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# The Dixit-Stiglitz economy with a ‘small group’ of firms: A simple and robust equilibrium markup formula

Claude d’Aspremont*   Rodolphe Dos Santos Ferreira†

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**Abstract**

In a general version of Dixit-Stiglitz two-sector economy, we present three variants of the concept of oligopolistic equilibrium in price-quantity pairs (d’Aspremont and Dos Santos Ferreira, 2016) integrating income feedback effects in three different ways. For the first two variants (Ford effects ignored or restricted to profits), a single and simple equilibrium markup formula is derived involving, for each firm, a conduct parameter indicating its degree of competitive toughness. Different specifications of these conduct parameters lead to different oligopolistic equilibria in prices and/or in quantities. In particular in the standard Dixit-Stiglitz economy, we show, that the first order conditions of a symmetric oligopolistic price equilibrium correspond to a unique degree of competitive toughness in the general markup formula, This degree is decreasing (and the markup increasing) as more feedback effects are taken into account by firms. On the contrary, for the third variant, introducing full Ford effects leads to lower markups and higher competitive toughness in the standard Dixit-Stiglitz economy and under conditions ensuring the equilibrium markup to remain in tne right interval,

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# 1 Introduction

In this note we revisit the celebrated simple general equilibrium model that has been introduced 40 years ago by Avinash K. Dixit and Joseph E. Stiglitz. There are two sectors in this model, an imperfectly competitive sector, with firms competing in prices, each producing a differentiated commodity, and a perfectly competitive sector with one numeraire good, interpreted either as leisure time or as the aggregation of all the other goods in the economy. The utility function of the representative consumer is separable, with a sub-utility aggregating the quantities of the differentiated goods. The most popular version of the model, introduced in section I of Dixit and Stiglitz (1977), assumes a homothetic utility function and a symmetric CES sub-utility so that the demand faced by each firm is affected by rival firms prices only through a price index of all differentiated goods and through the consumer income. Imposing the Chamberlinian 'large group' assumption, Dixit and Stiglitz (1977) could neglect both these indirect effects and take only into account, for each monopolistic firm, the direct effect on profit of varying its own price.

The 'large group' assumption greatly simplifies the model, but has the inconvenient and paradoxical consequence of neutralizing its general equilibrium structure: competition among the producers of differentiated goods is confined to their own sector, and even loses its strategic dimension as soon as the analysis is limited to the case of a symmetric CES sub-utility, with the markup exclusively approximated by the reciprocal of the single CES parameter. We can however keep the nice general equilibrium structure of the model, while relaxing the 'large group' assumption. In Yang and Heijdra (1993), for instance, the price-index feedback effect of each firm pricing decisions is taken into account when computing its optimal profit. But this is only going halfway, since consumer income is still taken as a parameter adjusted at equilibrium, ignoring the so-called "Ford effects" (see d'Aspremont et al., 1989 and 1990), namely the feedback effects of strategic decisions going through consumer income. Hart (1985) calls this the "no feedback effects" assumption. It was already used by Marschak and Selten (1974). As argued in d'Aspremont et al. (1996), when there is only a 'small group' of firms, and if one insists on an 'objective' demand approach,[1] there is no more reason to fix parametrically the consumer income than to fix the price index. But consumer income includes several components: wage income in both sectors and distributed profit in the imperfectly competitive sector (profit in the competitive sector is nil). Ford effects can be introduced for some or for all components. In d'Aspremont et al. (1996) reconsideration of Dixit-Stiglitz basic model,[2] firms are assumed only to recognize feedback effects going through distributed profits in the imperfectly competitive sector. We go

[1] See Nikaido (1975).

[2] In this article we also consider an enlarged model where the numeraire is interpreted as the aggregation of all the other goods in the economy but labor is introduced as an additional good. The Dixit-Stiglitz model becomes a partial equilibrium model, the equilibrium in the labor market being determined at a preliminary stage. Applications of this enlarged model are to be found in Weitzman (1985), Blanchard and Kiyotaki (1987) and d'Aspremont et al. (1990).

beyond that in the present note and consider different kinds of Ford effects.

We use the most general version of Dixit and Stiglitz (1977) two-sector economy. In particular we relax the CES assumption, do not require symmetry and allow for a small group of firms[3]. We use a concept of oligopolistic equilibrium (d'Aspremont and Dos Santos Ferreira, 2016) where firms behave strategically in price-quantity pairs, maximizing profit under two constraints, on market share and on market size. This allows for a continuum of competition regimes (including price-competition and, at the limit, monopolistic competition) taking strategic interactions into account. A simple (relative) markup formula is derived from the equilibrium first order conditions: each firm equilibrium markup is equal to the weighted harmonic mean of the reciprocals of the intrasectoral and intersectoral elasticities of substitution. Also involved will be a conduct parameter for each firm, derived from the Lagrange multipliers associated with the two constraints in the program of that firm and interpreted as a measure of the *competitive toughness* displayed towards its rival oligopolists at equilibrium. A first result is to show that this formula is robust to the integration of Ford effects, when firms are assumed only to recognize feedback effects going through the distributed profits.

Then, coming back to the price-competition regime and a CES sub-utility, we show that one recovers Dixit-Stiglitz equilibrium conditions. That is, in the symmetric case, the same markup formula can be applied to monopolistic competition. This is a limit case where all the weight is put on the intrasectoral elasticity of substitution. But there are two ways to reach this limit case. One, traditional, is to make the Chamberlinian large group assumption. The other, still possible with a small number of firms, is to assume that each firm has maximal competitive toughness (equal to 1) instead of being insignificant. This gives another view of monopolistic competition, somewhat like Bertrand competition gives another view of perfect competition. Then, allowing for strategic interactions only through the price index, we show that the Yang and Heijdra (1993) conditions can be recovered. A simple formula is obtained for the equilibrium markup and the corresponding competitive toughness is easily derived, equal to 1/2 in the basic model. The introduction of Ford effects is more intricate when firms behave strategically in prices only, since a price deviation by one firm modifies in that case the profits of all its competitors, contrary to what happens when their price-quantity pairs are fixed by the Nash conjecture, as first assumed.

Finally, we consider, in both the general model of price-quantity competition and in its application to price competition in a standard Dixit-Stiglitz economy, the case where firms take into account all feedback effects, through wages and profits. A new still simple markup formula is derived from the equilibrium first order conditions which, under homothetic utility, appears to be again the expression of a weighted harmonic mean of the reciprocals of the intrasectoral elasticity of substitution and of a redefined elasticity of intersectoral

[3] Going beyond the basic version of Dixit-Stiglitz has generally been limited to relaxing the CES assumption or symmetry, but maintaining monopolistic competition. See *e.g.* Krugman (1979), Behrens and Murata (2007), Zhelobodko et al. (2012).

substitution.

Our general model and the corresponding equilibrium concept are presented in section 2. The equilibrium markup formulae for this model and for its Dixit-Stiglitz-like variant are then established when Ford effects are ignored or restricted to profits (section 3) and when they extend to wage income (section 4). We conclude in section 5.

# 2 Oligopolistic competition in the Dixit-Stiglitz economy

Following Dixit and Stiglitz (1977), we consider a two-sector economy. The first sector is imperfectly competitive and produces $n$ differentiated goods under constant unit costs and non-negative fixed costs. The second sector is perfectly competitive and produces a homogeneous good, taken as the numeraire, again under a constant unit cost, but without fixed costs. This good can be viewed as the result of the aggregation of all the goods competitively supplied in the economy, outside the first sector. As in our previous work (d'Aspremont and Dos Santos Ferreira, 2016), we shall use this simple general equilibrium model to distinguish two types of strategic interactions between producers of the differentiated goods: one intrasectoral, when firms compete for their market shares, the other intersectoral, when firms compete for the size of their market relative to the size of the market for the numeraire good.

## 2.1 Consumer behavior

We suppose a representative consumer with separable utility function $U(X(\mathbf{x}), z)$, where $\mathbf{x} \in \mathbb{R}^n_+$ is the vector of the consumed quantities of the $n$ differentiated goods (sold at prices $\mathbf{p} \in \mathbb{R}^n_{++}$) and $z \in \mathbb{R}_+$ the consumed quantity of the numeraire good. The utility function $U : \mathbb{R}^2_+ \to \mathbb{R}$ and the aggregator function $X : \mathbb{R}^n_+ \to \mathbb{R}_+$ are assumed increasing and strongly quasi-concave (except, for $X$, in the linear and Leontief limit cases and, for $U$, in the case of quasilinearity in $z$). Notice that, apart from separability, which is an essential ingredient of the Dixit-Stiglitz model, we only impose standard assumptions on the utility function, in particular without requiring homotheticity, additivity or symmetry[4]. We further assume an inelastic supply of $L$ units of labor and a wage equal to 1, the unit cost in the competitive sector. Income is equal to the sum of wages and profits, namely $Y = L + \Pi$, where $\Pi$ is the profit from the imperfectly competitive sector (the profit of the other sector being necessarily zero).

The consumer program can be decomposed into two stages. First, the consumer minimizes the expenditure on the composite good $X(\mathbf{x})$ by choosing the

[4]Bertoletti and Etro (2017) also consider general preferences and non-identical firms in a model of imperfect and monopolistic competition, deriving explicit solutions at least for some types of asymmetric preferences. However, their model has only one sector, and the concept of elasticity of (intrasectoral) substitution they use to characterize equilibria differs from the one we introduce below, except in particular cases.

appropriate quantity $x_i$ of each differentiated good $i$, while ensuring at least some level $\underline{X}$ of the aggregate:

$$\min_{x\in\mathbb{R}^n_+} \{\mathbf{px} \,| X(\mathbf{x}) \geq \underline{X}\} \equiv e(\mathbf{p},\underline{X}). \tag{1}$$

This defines the expenditure function $e$. The following dual conditions follow:[5]

$$\begin{aligned} p_i &= \partial_X e(\mathbf{p},\underline{X})\, \partial_i X(\mathbf{x}) && \text{(first order condition)} \quad (2)\\ x_i &= \partial_{p_i} e(\mathbf{p},\underline{X}) \equiv H_i(\mathbf{p},\underline{X}) && \text{(Shephard's lemma).} \quad (3)\end{aligned}$$

By these two conditions, the *budget share* of good $i$ in the expenditure for the differentiated goods is

$$\xi_i(\mathbf{p},\underline{X}) \equiv \frac{p_i x_i}{e(\mathbf{p},\underline{X})} = \epsilon_X e(\mathbf{p},\underline{X})\, \epsilon_i X(\mathbf{x}) = \epsilon_{p_i} e(\mathbf{p},\underline{X}). \tag{4}$$

The function $H_i$ is the *Hicksian demand* for good $i$. It expresses the conditions constraining the producers of the differentiated goods in their competition for market share, against each other. A fundamental concept in this context is the *intrasectoral elasticity of substitution* of good $i$, which we take as the elasticity, in absolute value, of its market share $x_i/\underline{X}$ with respect to the marginal rate of substitution $\partial_i X(\mathbf{x})$, when the bundle of differentiated goods is $\mathbf{x}$:

$$s_i \equiv -\frac{d(x_i/X(\mathbf{x}))}{d(\partial_i X(\mathbf{x}))}\frac{\partial_i X(\mathbf{x})}{x_i/X(\mathbf{x})} = \frac{1-\partial_i X(\mathbf{x})\, x_i/X(\mathbf{x})}{-\partial^2_{ii} X(\mathbf{x})\, x_i/\partial_i X(\mathbf{x})}. \tag{5}$$

For a derivation of this formula and, more generally, for a detailed discussion of our definition of $s_i$, see d'Aspremont and Dos Santos Ferreira (2016, Appendix). It is shown in this appendix that an alternative definition in terms of prices, with $x_i$ given by Shephard's lemma (3) and the marginal rate of substitution replaced by the relative price $p_i/\underline{P}$ ($\underline{P}$ being the shadow price $\partial_X e(\mathbf{p},\underline{X})$ of the composite good), leads to the equivalent formula for $s_i$:

$$s_i \equiv -\frac{d(x_i/\underline{X})}{d(p_i/\underline{P})}\frac{p_i/\underline{P}}{x_i/\underline{X}} = \frac{-\epsilon_{p_i} H_i(\mathbf{p},\underline{X})}{1-[\epsilon_i X(\mathbf{x})][\epsilon_X H_i(\mathbf{p},\underline{X})]}. \tag{6}$$

In the second stage, the consumer maximizes utility $U(\underline{X}, z)$ by choosing, under the budget constraint, the quantities $\underline{X}$ of the composite good and $z$ of the numeraire:

$$\max_{(\underline{X},z)\in\mathbb{R}^2_+} \{U(\underline{X}, z) \,| e(\mathbf{p},\underline{X}) + z \leq Y\}. \tag{7}$$

The solution defines the *Marshallian demand* $\underline{X} = D(\mathbf{p}, Y)$ for the composite good and the demand $z = Y - e(\mathbf{p}, D(\mathbf{p}, Y))$ for the numeraire good. The former expresses the conditions constraining the producers of differentiated goods

[5] We denote $\partial_{x_i} F(\mathbf{x}, Y) \equiv \partial F(\mathbf{x}, Y)/\partial x_i$, $\partial_Y F(\mathbf{x}, Y) \equiv \partial F(\mathbf{x}, Y)/\partial Y$ and also $\partial_i F(\mathbf{x}) \equiv \partial F(\mathbf{x})/\partial x_i$ when there is no ambiguity. Similarly, $\partial^2_{ij} F(\mathbf{x}) \equiv \partial^2 F(\mathbf{x})/\partial x_i \partial x_j$ and $\epsilon_i F(\mathbf{x}) \equiv \partial_i F(\mathbf{x})\, x_i/F(\mathbf{x})$.

in their competition for market size, against the other sector. We use in this context the fundamental concept of *intersectoral elasticity of substitution* of good $i$ as the elasticity, in absolute value, of its share $x_i/Y$ in aggregate consumption with respect to its relative price $p_i/1 = p_i$, referring to the elasticity $\epsilon_{p_i} D(\mathbf{p}, Y)$ of the Marshallian demand (rather than to the elasticity $\epsilon_{p_i} H_i(\mathbf{p}, \underline{X})$ of the Hicksian demand, which expresses a mere market share adjustment):

$$\sigma_i \equiv -\left.\frac{d(x_i/Y)}{dp_i}\right|_{X(\mathbf{x})=D(\mathbf{p},Y)} \frac{p_i}{x_i/Y} = \frac{-\epsilon_{p_i} D(\mathbf{p}, Y)}{\epsilon_i X(\mathbf{x})} \tag{8}$$

(see d'Aspremont and Dos Santos Ferreira, 2016, Appendix).

An alternative concept of intersectoral elasticity of substitution of good $i$, used in d'Aspremont *et al.* (1996), refers instead to the elasticity, say $\widehat{\sigma}_i$, in absolute value, of the ratio $x_i/z$ with respect to the corresponding relative price $p_i/1 = p_i$. As $x_i/z = (x_i/Y)(Y/z)$, so that $\epsilon_{p_i}(x_i/z) = \epsilon_{p_i}(x_i/Y) - \epsilon_{p_i}(z/Y)$, and using equation (4), we obtain

$$\begin{aligned} \widehat{\sigma}_i &= -\left(\frac{\epsilon_{p_i} D(\mathbf{p}, Y)}{\epsilon_i X(\mathbf{x})} - \frac{\epsilon_{p_i}(Y - e(\mathbf{p}, D(\mathbf{p}, Y)))}{\epsilon_i X(\mathbf{x})}\right) \\ &= \underbrace{\frac{-\epsilon_{p_i} D(\mathbf{p}, Y)}{\epsilon_i X(\mathbf{x})}}_{\sigma_i} - \frac{e(\mathbf{p}, \underline{X})/Y}{1 - e(\mathbf{p}, \underline{X})/Y} \underbrace{\frac{\xi_i(\mathbf{p}, \underline{X})}{\epsilon_i X(\mathbf{x})}}_{\epsilon_X e(\mathbf{p}, \underline{X})} \left(1 - \underbrace{\frac{-\epsilon_{p_i} D(\mathbf{p}, Y)}{\epsilon_i X(\mathbf{x})}}_{\sigma_i}\right). \end{aligned} \tag{9}$$

If the aggregator $X$ is homogeneous of degree 1, $\epsilon_X e(\mathbf{p}, \underline{X}) = 1$, and the expression for $\widehat{\sigma}$ (uniform across differentiated goods) simplifies to

$$\widehat{\sigma} = \frac{\sigma - e(\mathbf{p}, \underline{X})/Y}{1 - e(\mathbf{p}, \underline{X})/Y}, \tag{10}$$

an expression we shall use in section 4.

Two additional elasticities will be used in our analysis, for which we now introduce simplifying notations. The elasticity $\alpha_i \equiv \epsilon_i X(\mathbf{x})$ measures the impact of a variation in the quantity of good $i$ on the volume of the composite good. The elasticity $\beta_i \equiv \epsilon_X H_i(\mathbf{p}, \underline{X})$ measures the reverse impact of a variation in the quantity of the composite good on the demand for its component $i$, at given prices $\mathbf{p}$. The product of these two elasticities, which appears in the multiplier $1/(1 - \alpha_i \beta_i)$ applied to the elasticity of the Hicksian demand in the formula of equation (6), measures the intensity of the feedback originating in a variation in the quantity of good $i$ and going through the volume of the composite good.

For easier reference, we recall the expressions for these four elasticities in the following table:

| | |
|---|---|
| intrasectoral substitution: | $s_i = \frac{1-\alpha_i}{-\epsilon_i \partial_i X(\mathbf{x})} = \frac{-\epsilon_{p_i} H_i(\mathbf{p}, X(\mathbf{x}))}{1-\alpha_i \beta_i}$ |
| intersectoral substitution: | $\sigma_i = \frac{-\epsilon_{p_i} D(\mathbf{p}, Y)}{\alpha_i}$ |
| impact of $x_i$ on $X$ | $\alpha_i \equiv \epsilon_i X(\mathbf{x})$ |
| impact of $X$ on $x_i$ | $\beta_i \equiv \epsilon_X H_i(\mathbf{p}, X(\mathbf{x}))$. |

**Table 1**

## 2.2 Firms competitive behavior and oligopolistic equilibria

In the imperfectly competitive sector, each firm $i$ belonging to what may be a *small group* of oligopolists, produces a single component of the composite good under constant positive unit cost $c_i$ and a non-negative fixed cost $\phi_i$. As in d'Aspremont and Dos Santos Ferreira (2016) we suppose that firms behave strategically in price-quantity pairs, $(p_i, x_i) \in \mathbb{R}_+^2$ for each firm $i = 1, ..., n$, under two admissibility constraints, corresponding to the two types of strategic interactions.

$$\begin{aligned} x_i &\leq H_i\left((p_i, \mathbf{p}_{-i}), X(x_i, \mathbf{x}_{-i})\right), \\ X(x_i, \mathbf{x}_{-i}) &\leq D\left((p_i, \mathbf{p}_{-i}), Y\right). \end{aligned}$$

The first is a *constraint on market share* and refers to the first stage of the consumer's utility maximization. The second is a *constraint on market size* and refers to the second stage of the consumer's utility maximization. The constraint on market share emphasizes the conflictual side of competition between oligopolists, whereas the constraint on market size reflects their common interest as a sector.

The corresponding equilibrium concept is based on the concept of oligopolistic equilibrium introduced in d'Aspremont *et al.* (2007) and d'Aspremont and Dos Santos Ferreira (2010). The following definition though distinguishes three variants of the concept, (i) one attributing to oligopolists a myopic income-taking behavior, so that income $Y$ is treated parametrically and supposed to be fixed at its equlibrium value, (ii)-(iii) the two others admitting a more far-sighted behavior of oligopolists, taking into account income feedback effects (the so-called "Ford effects"), which are either (ii) restricted to its profit component or (iii) extensive to the wage component.

**Definition 1** *An* oligopolistic equilibrium *is a $n$-tuple of pairs $(p_i^*, x_i^*)_{i=1,...,n} \in \mathbb{R}_+^{2n}$ such that, for any $i$,*

$$\begin{aligned} (p_i^*, x_i^*) \in \arg & \max_{(p_i,x_i)\in\mathbb{R}_+^2} (p_i - c_i)\, x_i \qquad (11) \\ \text{s.t.} \qquad x_i &\leq H_i\left(\left(p_i, \mathbf{p}_{-i}^*\right), X\left(x_i, \mathbf{x}_{-i}^*\right)\right) \\ \text{and } X\left(x_i, \mathbf{x}_{-i}^*\right) &\leq D\left(\left(p_i, \mathbf{p}_{-i}^*\right), Y\right), \end{aligned}$$

$$\begin{aligned} \text{where} \quad \text{(i) } Y \equiv \; & Y^* \equiv L + \sum_{j=1}^{n} \left(\left(p_j^* - c_j\right) x_j^* - \phi_j\right) \text{ (without Ford effects)}, \\ \text{or} \quad \text{(ii) } Y \equiv \; & L + \sum_{j\neq i} \left(\left(p_j^* - c_j\right) x_j^* - \phi_j\right) \\ & + \left((p_i - c_i)\, x_i - \phi_i\right) \text{ (with Ford effects restricted to profits)}, \\ \text{or} \quad \text{(iii) } Y \equiv \; & z^* + \sum_{j\neq i} p_j^* x_j^* + p_i x_i \text{ (with full Ford effects)}. \end{aligned}$$

*In addition, we require the profits to be non-negative, namely* $\left(p_j^* - c_j\right) x_j^* - \phi_j \geq 0$ *for each* $j$*, and the consumer to be non-rationed.*

Conditions (i) and (ii) decompose income into wages $L$ and profits $\Pi = \sum_{j=1}^{n} \left(\left(p_j - c_j\right) x_j - \phi_j\right)$, whereas condition (iii) decomposes income into wage income $z$ generated in the competitive sector and income $\sum_{j=1}^{n} p_j x_j$ distributed by the producers of differentiated goods. Non-rationing of consumers at equilibrium implies full employment, so that $z^* = L - \sum_{j=1}^{n} \left(c_j x_j^* + \phi_j\right)$.

# 3 Equilibrium markups when Ford effects are absent or restricted

We next show that an oligopolistic equilibrium, whether without Ford effects or with Ford effects restricted to profits, is characterized by the same simple expression for each firm $i$. More explicitly, the *equilibrium (relative) markup* $\mu_i^* \equiv \left(p_i^* - c_i\right)/p_i^*$ (*i.e.* the Lerner index for the degree of monopoly power of firm $i$), derived from the first order conditions, is expressed as the weighted harmonic mean of the reciprocals of the two elasticities of substitution $s_i^*$ and $\sigma_i^*$ at that equilibrium. The weights of this mean involve a conduct parameter[6] $\theta_i^* \in [0,1]$, equal to the Lagrange multiplier associated with the constraint on firm $i$ market share, divided by the sum of the two multipliers. This parameter may be interpreted as a measure of the *competitive toughness* displayed at some equilibrium by firm $i$ towards its rival oligopolists, and used to identify different regimes of competition, in particular price competition, the regime assumed by Dixit and Stiglitz (1977).

## 3.1 The general markup formula

The next proposition establishes the equilibrium markup formula.

**Proposition 1** *Let* $(p_i^*, x_i^*)_{i=1,\ldots,n} \in \mathbb{R}_{++}^{2n}$ *be an oligopolistic equilibrium (i) without Ford effects or (ii) with Ford effects restricted to profits. Then the equilibrium markup* $\mu_i^* = \left(p_i^* - c_i\right)/p_i^*$ *of each firm* $i$ *is given by*

$$\mu_i^* = \frac{\theta_i^* \left(1 - \alpha_i^* \beta_i^*\right) + \left(1 - \theta_i^*\right) \alpha_i^*}{\theta_i^* \left(1 - \alpha_i^* \beta_i^*\right) s_i^* + \left(1 - \theta_i^*\right) \alpha_i^* \sigma_i^*}, \tag{12}$$

*for some* $\theta_i^* \in [0,1]$.

**Proof.** We start by making dimensionally homogeneous the two constraints in the program of firm $i$, rewriting them in terms of the two ratios:

$$\frac{x_i}{H_i\left(\left(p_i, \mathbf{p}_{-i}^*\right), X\left(x_i, \mathbf{x}_{-i}^*\right)\right)} \leq 1 \text{ and } \frac{X\left(x_i, \mathbf{x}_{-i}^*\right)}{D\left(\left(p_i, \mathbf{p}_{-i}^*\right), Y\right)} \leq 1. \tag{13}$$

[6] This is the terminology used in the empirical industrial organization literature (see Bresnahan, 1989).

The first-order necessary conditions for profit maximization at $(p_i^*, x_i^*)$ under constraints (13) (holding as equalities at equilibrium because of the no-rationing condition) can then be expressed, for non-negative Lagrange multipliers $\lambda_i^*$ and $\nu_i^*$, as[7]

$$x_i^* = \lambda_i^* \frac{-\partial_{p_i} H_i^*}{H_i^*} + \nu_i^* \left( \frac{-\partial_{p_i} D^*}{D^*} + \frac{-\partial_Y D^*}{D^*} \partial_{p_i} Y^* \right) \tag{14}$$

and

$$p_i^* - c_i = \lambda_i^* \left( \frac{1 - \partial_X H_i^* \partial_i X^*}{H_i^*} \right) + \nu_i^* \left( \frac{\partial_i X^*}{X^*} - \frac{\partial_Y D^*}{D^*} \partial_{x_i} Y^* \right). \tag{15}$$

By dividing both hand sides of the second equality by the corresponding hand sides of the first, and then multiplying them by $x_i^*/p_i^*$, we obtain the following formula, in terms of elasticities, for the markup of firm $i$ at the equilibrium $(p_i^*, x_i^*)_{i=1,\dots,n}$:

$$\frac{p_i^* - c_i}{p_i^*} = \frac{\lambda_i^* \left(1 - \epsilon_X H_i^* \epsilon_i X^*\right) + \nu_i^* \left(\epsilon_i X^* - \epsilon_Y D^* \epsilon_{x_i} Y^*\right)}{\lambda_i^* \left(-\epsilon_{p_i} H_i^*\right) + \nu_i^* \left(-\epsilon_{p_i} D^* - \epsilon_Y D^* \epsilon_{p_i} Y^*\right)}. \tag{16}$$

Denoting $\theta_i \equiv \lambda_i / (\lambda_i + \nu_i)$ and referring to Table 1, we can rewrite the equilibrium markup formula as

$$\frac{p_i^* - c_i}{p_i^*} = \frac{\theta_i^* \left(1 - \alpha_i^* \beta_i^*\right) + \left(1 - \theta_i^*\right) \left(\alpha_i{}^* - \epsilon_Y D^* \epsilon_{x_i} Y^*\right)}{\theta_i^* \left(1 - \alpha_i^* \beta_i^*\right) s_i^* + \left(1 - \theta_i^*\right) \left(\alpha_i{}^* \sigma_i^* - \epsilon_Y D^* \epsilon_{p_i} Y^*\right)}. \tag{17}$$

When there are no Ford effects (case (i)), as $\epsilon_{x_i} Y^* = \epsilon_{p_i} Y^* = 0$, we immediately obtain the formula given by the proposition. Otherwise, in case (ii),

$$\epsilon_{x_i} Y^* = \frac{p_i^* x_i^*}{Y^*} \frac{p_i^* - c_i}{p_i^*} \text{ and } \epsilon_{p_i} Y^* = \frac{p_i^* x_i^*}{Y^*},$$

so that, if we multiply both hand sides of equation (17) by the denominator of the right-hand side, the same term in $(p_i^* - c_i)/p_i^*$ appears on both hand sides and can be eliminated. So, we are back to the formula given in the proposition. ■

It is not difficult to see why cases (i) and (ii) lead to the same expression for the first order condition. Consider the general structure of the program of firm $i$, expressed as the maximization of the Lagrangian

$$\max_q f(q) - \lambda g(q) - \nu h(q, Y(f(q))), \tag{18}$$

where $q$ is the pair of strategy variables, $f$ is the objective function, $g(q) \leq 0$ and $h(q, Y(f(q))) \leq 0$ the two constraints, and $\lambda$ and $\nu$ the corresponding Lagrange multipliers. The strategies of other firms, implicit in the functions $g$ and $h$, are here omitted for simplicity of notation. The crucial point is that $Y$

[7] For shortness, we use for equilibrium values the notations $F^* \equiv F(\mathbf{x}^*)$ and $\partial_i F^* \equiv \partial_i F(\mathbf{x}^*)$.

depends upon the strategy pair $q$ only through the objective function $f$. As a consequence, the first order condition for an interior solution is

$$[1-\nu\partial_Y h(q,Y(f(q)))Y'(f(q))]\,\partial f(q)=\lambda\partial g(q)+\nu\partial_q h(q,Y(f(q)))\,,\quad (19)$$

where the gradient $\partial f(q)$ is multiplied by 1 when Ford effects are ignored (since $Y'(f(q))=0$), and by a positive constant when they are not (when $Y'(f(q))=1$). Thus, taking into account restricted Ford effects only changes proportionately the two Lagrange multipliers without modifying the first order condition.

Under both conditions (i) and (ii) of the oligopolistic equilibrium concept, at any equilibrium $(p_i^*,x_i^*)_{i=1,\dots,n}$, the relative markup of each firm $i$ is a weighted harmonic mean of the reciprocals of the intrasectoral elasticity of substitution $s_i^*$ and of the intersectoral elasticity of substitution $\sigma_i^*$. The corresponding weights involve, for each firm $i$, the elasticities $\alpha_i^*$ and $\beta_i^*$ measuring the two reciprocal effects of quantity variations of good $i$ and of the composite good, as well as the conduct parameter resulting from the Lagrange multipliers associated with the two constraints. In particular, the relative weight on the intersectoral elasticity of substitution $\sigma_i^*$ is an increasing function of the impact $\alpha_i^*\equiv\epsilon_i X(\mathbf{x}^*)$ on the consumption of the composite good of a deviation in the quantity of good $i$ from its equilibrium value. In the limit situation of Chamberlin's 'large group', when this impact is negligible ($\alpha_i^*\simeq 0$), the markup equals the reciprocal of $s_i^*$, so that market power is entirely ruled by competition within the sector. Our formula points however out to an alternative for the same result, namely when firm $i$ reaches the highest possible competitive toughness ($\theta_i^*=1$) against its rivals in the sector. This situation may be the result of the conjecture by firm $i$ that the market share constraint is the only binding and, when concerning all firms, may be viewed as an instance of the so-called 'Bertrand paradox', since two firms (if $n=2$) prove enough for the competitive ('large group') result to obtain.

More generally, our concept of oligopolistic equilibrium leads to existence of a large set of equilibria, parameterized by the vector of conduct parameters $\boldsymbol{\theta}^*$.[8] Equilibrium selection may be performed by particular producers' conjectures about their competitors' behavior, for instance the conjecture that they stick to the choice of only one strategy type (price or quantity), while adjusting the other. As shown in d'Aspremont and Dos Santos Ferreira (2016, 3.1 and 3.4), we obtain, in this case and when disregarding Ford effects, standard price equilibria (oligopolistic equilibria parameterized by $\boldsymbol{\theta}^*=(1/(1+\beta_i^*))_{i=1,\dots,n}$) or standard quantity equilibria (oligopolistic equilibria parameterized by $\boldsymbol{\theta}^*=(1/(1+s_i^*/\sigma^*))_{i=1,\dots,n}$ if $X$ is homothetic).

[8] We abstain from tackling existence problems in this short note. An oligopolistic equilibrium does not always exist for any parameter value $\boldsymbol{\theta}\in[0,1]^n$ (see subsection 2.4 in d'Aspremont and Dos Santos Ferreira, 2016).

## 3.2 Dixit-Stiglitz revisited

In this subsection, we move closer to the Dixit-Stiglitz basic model, by restricting our analysis, first to the economy introduced in section I of Dixit and Stiglitz (1977), with a homothetic utility function $U$ and a symmetric CES aggregator function $X$, plus uniform costs for all producers, and second to the regime of price competition. As to the first restriction, homogeneity of degree 1 of $X$ implies $\beta_i^* = 1$ and $\sigma_i^* = \sigma^*$ for any $i$, and constancy of the elasticity of substitution translates into $s_i^* = s$ for any $i$. The equilibrium markup formula then simplifies to

$$\mu_i^* = \frac{\theta_i^* (1 - \alpha_i^*) + (1 - \theta_i^*) \alpha_i^*}{\theta_i^* (1 - \alpha_i^*) s + (1 - \theta_i^*) \alpha_i^* \sigma^*}. \tag{20}$$

More explicitly, we obtain in this economy the following specifications for the demand functions

$$\begin{aligned} H_i (\mathbf{p}, D(\mathbf{p}, Y)) &= \left(\frac{p_i}{P(\mathbf{p})}\right)^{-s} D(\mathbf{p}, Y), \text{ with } P(\mathbf{p}) = \left(\sum_j p_j^{1-s}\right)^{1/(1-s)}, \\ D(\mathbf{p}, Y) &= \frac{\gamma(P(\mathbf{p}))}{P(\mathbf{p})} Y, \end{aligned} \tag{21}$$

where $P(\mathbf{p})$ is the price index for the set of differentiated goods and $\gamma(P(\mathbf{p}))$ is the budget share of the imperfectly competitive sector in the whole economy. These specifications lead in particular to the elasticities

$$\alpha_i (\mathbf{p}) = \epsilon_i P(\mathbf{p}) = \frac{p_i^{1-s}}{\sum_j p_j^{1-s}} \text{ and } \sigma(\mathbf{p}) = 1 - \epsilon_P \gamma(P(\mathbf{p})). \tag{22}$$

As to the second restriction, to the regime of price competition, we first transpose to oligopolistic *price* equilibria the analysis performed in the previous subsection.

**Definition 2** *An* oligopolistic price equilibrium *is a n-tuple of prices* $(p_i^*)_{i=1,\ldots,n} \in$

$\mathbb{R}_+^n$ *such that, for any* $i$,

$$
\begin{aligned}
p_i^* \in \arg & \max_{p_i \in \mathbb{R}_+} (p_i - c_i) x_i & (23)\\
\textit{s.t. } x_i &\equiv H_i\left(\left(p_i, \mathbf{p}_{-i}^*\right), D\left(\left(p_i, \mathbf{p}_{-i}^*\right), Y\right)\right), \\
\textit{where (i) } Y &\equiv Y^* \equiv L + \sum_{j=1}^{n} \Pi_j^* \textit{ (without Ford effects)}, \\
\textit{with } \Pi_j^* &\equiv \left(p_j^* - c_j\right) H_j\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right) - \phi_j, \\
\textit{or (ii) } Y &\equiv L + \sum_{j=1}^{n} \left[\left(p_j^* - c_j\right) H_j\left(\left(p_i, \mathbf{p}_{-i}^*\right), D\left(\left(p_i, \mathbf{p}_{-i}^*\right), Y\right)\right) - \phi_j\right] \\
& \textit{(with Ford effects restricted to profits)}, \\
\textit{or (iii) } Y &\equiv z^* + \sum_{j \neq i} p_j^* H_j\left(\left(p_i, \mathbf{p}_{-i}^*\right), D\left(\left(p_i, \mathbf{p}_{-i}^*\right), Y\right)\right) \\
& + p_i H_i\left(\left(p_i, \mathbf{p}_{-i}^*\right), D\left(\left(p_i, \mathbf{p}_{-i}^*\right), Y\right)\right) \textit{ (with full Ford effects)}.
\end{aligned}
$$

*In addition, we require the profits to be non-negative:* $\left(p_j^* - c_j\right) H_j\left(\mathbf{p}^*, D\left(\mathbf{p}^*, Y^*\right)\right) - \phi_j \geq 0$ *for each* $j$.

When Ford effects are neglected (case (i)), since we know that the price equilibrium markup is characterized by the conduct parameter value $\theta_i^* = 1/\left(1 + \beta_i^*\right)$ for every $i$, and since homotheticity of $U$ implies $\beta_i^* = 1$, we have $\theta_i^* = 1/2$ for any $i$ (even when the equilibrium is asymmetric, because of non-uniformity of production costs). Consequently, the equilibrium markup formula simplifies to

$$\mu_i^* = \frac{1}{\left(1 - \alpha_i^*\right) s + \alpha_i^* \sigma^*} \tag{24}$$

and, with symmetry, to

$$\mu^* = \frac{1}{(1 - 1/n) s + (1/n) \sigma^*}. \tag{25}$$

This is the expression resulting from the approach of Yang and Heijdra (1993, eq. (4)), which allows for a significant impact $\alpha_i^*$ of differentiated good $i$ on the composite good, while excluding Ford effects.

Case (ii), with Ford effects restricted to profits, does not lead to the same result as in Proposition 1. Indeed, when strategies are price-quantity pairs, the Nash conjecture implies that deviations by firm $i$ only affect its own profit. By contrast, deviations by firm $i$ under price competition do affect profits of all firms. Should we suppose firm $i$ to conjecture, when deviating, that the profits of other firms remain equal to their equilibrium value $\sum_{j \neq i} \Pi_j^*$, then case (ii) would lead, as in Proposition 1, to the same result as case (i), hence to formula (24) in the present context. Taking into account the effect of a price deviation on the rivals' profits, we can still obtain a formula for the equilibrium markup, as stated in the following proposition. We shall further assume, following Dixit and Stiglitz, that $\max\{\sigma^*, 1\} < s$.

**Proposition 2** *Let $p^* \in \mathbb{R}_{++}$ be the price of a symmetric oligopolistic price equilibrium with Ford effects restricted to profits, in a standard Dixit-Stiglitz economy. Then the equilibrium markup $\mu^* = (p^* - c)/p^*$ of each firm $i$ is implicitly and uniquely given by*

$$\mu^* = \frac{(1-\mu^*\gamma^*)(1-1/n) + (1/n)}{(1-\mu^*\gamma^*)(1-1/n)s + (1/n)\sigma^*}. \tag{26}$$

*If, in addition, $1 \leq \sigma^* < s$, then $\mu^* \in [1/s, 1/\sigma^*]$.*

**Proof.** By Definition 2 and equations (21) and (**??**, the first order condition for maximization of the profit of firm $i$, expressed in terms of elasticities and using symmetry, is

$$\frac{1}{\mu^*} - \left(\left(1-\frac{1}{n}\right)s + \frac{1}{n}\sigma^*\right) + \epsilon_{p_i}Y^* = 0. \tag{27}$$

Without Ford effects, $\epsilon_{p_i}Y^* = 0$, and we obtain formula (25). Otherwise, by Definition 2 (ii), we have

$$Y = \frac{L - n\phi}{1 - \sum_{j=1}^{n}\left[(p_j - c)\,p_j^{-s}\frac{\gamma(P(\mathbf{p}))}{P(\mathbf{p})^{1-s}}\right]}, \tag{28}$$

hence

$$\epsilon_{p_i}Y^* = \frac{1-\mu^*\sigma^*}{1-\mu^*\gamma^*}\frac{\gamma^*}{n}, \tag{29}$$

so that the first order condition writes

$$\frac{1}{\mu^*} - s + (s-\sigma^*)\frac{1}{n} + \frac{1/\mu^* - \sigma^*}{1/\mu^* - \gamma^*}\frac{\gamma^*}{n} = 0, \tag{30}$$

an equation equivalent to the formula to be proven. This formula is a harmonic mean of $1/s$ and $1/\sigma^*$, provided there is a solution $\mu^* \in [0,1]$ for this equation. Equation (30) is equivalent to the quadratic equation

$$F(\mu^*) \equiv \mu^{*2}\left(1-\frac{1}{n}\right)s\gamma^* - \mu^*\left[\left(1-\frac{1}{n}\right)(s+\gamma^*) + \frac{1}{n}\sigma^*\right] + 1 = 0, \tag{31}$$

so that we have:

$$\begin{aligned} F(1/s) &= (1-\sigma^*/s)/n > 0, \\ F(1/\sigma^*) &= \left(1-\frac{1}{n}\right)\left[(1/\sigma^*)^2 s\gamma^* - (1/\sigma^*)(s+\gamma^*) + 1\right] < 0, \end{aligned}$$

if $\gamma^* < \sigma^* < s$. Thus, $F$ has a unique root $\mu^*$ such that $1/s < \mu^* < 1/\sigma^* \leq 1$. ∎

Equation (30) in the preceding proof can be shown to be equivalent to equation (10) in d'Aspremont *et al.* (1996), where (restricted) Ford effects have been added to the effect of a deviation in $p_i$ on the price index $P\left(p_i, \mathbf{p}^*_{-i}\right)$.[9]

The markup in oligopolistic price equilibria without Ford effects was expressed as a harmonic mean of $1/s$ and $1/\sigma^*$, so that it is associated with a general expression for $\theta^*$ ($1/2$ in the present homothetic context). With Ford effects restricted to profits, we still obtain a harmonic mean of $1/s$ and $1/\sigma^*$, but with weights that are only implicitly defined. The markup $\mu^*$ is now determined by a quadratic (not anymore a linear) equation, and we lose a general expression for $\theta^*$, which remains implicit. Take indeed the equilibrium markup formula determined by Proposition 1 and applied to the present standard Dixit-Stiglitz economy:

$$M\left(\theta\right) = \frac{\theta\left(1 - 1/n\right) + \left(1 - \theta\right)\left(1/n\right)}{\theta\left(1 - 1/n\right)s + \left(1 - \theta\right)\left(1/n\right)\sigma^*}. \tag{32}$$

$M\left(\theta\right)$ is a harmonic mean of $1/s$ and $1/\sigma^*$, increasing from $1/s$ to $1/\sigma^*$ as $\theta$ decreases from 1 to 0. By continuity, there is a value $\theta^*$ for which $M\left(\theta^*\right) = \mu^*$, the equilibrium markup given by the formula of Proposition 2. For $\theta^* = 1/2$, we get the markup $M\left(1/2\right) = 1/\left(\left(1 - 1/n\right)s + \left(1/n\right)\sigma^*\right)$ of the oligopolistic price equilibrium without Ford effects. In the formula of Proposition 2, the relative weight on $1/s$ is decreased through its multiplication by $1 - \mu^*\gamma^* < 1$, implying that the markup is increased when Ford effects are taken into account and that $\theta^*$ must be smaller than $1/2$. Firms adopt in this case a strategic behavior which is softer towards their rivals within the sector, as they give more importance to the interaction with the other sector (and the more so the higher the budget share $\gamma^*$ of their own sector). This translates into a lower competitive toughness as measured by $\theta^*$.

# 4 Equilibrium markups with full Ford effects

In order to tackle case (iii), corresponding to full Ford effects, which extend to wages, we come back to the general case of oligopolistic equilibria covered by Definition 1, before considering the Dixit-Stiglitz basic model.

## 4.1 A modified general markup formula

As we will now see, the general formula obtained for the equilibrium markup is modified, while remaining easy to interpret.

**Proposition 3** *Let* $\left(p^*_i, x^*_i\right)_{i=1,\ldots,n} \in \mathbb{R}^{2n}_{++}$ *be an oligopolistic equilibrium (iii) with full Ford effects. Then the equilibrium markup* $\mu^*_i = \left(p^*_i - c_i\right)/p^*_i$ *of each*

[9] Equation (10) in d'Aspremont *et al.* (1996) concerns the equilibrium demand elasticity $\varepsilon^*$, which is the reciprocal of the equilibrium markup $\mu^*$, and involves a differently defined intersectoral elasticity of substitution, corresponding to our $\widehat{\sigma}$ as defined by equation (10).

*firm $i$ is given by*

$$\mu_i^* = \frac{\widehat{\theta}_i^* \left(1 - \alpha_i^* \beta_i^*\right) + \left(1 - \widehat{\theta}_i^*\right) \alpha_i^* \left(1 - \left(\eta_i^* / \alpha_i^*\right) \epsilon_Y D^*\right)}{\widehat{\theta}_i^* \left(1 - \alpha_i^* \beta_i^*\right) s_i^* + \left(1 - \widehat{\theta}_i^*\right) \alpha_i^* \left(\sigma_i^* - \left(\eta_i^* / \alpha_i^*\right) \epsilon_Y D^*\right)}, \tag{33}$$

*with $\eta_i^* \equiv p_i^* x_i^* / Y^*$ the budget share of good $i$ in the* whole *expenditure, for some $\widehat{\theta}_i^* \in [0, 1]$.*

**Proof.** The formula given in the proposition stems directly from formula (17) in the proof of Proposition 1, established on the basis of the first order conditions for the same producer's program except for the account taken of Ford effects. Here, in case (iii), $\epsilon_{x_i} Y^* = \epsilon_{p_i} Y^* = p_i^* x_i^* / Y^* = \eta_i^*$, so that the proof is complete. ■

The expression obtained for the equilibrium markup, although similar to the one formulated in Proposition 1, is no more a harmonic mean of $1/s_i^*$ and $1/\sigma_i^*$. Its reciprocal is an affine combination of the two elasticities and 1 (the sum of the weights equals 1), with a negative weight on 1, so that its value is larger than the arithmetic mean of $s_i^*$ and $\sigma_i^*$ (with the same relative weights). In other words, the equilibrium markup is smaller after switching from restricted to full Ford effects. This is the consequence of the reactivity to strategy deviations by firm $i$ of wage income (fixed as $L$ in case (ii), and now varying with $c_i x_i$).

This means that the equilibrium markup $\mu_i^*$ could fall below the minimum of $1/s_i^*$ and $1/\sigma_i^*$. However, whenever the equilibrium value of $\eta_i^* \epsilon_Y D^*$ is not too high, so that the Ford effect remains moderate, we can still obtain: $\min\{1/s_i^*, 1/\sigma_i^*\} \leq \mu_i^* \leq \max\{1/s_i^*, 1/\sigma_i^*\}$. Then, by continuity, there will be $\theta_i^*$ such that $\mu_i^*$ can be written as a harmonic mean of $1/s_i^*$ and $1/\sigma_i^*$, as in Proposition 1.

When the utility $U$ is homothetic and the aggregator $X$ homogeneous of degree 1, $\epsilon_Y D^* = \epsilon_X e^* = 1$, so that, by equation (4), $\eta_i^* / \alpha_i^* = e\left(\mathbf{p}, \underline{X}\right) / Y^* \equiv \gamma^*$. We thus obtain in this case $\mu^*$ as a weighted harmonic mean of $1/s^*$ and $1/\widehat{\sigma}^*$, with $\widehat{\sigma}^*$as defined by equation (10). Taking as the intersectoral elasticity of substitution the elasticity of $x_i/z$ rather than that of $x_i/Y$ is the appropriate choice when utility is homothetic and when full Ford effects are at stake. The former concept (leading to $\widehat{\sigma}^*$) captures the full Ford effects, allowing to obtain the simplicity of the formula established when these effects are absent:

$$\mu_i^* = \frac{\widehat{\theta}_i^* \left(1 - \alpha_i^*\right) + \left(1 - \widehat{\theta}_i^*\right) \alpha_i^* \left(1 - \gamma^*\right)}{\widehat{\theta}_i^* \left(1 - \alpha_i^*\right) s^* + \left(1 - \widehat{\theta}_i^*\right) \alpha_i^* \left(1 - \gamma^*\right) \widehat{\sigma}^*}. \tag{34}$$

## 4.2 Back to Dixit-Stiglitz

In section 3, we only considered the oligopolistic price equilibrium without Ford effects (case (i)) or with Ford effects restricted to profits (case (ii)). The following proposition covers case (iii), with full Ford effects, in the standard Dixit-Stiglitz economy.

**Proposition 4** *Let $p^* \in \mathbb{R}_{++}$ be the price of a symmetric oligopolistic price equilibrium with full Ford effects, in a standard Dixit-Stiglitz economy. Then the equilibrium markup $\mu^* = (p^* - c) / p^*$ of each firm $i$ is implicitly and uniquely given by*

$$\mu^* = \frac{1}{(1 - 1/n)\, s + (1/n)\, \widehat{\sigma}^*}, \tag{35}$$

*with $\widehat{\sigma}^* = (\sigma^* - \gamma^*) / (1 - \gamma^*)$.*

**Proof.** The first order condition for maximization of firm $i$ profit is given by (27) in the proof of Proposition 2:

$$\frac{1}{\mu^*} - \left( \left( 1 - \frac{1}{n} \right) s + \frac{1}{n} \sigma^* \right) + \epsilon_{p_i} Y^* = 0.$$

Using (iii) in Definition 2 and equations (21) and (22), we have: $Y = z^* + \gamma (P (\mathbf{p})) Y = z^* / (1 - \gamma (P (\mathbf{p})))$, so that

$$\epsilon_{p_i} Y^* = \frac{\gamma^*}{1 - \gamma^*} \frac{1 - \sigma^*}{n}, \tag{36}$$

implying the formula stated in the proposition. ■

The equilibrium markup is a weighted harmonic mean of $1/s$ and $1/\widehat{\sigma}^*$. It remains in the interval $[1/s, 1/\sigma^*]$ iff $1 < \sigma^* < (1 - \gamma^*)\, s + \gamma^*$, a stronger and stronger constraint on $\sigma^*$ as the budget share $\gamma^*$ of the imperfectly competitive sector increases. Being in the interval $[1/s, 1/\sigma^*]$ ensures, by the same argument as in subsection 3.2, that there exists $\theta^*$ such that $M (\theta^*) = \mu^*$ for $M$ as defined by (32), the formula of Proposition 1 applied to the Dixit-Stiglitz economy. Also, as $(1 - \gamma^*) / (\sigma^* - \gamma^*) < 1/\sigma^*$ for $\sigma^* > 1$, $\mu^* < M (1/2)$, implying that $\theta^* \in (1/2, 1)$. Contrary to what happens when switching from price equilibria without Ford effects to price equilibria with restricted Ford effects, introducing full Ford effects leads to lower markups. Firms are now confronted with an elastic labour residual supply which diminishes their market power. This effect increases with the budget share of the imperfectly competitive sector.

The conduct parameter $\theta^*$, although uniquely determined, remains implicit. Alternatively, we may refer to the modified formula (34) applied to a symmetric price equilibrium in a standard Dixit-Stiglitz economy, and determine, by identification of the weights, an explicit value for the corresponding conduct parameter $\widehat{\theta}^*$:

$$\widehat{\theta}^* = \frac{1 - \gamma^*}{2 - \gamma^*} \in [0, 1/2]\,. \tag{37}$$

This value corresponds, for case (iii) with full Ford effects, to $\theta^* = 1/2$ in case (i), when there are no Ford effects. Both allow to characterize a price equilibrium by referring to general formulae established for oligopolistic equilibria where firms compete in prices and quantities.

## 5 Conclusion

Starting from the most general version of Dixit-Stiglitz two-sector economy, adding only standard assumptions on the utility function (apart from separability), and allowing for a small number of (non-identical) firms, we have presented three variants of a general concept of oligopolistic equilibrium in price-quantity pairs. Each firm maximizes profit under two admissibility constraints, one intrasectoral (on market share), the other intersectoral (on market size) while taking into account income feedback effects in three different ways. For the first two variants (Ford effects ignored or restricted to profits), a single and simple equilibrium markup formula has been derived, where markup appears as a weighted harmonic mean of the reciprocals of intrasectoral elasticity of substitution $s_i^*$ and intersectoral elasticity of substitution $\sigma_i^*$ (both specific to a particular equilibrium and a particular commodity). The weights involve, for each firm, a conduct parameter indicating its degree of competitive toughness at that equilibrium. Different specifications of these conduct parameters lead to different oligopolistic equilibria in prices and/or quantities. In particular, in the standard Dixit-Stiglitz economy, we have shown that the first order conditions of a symmetric oligopolistic price equilibrium correspond to a unique degree of competitive toughness in the general markup formula, monopolistic competition (even without imposing the large group assumption) being associated with the maximal degree $\theta^* = 1$, price competition without Ford effects with $\theta^* = 1/2$, price competition with Ford effects restricted to profits with some $\theta^* \in (0, 1/2)$.

Hence, firms strategic behavior towards their competitors is softer, as they take into account more price and income feedback effects. However, having firms take into account full Ford effects makes a difference. In the standard Dixit-Stiglitz economy, under conditions ensuring the equilibrium markup to remain in the interval $[1/s, 1/\sigma^*]$ (such condition is required since the general markup formula is modified), switching from Ford effects restricted to profits to full Ford effects translates into lower market power and higher competitive toughness, *i.e.* $\theta^* \in (1/2, 1)$: including wage income feedback effects implies in this context that the oligopolistic firms cease to face a perfectly inelastic labour supply due to the interaction with the competitive sector.

This conclusion challenges our general assumption of a given, perfectly inelastic, labor supply. As justly emphasized by Parenti et al. (2017) in that respect, most widespread assumptions made in oligopoly models are extreme, either that the labour supply is perfectly elastic, or that it is perfectly inelastic[10]. Clearly, it would be worthwhile to see how a less extreme assumption than a perfectly inelastic labour supply would affect our results.

Another important extension of our model would be to introduce free entry. The main issue raised 40 years ago by Dixit and Stiglitz seminal paper was to determine the equilibrium number of differentiated products under free entry and to compare this number to the (constrained) optimal one. However, introducing price and income feedback effects and, more generally, allowing for

[10] They also assume a perfectly inelastic labor supply but in a one sector model in which firms treat income parametrically.

strategic interactions within a small group of firms may change substantially the analysis. In the Dixit-Stiglitz approach the (unique) equilibrium number of active firms is associated with zero profits and such that any entrant would make a loss at the going price index. This is fully compatible with Dixit-Stiglitz approximation, which assumes a large group of firms ignoring the price index feedback effect. If such feedback effects are introduced, the zero profit result does not hold anymore and, as shown in d'Aspremont et al. (2000), the equilibrium number of active firms can be indeterminate and their profit positive. It would be interesting to pursue such investigation in the oligopolistic equilibrium approach we have adopted here.

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