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The Benefits of Cooperation Under Uncertainty: the Case of Climate Change

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Abstract This article presents an analysis of the behaviour of countries defining their climate policies in an uncertain context. The analysis is made using the S-CWS model, a stochastic version of an integrated assessment growth model. The model includes a stochastic definition of the climate sensitivity parameter. We show that the impact of uncertainty on policy design critically depends on the shape of the damage function. We also examine the benefits of cooperation in the context of uncertainty: We highlight the existence of an additional benefit of cooperation, namely risk reduction.

Keywords Cooperation · Uncertainty · Climate change · Integrated assessment model

JEL Classification C71 · C73 · D9 · D62 · F42 · Q2

1 Introduction

There exist huge uncertainties surrounding the magnitude and the speed of climate change. How such uncertainties shape climate policies is the main question addressed in this paper.

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The recent literature in economic theory has highlighted the consequences for climate policy of the uncertainties concerning the severity of climate change. Gollier [21] has pointed out that a prudent society should discount the future at a lower rate when it faces uncertainty. This means that a larger investment to prevent future losses (like climate damages) is socially desirable. The finding was discussed and confirmed by several later contributions (e.g. [2, 17]). Weitzman [40] even suggested that if the distribution of damages exhibits a 'fat-tail', policy recommendations may be radically altered. In the expected utility framework, this induces (infinitely) negative discount rates urging prompt action to avoid climate change (see also [2, 17]). Despite these theoretical results, the impact of uncertainty on climate policy design has hardly been studied in integrated assessment models (IAMs). These models combine scientific and socioeconomic aspects of climate change to assess climate change policies. Several of such models use optimal economic growth modelling, in line with the original model of Ramsey [29], for instance DICE [25], MERGE [23], OMEGA [19], PAGE [33], WITCH [10], DEMETER [38] or CWS [13, 15, 36].

Most of the uncertainty analyses carried out have taken the form of sensitivity analysis, in particular Monte Carlo analyses (see, e.g. [28, 33]). The aim was to obtain probability ranges for temperature increase or damages. But such analyses suppose that policy is designed after the uncertainty is resolved. By contrast, it is of interest to study how dealing with uncertainty reshapes policies and changes incentives to cooperate in an expected utility framework similar to the one used in the theoretical contribution by Gollier [21] and Weitzman [40]. To the best of our knowledge, the only contribution of this kind is the one by Nordhaus and Popp [26]. Still, these authors focus on the value of information rather than on the design of climate policies. The second question addressed in this paper is to assess to what extent cooperation can contribute to the protection against high climate damages. It is well-known that cooperation is welfare improving at the global level, but does it allow a reduction of risk? If it is the case, then it means that cooperation brings out a side benefit.[1]

In the present article, we propose to model decision making under uncertainty in an IAM: the ClimNeg world simulation (CWS) model. To do so, the CWS model had to be adapted to deal with uncertainty. There exist several techniques to do so, and they can be split into two categories: anticipative and non-anticipative. The first category assumes that the decision makers have perfect knowledge. The second category assumes that the decision makers take the decisions that are best adapted to different possible realizations of uncertainty. Sensitivity analyses fall in the first category. In this paper, we deal with the second category of techniques, which encompasses stochastic programming [9], dynamic programming [6] or more recent techniques such as robust optimization [8] or programming techniques using affine or step decision rules [7, 35].

The usual growth models *à la Ramsey* are written in a programming or algebraic modelling language as linear programming problems.[2] Linear programming is a very powerful optimization technique that can easily handle uncertain components. One can easily define the deterministic equivalent of a stochastic problem, especially if this problem is a linear programming problem. This step can even be automatized, as in [34].

Stochastic programming is the technique we have chosen to handle uncertain components in the CWS model, but other tools have already been used in the field. The use of stochastic modelling in IAMs has been proposed by [1] with a model derived from DICE in which the uncertain parameter is revealed at a given time period and discretized in three values. In [42], the authors use stochastic differential equations and meta-modelling. In [4], an optimal timing of climate policies is found using dynamic programming to represent a two-step process (revelation of the true climate sensitivity value and availability of a backstop technology) in which the order of the steps is not known in advance. Stochastic programming technique has been used in only one model, WITCH, where the efficiency of the clean technology [11] or the CO_2 concentration target in 2100 [12] could be defined as uncertain and resolved during the horizon at a given time period.[3]

In this paper, we shall present a stochastic version of the CWS model, called S-CWS. This version deals with a larger number of values for the uncertain parameter than what has been already done in similar models, such as WITCH. This better discretization will provide optimal climate policies that are more robust. This is the methodological contribution of the paper. The uncertain parameter will be the climate sensitivity, and the countries will look for a unique policy to cope with the different possible values of the uncertain parameter. As for economic analysis, our objective is twofold. First, we study how uncertainty alters the chosen course of action. Second, we highlight the consequences of uncertainty on the benefits from climate international cooperation. The idea is to test to what extent countries may be able to cope with uncertainty more efficiently under an international cooperation regime than alone.

The paper is organized as follows: In the next section, we provide the theoretical background about policy design under uncertainty. In Section 3, we describe the stochastic version of the CWS model. In Section 4, we discuss some findings obtained with the simulation model. In Section 5, we provide some conclusive remarks.

2 Policy Design Under Uncertainty

2.1 Choice Under Uncertainty

The economic theory has built a well-accepted framework to model how choices should be made under uncertainty: the expected utility model.[4] The model was initially developed by von Neumann and Morgenstern [39]. It is based on the idea that decision makers seek to maximize the expected value of their utility.

Assume for instance that uncertainty is represented by S possible states of nature, denoted $\{1, \ldots, s, \ldots, S\}$, which occur with respective probabilities $(p_1, \ldots, p_s, \ldots, p_S)$. Let x_s be the payoff obtained

[1] Here we do not raise the issue of countries' incentive for cooperation. For such an analysis, see, e.g. [13].

[2] The objective function is linear if the agent is risk neutral, but the constraints are always linear in the decision variables.

[3] The parameter value is discretized in three values.

[4] Recently, the expected utility model has been challenged in the context of uncertainty, and several alternatives have been proposed to represent ambiguity aversion and probabilities misperceptions. For the role of these models in the case of climate change, see for instance [5, 16, 32]. However, the expected utility model is a useful benchmark which has hardly been studied in IAMs. It is natural to take this approach as a starting point.

if the state of the world s realizes. The decision maker maximizes $\sum_s p_s u(x_s)$. When the function u is concave, the decision maker is said to be *risk-averse*: She dislikes risk.

In this paper, we introduce the expected utility methodology in a multi-country integrated assessment model of climate change. The payoff $x_{i,s}$ for a country $i \in \{1, \ldots N\}$ in state of the world s is the discounted sum of total present and future consumption, $x_{i,s} = \sum_{t=1}^{T} \frac{Z_{i,t,s}}{(1+\rho)^{t-1}}$, where $Z_{i,t,s}$ is total consumption in country i in period t and state of the world s and ρ is the discount rate. We assume that consumption cannot be known with certainty, so that $Z_{i,t,s}$ must be indexed by the state of the world, s. Every country i therefore chooses the course of action that maximizes

$$W_i = u^{-1}\left(\sum_s p_s u\left(\sum_{t=1}^{T} \frac{Z_{i,t,s}}{(1+\rho)^{t-1}}\right)\right). \tag{1}$$

A distinctive feature of the objective function W_i is that we take a transform u^{-1} of the expected utility. This means that we take what is called the 'certainty equivalent' of the expected utility, that is, the utility a country would reach in the absence of uncertainty. The reason for this modelling choice is that we want to have some coherence between our stochastic results and those obtained in the usual deterministic case, so that we are able to easily compare our results. Note indeed that if we were sure of the state of the world s, we would take $p_s = 1$ and $p_{s'} = 0$ for all $s' \neq s$, so that the above objective function would become $W_i = \sum_{t=1}^{T} \frac{Z_{i,t}}{(1+\rho)^{t-1}}$.[5] This is precisely the objective function used in the deterministic version of the CWS model [15, 36].

We assume that the function $u(.)$ in the above expression has the form

$$u(x) = \frac{x^{1-\eta}}{1-\eta},$$

where $\eta > 0$ is the coefficient of relative risk aversion. A higher η means that the society is more averse to risk: It is willing to pay more to avoid a risk. Throughout the paper, we will take $\eta = 2$, which is a quite standard value in the literature (see [3, 20] for instance).

2.2 Implications for Climate Change Policies

A key feature of the expected utility model is that welfare can be decomposed in two elements: the expected payoff and the amount of risk a decision maker bears. There is a potential trade-off between these two dimensions: In order to obtain a sure outcome, the decision maker is ready to give up some of her expected payoff. The amount she is ready to give up precisely measures her risk aversion. It is called the *risk premium*. It is of interest to determine whether a policy can improve welfare in one of the two dimensions (expected payoff and risk premium) or both.

In the context of climate change, countries must take decisions under large uncertainties about the dynamics of the climatic system. We assume that uncertainty is never resolved. By assuming that countries do not adapt their plan to observed shocks, we implicitly consider that there are other (unmodelled) forms of uncertainties affecting the economy so that policy makers are unable to appropriately update their beliefs.[6] In this context, the objective for a country is to find the policy that, if applied in all possible states of the world, will maximize her expected utility. This means that the policy must maximize the expected payoff without increasing uncertainty too much.

When there are uncertainties about how the climate reacts to greenhouse gases (GHG) concentrations, introducing risk aversion may provide an additional rationale for emission reduction. Indeed, if climate uncertainty keeps the expected damage constant while increasing risk, countries are ready to abate more to avoid the risk. One issue though is that the uncertainty about the climatic model does not translate directly into an uncertainty about the payoff. In particular, the damages corresponding to the expected parameters values of the climatic model are not necessarily the expected damages taking into account all possible values of this parameter.

We hence cannot predict unambiguously whether the policy using 'best guess' estimates of the parameters will be more or less restrictive than the policy arising from maximizing the expected utility. We shall indeed see in Section 4.1 that the damage corresponding to the expected parameter value of the climatic model may be higher than the expected damages, yielding more abatement and less emissions. If the climate uncertainty would not affect the expected damages, the opposite would be true because of risk aversion.

In a strategic framework, introducing uncertainty also has an impact on potential climate agreements. Indeed, cooperation is efficient in reducing uncertainty. Cooperation will both imply an efficient sharing of

[5] In contrast, not taking the transform u^{-1} would yield the objective function $W_i = u\left(\sum_{t=1}^{T} \frac{Z_{i,t,s}}{(1+\rho)^{t-1}}\right)$ in the deterministic case, which looks quite unusual.

[6] For a modelling of climate change policies with update of beliefs, see [11, 12], or [14] for decision patterns using 'model predictive control'.

emissions to limit the expected climate change impacts and an efficient risk sharing between the regions. One of our objectives is to study the impact of the additional gain from cooperation, namely risk reduction.

3 Dealing with Uncertainty by Using Stochastic Optimization

In order to design an appropriate climate policy to face climate sensitivity uncertainty, we apply a technique of optimization under uncertainty to extend the ClimNeg world simulation model. This stochastic version is labelled S-CWS. In this section, we shall present the introduction of uncertainty in the climate sensitivity parameter. Then, we shall explain how the CWS model has been adapted for stochastic computation.

3.1 Uncertainty About Climate Sensitivity

Climate sensitivity is a parameter in climate science surrounded by huge uncertainties. Climate sensitivity measures how much global warming can be expected in equilibrium after a doubling of GHGs concentration in the atmosphere. The Fourth Assessment Report of the Intergovernmental Panel on Climate Change (IPCC; [27]) compiled 18 recent studies about the distribution of the climate sensitivity parameters. Most studies highlight that there exist large uncertainties about this parameter: This uncertainty seems inherently very difficult to reduce. In particular, the possibility of high values of the climate sensitivity parameter cannot be ruled out ([27], Chap. 10, pp. 798–799). There is little hope that the precise value can be learned with sufficient accuracy in the near future because small uncertainties about various feedback mechanisms translate into large uncertainties in climate sensitivity [30].

Many estimates of this parameter lie in the region around 4, which means a four-degree expected increase in the average earth surface temperature for a doubling in GHG concentration in the long term. Most climate models generate an asymmetric distribution around this value. As explained by Roe and Baker [30], a distribution with a fat tail is a natural outcome of uncertainty about the various feedback processes whereby higher temperatures raise the level of radiative forcing. It is natural to portray the uncertainty about such feedbacks as uncertainty about climate sensitivity. This route is also suggested by Weitzman [40].

We generate the distribution of the climate sensitivity parameter following the same reduced-form approach as in [30]. In their notation, we set $f = 0.77$ and $\sigma_f = 0.189$ (f and σ_f are, respectively, the mean and the standard deviation of the distribution of the total feedback factor). We obtain a probability distribution function (pdf) similar in shape to the one reported in [31]. The mean and median of this theoretical distribution are 4.16 and 3.81, respectively. Figure 1 plots the pdf used in our simulations.[7]

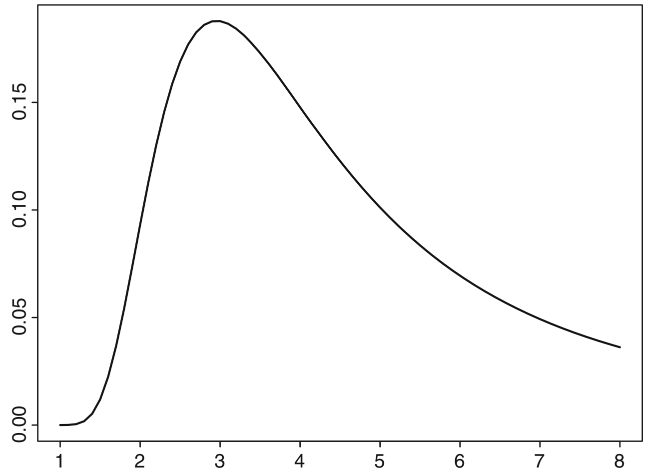

Fig. 1 Probability density function for climate sensitivity (x-axis) built from Roe and Baker [30]

3.2 A Stochastic Version of the CWS Model

The stochastic version of the CWS model (S-CWS) consists in an integrated assessment model of climate change and optimal growth, adapted for coalitional analysis from [25]. It encompasses economic, climatic and impact dimensions in a worldwide intertemporal setting. As a Ramsey-type model (see [29]), economic growth is driven by population growth, technological change and capital accumulation. The time dimension is discrete, indexed by t, finite, but very long.[8]

The world is split into 18 countries[9] (for the list, see Table 4 in "Appendix"). In each country $i = 1, \ldots n$, gross output $Y_{i,t}$ is given by a Cobb–Douglas production function combining capital and labour. Population change is exogenous. Capital accumulation results from an endogenous gross investment $I_{i,t}$ reduced by exogenous scrapping. Technical progress is Hicks neutral. Carbon emissions stem from global output with an emission coefficient which can be reduced by national

[7] Many other pdf are available in the literature review carried out by the IPCC report [27], for instance [18, 22, 24]. We have performed some sensitivity analysis, which showed that our numerical results are robust to the choice of the pdf.

[8] Specifically, the simulation horizon is 2330.

[9] For short, we use the term country to denote the regions/countries of the S-CWS model.

policies, $\widetilde{\sigma}_{i,t} = (1 - \mu_{i,t})\sigma_{i,t}$, where $\mu_{i,t} \in [0, 1]$ stands for the carbon abatement rate and $\sigma_{i,t}$ is the exogenous carbon intensity of the economy. Abatement costs are given by an increasing and convex cost function $C_i(\mu_{i,t})$.

GHG concentration, through a simplified carbon cycle, yields a global mean temperature expressed as temperature change with respect to the pre-industrial level, T_t^{E}. A key equation of the climatic model is the one which describes the dynamics of temperature.[10] This is the equation where the uncertain climate sensitivity parameter $T2X_s$ enters, so that temperature increase depends on the state of the world s:

$$T^E_{t+1,s} = \frac{T^E_{t,s}}{1 + c_1\left(\frac{F2X}{T2X_s}\right) + c_1 c_3} + c_1\left(F_{t+1} + c_3 T^{\mathrm{L}}_{t,s}\right),$$

where $F2X$, F_{t+1} and $T^{\mathrm{L}}_{t,s}$ stand for carbon forcing, radiative forcing and temperature change in lower ocean, respectively. It is possible to show that the relationship between $T^{\mathrm{E}}_{t+1,s}$ and $T2X_s$ is concave for $T2X_s \geq 1$.

Starting from the climate module of the model, the uncertainty is transmitted into the countries' payoffs through the damage functions:

$$D_i\left(T^E_{t,s}\right) = Y_{i,t}\left[\theta_{i,1} T^E_{t,s} + \theta_{i,2}\left(T^E_{t,s}\right)^{\theta_3}\right]. \tag{2}$$

The damage functions are increasing and convex in temperature change.[11] Finally, consumption $Z_{i,t,s}$ is given by the gross output minus investment, abatement costs and damage costs:

$$Z_{i,t,s} = Y_{i,t} - I_{i,t} - C_i(\mu_{i,t}) - D_i\left(T^{\mathrm{E}}_{t,s}\right).$$

As a result, countries' payoffs are stochastic.

This economic model is converted into a 18-player game by letting the countries be the players, whose strategies are the decision variables $I_{i,t}$ and $\mu_{i,t}$ over the entire period 2000–2300. The S-CWS model is used to determine paths of investment $(I_{i,t})$ and emissions (through $\mu_{i,t}$) in the face of uncertainty. Stated otherwise, the aim is to find paths of policy instruments that, if applied in *any* realization of the uncertain parameter, would maximize an objective function. The value of the objective function of a country-player i is defined as follows:

$$W_i = u^{-1}\left(\int u\left(\sum_{t=0}^{T} \frac{Z_{it}(\tilde{\xi})}{(1+\rho)^{t-1}}\right) dD(\tilde{\xi})\right), \tag{3}$$

where $u(x) = \frac{x^{1-\eta}}{1-\eta}$ is the utility function, with η the degree of risk aversion and ρ the discount rate. This objective function is to be maximized according to the decision variables, where $\tilde{\xi}$ is a continuous random variable of probability function D defined on a given probability space. Let us approximate the continuous variable $\tilde{\xi}$ by a discrete one denoted by ξ. We are now able to define the objective function of the deterministic equivalent of the stochastic objective 3, which is nothing more than Eq. 1, where we assumed that the *states of the world* ξ_s were known and countable $(s = 1, \dots S)$.

The players–countries' strategies are specified according to two alternative scenarios. First is the *Nash equilibrium* scenario, which is the joint outcome of each country maximizing its welfare taking the actions of the others as given.[12] Second is the *Cooperative* scenario where all countries act jointly so as to maximize the world welfare. This scenario is Pareto efficient.[13] The two scenarios are formally defined as follows (the constraints 5–16 of the model can be found in the "Appendix"):

- *Cooperative scenario (COOP)*:
  $\left(\mu^{\mathrm{COOP}}_{i,t}, I^{\mathrm{COOP}}_{i,t}\right)_{\substack{i=1,\dots N\\ t=0,\dots T}}$ that solves:
  Max $W = \sum_i W_i$, subject to Eqs. 5–16.
- *Nash equilibrium scenario (NASH)*:
  $\left(\mu^{\mathrm{NASH}}_{i,t}, I^{\mathrm{NASH}}_{i,t}\right)_{\substack{i=1,\dots N\\ t=0,\dots T}}$ that solves, for each $i = 1, \dots N$:
  Max W_i, subject to Eqs. 5–16, with $E_{j,t} = E^{\mathrm{NASH}}_{j,t}, \forall j \neq i, t = 0, \dots T$.

Solving a problem for $S = 1$ boils down to assuming that the value of the variable ξ (which is a parameter in this case) is known beforehand. So this is a pure deterministic framework. Such a solution is also named *See and Act* in [26, 41]. Solving a problem with $S > 1$ boils down to assume that uncertainty is not resolved until the last period. As explained in Section 2.2, we indeed assume that our problem has no recourse,

[10] For the list of variables and the complete description of the model, see the "Appendix".

[11] Values for those polynomials have been updated from the DICE-2010 model.

[12] In the terminology of dynamic noncooperative games, this is an open loop Nash equilibrium. Closed loop or feedback Nash equilibria have also been introduced in dynamic core-stability analysis in [37], albeit with a simpler model.

[13] A third kind of scenario can also be computed, namely the *partial agreement Nash equilibria with respect to a coalition* scenarios (PANEs). Each PANE is the outcome of a subset of countries maximizing jointly their welfare, while the others act individually (there are as many such scenarios considered as there are coalitions). See [15, 36] for applications with PANEs.

so that the sequence of the process is the following: (a) decisions are taken and then (b) after the last period uncertainty is resolved. The solution is named *Act and See* in [26].

The larger S, the better the numerical approximation. However, enlarging S increases the size of the computation problem: Stochastic programming suffers from the curse of dimensionality [6]. If the deterministic problem is already a large dimensional problem, then it is almost impossible to *stochastize* it. The CWS model is a rather small-scale model, so we were rather confident being able to define a problem with a larger S than what has been done in the literature, i.e. S greater than 3.[14] Indeed, the S-CWS model includes a description of the climate sensitivity parameter in seven values, which is the maximum number manageable with the current version of the GAMS-CONOPT software.

Nevertheless, even if S is large, there still exists a discrepancy between the approximated (discrete) distribution and the real (continuous) one. This is why it is important to validate the results of the optimization (or prediction phase) in what we shall call a *validation phase*.[15]

We define two distinct sets of possible outcomes of the random variable ξ: (a) a large set $\mathcal{V}$ for validation set which is supposed to be the real distribution (or a very close approximation) and (b) a smaller set $\mathcal{C}$ for computation set (or optimization set) that can be included in $\mathcal{V}$.[16] In practice, the sets could be defined by a Monte Carlo random draw. The output of the optimization phase consists in a set of optimal values $I^*_{i,t}$ and $\mu^*_{i,t}$ for the decision variables and the associated value W^* of the objective function $W = \sum_i W_i$ at the optimum. We call this value *prediction*. The value W^* is of limited interest because it is computed on the basis of a small set, $\mathcal{C}$. To *validate* the optimal policy found, we propose to compute[17] a more robust value, that is, the mean of the objective function if we apply the optimal policy on a large set $\mathcal{V}$ of outcomes of the random variable.[18] This mean value is denoted by $W^{\mathcal{V}}$ and is the result of the validation phase:

Table 1 Computation set: value and probability of elements

s	$T2X_s$	p_s
1	1.7625000	0.03101
2	2.5062400	0.23156
3	3.4310000	0.26034
4	4.4188400	0.18879
5	5.4189100	0.12881
6	6.4214700	0.0896
7	7.4690200	0.06989

$$W^{\mathcal{V}} = \sum_{i=1}^{S_{\mathcal{V}}} p_i W(\mu^*, I^*, \xi_i) \tag{4}$$

Based on the probability distribution presented in Section 3.1, the computation set $\mathcal{C}$, with $S_{\mathcal{C}} = 7$, is as defined in Table 1. The validation set $\mathcal{V}$ is defined on the same basis but with a larger cardinality, $S_{\mathcal{V}} = 71$. The values for $T2X_s$ range from 1.0 to 8.0 with a step of 0.1. The associated probabilities are presented in the "Appendix" in Table 13.

4 Computation Results

4.1 How Risk Aversion Shapes Climate Policies

The first set of results will show how the policies carried out in the NASH and COOP scenarios change when policy makers are risk averse. In other words, we shall compare the value of the policy instruments under the NASH and COOP scenarios when policy makers are risk averse with the ones in the deterministic case. Table 2 displays all figures in difference with respect to the deterministic case. They are shown for two values of the exponent θ_3 of the damage function (see Eq. 2 above), and the rationale for this will appear soon.

Let us start with the benchmark case where the damage parameter θ_3 is 2.0.[19] What Table 2 shows is that the average abatement rate μ_i is lower, whatever the scenario. As a result, GHG emissions are higher (+0.40% in NASH and +0.05% in COOP) and cumulated damages larger in both scenarios (+0.05% in NASH and

[14] The CWS model has been initially developed for coalition analyses that need a huge number of model runs. The limited size of the generated problem was then a main constraint. In that sense, CWS is different from other growth models such as WITCH or DICE that are more detailed, but also less manageable.

[15] This phase could also be used to contrast several approaches, for instance models with different values of S.

[16] In a multistage context, the computation set should be aggregated in a tree to be exploited by the model.

[17] There is no optimization, only computation using the optimal policy found.

[18] Empirically, it is well-known that the result of stochastic programming optimization, or prediction value, is very optimistic and, in a sense, not realistic and that the policy found is very sensitive, not robust.

[19] $\theta_3 = 2.0$ is the benchmark value in the CWS model as well as in many IAMs.

Table 2 Differences between stochastic and deterministic policies (expressed w.r.t. deterministic policy)

	Damage parameter $\theta_3 = 2.0$		Damage parameter $\theta_3 = 2.7$	
	NASH	COOP	NASH	COOP
Abatement rate (in 2200, in point of %)	−0.02	−0.18	0.70	0.65
GHG emission level (in 2200)	0.40%	0.05%	−1.75%	−3.18%
Damages (cumulated, discounted)	0.05%	0.37%	−1.79%	−2.86%
Consumption (cumulated, discounted)	0.00%	0.00%	0.09%	0.00%
Temperature (in 2200, in °C)	0.00	0.01	−0.03	−0.04
Welfare	0.00%	0.00%	0.09%	0.00%

+0.37% in COOP). In the meantime, global output is enhanced, so that cumulated discounted consumption is left unchanged. Surprisingly, welfare remains almost unchanged when risk is taken into account. This is due to the almost unchanged level of expected consumption combined with a very small effect of risk (small risk premia). To sum up, the two striking results coming out from the first two columns of Table 2 are that (a) the world emission level is higher and (b) positive and negative effects cancel out, so that global welfare is unchanged.

Such a result was rather unexpected. What would have been naturally expected is a lower emission level and a gain in welfare under risk aversion. To understand this counter-intuitive result, we need to compute a sensitivity analysis. This sensitivity analysis consists in setting the exponent parameter of the damage functions θ_3 to 2.7 in all countries (instead of 2.0). In this case, the results look quite different, as shown in Table 2. Now, GHG emissions are lower under risk aversion (−1.75% in NASH and −3.18% in COOP, in 2200) because of much stronger abatement rates. This leads to a smaller temperature increase and damages reduced by 1.79% and 2.86%, respectively. These results are now in line with the intuition: When policy makers are risk averse, abatement efforts are much stronger and, as a result, climate change is curbed.

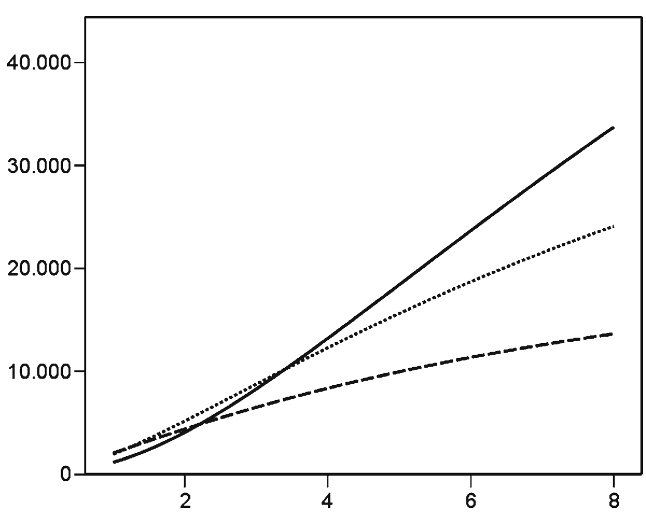

Fig. 2 Relation between climate sensitivity (x-axis) and cumulated discounted damages (y-axis) under NASH for three damage parameters: θ_3 =1.5 (*dashed line*), θ_3 =2.0 (*dotted line*) and θ_3 =2.7 (*solid line*)

The rationale for this unexpected outcome was actually provided in Section 2.2. Indeed, the uncertainty about the climate sensitivity parameter does not readily translate into damages. In particular, damages at the expected value of the climate sensitivity parameter are larger than expected damages. This comes from the fact that, in a more complex model as S-CWS, the relationship between the climate sensitivity and the damages can be non-linear, as illustrated in Fig. 2. The shape of the relationship depends on the interplay between two features. On the one hand, the stronger the convexity of the damage function, the stronger the incentive for risk averse policy makers to avoid a too large temperature increase and thus their incentive to curb GHG emissions. But, on the other hand, the impact of larger GHG emissions on temperature increase depends on the radiative forcing of emissions, and this radiative forcing has a concave shape. So the combination of these two effects (i.e. the link between global emissions and damages) may well be either concave or convex. If it is convex, then the intuition applies, namely that abatement efforts will be stronger under risk aversion. But if it is concave, then the contrary applies.[20] The S-CWS model reveals that the former comes out when $\theta_3 = 2.0$ and the latter when $\theta_3 = 2.7$.

Let us remark that the potential existence of a concave relationship between climate sensitivity and

[20]Indeed, if the relationship between global emissions and damages is convex, we know by Jensen's inequality that damages from expected emissions are lower than expected damages from emissions. Therefore, there are additional incentives to reduce emissions when risk is explicitly taken into account. The opposite is true when the relation is concave.

Table 3 Differences between COOP and NASH (expressed w.r.t. NASH)

	Damage parameter $\theta_3 = 2.0$	Damage parameter $\theta_3 = 2.7$
Abatement rate (in 2200, in point of %)	16.85	31.87
GHG emission level (in 2200)	−27.99%	−60.85%
Damages (cumulated, discounted)	−24.66%	−62.57%
Consumption (cumulated, discounted)	0.35%	2.71%
Temperature (in 2200, in °C)	−0.81	−1.78
Welfare	0.35%	2.71%

temperature contrasts with the analysis of Weitzman [40]. This comes from the fact that Weitzman considers a simplified model where the climate dynamics is at a stationary equilibrium, so that temperature increase (from doubling GHG concentration) is exactly the value of the climate sensitivity parameter. Our dynamic analysis suggests that taking into account climate dynamics is likely to question the mechanism highlighted by Weitzman.

4.2 The Benefits of Cooperation

The benefits of cooperation in the case of climate change are well established in the literature. They have been illustrated with the deterministic version of the CWS model in [15] and [36]. The purpose of this section is to ask whether side benefits can be expected from cooperation when policy makers are risk averse. We have learnt from the previous section that a key element in the analysis is the convex/concave relationship between climate sensitivity and damages. So we shall perform our analysis again for two values of the parameter θ_3, 2.0 and 2.7. The results are displayed in Table 3.

It first appears from Table 3 that cooperation has a strong impact on global GHG emissions: They are reduced (in 2200) by 27.9% when $\theta_3 = 2.0$ and by 60.8% when $\theta_3 = 2.7$, with respect to NASH. Global welfare is thus increased by 0.35% and 2.71%, respectively.

Figure 3 displays the density curves (or pdf) of the global temperature increase in the two scenarios and for the two values of the damage parameter. It clearly appears that the distribution shifts left under COOP, but one can also see that the shape is changed: Cooperation shrinks the range of the density curve. In other words, the world becomes able to prevent itself from too high temperature increases. The probability range at 90% is reduced roughly by one third. While (with $\theta_3 = 2.7$) there exists a 90% chance for the temperature increase to lie between +2.9°C and +7.1°C in 2200 for NASH, this range becomes +2.0°C and +4.7°C under COOP. The mean value drops from +5.1°C to +3.3°C. How does this shift of the density curves in temperature affect the economy? We have seen in Table 3 that consumption is increased and so does welfare in the COOP scenario. This shows that some of these beneficial effects come from a more efficient management of the uncertainty related to climate damages.

The density curves for global damages are displayed in Fig. 4. The figure shows how risk aversion reshapes the density curve of damages. It appears that cooperation is effective at shrinking the uncertainty range related to damages. In other words, under cooperation there is a high probability for the world to avoid dramatic climate damages. So an important benefit from cooperation, in addition to the reduction of expected damages, is to narrow the uncertainty range.

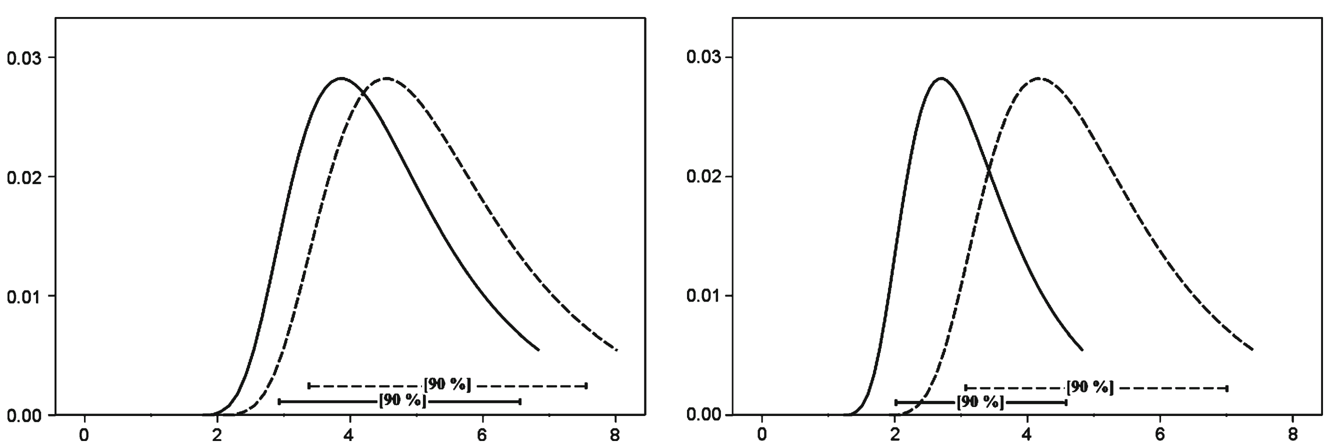

Fig. 3 Empirical density curves of temperature increase in 2200 (*x*-axis) under NASH (*dotted line*) and COOP (*solid line*) for $\theta_3 = 2.0$ (*left*) and $\theta_3 = 2.7$ (*right*)

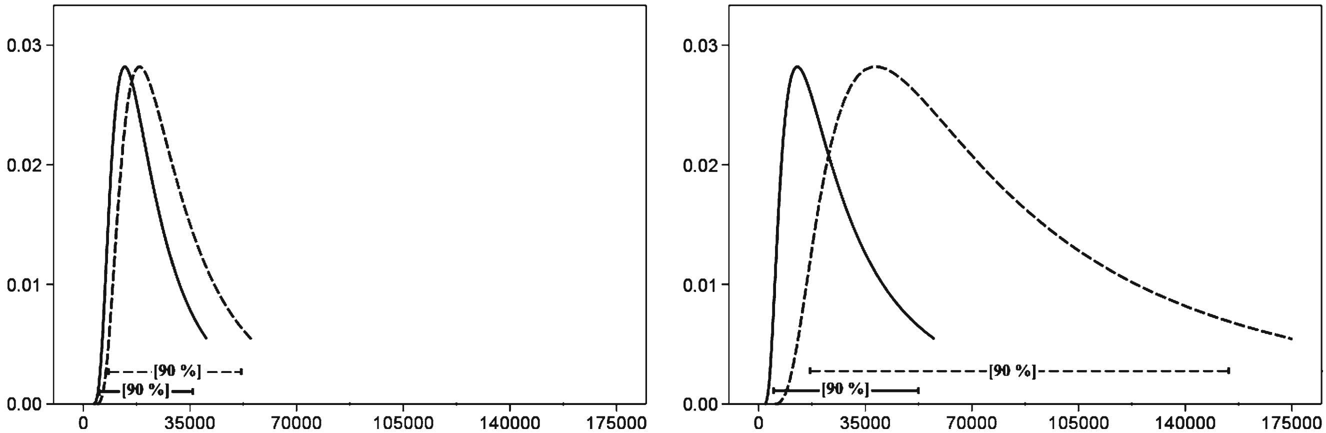

Fig. 4 Empirical density curves of climate damages in 2200 (x-axis) under NASH (*dotted line*) and COOP (*solid line*) for $\theta_3 = 2.0$ (*left*) and $\theta_3 = 2.7$ (*right*)

Fig. 5 Damages in 2200 for COOP (*unfilled*) and NASH (*filled*), in percent of GDP, with $\theta_3 = 2.7$; the *bar* stands for 90 pc.; the *dot* is the mean value (for country label, see Table 4 in "Appendix")

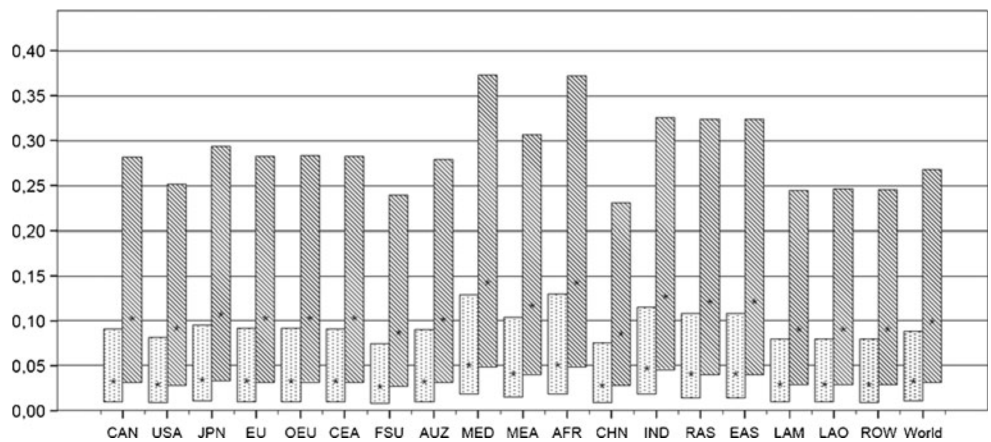

Fig. 6 Damages in 2200 for COOP (*unfilled*) and NASH (*filled*), in percent of GDP, with $\theta_3 = 2.0$; the *bar* stands for 90 pc.; the *dot* is the mean value (for country label, see Table 4 in "Appendix")

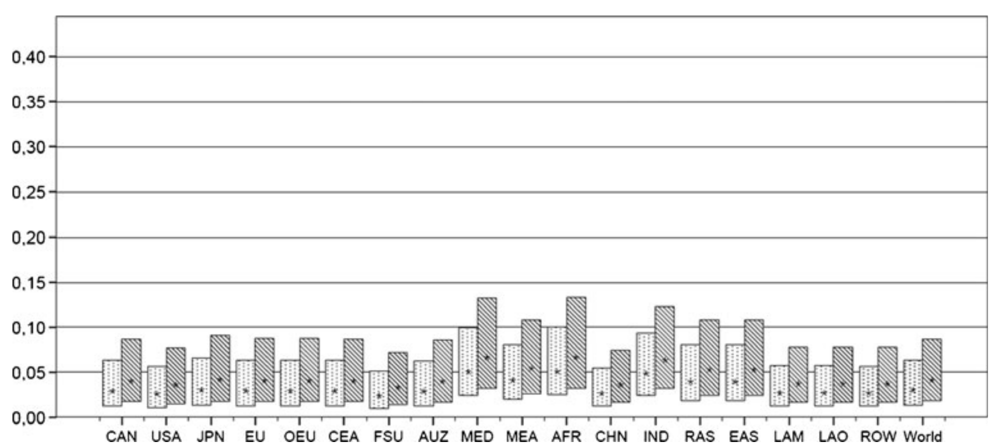

Figure 5 shows how damages at a probability range of 90% change between NASH and COOP at the country level, as expressed in percent of GDP, for $\theta_3 = 2.7$. At the world level (see the last bar on the right side of the figure), the expected damage goes from 10% under NASH to less than 3% of GDP under COOP. But the impact of cooperation on the probability range of damages is also striking. Under NASH, there is 90% chance for damages to lie between 3% and 28% of GDP in 2200. Under COOP, this 90% interval ranges between 1% and 9% of GDP. Put differently (and using the cumulative distribution), the probability to have damages larger than 8% of GDP is 62.6% under NASH, while this probability becomes 8.5% under COOP. So the sharp reduction in climate risks is a key benefit of cooperation.

When looking at the country level, the most vulnerable countries are Africa, Mediterranean countries, India, Asia (EAS and RAS) and Middle East countries, with expected damages in 2200 lying between 10% and 15% of GDP in NASH and between 4% and 5% of GDP in COOP. For these countries, the benefits associated to a reduction in the range of uncertainty are also strong. The reduction of the range is about 25 percentage points for Africa, while it is about 13 percentage points for the whole world.

A similar analysis can be made for the case $\theta_3 = 2$, illustrated in Fig. 6. We know that in that case, the impact of risk aversion is weaker, so that the results are less striking. We still find that cooperation reduces both expected damage and the size of uncertainty. At the world level, expected damages represent 4% of GDP under NASH but only 3% of GDP under COOP. There is a 95% chance that damages are less than 9% of GDP under NASH. There is a 95% chance that they are less than 6.5% of GDP under COOP.

The list of the most vulnerable countries is the same in the case $\theta_3 = 2$ as in the case $\theta_3 = 2.7$. But we can see that the benefits of cooperation in terms of risk reduction are less pronounced for these countries when $\theta_3 = 2$.

Finally, the estimates of the probability ranges for damages can be compared with those provided in the Stern review [33]. The Stern review indeed obtained a 90% confidence interval range for damages in 2200 as % of GDP of 0.5–12 in its baseline. In the case $\theta_3 = 2$, we obtain a much smaller range: 1–6.5 in the COOP scenario. The reason for the discrepancy is that the Stern review considers a business as usual (no policy) scenario, while we assume that policy makers play a Nash policy and try to reduce the range of uncertainty. In the case $\theta_3 = 2.7$, our range is similar to the one obtained by Stern because damages are more severe and much more difficult to avoid.

5 Conclusion

In this paper, we have analysed how taking uncertainty into account in an IAM impacts policy design and incentives to cooperate for different countries. It appears that uncertainty does not change very much policy recommendations. But this finding crucially depends on the shape of the damage function.

Our analysis indeed reveals that the impact of risk aversion can be very different depending on the shape of the damage function. The degree of convexity of this function plays a crucial role in determining in which direction uncertainty changes policy recommendations. Sensitivity analysis is usually not performed on the exponent of the damage function, nor is it performed on the exact form of this function. Our results suggest that much more must be learnt on the damage functions if we want to be more confident in the model's results. This concurs with Weitzman's contention that more research is needed on the shape of the damage function [40]. Besides, in a multi-country setting, it may well be that the shape of the damage function differs from one country to another. This may yield to very different policies being followed. Taking this into account may enrich the strategic analysis of the climate change problem. This presupposes that more empirical research be carried out to calibrate damage functions at the regional level.

We have highlighted that all countries have two incentives to join a grand coalition (COOP scenario), namely the reduction in expected damages and the reduction in risk. When uncertainty is taken into account, new reasons to cooperate thus appear. A natural avenue for further research would be to study at the country level the incentives to join an agreement (whether a grand coalition or a partial agreement between some countries). From the modelling point of view, we have highlighted that it is possible to handle a stochastic IAM with a finer discretization. This development may open new horizons such as multistage stochastic problem with recourse modelling. Exploring those two issues will be the matter of another paper.

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Appendix

Regions of the CWS Model

Table 4 Regions of the CWS model

Label	Name of the region	Composition
CAN	Canada	
USA	USA	
JPN	Japan	Japan, South Korea
EU	European Union	EU15
OEU	Other Europe	Iceland, Norway, Switzerland
CEA	Central Eastern Associates	Bulgaria, Cyprus, Czech Republic, Estonia Hungary, Latvia, Lithuania, Malta, Poland, Romania, Slovakia, Slovenia
FSU	Former Soviet Union	Armenia, Azerbaijan, Belarus, Georgia, Kazakhstan, Kyrgyzstan, Moldova, Russian Federation, Tajikistan, Turkmenistan, Ukraine, Uzbekistan
AUZ	Australasia	Australia, New Zealand
MED	Mediterranean	Algeria, Egypt, Israel, Lebanon, Morocco, Syria, Tunisia, Turkey
MEA	Middle East	Bahrain, Iran, Jordan, Kuwait, Oman, Saudi Arabia, United Arab Emirates, Yemen
AFR	Africa	Angola, Benin, Botswana, Burkina Faso, Burundi, Cameroon, Cape Verde, Central African Republic, Chad, Comoros, Congo, Democratic Republic of Congo, Djibouti, Equatorial Guinea, Eritrea, Ethiopia, Gabon, Gambia, Ghana, Guinea, Guinea Bissau, Ivory Coast, Kenya, Lesotho, Madagascar, Malawi, Mali, Mauritania, Mauritius, Mozambique, Namibia, Niger, Nigeria, Reunion, Rwanda, Senegal, Sierra Leone, South Africa, Sudan, Swaziland, Tanzania, Togo, Uganda, Zambia, Zimbabwe
CHN	China	
IND	India	
RAS	Rest of Asia	Bangladesh, Cambodia, Laos, Mongolia, Nepal, Pakistan, Papua New Guinea, Sri Lanka
EAS	Eastern Asia	Indonesia, Malaysia, Philippines, Singapore, Thailand, Vietnam
LAM	Latin America	Mexico, Brazil, Venezuela, Peru, Argentina, Chile, Uruguay, Paraguay
LAO	Latin America Other	Bolivia, Colombia, Costa Rica, Dominican Republic, Ecuador, El Salvador, Guatemala, Haiti, Honduras, Jamaica, Nicaragua, Panama, Trinidad and Tobago, Bahamas, Belize, Guyana, Suriname
ROW	Rest of the World	Afghanistan, Cuba, Libya, Iraq, … ~25 countries (not assignable or with incomplete data)

Statement of the CWS Model

Variables and Parameters Definitions

Table 5 Names and units of variables

E	Carbon emissions (billion tons of carbon per year)
μ	Carbon emission abatement (%)
M^{AT}	Atmospheric carbon concentration (billion tons of carbon)
M^{UO}	Upper ocean and vegetation carbon concentration (billion tons of carbon)
M^{LO}	Lower ocean carbon concentration (billion tons of carbon)
F	Radiative forcing (Watt per m^2)
T^{L}	Temperature change lower ocean (°C compared to 1,800)
T^{E}	Temperature change atmosphere (°C compared to 1,800)
Y	Production (billion US $2,000)
Z	Consumption (billion US $2,000)
I	Investment (billion US $2,000)
C	Abatement costs (billion US $2,000)
D	Damage costs (billion US $2,000)
K	Capital stock (billion US $2,000)

Table 6 Names and units of parameters

$Y_{2,000}$	GDP values 2,000 in million US $2,000 in market exchange value
$L_{2,000}$	Population 2,000 (million people)
$E_{2,000}$	CO2 emissions 2,000 from fossil fuel in gtC
A_0	Initial productivity 2,000
A_{T}	Regional asymptotic productivity
A_0^{G}	Initial regional productivity growth rate per decade (2000–2010)
σ_0	Initial emission–GDP ratio 2,000 (kg of carbon per US$)
σ_{T}	Regional asymptotic emission–GDP ratio (kg of carbon per US$)
σ_0^{G}	Initial regional emission–GDP growth rate per decade (2000–2010)
L_0^{G}	Initial regional population growth rate per decade (2000–2010)

Constraints

The index $i = 1, \ldots N$ stands for region/country.

$$Y_{i,t} = A_{i,t} K_{i,t}^{\alpha} L_{i,t}^{1-\alpha} \quad (5)$$

$$Y_{i,t} = Z_{i,t,s} + I_{i,t} + C_i(\mu_{i,t}) + D_i\left(T_{t,s}^{E}\right) \quad (6)$$

$$K_{i,t+1} = (1-\delta_K)^{10} K_{i,t} + 10 I_{i,t}\text{ , with } K_{i,0} \text{ given} \quad (7)$$

$$E_{i,t} = \sigma_{i,t}(1-\mu_{i,t})Y_{i,t}, \text{with } \mu \in (0,1) \quad (8)$$

$$C_i(\mu_{i,t}) = -Y_{i,t}\, c_i \left[(1-\mu_{i,t}) log(1-\mu_{i,t}) + \mu_{i,t}\right] \quad (9)$$

$$M_{t+1}^{\mathrm{AT}} = M_t^{\mathrm{AT}} + 10\left(M_t^{\mathrm{AT}} b_{11} + \sum_{j=1}^{n} E_{j,t} + M_t^{\mathrm{UO}} b_{21}\right) \quad (10)$$

$$M_{t+1}^{\mathrm{UO}} = M_t^{\mathrm{UO}} + 10\left(M_t^{\mathrm{AT}} b_{12} + M_t^{\mathrm{UO}} b_{22} + M_t^{\mathrm{LO}} b_{32}\right) \quad (11)$$

$$M_{t+1}^{\mathrm{LO}} = M_t^{\mathrm{LO}} + 10\left(M_t^{\mathrm{LO}} b_{33} + M_t^{\mathrm{UO}} b_{23}\right) \quad (12)$$

$$F_t = F2X\left(\frac{\log\left(M_t/M_0\right)}{\log(2)}\right) + RFOgas_t \quad (13)$$

$$T_{t+1,s}^{E} = \frac{T_{t,s}^{E}}{1 + c_1(F2X/T2X_s) + c_1 c_3} + c_1\left(F_{t+1} + c_3 T_{t,s}^{\mathrm{L}}\right), \text{with } T_0^E \text{ given} \quad (14)$$

$$T_{t+1,s}^{\mathrm{L}} = T_{t,s}^{\mathrm{L}} + c_4\left(T_{t,s}^{E} - T_{t,s}^{\mathrm{L}}\right), \text{with } T_0^{\mathrm{L}} \text{ given} \quad (15)$$

$$D_i\left(T_{t,s}^{E}\right) = Y_{i,t}\left[\theta_{i,1} T_{t,s}^{E} + \theta_{i,2}\left(T_{t,s}^{E}\right)^{\theta_3}\right] \quad (16)$$

Parameters Values

Table 7 General scalars

δ_K	Depreciation rate of capital per year	0.10
η	Risk aversion	2.0
α	Capital elasticity in output	0.25
ρ	Discount rate	0.02

Table 8 Asymptotic values

A_{T}	Regional asymptotic productivity	20
σ_{T}	Regional asymptotic emission–GDP ratio (kg of carbon per US$)	0.020

Table 9 Parameters for abatement cost and damage functions

	$Y_{2,000}$	$L_{2,000}$	$E_{2,000}$	A_0	A_0^{G}	σ_0	σ_0^{G}	L_0^{G}
CAN	714.458	30,769	0.142	1,445	1.045	0.199	0.029	1.484
USA	9,764.800	285,003	1.581	1,934	1.309	0.162	0.031	1.328
JPN	4,649.615	150,035	0.345	1,794	1.132	0.074	0.020	0.480
EU	8,027.668	377,335	0.891	1,353	1.397	0.111	0.023	0.425
OEU	421.584	11,928	0.023	1,980	1.523	0.055	0.023	0.259
CEA	402.052	68,676	0.195	514	5.333	0.485	0.070	−0.161
FSU	352.493	282,353	0.617	161	4.265	1.750	0.057	−0.025
AUZ	452.338	22,937	0.100	1,278	1.034	0.221	0.028	1.400
MED	557.409	231,016	0.167	264	2.401	0.300	0.053	2.021
MEA	443.778	119,994	0.227	364	2.449	0.512	0.053	1.957
AFR	338.556	640,874	0.151	85	2.704	0.446	0.052	1.523
CHN	1,198.480	1,282,022	0.928	130	6.084	0.774	0.092	0.931
IND	460.189	1,016,938	0.282	75	4.256	0.613	0.074	1.594
RAS	152.075	348,978	0.044	73	2.370	0.289	0.054	2.159
EAS	1,089.013	477,183	0.336	254	2.542	0.309	0.050	1.544
LAM	1,740.755	382,068	0.299	426	1.614	0.172	0.038	1.607
LAO	225.167	120,851	0.054	218	2.855	0.240	0.056	1.710
ROW	43.765	131,688	0.046	60	4.159	1.051	0.064	0.773

Table 10 Parameters for abatement cost and damage functions

	θ_1	θ_2	c
CAN	0.0000616	0.0015639	−0.054
USA	0.0000000	0.0014167	−0.033
JPN	0.0000053	0.0016178	−0.019
EU	0.0000452	0.0015911	−0.032
OEU	0.0000452	0.0015911	−0.028
CEA	0.0000452	0.0015911	−0.079
FSU	0.0000011	0.0013047	−0.12
AUZ	0.0000616	0.0015639	−0.045
MED	0.0034448	0.0019832	−0.188
MEA	0.0027937	0.0015860	−0.104
AFR	0.0034448	0.0019832	−0.13
CHN	0.0009018	0.0012589	−0.162
IND	0.0043852	0.0016913	−0.096
RAS	0.0017551	0.0017468	−0.112
EAS	0.0017551	0.0017468	−0.089
LAM	0.0006090	0.0013461	−0.069
LAO	0.0006090	0.0013461	−0.107
ROW	0.0006090	0.0013461	−0.063

Table 11 Parameter values carbon cycle

M_0	Initial atmospheric CO_2 concentration in 1,800 (btC)	590
M_0^{AT}	Initial atmospheric CO_2 concentration in 2,000 (btC)	783
M_0^{UO}	Initial upper ocean and vegetation CO_2 concentration in 2,000 (btC)	807
M_0^{LO}	Initial lower ocean CO_2 concentration in 2,000 (btC)	19,238
b_{11}	Carbon cycle transition matrix coefficient	−0.033384
b_{22}	Carbon cycle transition matrix coefficient	−0.039103
b_{33}	Carbon cycle transition matrix coefficient	−0.000422
b_{12}	Carbon cycle transition matrix coefficient	0.033384
b_{21}	Carbon cycle transition matrix coefficient	0.027607
b_{23}	Carbon cycle transition matrix coefficient	0.011496
b_{32}	Carbon cycle transition matrix coefficient	0.000422

Table 12 Parameter values temperature cycle

T_0^L	Initial temperature change lower ocean (°C compared to 1,800)	0.108
T_0^E	Temperature change atmosphere (°C compared to 1,800)	0.622
$F2X$	Forcing with a carbon concentration doubling	3.800
c_1	Coefficient for upper level	1.7
c_3	Transfer coefficient upper to lower level	0.794
c_4	Transfer coefficient for lower level	0.03609

Probability Distribution

Table 13 Validation set: probability of elements

s	$T2X_s$	π_s	s	$T2X_s$	π_s	s	$T2X_s$	π_s
1	1.0	0.00001	26	3.5	0.02599	51	6.0	0.01044
2	1.1	0.00001	27	3.6	0.02528	52	6.1	0.01007
3	1.2	0.00007	28	3.7	0.02454	53	6.2	0.00972
4	1.3	0.00027	29	3.8	0.02377	54	6.3	0.00938
5	1.4	0.00079	30	3.9	0.02299	55	6.4	0.00906
6	1.5	0.00179	31	4.0	0.02220	56	6.5	0.00875
7	1.6	0.00338	32	4.1	0.02142	57	6.6	0.00846
8	1.7	0.00553	33	4.2	0.02065	58	6.7	0.00817
9	1.8	0.00814	34	4.3	0.01989	59	6.8	0.00790
10	1.9	0.01103	35	4.4	0.01916	60	6.9	0.00765
11	2.0	0.01400	36	4.5	0.01844	61	7.0	0.00740
12	2.1	0.01687	37	4.6	0.01774	62	7.1	0.00716
13	2.2	0.01952	38	4.7	0.01707	63	7.2	0.00694
14	2.3	0.02185	39	4.8	0.01642	64	7.3	0.00672
15	2.4	0.02381	40	4.9	0.01580	65	7.4	0.00651
16	2.5	0.02538	41	5.0	0.01520	66	7.5	0.00632
17	2.6	0.02658	42	5.1	0.01462	67	7.6	0.00612
18	2.7	0.02742	43	5.2	0.01407	68	7.7	0.00594
19	2.8	0.02794	44	5.3	0.01354	69	7.8	0.00577
20	2.9	0.02819	45	5.4	0.01304	70	7.9	0.00560
21	3.0	0.02821	46	5.5	0.01255	71	8.0	0.00541
22	3.1	0.02803	47	5.6	0.01209			
23	3.2	0.02768	48	5.7	0.01165			
24	3.3	0.02721	49	5.8	0.01123			
25	3.4	0.02664	50	5.9	0.01082			

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