sigma) < 0$, *the pass-through rate is smaller than* 1 because varieties becomes more differentiated, which relaxes competition. On the contrary, when $\mathcal{E}_x(\sigma) > 0$, the markup and the mass of firms decrease because varieties get less differentiated. In other words, competition becomes so tough that p^* decreases more than proportionally with c. In this event, *the pass-through rate exceeds* 1.

Under homothetic preferences, ($\mathcal{E}_x(\sigma) = 0$), $\bar{p}(N)$ is given by

$$\bar{p}(N) = \frac{c}{1 - \varphi(N)} \Longrightarrow m(N) = \varphi(N).$$

As a consequence, (25) does not involve c as a parameter. This implies that a technological shock does affect the number of firms. In other words, the markup remains the same regardless of the productivity shocks, thereby implying that *under homothetic preferences the pass-through rate equal to* 1.

The impact of technological shocks on firms' size leads to ambiguous conclusions. For example, under quadratic preferences, q^* may increase and, then, decreases in response to a steadily drop in c.

The following proposition comprises a summary.

Proposition 6. *If the marginal cost of all firms drops,* (i) *the market price decreases and* (ii) *the markup and number of firms increase if and only if* (31) *holds at the SFE.*

This proposition has an important implication. If the data suggest a pass-through rate smaller than 1 (Berman et al., 2011), then it must be that $\mathcal{E}_x(\sigma) < 0$. In this case, (29) must hold while (26) is satisfied when $\mathcal{E}_N(\sigma)$ exceeds -1, thereby a bigger or richer economy is more competitive and more diversified than a smaller or poorer one. Note that (26) does not restrict the domain of admissible values of $\mathcal{E}_x(\sigma)$ for a pass-through rate to be smaller than 1, whereas (26) requires that $\mathcal{E}_x(\sigma)$ cannot exceed $(1 - 1/\sigma)(1 + \mathcal{E}_N(\sigma))$. Recent empirical evidence shows that the pass-through generated by a commodity tax or by trade costs need not be smaller than 1 (Martin, 2012; Weyl and Fabinger, 2013). Our theoretical argument thus concurs with the inconclusive empirical evidence: the pass-through rate may exceed 1, although it is more likely to be less than 1.

Let us summarize our main results. We have found necessary and sufficient conditions for the existence and uniqueness of a SFE (Proposition 3), and provided a complete characterization of the effect of a market size or productivity shock (Propositions 4 to 6). Observing that (16) implies (26), (17) and (30), we may conclude as follows: *if* (16) *holds, a unique SFE exists* (Proposition 3), *a larger market or a higher income leads to lower markups, bigger firms and a larger number of varieties* (Propositions 4 and 5), *and the pass-through rate is smaller than* 1 (Proposition 6).

Local conditions. It is legitimate to ask what Proposition 3 becomes when (26) does not hold for all $x > 0$ and $N > 0$. In this case, there may exist several stable SFEs, so that Propositions 4-6 hold true for small shocks at any stable SFE. Of course, when there is multiplicity of equilibria, different patterns may arise at different equilibria because the functions $\mathcal{E}_x(\sigma)$ and $\mathcal{E}_N(\sigma)$ need not behave in the same way at each stable equilibrium.

4 Heterogeneous firms

Our approach is also well suited to cope with Melitz-like heterogeneous firms. In what follows, we consider the one-period framework used by Melitz and Ottaviano (2008), the mass of potential firms being given by $\mathcal{N}$. Prior to entry, risk-neutral firms face uncertainty about their marginal cost while entry requires a sunk cost F_e. Once this cost is paid, firms observe their marginal cost drawn randomly from the continuous probability distribution $\Gamma(c)$ defined over $\mathbb{R}_+$. After observing its type c, each entrant decides to produce or not, given that an active firm must incur a fixed production cost F. Under such circumstances, the mass of entrants, N_e, typically exceeds the mass of operating firms, N. Because preferences are assumed to be symmetric, firms sharing the same marginal cost c behave in the same way and earn the same profit at equilibrium. As a consequence, we may refer to any variety/firm by its c-type only.

The equilibrium conditions are as follows:

(i) the profit-maximization condition for c-type firms:

$$\max_{x_c} \pi_c(x_c, \mathbf{x}) \equiv \left[\frac{D(x_c, \mathbf{x})}{\lambda} - c\right] L x_c - F; \tag{32}$$

(ii) the zero-profit condition for the cutoff firm:

$$(p_{\hat{c}} - \hat{c}) q_{\hat{c}} = F,$$

where $\hat{c}$ is the cutoff cost. As shown in Section 2.2, firms are sorted out by decreasing order of productivity, which implies that the mass of active firms is equal to $N \equiv N_e \Gamma(\hat{c})$;

(iii) the product market clearing condition:

$$q_c = L x_c$$

for all $c \in [0, \hat{c}]$;

(iv) the labor market clearing condition:

$$N_e\left[F_e+\int_0^{\hat{c}}(cq_c+F)\mathrm{d}\Gamma(c)\right]=yL,$$

where N_e is the number of entrants;

(v) firms enter the market until their expected profits net of the entry cost F_e are zero:

$$\int_0^{\hat{c}}\pi_c(x_c,\mathbf{x})\mathrm{d}\Gamma(c)=F_e.$$

Since the distribution Γ is given, the profit-maximization condition implies that the equilibrium consumption profile is entirely determined by the set of active firms, which is fully described by $\hat{c}$ and N_e. In other words, a variety supplied by an active firm can be viewed as a point in

$$\Omega\equiv\{(c,\nu)\in\mathbb{R}_+^2\mid c\leq\hat{c};\nu\leq N_e\gamma(c)\}.$$

In the case of homogeneous firms, because the variable N is sufficient to describe the set of active firms, we have $\Omega=[0,N]$.

Very much as in 3.1 where N is treated parametrically, we assume for the moment that $\hat{c}$ and N_e are given, and consider the game in which the corresponding active firms compete in quantities. Because we work with general preferences, the equilibrium outcome cannot be obtained by solving the game pointwise. Indeed, the profit-maximizing output of a c-type firm depends on what the other types of firms do. Furthermore, because all the c-type firms sell at the same price that depends on c, the consumption of a variety is c-specific. In what follows, we assume that, for any $\hat{c}$ and N_e, this game has a Nash equilibrium $\bar{\mathbf{x}}(\hat{c},N_e)$.[4] The counterpart of $\bar{\mathbf{x}}(\hat{c},N_e)$ in the case of symmetric firms is $\bar{x}(N)$ (see (19)). The corresponding operating profits are defined as follows:

$$\bar{\pi}_c(\hat{c},N_e)\equiv\left[\frac{D\left(\bar{\mathbf{x}}(\hat{c},N_e)\right)}{\lambda\left(\bar{\mathbf{x}}(\hat{c},N_e)\right)}-c\right]L\bar{x}_c(\hat{c},N_e).$$

The perfect sorting of firms means that $\bar{\pi}_c(\hat{c},N_e)$ decreases with c. We now impose an additional condition that implies that firms face a more competitive market when the number of active firms rises.

(**B**) *The equilibrium profit of each firm's type decreases in $\hat{c}$ and N_e.*

The intuition behind this assumption is easy to grasp: a larger number of entrants or a higher cutoff leads to lower profits, for the mass of active firms rises. When firms

[4]In Appendix 3, we show that such an equilibrium exists under mild assumptions.

are symmetric, the equilibrium operating profits depend only upon the number N of active firms (see (23)). Thus, (**B**) amounts to assuming that these profits decrease with N. Proposition 3 shows that this is so if and only if (26) holds. When firms are heterogeneous, it is difficult to find what the condition (26) becomes. However, (**B**) can be shown to hold for specific classes of preferences. For example, consider the CES case where the equilibrium operating profits are given by

$$\bar{\pi}_c(\hat{c}, N_e) = \frac{yL}{\sigma N_e} \frac{c^{1-\sigma}}{\int_0^{\hat{c}} z^{1-\sigma} \mathrm{d}\Gamma(z)}, \tag{33}$$

which implies (**B**). More generally, using Zhelobodko et al. (2012), it can be shown that any additive preference satisfying (**A**) also satisfies (**B**).

A free-entry equilibrium with heterogeneous firms is defined by a pair $(\hat{c}^*, N_e^*)$ that satisfies the zero-expected-profit condition for each firm

$$\int_0^{\hat{c}} [\bar{\pi}_c(\hat{c}, N_e) - F] \, \mathrm{d}\Gamma(c) = F_e, \tag{34}$$

as well as the cutoff condition

$$\bar{\pi}_{\hat{c}}(\hat{c}, N_e) = F. \tag{35}$$

Thus, regardless of the nature of preferences and the distribution of marginal costs, the heterogeneity of firms amounts to replacing the variable N by the two variables $\hat{c}$ and N_e because $N = \Gamma(\hat{c})N_e$ when $\bar{x}(N)$ is replaced by $\bar{\mathbf{x}}(\hat{c}, N_e)$. As a consequence, the complexity of the problem goes from one to two dimensions.

Dividing (34) by (35) yields the following new equilibrium condition:

$$\int_0^{\hat{c}} \left[\frac{\bar{\pi}_c(\hat{c}, N_e)}{\bar{\pi}_{\hat{c}}(\hat{c}, N_e)} - 1 \right] \mathrm{d}\Gamma(c) = \frac{F_e}{F}. \tag{36}$$

Let $\hat{c} = g(N_e)$ be the locus of solutions to (35) and $\hat{c} = h(N_e)$ the locus of solutions to (36).[5] A free-entry equilibrium is thus an intersection point of the two loci $\hat{c} = g(N_e)$ and $\hat{c} = h(N_e)$ in the $(N_e, \hat{c})$-plane. As implied by (**B**), $g(N_e)$ is downward-sloping in the $(N_e, \hat{c})$-plane. Furthermore, it is shifted upward when L rises. As for $h(N_e)$, it is independent of L but its slope is a priori undetermined. Yet, we will see that the sign of the slope of $h(N_e)$ is critical for the impact of a population hike on the cutoff cost.

[5] We give below sufficient conditions for the left-hand side of (36) to be monotone in $\hat{c}$ and N_e, two conditions that guarantee that the locus $\hat{c} = h(N_e)$ is well defined.

Three cases may arise. First, if the locus $h(N_e)$ is upward-sloping, there exists a unique free-entry equilibrium, and this equilibrium is stable. Furthermore, both N_e^* and $\hat{c}^*$ increase with L (see Figure 1a). Let us now rotate the locus $h(N_e)$ in a clockwise manner, so that $h(N_e)$ gets less and less steep. The second case, which corresponds to the CES preferences, arises when $h(N_e)$ is horizontal, which implies that N_e^* rises with L while $\hat{c}^*$ remains constant.

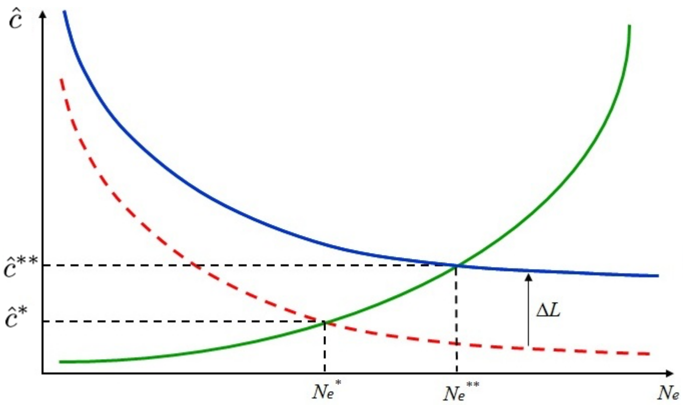

Figure 1a

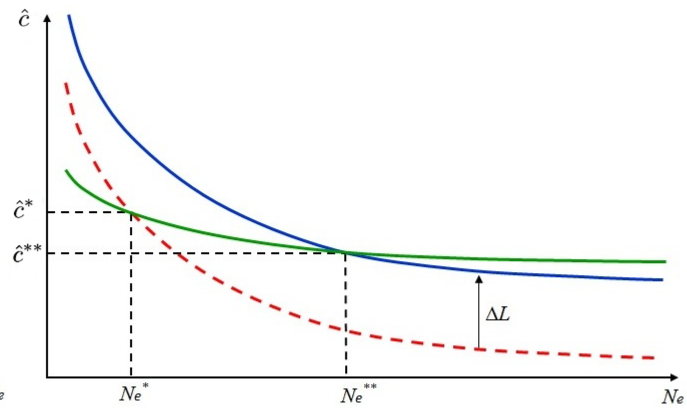

Figure 1b

Cutoff and market size

Last, when $h(N_e)$ is downward-sloping, two subcases must be distinguished. In the first one, $h(N_e)$ is less steep than $g(N_e)$. As a consequence, there still exists a unique free-entry equilibrium. This equilibrium is stable and such that N_e^* increases with L, but $\hat{c}^*$ now decreases with L (see Figure 1b). In the second subcase, $h(N_e)$ is steeper than $g(N_e)$, which implies that the equilibrium is unstable because $h(N_e)$ intersects $g(N_e)$ from below. In what follows, we focus only upon stable equilibria.

In sum, we end up with the following properties of the free-entry equilibrium:

Proposition 7. *Assume* **(B)**. *Then, the equilibrium mass of entrants always increases with L, while the equilibrium cutoff cost decreases with L if and only if $h(N_e)$ is downward-sloping.*

When firms are symmetric, we have seen that the sign of $\mathcal{E}_N(\sigma)$ plays a critical role in comparative statics. The same holds here. The difference is that the mass of active firms is now determined by $\hat{c}$ and N_e. As a consequence, understanding how the mass of active firms responds to a population hike requires studying the way the left-hand side of (36) varies with $\hat{c}$ and N_e. To this end, we rewrite $\bar{\pi}_c(\hat{c}, N_e)$ in terms of the elasticity of substitution among c-type varieties:

$$\bar{\pi}_c(\hat{c}, N_e) = \frac{c}{\sigma_c(\hat{c}, N_e) - 1} L\bar{x}_c(\hat{c}, N_e),$$

where $\sigma_c(\hat{c}, N_e)$ is the equilibrium value of the elasticity of substitution between any two varieties supplied by c-type firms:

$$\sigma_c(\hat{c}, N_e) \equiv \bar{\sigma}[\bar{x}_c(\hat{c}, N_e), \bar{\mathbf{x}}(\hat{c}, N_e)].$$

Note that σ_c is now *cost-specific*, the reason being that firms facing different marginal costs price at different points of their demand schedules and, therefore, react differently to a hike or drop in the number of active firms. By contrast, two firms sharing the same cost sell at the same price, so that they react in the same way. In other words, varieties are symmetric among c-type firms but asymmetric between c_i-type and c_j-type firms when c_i and c_j differ. As a result, *heterogeneous firms supplying symmetric varieties behave like homogeneous firms selling varieties whose degree of differentiation varies with their type* c.

The elasticity of substitution being the keystone of our approach, it is legitimate to ask whether imposing some simple conditions on σ_c (similar to those used in Section 3) can tell us something about the slope of $h(N_e)$.

Using the envelope theorem and the profit-maximization condition (32), we obtain:

$$\mathcal{E}_c\left(\bar{\pi}_c(\hat{c}, N_e)\right) = 1 - \sigma_c(\hat{c}, N_e), \tag{37}$$

which shows how the function $\sigma_c(\hat{c}, N_e)$ affects the operating profits of a c-type firm. Since

$$-\int_c^{\hat{c}} \frac{\mathcal{E}_z\left(\bar{\pi}_z(\hat{c}, N_e)\right)}{z} \mathrm{d}z = -\int_c^{\hat{c}} \frac{\partial \ln \bar{\pi}_z(\hat{c}, N_e)}{\partial z} \mathrm{d}z = \ln \bar{\pi}_c(\hat{c}, N_e) - \ln \bar{\pi}_{\hat{c}}(\hat{c}, N_e),$$

it follows from (37) that

$$\ln\left[\frac{\bar{\pi}_c(\hat{c}, N_e)}{\bar{\pi}_{\hat{c}}(\hat{c}, N_e)}\right] = \int_c^{\hat{c}} \frac{\sigma_z(\hat{c}, N_e) - 1}{z} \mathrm{d}z.$$

Accordingly, the equilibrium condition (36) may rewritten as follows:

$$\int_0^{\hat{c}} \left[\exp\left(\int_c^{\hat{c}} \frac{\sigma_z(\hat{c}, N_e) - 1}{z} \mathrm{d}z \right) - 1 \right] \mathrm{d}\Gamma(c) = \frac{F_e}{F}. \tag{38}$$

Therefore, if $\sigma_c(\hat{c}, N_e)$ increases with $\hat{c}$, the left-hand side of (36) increases with $\hat{c}$. Intuitively, when $\hat{c}$ increases, the mass of firms rises as less efficient firms stay in business, which intensifies competition and lowers markups. In this case, the selection process is tougher. This is not the end of the story, however. Indeed, the competitiveness of the market also depends on how N_e affects the degree of differentiation across varieties.

The expression (38) shows that the left-hand side of (36) increases with N_e if and only if $\sigma_c(\hat{c}, N_e)$ increases in N_e. This amounts to assuming that, for any given cutoff, the relative impact of entry on the low-productivity firms (i.e., the small firms) is larger than the impact on the high-productivity firms, the reason being that $\mathcal{E}_c\left(\bar{\pi}_c(\hat{c}, N_e)\right)$ decreases in N_e if and only if $\sigma_c(\hat{c}, N_e)$ increases in N_e.

When $\sigma_c(\hat{c}, N_e)$ increases both with $\hat{c}$ and N_e, the locus $h(N_e)$ is downward-sloping. Indeed, when N_e rises, so does the left-hand side of (38). Hence, since $\sigma_c(\hat{c}, N_e)$ also increases with $\hat{c}$, it must be that $\hat{c}$ decreases for (38) to hold. As a consequence, we have:

Proposition 8. *Assume* (**B**). *If $\sigma_c(\hat{c}, N_e)$ increases with $\hat{c}$ and N_e, then $\hat{c}^*$ decreases with L. If $\sigma_c(\hat{c}, N_e)$ increases with $\hat{c}$ and decreases with N_e, then $\hat{c}^*$ increases with L.*

Given $\hat{c}$, we know that the number of active firms N is proportional to the number of entrants N_e. Therefore, assuming that $\sigma_c(\hat{c}, N_e)$ increases with N_e may be considered as the counterpart of (17), which is one of our most preferred assumptions in the case of symmetric firms. Indeed, as shown in Section 2.3, (17) can be reformulated as follows: $\sigma(\bar{x}(N), N)$ increases with N. In this case, the pro-competitive effect generated by entry exacerbates the selection effect across firms. In response to a hike in L, the two effects combine to induce the exit of the least efficient active firms. This echoes Melitz and Ottaviano (2008) who show that a trade liberalization shock gives rise to a similar effect under quadratic preferences. However, since N_e^* and $\hat{c}^*$ move in opposite directions, it is hard to predict how L affects the equilibrium mass N^* of active firms. Indeed, how strong are the entry and selection effects depends on the elasticity of substitution and the distribution $G(c)$.

Repeating the above argument shows that, when $\sigma_c(\hat{c}, N_e)$ increases with $\hat{c}$ but decreases with N_e, the locus $h(N_e)$ is upward-sloping. When $\sigma_c(\hat{c}, N_e)$ decreases with N_e, two opposing effects are at work. On the one hand, entry fosters product differentiation, and thus relaxes competition. This invites new firms to enter and allows less efficient firms to stay in business. On the other hand, the corresponding hike in $\hat{c}$ tends to render varieties closer substitutes. What our result shows is that the former effect dominates the latter. Hence, the equilibrium mass of active firms $N^* = \Gamma(\hat{c}^*)N_e^*$ unambiguously rises with L.

The impact of population size on the number of entrants is thus unambiguous: a larger market invites more entry.[6] By contrast, the cutoff cost behavior depends on how the elasticity of substitution varies with N_e.[7] All of this shows that the interaction between the entry and selection effects is non-trivial.

Finally, note that what we said in 3.2 about local rather global conditions equally applies here. Indeed, when $\sigma_c(\hat{c}, N_e)$ increases with $\hat{c}$ and N_e in an neighborhood of the equilibrium, the above argument can be repeated to show that $\hat{c}^*$ decreases with L for small changes in L.

Pass-through. We show here that the result on complete pass-through under homothetic preferences still holds when firms are heterogeneous. The first-order condition for a c-type firm is given by

$$\frac{p_c - c}{p_c} = \bar{\eta}(x_c, \mathbf{x}). \tag{39}$$

Consider a proportionate drop in marginal costs by a factor $\mu > 1$, so that the distribution of marginal costs is given by $G(\mu c)$. We first investigating the impact of μ on firms' operating profits when the cutoff $\hat{c}$ is unchanged. The cutoff firms now have a marginal cost equal to $\hat{c}/\mu$. Furthermore, under homothetic preferences, $\bar{\eta}(x_c, \mathbf{x})$ does not depend on the income Y and is positive homogeneous of degree 0. Therefore, (39) is invariant to the same proportionate reduction in c. As a consequence, the new price equilibrium profile over $[0, \hat{c}]$ is obtained by dividing all prices by μ.

We now show that the profits of the $\hat{c}$-type firms do not change in response to the drop in cost, so that the new cutoff is given by $\hat{c}/\mu$. Indeed, both marginal costs and prices are divided by μ, while homothetic preferences imply that demands are shifted upwards by the same factor μ. Therefore, the operating profit of the $\hat{c}$-type firms is

[6] By reformulating the game with price-setting firms, the same comparative statics analysis can be undertaken to study how the per capita income affects the market outcome.

[7] Results are ambiguous when $\sigma_c(\hat{c}, N_e)$ decreases with $\hat{c}$. In this case, the left-hand side of (38) may be non-monotone in $\hat{c}$. As a result, the mapping $h(N_e)$ may cease to be single-valued, which potentially leads to the existence of multiple equilibria. However, note that at any specific equilibrium, the behavior of $\hat{c}$ with respect to L depends solely on whether $h(N_e)$ is locally upward-sloping or downward-sloping.

unchanged because

$$\left(\frac{p_{\hat{c}}}{\mu} - \frac{\hat{c}}{\mu}\right) L\mu x_{\hat{c}} = \left(p_{\hat{c}} - \hat{c}\right) L x_{\hat{c}} = F.$$

In sum, regardless of the cost distribution, *under homothetic preferences the equilibrium price distribution changes in proportion with the cost distribution*, thereby leaving unchanged the distribution of equilibrium markups, as in Proposition 6.

Heterogeneous firms or asymmetric preferences. The assumption of symmetric preferences puts a strong structure on substitution between variety pairs. We show that, without affecting the nature of our results, this assumption can be relaxed to capture a more realistic substitution pattern. Indeed, we have seen that varieties sharing the same marginal cost c may be viewed as symmetric, whereas varieties produced by c_i-type and c_j-type firms are asymmetric when c_i and c_j differ. As a consequence, *a model with heterogeneous firms supplying symmetric varieties is isomorphic to a model with homogeneous firms selling varieties whose degree of differentiation varies with their type* c.

We illustrate this by means of a simple example in which preferences are asymmetric in the following way: the utility functional $\mathcal{U}(\mathbf{x})$ is given by

$$\mathcal{U}(\mathbf{x}) = \tilde{\mathcal{U}}(\mathbf{a} \cdot \mathbf{x}), \tag{40}$$

where $\tilde{\mathcal{U}}$ is a symmetric functional that satisfies (2), $\mathbf{a}$ a weight function, and $\mathbf{a} \cdot \mathbf{x}$ is defined pointwise by $(\mathbf{a} \cdot \mathbf{x})_i \equiv a_i x_i$ for all $i \in [0, \mathcal{N}]$. If $x_i = x_j$, $a_i > a_j$ means that all consumers prefer variety i to variety j, perhaps because the quality of i exceeds that of j.

The preferences (40) can be made symmetric by changing the units in which the quantities of varieties are measured. Indeed, for any $i, j \in [0, \mathcal{N}]$ the consumer is indifferent between consuming a_i/a_j units of variety i and one unit of variety j. Therefore, by using the change of variables $\tilde{x}_i \equiv a_i x_i$ and $\tilde{p}_i \equiv p_i/a_i$, we can reformulate the consumer's program as follows:

$$\max_{\tilde{\mathbf{x}}} \tilde{\mathcal{U}}(\tilde{\mathbf{x}}) \qquad \text{s.t.} \int_0^N \tilde{p}_i \tilde{x}_i \mathrm{d}i \le Y.$$

In this case, by rescaling of prices, quantities and costs by the weights a_i, the model now involves symmetric preferences but heterogeneous firms. Anticipating the discussion of Section 4, it is readily verified that the equilibrium of the former model is the same as the one of a latter model when both c_i and $1/a_i$ have the same cumulative distribution. In other words, there is a one-to-one mapping between models with symmetric preferences and heterogeneous firms, and models with asymmetric preferences of type (40) and symmetric firms. In this case, the elasticity of substitution is a-specific and there exists a cut-off variety $\hat{a}$ such that market forces select only the varieties that have a weight exceeding $\hat{a}$. Such an approach allows capturing features of spatial models of product differentiation, as in Di Comite et al. (2014).

5 Monopolistic versus oligopolistic competition

It should be clear that Propositions 4-6 have the same nature as results obtained in similar comparative analyses conducted in oligopoly theory (Vives, 1999). They may also replicate less standard anti-competitive effects under some specific conditions (Chen and Riordan, 2008).

The markup (22) stems directly from preferences through the sole elasticity of substitution because we focus on monopolistic competition. However, in symmetric oligopoly models the markup emerges as the outcome of the interplay between preferences *and* strategic interactions. Nevertheless, at least to a certain extent, both settings can be reconciled.

To illustrate, consider the case of an integer number N of quantity-setting firms and of an arbitrary symmetric utility $U(x_1, ..., x_N)$. Using Appendix 2 and symmetrizing yields the elasticity of substitution $s(x, N)$ given by

$$\frac{1}{s(x,N)} = -\frac{xU_{ii}(x,...,x)}{U_i(x,...,x)} + \frac{xU_{ij}(x,...,x)}{U_i(x,...,x)}. \tag{41}$$

Assuming that firms do not manipulate consumers' income through profit redistribution, firm i's profit-maximization condition can be shown to yield the following markup at a symmetric outcome:

$$\frac{p-c}{p} = \frac{1}{N} + \left(1 - \frac{1}{N}\right)\frac{1}{s(x,N)}. \tag{42}$$

Unlike the profit-maximization condition (42), product and labor market balance, as well as the zero-profit condition, do not depend on strategic considerations. Since

$$\frac{p-c}{p} = \frac{1}{\sigma(x,N)} \tag{43}$$

under monopolistic competition, comparing (43) with (42) shows that *the set of Cournot symmetric free-entry equilibria is the same as the set of equilibria obtained under monopolistic competition if and only if* $\sigma(x, N)$ *is given by*

$$\frac{1}{\sigma(x,N)} = \frac{1}{N} + \left(1 - \frac{1}{N}\right)\frac{1}{s(x,N)},$$

where N is now a continuum. Clearly, the same holds when firms are Bertrand competitors. As a consequence, by choosing appropriately the elasticity of substitution as a function of x and N, our model of monopolistic competition is able to replicate not only the direction of comparative static effects generated in symmetric oligopoly models under free entry, but also their magnitude.[8] Therefore, we may conclude that *monopolistic competition mimics oligopolistic competition*. However, the conditions for the existence of an equilibrium under either market structure need not be the same because the above arguments relies on the sole first-order conditions.

6 Concluding remarks

We have shown that monopolistic competition can be modeled in a much more general way than what is typically thought. Using the concept of elasticity of substitution, we have provided a complete characterization of the market outcome and of all the comparative statics implications in terms of prices, firm size, and mass of firms/varieties. Somewhat ironically, the concept of elasticity of substitution, which has vastly contributed to the success of the CES model of monopolistic competition, keeps its relevance in the case of general preferences, both for symmetric and heterogeneous firms. The fundamental difference is that the elasticity of substitution ceases to be constant and now varies with the key-variables of the setting under study. We take leverage on this to make clear-cut predictions about the impact of market size and productivity shocks on the market outcome.

Furthermore, we have singled out our most preferred set of assumptions and given a disarmingly simple sufficient condition for all the desired comparative statics effects to hold true. But we have also shown that relaxing these assumptions does not jeopardize the tractability of the model. Future empirical studies should shed light on the plausibility of the assumptions discussed in this paper by checking their respective implications.

[8]Note that Cournot equilibrium under the CES is isomorphic to a SFE under non-CES homothetic preference à la Kimball (1995).

It would be unreasonable, however, to expect a single set of conditions to be universally valid.

Finally, our framework is able to mimic a wide range of strategic effects usually captured by oligopoly models, and it does so without encountering several of the difficulties met in general equilibrium under oligopolistic competition. We would be the last to say that monopolistic competition is able to replicate the rich array of findings obtained in industrial organization. However, it is our contention that models such as those presented in this paper may help avoiding several of the limitations imposed by the partial equilibrium analyses of oligopoly theory. Although we acknowledge that monopolistic competition is the limit of oligopolistic equilibria, we want to stress that monopolistic competition may be used in different settings as a substitute for oligopoly models when these ones appear to be unworkable.

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Appendix

Appendix 1. Proof of Proposition 1.

It is readily verified that the inverse demands generated by preferences (6) are given by $D(x_i, \mathbf{x}) = u'(x_i)$. The uniqueness of the marginal utility implies that preferences are additive. This proves part (i).

Assume now that $\mathcal{U}$ is homothetic. Since a utility is defined up to a monotonic transformation, we may assume without loss of generality that $\mathcal{U}$ is homogenous of degree 1. This, in turn, signifies that $D(x_i, \mathbf{x})$ is homogenous of degree 0 with respect to $(x_i, \mathbf{x})$. Indeed, because $t\mathcal{U}(\mathbf{x}/t) = \mathcal{U}(\mathbf{x})$ holds for all $t > 0$, (2) can be rewritten as follows:

$$\mathcal{U}(\mathbf{x}+\mathbf{h}) = \mathcal{U}(\mathbf{x}) + \int_0^N D\left(\frac{x_i}{t}, \frac{\mathbf{x}}{t}\right) h_i \, di + \circ \left(||\mathbf{h}||_2\right). \tag{A.1}$$

The uniqueness of the marginal utility together with (A.1) implies

$$D\left(\frac{x_i}{t}, \frac{\mathbf{x}}{t}\right) = D(x_i, \mathbf{x}) \text{ for all } t > 0$$

which shows that D is homogenous of degree 0. As a result, there exists a functional Φ belonging to $L_2([0, \mathcal{N}])$ such that $D(x_i, \mathbf{x}) = \Phi\left(\mathbf{x}/x_i\right)$. Q.E.D.

Appendix 2. We use an infinite-dimensional version of the definition proposed by Nadiri (1982, p.442). Setting $D_i = D(x_i, \mathbf{x})$, the elasticity of substitution between varieties i and j is given by

$$\bar{\sigma} = -\frac{D_i D_j (x_i D_j + x_j D_i)}{x_i x_j \left[D_i' D_j^2 - \left(\frac{\partial D_i}{\partial x_j} + \frac{\partial D_j}{\partial x_i}\right) D_i D_j + D_j' D_i^2\right]}.$$

Since $\mathbf{x}$ is defined up to a zero measure set, it must be that

$$\frac{\partial D_i(x_i, \mathbf{x})}{\partial x_j} = \frac{\partial D_j(x_j, \mathbf{x})}{\partial x_i} = 0$$

for all $j \neq i$. Therefore, we obtain

$$\bar{\sigma} = -\frac{D_i D_j (x_i D_j + x_j D_i)}{x_i x_j \left(D_i' D_j^2 + D_j' D_i^2\right)}.$$

Setting $x_i = x_j = x$ implies $D_i = D_j$, and thus we come to $\bar{\sigma} = 1/\bar{\eta}(x, \mathbf{x})$. Q.E.D.

Appendix 3. We first show that, when the set Ω of active firms is given, there exists a unique utility-maximizing consumption profile $\bar{\mathbf{x}}(\mathbf{p}, Y)$. Recall that (i) a quasi-concave Frechet-differentiable functional is weakly upper-semicontinuous, and (ii) an upper-semicontinuous functional defined over a bounded weakly closed subset of a Hilbert space has a maximizer. Since the budget set is bounded and weakly closed, while $L_2(\Omega)$ is a Hilbert space, existence is proven. Uniqueness follows from the strict quasi-concavity of $\mathcal{U}$ and the convexity of the budget set. Plugging $\bar{\mathbf{x}}(\mathbf{p}, Y)$ into (4) – (5) and solving (4) for x_c, we obtain the Marshallian demand for a c-type variety, that is, $x_c = \mathcal{D}(p_c, \mathbf{p}, Y)$.

Assume now that $\overline{c}$ and N_e are given and consider the game in which firms compete in prices. We show below that a Nash equilibrium of the price game exists if the demand price elasticity

$$\bar{\varepsilon}(p_c, p, y) \equiv -\frac{\partial \mathcal{D}}{\partial p_c}\frac{p_c}{\mathcal{D}}$$

increases in p_c (i.e. (**A.bis**) holds) and decreases in the common price p charged by the other firms. Since each firm behaves as if it were a monopolist, the quantities sold at the price equilibrium $\bar{\mathbf{p}}$ determine a Nash equilibrium $\bar{\mathbf{x}}_c \equiv \mathcal{D}(\bar{p}_c, \bar{\mathbf{p}}, Y)$ of the quantity game.

The first-order condition for a c-type firm, conditional on the vector of prices $\mathbf{p}$ charged by the other firms, is given by

$$\frac{p_c - c}{p_c} = \frac{1}{\bar{\varepsilon}(p_c, \mathbf{p}, y)}. \tag{A.2}$$

Since the left- (right-)hand side of (A.2) continuously increases (decreases) with p_c for any given $\mathbf{p}$, there exists a unique $\hat{p}_c(\mathbf{p})$ that solves (A.2) for any given $\mathbf{p} \in L_2([0, \overline{c}])$ and $c \in [0, \overline{c}]$. Since $\bar{\varepsilon}$ decreases with $\mathbf{p}$, $\hat{p}_c(\mathbf{p})$ increases both in $\mathbf{p}$. Finally, since the left-hand side of (A.2) decreases in c, $\hat{p}_c(\mathbf{p})$ also increases in c.

Let $\mathcal{P}$ be the best-reply mapping from $L_2([0, \overline{c}])$ into itself:

$$\mathcal{P}(\mathbf{p}; c) \equiv \hat{p}_c(\mathbf{p}).$$

Since $\hat{p}_c(\mathbf{p})$ increases in $\mathbf{p}$, $\mathcal{P}$ is an increasing operator. To check the assumption of Tarski's fixed-point theorem, it suffices to construct a set $S \subset L_2([0, \overline{c}])$ such that (i) $\mathcal{P}S \subseteq S$, i.e. $\mathcal{P}$ maps the lattice S into itself and (ii) S is a complete lattice.

Denote by $\bar{p}$ the unique symmetric equilibrium price when all firms share the marginal cost $\overline{c}$ (see Section 2) and observe that $\bar{p} = \hat{p}_{\overline{c}}(\bar{\mathbf{p}})$, where $\bar{\mathbf{p}} \equiv \bar{p}1_{[0,\overline{c}]}$. Since $\hat{p}_c(\mathbf{p})$ increases in c, we have $\bar{p} \geq \hat{p}_c(\bar{\mathbf{p}})$ for all $c \in [0, \overline{c}]$ or, equivalently, $\bar{\mathbf{p}} \geq \mathcal{P}\bar{\mathbf{p}}$. Furthermore, because $\mathcal{P}$ is an increasing operator, $\mathbf{p} \leq \bar{\mathbf{p}}$ implies $\mathcal{P}\mathbf{p} \leq \mathcal{P}\bar{\mathbf{p}} \leq \bar{\mathbf{p}}$. In addition, $\mathcal{P}\mathbf{p}$ is an increasing function of c because $\hat{p}_c(\mathbf{p})$ increases in c. In other words, $\mathcal{P}$ maps the set S of all non-negative weakly increasing functions bounded above by $\bar{p}$ into itself. It remains to show that S is a complete lattice, i.e. any non-empty subset of S has a supremum and an infimum that belong S. This is so because pointwise supremum and pointwise infimum of a family of increasing functions are also increasing.

To sum-up, since S is a complete lattice and $\mathcal{P}S \subseteq S$, Tarski's theorem implies that $\mathcal{P}$ has a fixed point, which is a Nash equilibrium of the price game. Q.E.D.

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