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Random Encounters and Information Diffusion about Markets*

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Abstract

International openness enhances social interaction between citizens of different countries or regions and vice versa. Social exchanges, in turn, increase trade flows between countries and influence markets and prices. We analyze the increased mobility that follows from openness between two countries and its effects on market outcomes. The primary result of our analysis shows that at the limit, market prices tend to align with the duopoly solution. Nonetheless, this convergence can take two different paths depending on the size asymmetry between countries.

Keywords: *Vertically Differentiated Markets, Information Diffusion, Openness.*

JEL Classification: D42, D43, L1, L12, L13, L41.

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1 Introduction

The mobility of European citizens increased considerably following the opening of the national markets in Europe, and citizens located for instance in Belgium and in Italy met much more frequently and accordingly shared their personal consumption experiences.[1] This reveals that social meetings are a vehicle of diffusion of information about goods, their prices and their quality. When agents belong to different countries or regions, international openness increases the frequency of such meetings. And interpersonal exchanges, in turn, increase trade flows between countries, influencing national markets and prices.

In this paper, we analyze how such interpersonal non-market interactions, intensified by the opening of trade between two countries, affect market quantities and prices in both countries. More precisely, we initially consider two countries with different population densities. Each country has a national market on which a domestic firm sells a domestic good, and each such good is further assumed to be a vertically differentiated commodity with respect to the substitute commodity sold in the foreign country. As long as these markets remain strictly national, consumers in each country remain ignorant regarding the existence of the substitute commodity. There is no mutual influence between their respective markets: each national firm is a monopolist on its national market. Next, imagine that a common market is created beginning to link these two goods and thus resulting in the birth and outgrowth of mutual interactions among agents in the two asymmetrically sized countries. Adopting a random encounter model, as in Lazear (1995, 1999), mutual experiences of consumption habits are exchanged progressively and according to the number and frequency of these interactions. While several consumers in each country remain ignorant regarding the existence of the foreign product at the start, the exchange of consumption experiences among them reinforces the process of competition between the national and the foreign substitute. This process magnifies competition between the products, slowly transforming two national monopolies into a single duopoly market with vertically differentiated products.

How these monopoly markets initially metamorphose into a single common duopoly market is precisely the focus of this paper. In particular, we are interested in exploring how prices change along the sequence of equilibria generated by the dynamics of international interactions: does the common market drive initial monopoly prices towards the duopoly market solution with vertically differentiated products? This question can be precisely formulated and answered in the model proposed hereafter.

The above exercise aims to combine literature about markets with the growing research trend on the role played by social interactions in market shifts. In this study, we assume that the

[1] At least this was experienced by the coauthors of this paper !

existence and exchange possibilities of national products are progressively revealed to citizens of different countries by the individual relations developing among the citizens. These relations are enhanced through time due to the existence of a common market allowing consumers of different countries to share their consumption experiences. This view is typically studied in the field of behavioural microeconomics (see for instance, Lazear, 1999, or Bowles, 2004). Indeed, our paper brings together theoretical tools inspired by industrial organization (Gabszewicz and Thisse, 1979, Gabszewicz et al.. 1981), behavioural microeconomics (Camerer, 2003), and international trade (Mercenier and Schmitt, 1995; Boccard and Wauthy, 2006; Chatterjee and Raychaudhuri, 2004). However, the problem of information regarding the existence of goods, and their markets has been mostly neglected by the literature, which deals primarily with the price and quality of goods. By contrast, since we assume that knowledge concerning a good implies knowledge regarding its price and quality, our paper is primarily concerned with the effects of information diffusion about the existence of goods.

Other related papers are the followings. Vettas (1997) shows how a monopoly may use a low initial price to foster learning about its product among consumers who communicate through word-of-mouth. In a complementary paper, Vettas (1998) investigates the formation of new markets with endogeneous diffusion of information along both sides of the market. Diffusion of entry of new firms when consumers learn about the new goods through past consumption experience follows a *S-shaped* diffusion path. Caminal and Vives (1999) analyze price dynamics in a duopoly where consumers learn about the quality differentials. Authors find that despite price wars, consumers learn slowly and convergence to full information is also slow. An important contribution of our research in this literature is in providing a simple model in which meetings are combined with strategic pricing allowing to assess the type and the speed of convergence from monopolies to a duopoly after openness.

The main outcome of our analysis shows that market price tends to align with the duopoly solution at the limit. Surprisingly, this convergence can take two different paths. When country sizes are relatively similar, interpersonal meetings between consumers from each country are relatively frequent, and a significant size of the population in each country becomes aware of the other country's good. Therefore, competition between national goods intensifies quickly in the common market. It follows that market evolution from monopoly to duopoly occurs in the first period, while prices take time to adjust to their full information level. By contrast, when countries differ significantly in size, meetings of consumers are rare; thus, the diffusion of information regarding the existence of the foreign commodity is considerably slower. As a consequence, in spite of the opening of the two monopoly markets, competition does not succeed in driving the monopoly price to the duopoly price for a significant period of time, which creates a situation of "*nearly-*

monopoly". However, we show that there is a time period during which informed consumers are sufficiently numerous to turn the near-monopoly into a duopoly. Finally, we analyze how openness affect the price gap and the market demands for the high and low quality good.

The paper is organized as follows. The next section presents the model in autarky and under a full information hypothesis. The section also explains the information diffusion mechanism. Section 3 provides the market solution for period one, and Section 4 develops the multi-period market solution. Section 5 presents the study's conclusions.

2 The model

Consider a two-country-two-good model, where country $i = 1$ produces good 1 and country $i = 2$ good 2. Heterogeneous consumers in each country are indexed by θ and uniformly distributed over the interval $[a, b]$, with $a > 0$ and $b < \infty$. The parameter θ captures the consumers' heterogeneous willingness to pay for the good: the higher θ, the higher the utility obtained when consuming the good. Each consumer can either buy one unit of a given commodity or not buy at all. Formally, consumer's utility is given by

$$U(\theta) = \begin{cases} \theta u_i - p_i \text{ if he buys variant } i \\ 0 \text{ if he refrains from buying.} \end{cases} \tag{1}$$

where u_i denotes the variant of each good $i = 1, 2$ and p_i its market price. Let s denote the fraction of consumers living in country *one* and $(1 - s)$ that of country *two*.[2] Let denote C_1 the population of country *one* and C_2 the population of country *two*, where $C_1 = (b - a)s$ and $C_2 = (b - a)(1 - s)$.

Initially the consumers of each country are served by a monopoly (or by a national industry acting as a monopolist) selling its good at a price that *covers* the entire domestic market. The assumption of market coverage made in period 0, carries over in every following period.[3] At $t = 0$, the two countries are, then, autarchies producing a good consumed only in their own country. For simplicity, we assume zero costs of production.[4]

At period $t = 1$, the two governments decide to sign an agreement that opens the two countries to unrestricted trade and citizens' circulation. The reasons for this agreement are exogenous to the model. Starting from period 1, consumers have the chance to meet either domestic or foreign

[2] Note that the total population of the two countries is normalized to 1 so that s and $(1 - s)$ simply express the fraction of people living in country one and two, respectively. While consumers' population of the two countries is assumed different (except when $s = 0.5$), their preferences distribution (degree of heterogeneity), expressed by the support of consumers' willingness to pay for goods, is assumed equal in both countries. Assuming different supports in the two countries would not alter the qualitative results of the model.

[3] Admittedly, the assumption of covered market is made for simplicity. Uncovered markets make the analysis of price dynamics intractable.

[4] Introducing a production cost depending on product quality would make the analysis more cumbersome without improving the model intuitions.

consumers and share their consumption experience. We assume that these social interactions arise for any reason we may think of (work, friendship, schooling, romantic or simply vacations). Whatever the reason, when two people meet, we assume that they exchange information about the goods they consume. Only then, some consumers become acquainted to both goods and they acknowledge them as vertically differentiated in accordance to (1). We assume for simplicity that both goods are made available in both markets with zero transportation costs. Without loss of generality, we assume that good 1 produced by country *one* is of lower quality than good 2 produced by country *two*, namely $u_2 > u_1$. Openness improves personal meetings and may imply trade exchanges. In fact, if two consumers of two countries meet, they can decide to change their consumption habits and consume the foreign good. Once acquainted with the foreign good, these consumers may become 'ambassadors' of the foreign good among their co-nationals. By contrast, when two uninformed consumers, who are co-nationals meet, there is no information diffusion about the foreign good. This iterative process of information diffusion will be detailed in the following sections.

2.1 Market solution in autarky

Before the markets open, the two countries live in a regime of autarky. Under the assumption of fully covered market, the consumer located at a, i.e., $\theta = a$, will consume the good 1 only if

$$au_1 - p_1 \geq 0$$

from where we get the monopoly price at time $t = 0$, p_1^M, compatible with a fully covered market, as

$$p_1^M = au_1 \text{ and } p_2^M = au_2. \tag{2}$$

Before openness, populations do not mix and, hence, they only purchase the domestic good paying the monopoly price. The corresponding demand functions $\{D_1\}_{t=0}$ and $\{D_2\}_{t=0}$ for each firm are respectively:

$$\{D_1\}_{t=0} = s(b-a) \text{ and } \{D_2\}_{t=0} = (1-s)(b-a),$$

with profits

$$\{\Pi_1\}_{t=0} = au_1 s(b-a) \text{ and } \{\Pi_2\}_{t=0} = au_2(1-s)(b-a).$$

2.2 Open market solution under full information

If after the trade opening everyone living in country i *would meet instantaneously everyone* of country j, then all consumers would immediately become fully informed about the existence of the two goods. Consequently, the two markets would be segmented among the consumers of good one and good two where the marginal consumer (indifferent whether to buy good one or two) in each country would be:

$$\theta(p_1, p_2) = \frac{p_2 - p_1}{u_2 - u_1}.$$

Then, with perfectly informed consumers, the demand for good one, $D_1(p_1, p_2)$, and two, $D_2(p_1, p_2)$, would be, respectively:

$$D_1(p_1, p_2) = \frac{p_2 - p_1}{u_2 - u_1} - a \quad \text{and} \quad D_2(p_1, p_2) = b - \frac{p_2 - p_1}{u_2 - u_1},$$

yielding the following equilibrium prices at every period

$$p_1^* = \frac{1}{3}\left(u_2 - u_1\right)\left(b - 2a\right), \tag{3}$$

$$p_2^* = \frac{1}{3}\left(u_2 - u_1\right)\left(2b - a\right). \tag{4}$$

The positivity of full information prices (3)-(4) is implied by the condition $b > 2a$ that, we assume to hold. Then, the marginal consumer, would locate at

$$\theta^* = \frac{1}{3}\left(a + b\right).$$

Moreover, for the markets to be fully covered at the duopoly prices, we need to ensure that *all* consumers receive positive utility from purchasing both goods at the duopoly prices under full information. This occurs when the following condition holds:

$$\frac{u_1}{u_2} > \frac{b - 2a}{b + a}. \tag{5}$$

The corresponding profits for firms 1 and 2 at the duopoly solution obtain, respectively as:

$$\begin{aligned} \Pi_1^* &= \frac{1}{9}\left(u_2 - u_1\right)\left(2a - b\right)^2, \\ \Pi_2^* &= \frac{1}{9}\left(u_2 - u_1\right)\left(a - 2b\right)^2. \end{aligned}$$

However, since not *all consumers* meet at once, when the two countries open their markets, at every period there may exist consumers with different levels of knowledge about the two goods. In the next section, we analyze how the demands in each country evolve over time after trade openness.

2.3 Information diffusion under social interaction

We now describe the evolution of the demand functions of each good when the two countries open to meetings among agents living in different countries and to trade. In particular, at each period there exist three types of consumers in each country:

Definition 1 ***Uninformed,*** *the consumers who are only aware of the domestic good but ignore the existence of the foreign good.*

Definition 2 ***Informed,*** *the consumers who become informed of the existence of the foreign good by socially interacting with a foreign consumer or a domestic one who has already met a foreign consumer.*

Finally, using a definition borrowed from evolutionary game theory, we can define

Definition 3 ***Mutants,*** *the consumers who after becoming informed find convenient to update their consumption choice, switching from one good to another.*

Notice that trade openness and social interaction among consumers of the two countries bring two important consequences: *(i)* the two goods become available in both markets at negligible trade costs; *(ii)* mutants originating from social interactions modify the demand function of each firm.

We assume the following process of knowledge transmission among consumers. As in Lazear (1995, 1999), in each period, every consumer randomly meets one consumer who can therefore either be a foreign, or a domestic, consumer. Given the fraction of consumers s and $(1-s)$ in each country, the probability that a consumer of country *one* meets a domestic consumer at period 1, remaining uninformed of the existence of the other good, is simply given by

$$\Pr\left\{(i \in C_1) \cap (j \in C_1)\right\}_{t=1} = s^2.$$

Similarly, the probability that an individual of country 2 meets a domestic consumer, thus remaining uninformed of country *one* good, is given by

$$\Pr\left\{(i \in C_2) \cap (j \in C_2)\right\}_{t=1} = (1-s)^2.$$

Thus, the probability that the consumers of the two countries become *informed,* in period 1, is given by

$$\Pr\left\{(i \in C_1) \cap (j \in C_2)\right\}_{t=1} = 1 - s^2 - (1-s)^2 = 2s\,(1-s)\,.$$

A similar knowledge transmission process occurs in all subsequent periods t, with one new feature. The informed domestic mutants are now *ambassadors* of the foreign good in the domestic market. Accordingly, from period 2 on, information about foreign goods is transmitted at every meeting with a foreign consumer or a domestic mutant. In what follows, we analyze how the sets of informed and uninformed consumers in each country evolve over time. The population dynamics of these two subsets defines the demand for each good. For ease of exposition, we start with the analysis of the first period and then present the extension to any number of period t.

2.4 Open market solution in period one

At period $t = 1$, agents of different countries have the possibility to meet and, hence, mutants may appear within each country's population. We start by building the demand function faced by the firm producing good 1. Define period one prices as $p_1(1)$ and $p_2(1)$. All consumers of country one, whose willingness to pay lays in the interval $(a, \theta(p_1(1), p_2(1)))$, whether uninformed or informed, given their low willingness to pay, will continue to buy good one in period one. Their mass is given by $s[\theta(p_1(1), p_2(1)) - a]$, as in Figure. 1. In this figure, the vertical axis indicates the fraction of consumers living in each country, while the horizontal axis is the support of consumers' willingness to pay in country one and two.

The uninformed consumers in country one who could potentially buy good two but who do not meet anyone from country two (occurring with probability $s \cdot s$) are of mass $s^2[b - \theta(p_1(1), p_2(1))]$. These agents will, therefore, continue to buy good 1 at period one.

Finally, consumers of country two with a willingness to pay laying in interval $(a, \theta(p_1(1), p_2(1)))$, becoming informed of good 1 with probability $s(1-s)$, will become mutants switching from good 2 to good 1. Their mass is given by $s(1-s)(\theta(p_1(1), p_2(1)) - a)$. This is illustrated in Figure 1.

In particular, notice that *mutants* can appear only among individuals having a level of willingness to pay laying in the interval $(\theta - a)$ in country two (and $(b - \theta)$ in country one).

It follows that the demand function $D_1((p_1(1), p_2(1))$ for good 1 in period 1 is equal to

$$D_1((p_1(1), p_2(1)) = s(\theta(p_1(1), p_2(1)) - a) + s^2(b - \theta(p_1(1), p_2(1))) + s(1-s)(\theta(p_1(1), p_2(1)) - a). \quad (6)$$

Let us now turn to the demand function of firm 2. Similarly to the above, all consumers whose willingness to pay lays in the interval $[b - \theta(p_1(1), p_2(1))]$ in country *two*, whether uninformed or informed, will continue to demand good 2. Their mass is $(1 - s)\left[b - \theta(p_1(1), p_2(1))\right]$. The mass of consumers in country *two* who would consume good 1 but are uninformed is $(1 - s)^2\left[b - \theta(p_1(1), p_2(1))\right]$. Finally, the informed consumers of country 1 with willingness to pay in the interval $b - \theta(p_1(1), p_2(1))$, i.e. the mutants of country *one*, are of mass $s(1-s)\left[b - \theta(p_1(1), p_2(1))\right]$.

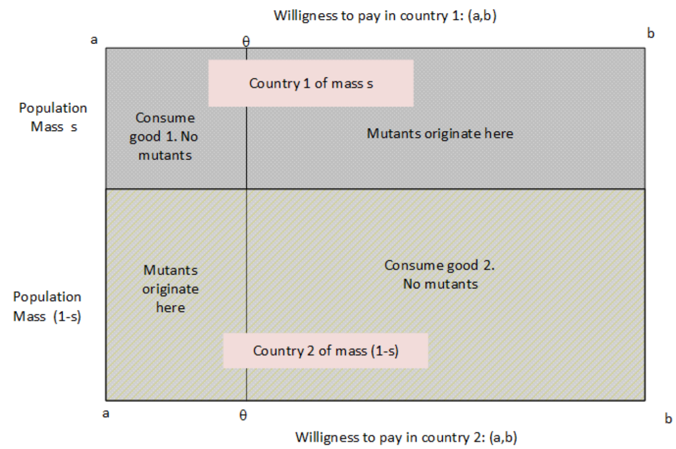

Figure 1: Willingness to pay and mutants in period 1.

Therefore the demand function $D_2(p_1(1), p_2(1))$ for good 2 in period $t = 1$ is equal to:

$$D_2(p_1(1), p_2(1)) = (1-s)(b-\theta(p_1(1), p_2(1))) + (1-s)^2(\theta(p_1(1), p_2(1)) - a) + s(1-s)(b-\theta(p_1(1), p_2(1))).$$

Thus, firms set prices to maximize profits $\Pi_1(p_1(1), p_2(1))$ and $\Pi_2(p_1(1), p_2(1)$

$$\begin{aligned} \Pi_1(p_1(1), p_2(1)) &= p_1(1) \cdot D_1(p_1(1), p_2(1)), \\ \Pi_2(p_1(1), p_2(1)) &= p_2(1) \cdot D_2(p_1(1), p_2(1)). \end{aligned}$$

Concavity of firms' profits functions with respect to their own prices, allows us to solve the system of FOCs for an interior maximum and to obtain the profit maximizing prices $p_1^*(1)$ and $p_2^*(1)$ as:

$$\begin{aligned} p_1^*(1) &= \frac{1}{6} \frac{(u_2 - u_1) \left[\left(1 + s^2\right) b - \left(1 - s^2 + 2s\right) a \right]}{s\,(1-s)}, \\ p_2^*(1) &= \frac{1}{6} \frac{(u_2 - u_1) \left[\left(2 - s^2\right) b - \left(2 - 2s + s^2\right) a \right]}{s\,(1-s)}. \end{aligned} \tag{7}$$

It is easy to check that the positivity of the profit maximizing prices simply follows from condition $b > 2a$, assumed above for prices positivity, in a duopoly with perfectly informed consumers.[5]

At period 1 the corresponding marginal consumer is, therefore, given by

$$\theta(p_1^*(1), p_2^*(1)) = \frac{1}{6} \frac{b - a\,(1-4s) - 2s^2\,(a+b)}{s\,(1-s)},$$

[5] Indeed, p_1^* is positive iff $b > a \left(1 - s^2 + 2s\right) / \left(1 + s^2\right)$. The coefficient $\left(1 - s^2 + 2s\right) / \left(1 + s^2\right)$ is strictly smaller than 2 for any $s \in (0, 1)$ and, therefore, $b > 2a$ is a more binding condition.

and the corresponding equilibrium profits are:

$$\Pi_1(p_1^*(1), p_2^*(1)) = \frac{(u_2 - u_1)\left(b - a + as^2 + bs^2 - 2as\right)^2}{18s\,(1-s)}$$
$$\Pi_2(p_1^*(1), p_2^*(1)) = \frac{(u_2 - u_1)\left(2a - 2b + as^2 + bs^2 - 2as\right)^2}{18s\,(1-s)}.$$

Let us define below two threshold values,

$$s' = \frac{1}{2}\frac{3b - 2a - \sqrt{5b^2 - 6ab + 2a^2}}{2b - a} \text{ and } s'' = \frac{\sqrt{2b^2 - 6ab + 5a^2} - a}{2b - 4a}.$$

Then, at period $t = 1$ the expression for marginal consumer $\theta(p_1^*, p_2^*)$ takes a value inside the interval $[a, b]$ if

$$s' < s < s''. \tag{8}$$

In addition, the above interval of values of s guarantees that $p_2^*(1)$ exceeds $p_1^*(1)$, as required in a model of vertical differentiation.

When the value of s is within the interval (8), we can use Figure. 2 to plot period 1 profit maximizing prices for firm 1 and 2 as a function of s

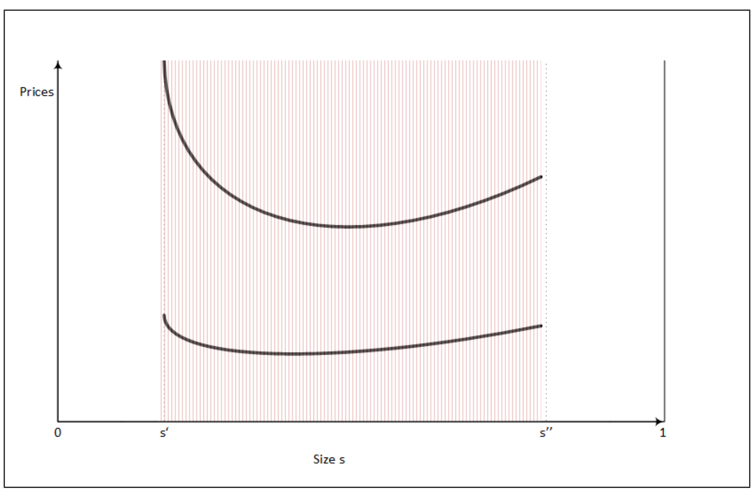

Figure 2: Equilibrium prices in period 1 as a function of s.

It is worth noticing two properties of the profit-maximizing prices. The first concerns the nonmonotone and convex behavior of both prices with respect to the level of countries' size asymmetry. The convexity of prices is caused by the frequency of meetings implied by the countries' size asymmetry. For high country size asymmetry, namely for very small or very large values of s, prices are high. This occurs because a high population asymmetry implies small chances of meeting foreign consumers and ultimately small chances for *mutants* to appear. Thus, for high size asymmetry, firms do not incur into dramatic reduction in the corresponding demand functions due to trade openness. By contrast, when the population asymmetry between the two countries is low (which occurs for intermediate values of s) chances for foreign consumers to meet is high and, hence, also the chances for *mutants* to appear. It follows that the market power of the two firms gets quickly reduced and their equilibrium prices become substantially smaller already in the first period of trade openness and social interaction.

When the two countries are exactly symmetric, namely for $s = 0.5$, the marginal consumer at period $t = 1$ exactly coincides with the one at the full information scenario. However, even so, the prices of the two goods are nevertheless higher than under full information, due to the existing informational frictions.

The second property of prices concerns the relationship between *price gap* and level of s. It can be noticed that when s is very small the difference between equilibrium prices is much larger than when s is big. When the mass of population living in the country producing the low quality good is large (s big), the price competition coming from the social interactions has a strong and immediate negative impact on prices. In particular, due to the large size of the low quality market, the price gap is squeezed down. By contrast, when s is very small, implying a very large high quality market, the price gap is big. This is not all. The price of the high quality good is very high due to the high relative size of its market. In turn, this enlarges the price gap. Importantly, these properties hold in every period and shape the dynamics of the market solutions.

It remains to clarify the type of market solution for values of s that do not belong to the interval (8). Two scenarios may occur: *(i)* $\theta(p_1^*(1), p_2^*(1)) = b$ in correspondence of $s < s'$ and *(ii)* $\theta(p_1^*(1), p_2^*(1)) = a$ in correspondence of $s > s''$. More specifically, in *(i)*, the country producing the low quality good is very small and the price gap associated to the duopoly solution, as we saw it in figure 2, is very high. This drives the position of the marginal consumer outside the interval $[a, b]$ exceeding the threshold b. As a consequence, no informed consumer will buy good 2 and rather prefer to buy good 1, whereas only *uninformed* consumers of country *two* will continue to buy good 2. This determines a corner solution in which $\theta(p_1^*(1), p_2^*(1)) = b$. In *(ii)* instead, the country producing the low quality good is very large and the price gap associated to the open market solution turns out to be smaller. Again, this drives the position of the marginal consumer outside the interval $[a, b]$ being smaller than the threshold a. Accordingly, no informed consumer will buy good 1 and rather prefer to buy good 2, whereas only *uninformed* consumers of country *one* will continue to buy good 1. Similarly to case *(i)*, this corresponds to a corner solution with $\theta(p_1^*(1), p_2^*(1)) = a$. We call this corner solutions *nearly-monopolies.* These solutions are reminiscent of "Diamond paradox": the introduction of search costs among consumers will raise equilibrium prices from competitive to monopoly levels (Diamond, 1971). In our framework, when size asymmetry is quite large, meeting someone informed is rare, hence equilibrium prices correspond to nearly-monopoly prices rather than duopoly prices.

3 Multi-period market solution

3.1 Knowledge Transmission over Time

To characterize the levels of prices set by the firms and the associated quantities sold over time, we need to specify the process of information transmission. The evolution of the mass of informed, uninformed and mutants allows to analyze the market equilibrium prices at each period t and their convergence, if any, for $t \to \infty$.

In particular, we denote the mass of *uninformed* consumers on good i at time t as U_i^t for $i = 1, 2$, and similarly, that of consumers becoming *informed* at time t, I_i^t. As explained above, the total mass of informed consumers about good 1 at period 1 is given by

$$I_1^0 + I_1^1 = s + (1-s)s = s\left(2-s\right).$$

By the same reasoning, the mass of *newly* informed consumers on good 1 at $t = 2$, denoted I_1^2 is,

$$I_1^2 = \Pr\left(\left\{i \in \left(I_1^0 \cup I_1^1\right)\right\} \cap \left\{j \in U_1^1\right\}\right) = s\left(2-s\right)\left(1-s\right)^2.$$

The cumulative mass of consumers who are informed on good 1 at period 2 is, therefore, given by

$$\begin{aligned} \mathcal{I}_1^2 &= I_1^1 + I_1^2 = s\left(2-s\right) + s\left(2-s\right)\left(1-s\right)^2 \\ &= s\left(2-s\right)\left(1+\left(1-s\right)^2\right), \end{aligned}$$

and the proportion of uninformed consumers of country *two* (who can potentially demand good 1) are:

$$\mathcal{U}_1^2 = 1 - \mathcal{I}_1^2 = \left(1-s\right)^4.$$

It is important to remind here that the information diffusion process takes into account the information transmission from all informed people regardless of their country origin. In particular, starting from period $t = 1$ *a consumer in country's i can learn about product j either if he meets a consumer of country j or if he meets an informed consumer of his own country.* In *Appendix* A, we develop the general expression for $\mathcal{I}_1^t$ and $\mathcal{U}_1^t$ at any period t that are, respectively:

$$\mathcal{I}_1^t = 1 - \left(1-s\right)^{2^t} \text{ and } \mathcal{U}_1^t = \left(1-s\right)^{2^t}.$$

In an analogous way, the set of informed and uninformed agents about good 2 at time t are equal to:

$$\mathcal{I}_2^t = 1 - s^{2^t} \text{ and } \mathcal{U}_2^t = s^{2^t}.$$

As a result, in any period t, the demand functions $D_1(p_1(t), p_2(t))$ and $D_2(p_1(t), p_2(t))$ for the two goods are:

$$D_1(p_1(t), p_2(t)) = \left(1 - (1-s)^{2^t}\right)(\theta(p_1(t), p_2(t)) - a) + s^{2^t}(b - \theta(p_1(t), p_2(t))), \tag{9}$$

$$D_2(p_1(t), p_2(t)) = \left(1 - s^{2^t}\right)(b - \theta(p_1(t), p_2(t))) + (1-s)^{2^t}(\theta(p_1(t), p_2(t)) - a). \tag{10}$$

3.2 Duopoly equilibrium

Let us now proceed with the characterization of an interior duopoly equilibrium with information frictions. Given the demands (9)-(10) of the two firms in every period, the duopoly equilibrium is simply given by the price profile (whenever interior) which maximizes the firms' profit functions at every period t:

$$\begin{aligned}\Pi_1(p_1(t), p_2(t)) &= D_1(p_1(t), p_2(t)) \cdot p_1(t), \\ \Pi_2(p_1(t), p_2(t)) &= D_2(p_1(t), p_2(t)) \cdot p_2(t).\end{aligned}$$

By the concavity of $\Pi_i(t)$ in $p_i(t)$ for $i = 1, 2$, the solution of the system of first order conditions yields the following unique equilibrium prices at every period t,

$$p_1^*(t) = \frac{1}{3}(u_2 - u_1)\frac{b - 2a + a(1-s)^{2^t} + bs^{2^t}}{1 - (1-s)^{2^t} - s^{2^t}}, \tag{11}$$

$$p_2^*(t) = \frac{1}{3}(u_2 - u_1)\frac{2b - a - a(1-s)^{2^t} - bs^{2^t}}{1 - (1-s)^{2^t} - s^{2^t}}, \tag{12}$$

Given the above duopoly prices (11)-(12), the equilibrium marginal consumer in any period t is simply obtained as:

$$\theta(p_1^*(t), p_2^*(t)) = \frac{1}{3}\frac{a + b - 2\left(a(1-s)^{2^t} + bs^{2^t}\right)}{1 - (1-s)^{2^t} - s^{2^t}}. \tag{13}$$

It is important now to recall that the above equilibrium prices are *duopoly* equilibrium prices if and only if they induce a marginal consumer whose location lies at the interior of the support of consumers' willingness to pay $[a, b]$. For period $t = 1$, we have defined two thresholds $s'$and s'' of country sizes that guarantee that the location of the marginal consumer at $t = 1$ lies inside $[a, b]$.

Similarly, for any period t, we can implicitly define two intertemporal thresholds $s'(t)$ and $s''(t)$ by solving $\theta(p_1^*(t), p_2^*(t)) - a = 0$ and $b - \theta(p_1^*(t), p_2^*(t)) = 0$ in s, correspondingly, for any t.

Using expression (13) and (9)-(10) equilibrium market shares obtain as

$$D_2(p_1^*(t), p_2^*(t)) = \frac{1}{3}\left(2b - a - bs^{2^t} - a(1-s)^{2^t}\right), \tag{14}$$

$$D_1(p_1^*(t), p_2^*(t)) = \frac{1}{3}\left(b - 2a + bs^{2^t} + a(1-s)^{2^t}\right). \tag{15}$$

with corresponding equilibrium profits given by

$$\begin{aligned}
\Pi_1^*(p_1^*(t), p_2^*(t)) &= \frac{1}{9}(u_2 - u_1)\frac{\left(b - 2a + a(1-s)^{2^t} + bs^{2^t}\right)^2}{1 - (1-s)^{2^t} - s^{2^t}}, \\
\Pi_2^*(p_1^*(t), p_2^*(t)) &= \frac{1}{9}(u_2 - u_1)\frac{\left(2b - a - a(1-s)^{2^t} - bs^{2^t}\right)^2}{1 - (1-s)^{2^t} - s^{2^t}}.
\end{aligned}$$

Looking at expressions (11)-(12), the positivity of equilibrium prices and the fully covered market assumption are guaranteed at any period by the condition on the support of consumers' preferences: $b > 2a$. We state the following proposition about duopoly prices.

Proposition 1. *Duopoly equilibrium prices* $p_1^*(t)$ and $p_2^*(t)$ *converge over time to their counterparts* p_1^* and p_2^* *in a duopoly with vertically differentiated goods and fully informed agents, namely*

$$\lim_{t\to\infty} p_1^*(t) = p_1^* \text{ and } \lim_{t\to\infty} p_2^*(t) = p_2^*.$$

Proof. *It immediately follows from manipulations of expressions* (11)-(12).

We now turn the attention to the *dynamics of the price gap* over time. Interestingly, this dynamics is determined by which country (either the biggest or the smallest) is producing which quality. On the one side, if the low quality is produced in the large country, the information about the existence of a cheap good will spread fast enlarging the market for the low quality good. At the same time, the information about the existence of a higher quality good will spread, but at a lower pace. On the other side, if the low quality good is produced in the smaller country, the information about the existence of a higher quality good will spread faster, increasing the number of mutants in the smaller country that switch to the higher quality good consumption. Such dynamics make the equilibrium prices relatively low in the former scenario, and high in the latest scenario. In Figure 3, we depict the price gap of period one (bold line) and period two (thin line) as a function of s. It can be noticed that, for s exceeding 0.5 (the intersection point) the price gap of period two exceeds the price gap of period one, and this gap continues to grow until it reaches the price gap of the full information duopoly, namely $1/3(u_2 - u_1)(a + b)$. In contrast, if s is lower than 0.5, the price gap of period one exceeds the price gap of period two, and this gap continues to fall until, once again, it reaches the price gap of the full information duopoly. Finally, if the size of the two countries is symmetric ($s = 0.5$), the price gap between firms is equal to the price gap of the duopoly under full information immediately after opening trade at $t = 1$ and remains at this level thereafter, even though optimal prices require some periods to converge to the duopoly prices under full information. This is expressed in the next proposition.

Proposition 2. *For $s < 0.5$, the price gap monotonically decreases in t and converges to the price gap of full information duopoly for $t \to \infty$, whereas, for $s > 0.5$, it monotonically increases in t and converges to the price gap of full information duopoly for $t \to \infty$. Finally, if countries' size is symmetric ($s = 0.5$), the price gap is equal to the price gap of full information duopoly from the first period onward.*

Proof. *It immediately follows from manipulations of expressions* (11)-(12).

Similarly, we can consider the behaviour of the location of the marginal consumer over time.

Proposition 3. *For $s = 0.5$, $\theta(p_1^*(t), p_2^*(t))$ converges to the full information duopoly θ^* already at $t = 1$; (ii) for $s < 0.5$, $\theta(p_1^*(t), p_2^*(t))$ is monotonically decreasing in t and converges to θ^* for $t \to \infty$; and finally, (iii) for $s > 0.5$, $\theta(p_1^*(t), p_2^*(t))$ is monotonically increasing in t and converges to θ^* for $t \to \infty$.*

Proof. The difference $\Delta = d^t(s) - d^*(s)$ between the price gap in period t and the price gap under full information can be expressed as $\Delta = \frac{1}{3}(u_2 - u_1)(b - a)\frac{(1-s)^{2^t} - s^{2^t}}{1-(1-s)^{2^t} - s^{2^t}}$, whose sign is positive (negative) for any t whether s is lower (higher) than 0.5. Furthermore, $\lim_{t\to\infty} \Delta = 0$. ■

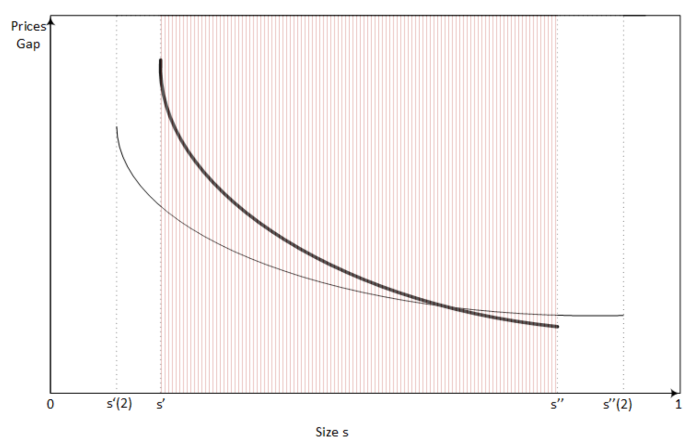

Figure 3: Price gap in period 1 (bold line) and period 2 (thin line) as a function of s.

Furthermore, the above expression for $\theta^*(t)$ lies inside the interval $[a,b]$ for $s \in (s'(t), s''(t))$. Clearly, the fact that $\theta(p_1^*(t), p_2^*(t))$ lies in $[a,b]$ embodies the condition that $p_2^*(t) > p_1^*(t)$, because otherwise, if $p_2^*(t)$ would be smaller than $p_1^*(t)$, no consumer would buy good 2 and $\theta(t)$, in turn, would not lie within the interval $[a,b]$.

The next proposition investigates the dynamics of the market demands for each firm.

Proposition 4. *The market demand for the high quality firm always increases with trade openness till it converges to corresponding market share in the perfect information duopoly. By contrast, the market demand of the low quality firm always decreases with trade openness till it converges to the corresponding market share in duopoly.*

Proof. *It immediately follows from manipulations of expressions* (14)-(15).

The result in the above proposition is to some extent surprising because as shown in Proposition 3, the dynamics of the marginal consumer $\theta(p_1^*(t), p_2^*(t))$ is not always the same and depends on the size asymmetry between the two countries. Then, why does the market share of the high quality always increases when $\theta(p_1^*(t), p_2^*(t))$ is increasing? We expect that as $\theta(p_1^*(t), p_2^*(t))$ increases, the length of the segment $b - \theta(p_1^*(t), p_2^*(t))$ decreases, lowering the demand for the high quality. This is true, but a second force is in place. *Time changes the number of consumers that get informed, favoring the high quality good.* Indeed, when $s > 0.5$, the informed consumers about the high quality good in t exceeds the number of uninformed ones about the low quality good: $\mathcal{I}_2^t = 1 - s^{2^t} > \mathcal{I}_1^t = 1 - (1-s)^{2^t}$. Furthermore, the uninformed about the high quality good in t, namely $\mathcal{U}_2^t = s^{2^t}$, decreases much faster than the uninformed about the low quality good $\mathcal{U}_1^t = (1-s)^{2^t}$. For this reason, the rise in the market size of the informed about good 2, i.e., $(1 - s^{2^t})(b - \theta(p_1^*(t), p_2^*(t)))$ overcomes the fall in the market share of the uninformed consumers about good 1, i.e., $(1-s)^{2^t}((\theta(p_1^*(t), p_2^*(t)) - a)$, implying an increase in the market share of the high quality good. Analogously, the sudden decrease in the market share of the uninformed consumers about good 2, i.e., $s^{2^t}((b - \theta(p_1^*(t), p_2^*(t)))$, offsets the increase in the market size of the informed about good 2, i.e., $(1-(1-s)^{2^t})(\theta(p_1^*(t), p_2^*(t)) - a)$, leading to a decreasing market share for the low quality good, regardless of the dynamics of $\theta(p_1^*(t), p_2^*(t))$.

Finally, we focus on the role played by countries' asymmetry on equilibrium prices. The relationship between every period equilibrium prices and countries' sizes is not obvious: prices are nonmonotone, they are convex in s, and the price gap is large for s small and small for s large. Nonmonotonicity and convexity are both due to the frequency of social interaction occurring in every period. The more symmetric countries' populations, the more frequent meetings, the more intense competition and, thus, the lower equilibrium prices. By contrast, the higher countries' size asymmetry, the smaller chances for mutants to appear and, therefore, the softer price competition leading to higher equilibrium prices. Using expressions (11)-(12) obtained in the duopoly equilibrium, we see that duopoly equilibrium prices at every period t are nonmonotone and convex in s. The equilibrium price $p_1^*(t)$ of good 1 ($p_2^*(t)$ of good 2) reaches its maximum value when the size of the country producing the low quality good is very large (very small).[6] At last, we see that in every period the price gap $d(t) \equiv (p_2^*(t) - p_1^*(t))$ decreases monotonically with the size s of the low quality country.

3.3 Nearly-monopoly equilibrium

A careful analysis of the equilibrium prices reveals that in our intertemporal setting, two substantially different equilibria may arise, one interior, that we have labelled as *duopoly equilibrium*, and another one, as *nearly-monopoly* equilibrium. Let us discuss here some of the features of nearly-monopoly equilibria.

The analysis of period one equilibrium prices has revealed that a duopoly equilibrium can arise only when $s \in (s'(t), s''(t))$, namely when the two countries are not "too asymmetric" in terms of their populations. Alternatively, the nearly-monopoly equilibrium can occur when $s \notin (s'(t), s''(t))$, namely when the two countries are extremely asymmetric in size. Two different types of nearly-monopoly equilibria may actually arise. The first occurs for $s < s'(t)$, namely when the low quality good is produced in a very small country. Then, using expression (13), the marginal consumer location falls at its highest bound b just because, by construction, cannot be higher than b. This implies that all consumers of country 1, whether informed or not, prefer to buy good 1. Accordingly, the firms' demand functions in period t write as

$$\begin{aligned} D_1(t) &= \mathcal{U}_2^t (b-a) = s^{2^t} (b-a), \\ D_2(t) &= \mathcal{I}_2^t (b-a) = \left(1 - s^{2^t}\right)(b-a). \end{aligned}$$

[6] Before converging to the full information counterparts, the denominators of expressions (11)-(12) are concave in s reaching a maximum for $s = 0.5$. In addition, the numerator of $p_1^*(t)$ can be either convex or concave in s depending on t, while the numerator of $p_2^*(t)$ is convex in s for any t. Both these facts directly imply that prices are nonmonotone in s for any t. From the property that the product of two convex functions is a convex function itself and that the ratio of 1 over a concave function is itself a convex function, directly proves that $p_2^*(t)$ is convex in s. Convexity of $p_1^*(t)$ is guaranteed by the condition $b > 2a$. Indeed, the sign of second derivative of $p_1^*(t)$ is given by the sign of the expression $-(1-s)^{2^t}\left(s + s^{2^t}\right) + 2(1-s)s^{2^t}$ which is negative for $s' < s < s''$ given $b > 2a$. Finally, since the numerator of $p_1^*(t)$ (respectively $p_2^*(t)$) assumes highest (lowest) values in the neighborhood of $s = 1$ ($s = 0$) coupled with the symmetry of the denominator around $s = 0.5$, leads to the result that the price of good 1 (good 2) reaches its maximum value when the size of the country producing the low quality good is very large (very small), for any given t.

Only the uninformed consumers of country 2 keep buying good 2, whereas all informed consumers prefer to buy good 1. The corresponding prices are given by monopoly prices (2) which are the highest fully covered market prices.

The second type of nearly-monopoly arises when $s > s''(t)$, namely, when the low quality good is produced in a very large country (compared to the country producing the high quality good). Then, using expression (13), the marginal consumer locates at the lower bound a because by construction cannot be smaller than a. This implies that all consumers of country 2, when informed, will buy good 1 and the demand functions for each firm at period t write as

$$\begin{aligned} D_1(t) &= \mathcal{I}_1^t (b-a) = \left(1-(1-s)^{2^t}\right)(b-a), \\ D_2(t) &= \mathcal{U}_1^t (b-a) = \left(1-s^{2^t}\right)(b-a). \end{aligned}$$

Given the above expressions, the nearly-monopoly prices are simply given by the monopoly prices (2). We summarize below these results.

Proposition 5 *At a given period t, for $s \notin (s'(t), s''(t))$, namely if the two countries size asymmetry is sufficiently high, equilibrium prices of the multi-period market coincide with the nearly-monopoly prices (2).*

Proposition 5 describes a situation in which either the size of country one or country two is proportionally too big, compared with that of the other country. In turn, this makes the price gap between the two goods either too big or two small for one of the goods to remain attractive for consumers. Only uninformed consumers continue to patronize the good produced by the larger country, whose mass progressively shrinks over time.

Thus, the model reveals that knowledge transmission among the two countries ultimately plays a balancing role in the market. When time goes by and the mass of informed consumers increases, the excessive number of people living in the bigger country and only purchasing the domestic good progressively shrinks, thus driving the price gap once again within a reasonable range, and allowing a duopoly market equilibrium to arise. This property of the nearly- monopoly equilibrium to be a transitory phenomenon is expressed in the next proposition.

Proposition 4 *Nearly-monopoly equilibria may only last for a finite number of periods after which the market transforms into a duopoly.*

Proof. The switch from the equilibrium that we denote *nearly-monopoly* to the duopoly equilibrium occurs by definition when the value of the marginal consumer $\theta(p_1^*(t), p_2^*(t))$ returns inside the interval $[a, b]$, meaning that, once again, both firms face the competition of the firm located in the other country. We proved above that $\lim_{t\to\infty} \{\theta(p_1^*(t), p_2^*(t))\}_t = \frac{1}{3}(a+b)$, meaning that over time, when the information on the existence of the two goods is fully unveiled to the consumers living in the two countries, the market ends up behaving as a full information duopoly, with the marginal consumer θ^* laying at the interior of the interval of consumers' willingness to pay $[a, b]$. In addition, the sequence $\{\theta(p_1^*(t), p_2^*(t))\}_t$ is either monotonically decreasing (increasing) in t for $s > 0.5$ ($s < 0.5$) or constant, for $s = 0.5$. Thus, there does necessarily exist a finite period $t < +\infty$ for which $\theta(p_1^*(t), p_2^*(t))$ reaches the interior of the interval $[a, b]$, either from the right or from the left (respectively for $s < s'$ and $s > s''$). This return inside interval $[a, b]$ brings to an end the phenomenon of nearly-monopoly. ■

Trade openness, therefore, always breaks national monopolies, either in a quick or in a slow pace, which ultimately depends on the fraction of people living in the two countries. A monopoly equilibrium arises when the size s is quite far from $s = 0.5$, meaning that the asymmetry in the two countries' populations is quite high. A very small size s causes two forces into place, which in turn determine that $p_2^*(t)$ is too high compared to $p_1^*(t)$. First, because of the very high number of consumers who know only the high quality good, the high quality firm is not under much competitive pressure from the bottom quality one and, thus, does not loose market share even when setting relatively high prices. Second, since s is very small, knowledge transmission arises slowly, keeping the market power of the high quality firm high. Proposition 4 shows that, as time goes by, the nearly-monopoly equilibrium can never persist. This is so because the increasing number of mutants switching from one market to the other over time, decreases the demand share of nearly-monopolists given by $(1-s)^{2^t}$ or s^{2^t}.

4 Concluding remarks

This paper introduces a multi-period model of vertical differentiation to explore the effect of encounters among individuals of different countries on their respective markets. In particular, we explore how prices change along the sequence of equilibria generated by individual interactions through time. Our analysis shows that market prices tend to align with the standard duopoly solution at the limit. However, this convergence can take two different paths according as population sizes are similar or very different. When country sizes are relatively similar, the evolution of a market from monopoly to differentiated duopoly is fast. By contrast, when countries sizes are very different, then meetings of their consumers are rare. Thus, competition does not succeed in driving the monopoly price to the duopoly price for a significant period of time (denoted near-monopoly). However, we show that there is always a time period during which informed consumers are sufficiently numerous to drive a market to the standard differentiated duopoly configuration.

In the present paper, we can investigate who loses and who wins from trade openness. Consumers certainly benefit from openness because now they have the chance to choose among vertically differentiated goods and they pay lower prices due to competition.

By contrast, openness has an ambiguous overall effect on firms' profits because exchange determines two contrasting effects on profits. On the one hand, openness enlarges markets served by each firm. Some consumers of country one now meet consumers of country two and become mutants consuming good 2, thus enlarging the market share of good 2. Yet, some consumers of country two will also meet consumers of country one and some may become mutants consuming good 1, increasing the demand for good 1. This market expansion effect increases competition between firms.

In general, the high quality firm, when located in the large country, tends to gain more from free trade. However, the low quality firm located in the small country might still benefit from trade, because it now sells in a larger market than the domestic one. This happens when the asymmetry in size between the countries is very significant. However, the opposite might happen to the low quality good firm operating in a country with relatively high population size. Then, the competition effect is larger than the market expansion effect leading to detrimental effects on the profits of the low quality firm. Thus the net effect depends on the particular market configuration.[7]

In the present paper, we have constructed a model in which the transmission of information only occurs between consumers of different countries. However, information could be evidently transmitted in alternative ways. For instance, advertising could reveal the existence of a product and its quality. Furthermore, firms could use prices in order of diffusing information about their good. By lowering their price, they could attract a larger set of new consumers. This would relate our paper to the recent literature in economics and management that studies 'market seeding' and information transmission through 'consumers/ambassadors' (Hinz *et al.*, 2011, Groeger and Buttle, 2013).

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Appendix A: Population dynamics

A.1. Informed and uninformed agents on good 1 over time

Let denote I_1^t the *new* proportion of *agents* who have become *informed* on good 1 at time t and U_1^t the proportion of those still remaining uninformed. Let us also denote $\mathcal{I}_1^1$ (resp. $\mathcal{U}_1^t$) the total fraction of all agents in the two country informed (resp. uninformed) on good 1 existence at time t.

At $t = 0$ only a proportion s of agents knows about good 1 (monopoly): $\mathcal{I}_1^0 = s$, $\mathcal{U}_1^0 = 1 - s$.

At $t = 1$, an additional proportion of agents become informed on good 1, and, therefore:

$$\mathcal{I}_1^1 = s + (1-s)s = s\,(2-s)\,,$$

and

$$\mathcal{U}_1^1 = 1 - (s + (1-s)s) = (1-s)^2\,.$$

At $t = 2$, the mass of newly informed agents on good 1 is:

$$I_1^2 = \Pr\left\{\left(\mathcal{I}_1^1 \cap \mathcal{U}_1^1\right)\right\} = s\,(2-s)\,(1-s)^2\,,$$

and, hence

$$\mathcal{I}_1^2 = \mathcal{I}_1^1 + I_1^2 = s\,(2-s) + s\,(2-s)\,(1-s)^2 = s\,(2-s)\,(1+(1-s)^2),$$

$$\mathcal{U}_1^2 = 1 - s\,(2-s)\,(1+(1-s)^2) = (1-s)^4\,.$$

At $t = 3$, the mass of agents becoming informed on good 1 is given by:

$$\begin{aligned} I_1^3 &= \Pr\left\{\left(\mathcal{I}_1^2 \cap \mathcal{U}_1^2\right)\right\} = \left(s\,(2-s)\,(1+(1-s)^2)\right)(1-s)^4 = \\ &= \left(s^2 - 2s + 2\right)(2-s)\,(s-1)^4\,s. \end{aligned}$$

and,

$$\begin{aligned} \mathcal{I}_1^3 &= \mathcal{I}_1^2 + I_1^3 = s\,(2-s)\,(1+(1-s)^2) + \left(s^2 - 2s + 2\right)(2-s)\,(s-1)^4\,s = \\ &= s\,(2-s)\left(s^2 - 2s + 2\right)\left(6s^2 - 4s - 4s^3 + s^4 + 2\right). \end{aligned}$$

Since

$$\mathcal{U}_1^3 = 1 - \left(s\,(2-s)\left(s^2 - 2s + 2\right)\left(6s^2 - 4s - 4s^3 + s^4 + 2\right)\right) = (1-s)^8\,,$$

the total proportion of informed agents at $t = 3$ on good 1 is obtained as

$$\mathcal{I}_1^3 = 1 - \mathcal{U}_1^3 = 1 - (1-s)^8 .$$

At $t = 4$, the proportion of newly informed consumers on good 1 is given by:

$$\begin{aligned} I_1^4 &= \Pr\left\{\left(\mathcal{I}_1^3 \cap \mathcal{U}_1^3\right)\right\} = (1-s)^8 \left(1-(1-s)^8\right) = \\ &= \left(6s^2 - 4s - 4s^3 + s^4 + 2\right)\left(s^2 - 2s + 2\right)(2-s)\, s\,(s-1) . \end{aligned}$$

which once again shows that

$$\begin{aligned} \mathcal{I}_1^4 &= \mathcal{I}_1^3 + I_1^4 = 1 - (1-s)^8 + \left(6s^2 - 4s - 4s^3 + s^4 + 2\right) \\ \left(s^2 - 2s + 2\right)(2-s)\, s\,(s-1)^8 &= s\,(2-s)\left(s^2 - 2s + 2\right) \times \\ &\quad \times \left(28s^2 - 8s - 56s^3 + 70s^4 - 56s^5 + 28s^6 - 8s^7 + s^8 + 2\right) \\ &\quad \times (6s^2 - 4s - 4s^3 + s^4 + 2), \end{aligned}$$

and

$$\begin{aligned} \mathcal{U}_1^4 &= 1 - s\,(2-s)\left(s^2 - 2s + 2\right) \times \\ &\quad \times \left(28s^2 - 8s - 56s^3 + 70s^4 - 56s^5 + 28s^6 - 8s^7 + s^8 + 2\right) \\ \times \left(6s^2 - 4s - 4s^3 + s^4 + 2\right) &= (1-s)^{16} . \end{aligned}$$

This implies that

$$\begin{aligned} \mathcal{U}_1^4 &= (1-s)^{16} , \\ \mathcal{I}_1^4 &= 1 - (1-s)^{16} . \end{aligned}$$

The above expressions show that the same information path occurs at every period and, in general, for good 1 the total fraction of informed and uninformed agents at any arbitrary period t is simply given by:

$$\mathcal{I}_1^t = 1 - (1-s)^{2^t} \text{ and } \mathcal{U}_1^t = (1-s)^{2^t} .$$

Morever, notice that, consistently, for any $s \in (0,1)$,

$$\begin{aligned} \mathcal{I}_1^\infty &= \lim_{t\to\infty}\left\{1 - (1-s)^{2^t}\right\} = 1 \text{ and} \\ \mathcal{U}_1^\infty &= \lim_{t\to\infty}\left\{(1-s)^{2^t}\right\} = 0. \end{aligned}$$

A.2. Informed and uninformed agents on good 2 over time

As above, let us denote $\mathcal{I}_2^t$ (resp $\mathcal{U}_2^0$) the total proportion of informed agents on good 2 at time t.

At $t = 0$ only a fraction $(1-s)$ of agents is aware of good 2 (monopoly) and, therefore, $\mathcal{I}_2^0 = 1-s$. $\mathcal{U}_2^0 = s$.

At $t = 1$: $\mathcal{I}_2^1 = \mathcal{I}_2^0 + I_2^1 = (1-s) + (1-s)s = 1-s^2$, $\mathcal{U}_2^1 = s^2$;

At $t = 2$, the proportion of newly informed consumers is given by:

$$I_2^1 = \Pr\left\{\left(\mathcal{I}_2^1 \cap \mathcal{U}_2^1\right)\right\} = s^2\left(1-s^2\right),$$

and

$$\begin{aligned}\mathcal{I}_2^2 &= \left(1-s^2\right) + s^2\left(1-s^2\right) = 1-s^4 \\ \mathcal{U}_2^2 &= 1-1-s^4 = s^4.\end{aligned}$$

At $t = 3$, the set of newly informed is given by:

$$I_2^3 = \Pr\left\{\left(\mathcal{I}_2^2 \cap \mathcal{U}_2^2\right)\right\} = \left(1-s^4\right)s^4,$$

and

$$\begin{aligned}\mathcal{I}_2^3 &= 1-s^4 + \left(1-s^4\right)s^4 = 1-s^8; \\ \mathcal{U}_2^3 &= 1-1-s^8 = s^8.\end{aligned}$$

At $t = 4$, the mass of newly informed consumers is given by:

$$I_2^4 = \Pr\left\{\left(\mathcal{I}_2^3 \cap \mathcal{U}_2^3\right)\right\} = \left(1-s^8\right)s^8,$$

so that

$$\mathcal{I}_1^4 = \mathcal{I}_2^3 + I_2^4 = 1-s^8 + \left(1-s^8\right)s^8 = 1-s^{16}$$

and

$$\mathcal{U}_2^4 = s^{16}.$$

Thus, in general, for good 2, the total fraction of informed and uninformed agents show a recurrent path and, in any period t is just equal to:

$$\mathcal{I}_2^t = 1-s^{2^t} \text{ and } \mathcal{U}_2^t = s^{2^t}.$$

Finally, notice that, again, consistently, for any $s \in (0,1)$:

$$\begin{aligned}
\mathcal{I}_2^{\infty} &= \lim_{t\to\infty} \left\{1 - s^{2^t}\right\} = 1 \\
\mathcal{U}_1^{\infty} &= \lim_{t\to\infty} \left\{s^{2^t}\right\} = 0.
\end{aligned}$$