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Does Technological Progress Affect the Location of  
Economic Activity?

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**DISCUSSION PAPER**

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**Does Technological Progress Affect the Location of  
Economic Activity?**

Takatoshi Tabuchi<sup>1</sup>, Jacque-François Thisse<sup>2</sup>  
and Xiwei Zhu<sup>3</sup>

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**Abstract**

We show that how technological innovations and migration costs interact to shape the space-economy. Regardless of the level of transport costs, rising labor productivity fosters the agglomeration of activities, whereas falling transport costs do not affect the location of activities. When labor is heterogeneous, the number of workers residing in the more productive region increases by decreasing order of productive efficiency when labor productivity rises. This process affects in opposite directions the welfare of those who have a lower productivity.

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# 1 Introduction

In developed and emerging economies alike, a few large cities attract firms paying high wages to their employees. At the same time, these large metropolitan areas, where housing costs are high, accommodate a growing number of high-skilled workers. An illustration of this is provided by the jacket of Moretti’s recent book *The New Geography of Jobs*, which shows a map of the United States suggesting that less than 20 American cities account for the bulk of the most innovative and productive activities. To be precise, Moretti (2012) observed that “a handful of cities with the “right” industries and a solid base of human capital keep attracting good employers and offering high wages, while those at the other extreme, cities with the “wrong” industries and a limited human capital base, are stuck with dead-end jobs and low average wages.” Similar maps and conclusions could be drawn for many other countries (see, e.g. Combes et al., 2008, for the case of France).<sup>1</sup>

In a path-breaking paper, Krugman (1991) proposed to explain the emergence of sizeable and lasting regional difference by the integration of markets brought about by the dramatic fall in transport costs that started with the Industrial Revolution.<sup>2</sup> Specifically, Krugman argued that manufacturing activities are dispersed across regions and countries when transport costs are high because local producers are protected against imported goods. As transport costs steadily decline, firms and consumers agglomerate in a handful of places where firms are able to better exploit increasing returns by supplying larger markets and exporting their output at low cost. In the benchmark case of two symmetric regions, the symmetric distribution of manufacturing firms breaks down when transport costs decrease sufficiently to reach a certain threshold. Once transport costs fall below this threshold, the manufacturing sector gets agglomerated in what becomes the core region, while the now-peripheral region is specialized in farming. This explanation has been embraced by a great number of authors under the heading of “new economic geography” (NEG).

We find it hard to believe that the sole collapse in transport costs was the reason for the uneven geographical distribution of activities which emerges in the aftermath of the Industrial Revolution, as well as for the spatial concentration of human capital. Given the massive role played by productivity gains in the process of economic development, we argue in this paper that, regardless of the level of transport costs, *a rising labor productivity explains why some places fare better than others, while the most able workers tend to cluster in a few places.*

To achieve our goal, we develop a parsimonious model with one sector—manufacturing—featuring increasing returns and monopolistic competition. Allen (2009) has convincingly argued that the relative scarcity of labor in Britain, where wages were remarkably high, had fostered the development of labor-saving technologies that permit the substitution of capital and energy for labor. For this reason, we find it reasonable as a first-order approximation to focus on labor as the main production factor. Since firms operate under

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<sup>1</sup>The geographical concentration of activities and the clustering of talents are not really new phenomena. The Industrial Revolution exacerbated regional disparities by an order of magnitude that was unknown before (Pollard, 1981). Less known, perhaps, is that cities grew predominantly through the concentration of human capital (Hohenberg and Lees, 1985).

<sup>2</sup>Bairoch (1997) observed that “between 1800 and 1910, it can be estimated that the lowering of the real average prices of transportation was of the order of 10 to 1.” Transport costs continued to fall after World War I. In the United States, Glaeser and Kohlhase (2004) noted that over the twentieth century, the costs of moving manufactured goods have declined by over 90 per cent in real terms.

increasing returns in our model, labor productivity is expressed through the marginal and fixed labor requirements needed by a firm to produce a variety of the manufactured good. In this context, *technological progress, or rising labor productivity, takes the form of steadily decreasing marginal or fixed requirements of labor*. Like Krugman who does not explain why transport costs fall, we will consider an exogenous technological progress that permits an increase in the output per worker. In other words, we are agnostic about the concrete form taken by the various innovations developed before, during and after the Industrial Revolution. Our model is thus consistent with different narrative approaches to technological progress.

Although we recognize that consumers are mobile, it is unquestionable that they bear positive costs when they change location. These costs are often considered a one-time expenditure but this view strikes us as being too extreme. Indeed, migration generates substantial non-pecuniary costs created by differences in languages/dialects, cultures and religions within and between nations, which have a lasting influence on individual well-being (Belot and Ederveen, 2012; Falck et al., 2012). Furthermore, temporary and return migration is evidence that migrants bear permanent social dislocation costs when they live away from their country or region of origin (Dustmann and Mestres, 2010). Last, migrants typically get a lower pay than local consumers who have a better tacit knowledge of social rules that make them more productive. Summarizing the state of the art, Collier (2013) asserts that “migrants tend to be less happy than the indigenous host population.”

The paradigmatic model of NEG focuses on a two-sector economy (manufacturing and agriculture) with two types of sector-specific labor (workers and farmers). What matters in Krugman (1991) is that the immobility of farmers plays the role of a dispersion force. Admittedly, before the Industrial Revolution the agricultural sector proper was large, but this is no longer in modern economies where the primary sector accounts for a very small share of the gross domestic product.<sup>3</sup> In this paper, we consider a new setting that fits better developed and emerging economies alike: the economy involves only the manufacturing sector, but workers are imperfectly mobile. By incentivizing consumers to stay put even when they may be guaranteed a higher living standard in other places, *migration costs* may be viewed as a force that fosters the stickiness of activities. In addition, labor productivity rises when the fixed labor requirement, the marginal labor requirement, or both, fall. The distribution of firms and consumers is thus determined by the interplay between labor productivity and migration costs.

Our main two findings may be summarized as follows. First, when labor productivity starts rising, the set of stable equilibria shrinks. In the limit, when one region is initially bigger than another—even by a trifle—all firms and consumers get agglomerated in the larger region. To put it differently, even in the absence of falling transport costs, *rising labor productivity is sufficient to explain why the manufacturing sector is agglomerated*. We thus provide a new and historically relevant explanation for the geographical concentration of economic activities that started with the Industrial Revolution. Observe that technological progress magnifies any interregional difference, whereas market integration reduces wage and price differentials between regions in NEG. By implication, a hike in the productivity of labor that leads to falling production costs is not equivalent to falling

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<sup>3</sup>The term “agriculture” should not be taken literally. This sector includes all production activities bound to the land, such as mining and farming, as well as activities using immobile inputs, such as tourism. In France, which is a major provider of touristic services, agriculture and tourism account together for less than 10 per cent of the French GDP in 2012.

transport costs. Note also that a lower marginal labor requirement leads to an increase in firms' output, but does not affect the number of firms. In contrast, a decreasing fixed labor requirement triggers the entry of new firms, which all firms keep producing the same output.

Second, *in the absence of technological progress in the manufacturing sector, falling transport costs do not trigger the agglomeration of firms.* The intuition is easy to grasp. Since interregional price and wage differences narrow down when transport costs fall, consumers have lower incentives to move. As a consequence, if the utility differential is not sufficiently large to spark consumers' migration when transport costs are high, this is even more so when transport costs are low.

How does this compare to NEG? When labor productivity is low, a wide range of distributions may be sustained as stable spatial equilibria, including the dispersed and agglomerated distributions. By contrast, when labor productivity is high, the set of equilibrium distributions that are (partially) dispersed is narrow. In addition, various configurations remain stable even when transport costs take on low values. For example, when firms and consumers are a priori dispersed, they will remain so even when markets are very integrated. These two results clash with what NEG tells us. The reason for such a major difference in results is to be found in the presence of migration costs. Regardless of the level of transport costs, positive migration costs always prevent a marginal change in locations from destabilizing an equilibrium distribution, the reason being that small welfare gains are not sufficient to compensate workers for the cost they bear when they move to the more prosperous region. Does this mean that migration costs must be absent when explaining the agglomeration of economic activities in a few regions? Happily enough, we will show that the answer is no.

Finally, recall that Pollard (1981) argued that, during the Industrial Revolution, the core regions attracted from the peripheral regions “some of their most active and adaptable labour.” Focussing on the contemporary period, Moretti (2012) asserts that “geographically, American workers are increasingly sorting along educational lines.” In an attempt to account for this fact, we assume that workers are heterogeneous in that they are endowed with different amounts of efficiency units of labor. Under such circumstances, we show that *the more efficient workers living in the less productive region move toward the more productive region by decreasing order of efficiency.* Thus, migration goes hand in hand with workers' productivity, an empirically well-documented fact (Docquier and Rapoport, 2012). As a consequence, interregional income and welfare differences reflect differences in the geographical distribution of skills and human capital (Glaeser and Maré, 2001; Combes et al., 2008; Moretti, 2011). Note that this process also affects the unskilled: the concentration of skilled brings about a welfare hike for the unskilled living in the same region and a welfare drop for those who stay in the periphery.

The paper is organized as follows. In the next section, we present the model and derive some preliminary results. In Section 3, we characterize the spatial equilibria and study their stability. Section 4 shows how technological progress leads to the emergence of a core-periphery structure, while Section 5 studies the concentration of human capital. Section 6 concludes.

## 2 The model and preliminary results

The economy is endowed with two regions, denoted  $r, s = 1, 2$ , a manufacturing (or tradable service) sector producing a horizontally differentiated good, one production factor (labor), and a population of consumers of mass  $L$ . Workers are imperfectly mobile because they bear a positive cost when they move from one region to the other.

The differentiated good is made available under the form of a continuum  $n$  of varieties. Consumers are endowed with one efficiency unit of labor and share the same preferences. The preferences of a consumer located in region  $r = 1, 2$  are given by the CES utility:

$$U_r = \left( \sum_s \int_0^{n_s} q_{sr}(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \quad (1)$$

where  $n_s$  is the number of varieties produced in region  $s = 1, 2$ ,  $q_{sr}(i)$  the consumption of variety  $i$  produced in region  $s$  and consumed in region  $r$ , and  $\sigma > 1$  the elasticity of substitution between any two varieties.

The budget constraint of a consumer located in region  $r$  is given by

$$\sum_s \int_0^{n_s} p_{sr}(i) q_{sr}(i) di = w_r,$$

where  $p_{sr}(i)$  is the price of variety  $i$  produced in region  $s$  and consumed in  $r$ , while  $w_r$  is the wage rate in region  $r$ .

Labor markets are competitive and local, thus implying that wages need not be equal between the two regions. The equilibrium wage in region  $r$  is determined by a bidding process in which the region  $r$ -firms compete for workers by offering them higher wages until no firm earns strictly positive profits. Thus, a firm's operating profits are equal to its wage bill.

The individual demand in region  $r$  for variety  $i$  produced in region  $s$  is then as follows:

$$q_{sr}(i) = \frac{p_{sr}(i)^{-\sigma}}{P_r^{1-\sigma}} w_r, \quad (2)$$

where the price index  $P_r$  that prevails in region  $r$  is given by

$$P_r \equiv \left( \sum_s \int_0^{n_s} p_{sr}^{1-\sigma}(i) di \right)^{\frac{1}{1-\sigma}}. \quad (3)$$

Firms operate under increasing returns and no scope economies. Thus, each firm produces a single variety and each variety is produced by a single firm, so that  $n_s$  is also the number of firms set up in region  $s$ . The production of a variety needs a fixed requirement of  $f > 0$  units of labor and a marginal requirement of  $c > 0$  units of labor. In this paper, technological progress means that  $f$ ,  $c$ , or both fall. The technology is identical in all locations - regions have no specific comparative advantage - and for all the varieties - firms are symmetric. Hence, we may drop the variety-index  $i$ .

Following the new trade literature, we assume iceberg transport costs:  $\tau_{rs} = \tau > 1$  units of a variety have to be shipped from region  $r$  for one unit of that variety to be available in region  $s \neq r$ , while transport costs are zero when a variety is sold in the

region where it is produced ( $\tau_{rr} = \tau_{ss} = 1$ ). Therefore, we have  $p_{rr} = p_r$  and  $p_{sr} = \tau p_s$ . If  $\lambda_s$  denotes the share of consumers living in region  $s$  (with  $\lambda_1 + \lambda_2 = 1$ ), for the demand  $\lambda_s L q_{rs}$  in region  $s$  to be satisfied, each firm in region  $r$  must produce  $\tau \lambda_s L q_{rs}$  units. The profits earned by a firm located in region  $r$  are thus given by

$$\pi_r = p_r L \left( \sum_s \lambda_s \tau_{rs} q_{rs} \right) - w_r \left( f + cL \sum_s \lambda_s \tau_{rs} q_{rs} \right). \quad (4)$$

Factorizing  $L$  in this expression shows that  $L$  plays the role of a scaling factor of  $f$ . Therefore, without loss of generality, we may assume that  $L = 1$ . In this case, a lower value of  $f$  is equivalent to a larger population size.

Given the individual demand (2), the profit-maximizing price is

$$p_r = \frac{\sigma c}{\sigma - 1} w_r. \quad (5)$$

Assuming free entry and exit in the manufacturing sector, profits (4) are zero in equilibrium:

$$(p_r - c w_r) \sum_s \lambda_s \tau_{rs} q_{rs} = w_r f. \quad (6)$$

Plugging (5) into (6) and solving for the total output  $q_r = \lambda_r q_{rr} + \tau \lambda_s q_{rs}$  yields

$$q_r^* = \frac{(\sigma - 1)f}{c}. \quad (7)$$

Last, labor market balance in region  $r$  implies

$$n_r \left( f + c \sum_s \lambda_s \tau_{rs} q_{rs} \right) = \lambda_r. \quad (8)$$

Using (7) and (8), we obtain:

$$n_r = \frac{\lambda_r}{\sigma f}. \quad (9)$$

The balance condition of the product market yields the wage equation in region  $r$ :

$$\sum_s \frac{\phi_{rs} \lambda_s w_s}{\sum_t \phi_{ts} \lambda_t w_t^{1-\sigma}} = w_r^\sigma, \quad (10)$$

where  $\phi_{rs} \equiv \tau_{rs}^{1-\sigma} \in [0, 1)$ . Choosing labor in region 2 as the numéraire, we have  $w_1 = w$  and  $w_2 = 1$ . Setting  $\lambda_1 \equiv \lambda \geq 1/2$  and  $\lambda_2 \equiv 1 - \lambda$ , for any given  $\lambda$  the wage equation (10) in the larger region may be rewritten as follows:

$$\lambda = \frac{w^\sigma - \phi}{w^\sigma - (w + 1)\phi + w^{1-\sigma}}, \quad (11)$$

where  $\phi \equiv \tau^{1-\sigma} \in [0, 1)$ .<sup>4</sup> The Walras law implies that trade between the two regions is balanced.

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<sup>4</sup>We show in the Appendix 1 that the denominator of (11) is positive.

Observe, first, that (11) implies that there is a one-to-one correspondence between  $\lambda$  and  $w$ . Therefore,  $w = 1$  when  $\lambda = 1/2$  while  $w = \phi^{-1/\sigma} > 1$  when  $\lambda = 1$ . Note the following difference with Krugman: nominal wages are equal at the dispersed and agglomerated configurations in the CP model; in our setting, the nominal wage is higher in the larger region. Furthermore, as shown by Krugman (1980), the right hand side of (11) increases in  $w$ , so that there exists a unique equilibrium wage  $w^*(\lambda) \geq 1$  for any given  $\lambda \geq 1/2$ . Even though the labor and product markets are more competitive in region 1 than in region 2, the nominal wage is higher in the former. In addition, the nominal wage prevailing in region 1 rises with the relative size  $\lambda$  of this region. Consequently, since profits are zero, the gross domestic product of the global economy rises monotonically together with the degree of agglomeration.

What is more, the interregional wage gap widens when the two regions become more asymmetric. Note, however, that the wage gap shrinks when  $\phi$  rises, that is, when the two regions get more integrated. This is because the interregional difference in prices get smaller when  $\phi$  increases, which fosters the interregional convergence of wages. In the limit, when the two markets are fully integrated ( $\phi = 1$ ), the size difference becomes immaterial and there is wage equalization ( $w^* = 1$ ).

Furthermore, using (3), (5) and (11) as well as the inequality  $w^* > 1$ , we get

$$P_1^{1-\sigma} - P_2^{1-\sigma} = K \frac{(w^\sigma - 1) w^{1-\sigma}}{w^\sigma - (w + 1) \phi + w^{1-\sigma}} > 0,$$

where  $K$  is a positive constant. It then ensues from this expression that  $P_1(\lambda) < P_2(\lambda)$ . Thus, even though wages are higher in region 1 than in region 2, the price index in the larger region is lower than that in the smaller one. Hence, consumers residing in the larger region enjoy both higher wages and lower prices than those located in the smaller region.<sup>5</sup> Moreover, although the natives and the migrants earn the same real wage in the larger region, the former are better-off than the latter who bear continued migration costs, as mentioned by Collier (2013).

Since the indirect utility of an individual living in region  $r$ , which is equal to her real wage, is given by

$$V_r(\lambda) = \frac{w_r(\lambda)}{P_r(\lambda)}, \quad (12)$$

$V_1(\lambda)$  exceeds  $V_2(\lambda)$  if and only if  $\lambda > 1/2$ . Let  $\Delta V(\lambda) \equiv V_1(\lambda) - V_2(\lambda)$  be the interregional utility differential. Since  $d\lambda/dw > 0$ , we obtain

$$\frac{d\Delta V(\lambda)}{d\lambda} = \frac{\partial \Delta V(\lambda)}{\partial \lambda} + \frac{\partial \Delta V(\lambda)}{\partial w} \frac{dw^*}{d\lambda} > 0, \quad (13)$$

which means that the utility differential increases with the size of the larger region. In other words, *the incentive to move from region 2 to region 1 gets stronger as the larger region grows in size*. It is worth stressing, however, that this incentive weakens as the

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<sup>5</sup>This result might come as a surprise to the reader because larger cities are often places where the cost of living is higher. To a large extent, this is because housing and some nontradables are much more expensive in large cities than in small ones (Suedekum, 2006). Housing and nontradables are absent in our model because the focus is on regions, not on cities. However, introducing commuting and land consumption put a break on the agglomeration process without affecting the nature of our main results.

two regional markets get more integrated, the reason being that the economic differences between regions fade away.

Thus, we have the following proposition.

**Proposition 1** *Assume any given distribution of firms and consumers such that  $\lambda > 1/2$ . Then, the real wage in the larger region exceeds that in the smaller region. Furthermore, the interregional gap widens when regions become more asymmetric.*

Even though the productivity of labor is the same in the two regions, the difference in market size is sufficient to explain why firms located in the larger region can pay a higher wage to their workers, a result supported by robust empirical evidence (Head and Mayer, 2011; Redding, 2011). Furthermore, since more varieties are produced in the larger region, the corresponding price index is lower, which also agrees with the empirical evidence provided by Handbury and Weinstein (2013) who observe that price level for food products falls with city size. Therefore, migration flows (if any) are unidirectional: *consumers move from the smaller to the larger region* but never from the larger to the smaller region.

Note that the above proposition implies that, in the absence of migration costs, all consumers migrate to the larger region. In other words, there is always agglomeration in a single region. This highlights the role of migration costs in the process shaping the space-economy.

### 3 Spatial equilibrium with homogeneous labor

Because the equilibrium wage  $w$  is uniquely determined by the wage equation (11), the interregional utility differential can be expressed as a function of  $\lambda$ :

$$\Delta V(\lambda) = \frac{\sigma - 1}{c f^{\frac{1}{\sigma-1}} \sigma^{\frac{\sigma}{\sigma-1}}} \left[ w (\phi - \lambda\phi + \lambda w^{1-\sigma})^{\frac{1}{\sigma-1}} - (1 - \lambda + \lambda\phi w^{1-\sigma})^{\frac{1}{\sigma-1}} \right]. \quad (14)$$

The decision made by a consumer to migrate relies on the utility differential  $\Delta V(\lambda)$  and the migration cost she bears when moving. As argued in the introduction, moving from one region to the other involves various psychological adjustments that adversely affect a migrant.

Migration cost is not a one-time expense: if consumers move to the other region, they incur a permanent cost to adjust to their new place. As a result, some people will stay put even when they may guarantee to themselves higher living standards in other places. Migration costs thus have the nature of a *dislocation cost*, which vanishes if consumers return to their place of origin or is equal to zero if they stay put. Such a cost is usually consumer-specific, which implies that different consumers bear different migration costs  $m(\theta) > 0$ , where  $m$  increases with  $\theta$ . Therefore, measuring  $m(\theta)$  in utility terms, the  $\theta$ -consumers choose to stay put if

$$|\Delta V(\lambda)| \leq m(\theta), \quad (15)$$

where the indirect utility differential is the same across all consumers whereas migration costs vary with consumers' type. In Section 5 we assume that workers are heterogeneous in their individual productivity. The analysis developed therein can be repeated mutatis

mutandis to show that consumers living in the smaller region move toward the larger region by increasing order of migration costs. Therefore, in region 2, as  $\theta$  rises migrants enjoy a decreasing utility level; only the marginal migrant has the same net utility level in both regions. For simplicity, we assume that migrants bear the same migration cost:  $m(\theta) = m > 0$ . Note also that a growing number of migrants often lower migration costs through network effects that reduce information asymmetries (see, e.g. Munshi, 2003). In this case,  $m$  is a decreasing function of  $\lambda$ . Accounting for such an effect weakens the dispersion force and makes our point stronger.

The expression (14) reveals the striking difference between a fall in  $c$  and a fall in  $\tau$ : for a given value of  $\lambda$ , the utility differential  $\Delta V(\lambda)$  rises when  $c$  decreases whereas  $\Delta V(\lambda)$  falls when  $\tau$  decreases, thus changing the nature of migration incentives. This is because market integration makes the two regions more similar in terms of prices and wages, whereas a rising labor productivity that equally affects both regions exacerbates existing regional disparities.

A consumer distribution  $\lambda^* \in [0, 1]$  is a *spatial equilibrium* if no consumer has an incentive to migrate away from the region where they are located:  $|\Delta V(\lambda^*)| \leq m$ . At such an equilibrium, consumers enjoy a level of well-being that varies with the region in which they reside, but the welfare gap is bounded above by  $m$ .

Since  $\Delta V(\lambda)$  increases with  $\lambda$  and  $\Delta V(1/2) = 0$ , the equation  $\Delta V(\lambda) = m$  has at most one solution  $\bar{\lambda} > 1/2$ , which increases with  $m$ . The function  $\Delta V(\lambda)$  being point symmetric,  $\Delta V(1/2 + x) = -\Delta V(1/2 - x)$ ,  $1 - \bar{\lambda}$  is the solution to  $\Delta V(\lambda) = -m$ . If  $|\Delta V(\lambda)| = m$  has no solution in  $(1/2, 1)$ , then migration costs are so high that no distribution exists that yields a positive utility gain net of migration costs. In other words, migration costs are large enough for *any* distribution to be a spatial equilibrium. From now on, we rule out this case by assuming that

$$\frac{\sigma - 1}{cf^{\frac{1}{\sigma-1}} \sigma^{\frac{\sigma}{\sigma-1}}} > m$$

for the equation  $\Delta V(\lambda) = m$  to have a solution in  $(1/2, 1)$ .

Two types of equilibria may arise. In the first one,  $\lambda^* \in [1 - \bar{\lambda}, \bar{\lambda}]$  so that firms and activities are partially dispersed. In this case, no consumers migrate because the utility gains do not exceed their mobility cost, thus implying that any  $\lambda \in [1 - \bar{\lambda}, \bar{\lambda}]$  is a spatial equilibrium. The second type of equilibrium involves the agglomeration of activities in a single region:  $\lambda^* = 0, 1$ . When  $\lambda^* = 0$ , we get  $w^* = \phi^{1/\sigma}$ , and thus  $\Delta V < 0$ ; when  $\lambda^* = 1$ , we get  $w^* = \phi^{-1/\sigma}$ , and thus  $\Delta V > 0$ . In either case, regardless of the values of the parameters of the economy no migration occurs. The reason for this is the absence of immobile farmers, who otherwise lead firms and consumers to leave the cluster when transport costs are high.

### 3.1 The set of stable spatial equilibria

When several equilibria exist, it is commonplace to use some stability concept to discriminate between the different equilibria. This requires the use of a specific adjustment process. In what follows, we use the myopic evolutionary dynamics of NEG:

$$\dot{\lambda} = \begin{cases} \lambda(1 - \lambda)(\Delta V(\lambda) + m) & \text{for } 0 \leq \lambda < 1 - \bar{\lambda} \\ 0 & \text{for } 1 - \bar{\lambda} \leq \lambda \leq \bar{\lambda} \\ \lambda(1 - \lambda)(\Delta V(\lambda) - m) & \text{for } \bar{\lambda} < \lambda \leq 1. \end{cases} \quad (16)$$

Of course, a spatial equilibrium is a steady-state of (16). Since the utility differential  $\Delta V(\lambda)$  must exceed  $m$  for  $\lambda > 1/2$  to increase, consumers stay put when the economic gains stemming from migrating are not sufficiently large to offset their own migration costs.

Ideally, migration generating continued costs should be studied in a dynamic framework where consumers maximizes the intertemporal value of their utility flows net of the various costs that negatively affect migrants at each period of their lifetime. Rather, we have chosen to use the same static setting as Krugman (1991) for the following two reasons. First, the static approach makes the analysis much simpler and facilitates the interpretation of the results. Second, when consumers have a low mobility, (16) may be considered as a good approximation of a model involving a forward-looking dynamics (Oyama, 2009). Since we focus on the impact of significant migration costs, we find it reasonable to believe the static approach to pin down the first-order results. By adding  $m$  to (16), we capture the idea that a consumer who moves from, say, region 2 to region 1 enjoys a permanent utility flow given by  $V_1(\lambda) > V_2(\lambda)$  and bears a continued cost equal to  $m$ .

The spatial equilibrium  $\lambda^*$  is said to be (asymptotically) *stable* when the adjustment process (16) leads the off-equilibrium consumers back to  $\lambda^*$ . The existence of positive migration costs implies that any distribution belonging to the interval  $(1 - \bar{\lambda}, \bar{\lambda})$  are stable spatial equilibria because this interval is an open set. In other words, migration costs stabilizes a wide range of distributions of activities. As  $m$  falls, the interval  $(1 - \bar{\lambda}, \bar{\lambda})$  narrows down; it is empty for  $m = 0$  because  $\bar{\lambda} = 1/2$ . When the equality holds in (15), there exist two other equilibria given by  $\lambda^* = 1 - \bar{\lambda}, \bar{\lambda}$ . However, both are unstable as shown by computing the derivative of the utility differential (13).<sup>6</sup> Last, since  $\Delta V < 0$  at  $\lambda^* = 0$  while  $\Delta V > 0$  at  $\lambda^* = 1$ , these two configurations are also stable equilibria.

To sum up, we present the next proposition.

**Proposition 2** *In the presence of migration costs, there exists a continuum of stable equilibria given by  $(1 - \bar{\lambda}, \bar{\lambda})$  and  $\lambda^* = 0, 1$ .*

Note the difference with the standard core-periphery (CP) model where the number of equilibria is finite while the only stable equilibria involve full dispersion or full agglomeration (Krugman, 1991; Robert-Nicoud, 2005). This is because migration costs act as a stabilizing force whose intensity is unaffected by the geographical distribution of activities. For example, the symmetric and agglomerated configurations on which NEG focuses are always stable equilibria here, thus destroying the main prediction of NEG saying that dispersion (agglomeration) prevails when transport costs are high (low).

Whereas the value  $\lambda_0 > 1/2$  has no impact on the final outcome in standard NEG models,  $\lambda_0$  now matters because the continued migration costs are borne only when consumers reside outside their place of origin. Indeed, given that  $\Delta V(\lambda) > 0$  for any  $\lambda > 1/2$ , two cases may arise. (i) If  $\Delta V(\lambda_0) \leq m$ , no consumer migrates. Hence,  $\lambda^* = \lambda_0$  is a stable spatial equilibrium. (ii) If  $\Delta V(\lambda_0) > m$ , some consumers migrate from region 2 to region 1. Since  $\Delta V(\lambda)$  rises with  $\lambda$ ,  $\Delta V(\lambda)$  increases when more consumers move to region 1. As a result, the inequality  $\Delta V(\lambda) > m$  is never reverse. In this event,  $\lambda^* = 1$  is a stable spatial equilibrium. In sum, for any given  $\tau \geq 1$  there exists a unique spatial equilibrium  $\lambda^*(\tau)$  given by  $\lambda_0$  or 1.

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<sup>6</sup>These two equilibria correspond the two asymmetric interior equilibria identified by Krugman (1991), which are unstable.

### 3.2 Do transport costs matter?

In the presence of multiple stable equilibria, it is hard to predict which equilibrium emerges. A common way out is to start from an arbitrary initial equilibrium  $\lambda_0 \in (1 - \bar{\lambda}, \bar{\lambda})$  and to study its evolutionary path by changing steadily a key-parameter of the model. The standard thought experiment of NEG focuses on the impact of falling transport costs on the distribution of the manufacturing sector. In what follows, we thus assume that the economy starts with sufficiently high values of  $\tau$  and study how the initial distribution  $\lambda_0 \geq 1/2$  reacts to steady decreases in  $\tau$ .

When  $\tau = 1$ ,  $\Delta V(\lambda) = 0$  regardless of the value of  $\lambda$ . Therefore, by continuity,  $\bar{\tau} > 1$  exists such that  $\Delta V < m$  for any  $\lambda$  and all  $1 < \tau < \bar{\tau}$ . In other words, when transport costs are very low, the set of stable spatial equilibria encompasses the unit interval. But what happens when  $\tau$  exceeds  $\bar{\tau}$ ? To answer this question, we have to find how  $\bar{\lambda}$  varies with  $\tau$ .

Figure 1 depicts the relationship between the equilibrium distributions and the level of transport costs. The interior of the shaded domain describes the continuum of dispersed equilibria satisfying (15), while the two bold horizontal lines describe the two agglomerated equilibria. As shown in the Appendix,  $\bar{\lambda}(\tau)$  increases when  $\tau$  falls provided that  $\sigma \geq \bar{\sigma} \equiv 1 + 1/\sqrt{2} \approx 1.71$ .<sup>7</sup> Since the empirical estimates of  $\sigma$  are all much larger than  $\bar{\sigma}$  (Anderson and van Wincoop, 2004), we may assume without much loss of generality that this condition holds. In this case, the interval  $(1 - \bar{\lambda}, \bar{\lambda})$  expands as  $\tau$  decreases. As a consequence, when the initial distribution  $\lambda_0$  belongs to  $(1 - \bar{\lambda}, \bar{\lambda})$  for some  $\hat{\tau}$ , this distribution remains a stable equilibrium for all  $\tau < \hat{\tau}$ . That is, when  $1/2 < \lambda_0 < \bar{\lambda}$ , there exists a unique spatial equilibrium given by  $\lambda^*(\tau) = \lambda_0$  for all  $\tau \geq 1$ ; see the dashed arrow in Figure 1. This is very different from the main finding of NEG where a steady decrease in  $\tau$  always moves the economy from dispersion to agglomeration (Krugman, 1991; Fujita et al., 1999).

Insert Figure 1 about here

The reason for this change in results may be explained as follows. Because differences between the interregional price and the wage gaps shrink when transport costs fall, the larger region becomes relatively less attractive. As a consequence, if the initial utility differential is not large enough to trigger consumers' migration, this holds true even more so when transport costs are lower because the cost-of-living difference has decreased. In addition, as illustrated by the shaded area of Figure 1, smaller transport costs allow sustaining a larger domain of spatial equilibria. In the limit, as said above, when  $\tau$  gets close to 1, the domain of spatial equilibria often encompasses the unit interval, which implies that any initial distribution of activities is a spatial equilibrium. To put it differently, when transport costs are positive but low enough, location no longer matters provided that the initial distribution is not too unbalanced. In sum, since Proposition 1 implies that no migration from the larger to the smaller region occurs, there is no force incentivizing consumers to migrate. Thus, contrary to the main prediction of NEG, we may conclude that *the integration of regional markets does not necessarily spark the agglomeration of the manufacturing sector*.

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<sup>7</sup>If  $\sigma < \bar{\sigma}$ ,  $\bar{\lambda}$  first decreases and then increases with falling transport costs. In this case,  $\lambda_0$  ceases to be a stable equilibrium at the first value of  $\tau$  such that  $\bar{\lambda} = \bar{\lambda}(\tau)$ . The equilibrium is then given by  $\lambda^* = 1$  for lower transport costs.

By contrast, if  $\lambda_0$  belongs to the upper (lower) non-shaded domain of Figure 1 while transport costs are high, the initial distribution is *not* a spatial equilibrium. If  $\lambda_0 > 1/2$ , Proposition 1 implies that the spatial equilibrium is unique and given by  $\lambda^* = 1$ . Indeed, owing to its size advantage, region 1 produces a much wider range of varieties than region 2, while high transport costs make these varieties much more expensive in region 2 than in region 1. As a consequence, the cost-of-living difference is large enough to trigger the relocation of consumers from region 2 to region 1. In this event, the larger region can be viewed as a “black hole” that accommodates the entire manufacturing sector (Fujita et al., 1999).

This is not the end of the story, however. Since

$$\text{sgn} \frac{\partial \Delta V(1)}{\partial \phi} = \text{sgn} \frac{\partial}{\partial \phi} \left( 1 - \phi^{\frac{2\sigma-1}{\sigma(\sigma-1)}} \right) < 0$$

a falling  $\tau$  makes  $\Delta V(1)$  smaller and smaller. The continued dislocation cost that consumers bear when they do not live in their place of origin vanishes when they go back to their place of origin. Therefore, when  $\Delta V(1)$  becomes smaller than  $m$ , those who had chosen to migrate from 2 to 1 return to their place of origin. That is, when  $\lambda_0$  exceeds  $\bar{\lambda}$ ,  $\bar{\tau}$  exists such that  $\lambda^*(\tau) = 1$  for  $\tau > \bar{\tau}$  and  $\lambda^*(\tau) = \lambda_0$  for  $\tau < \bar{\tau}$ ; see the solid arrows in Figure 1. In other words, by reducing interregional economic differences, market integration gives rise to return migrations. Note that, after their second move, those consumers enjoy a utility level  $V_2(\lambda_0)$  lower than  $V_1(\lambda_0)$  because region 1 is bigger. Nevertheless, they prefer to move back to their place of origin because this allows them to avoid incurring the dislocation cost  $m$ , which now exceeds the value of  $\Delta V(\lambda_0)$  because for  $\tau < \bar{\tau}$ . As a consequence, the larger region ceases to be a black hole.

All those results clash with what NEG tells us. Yet they are both intuitive and plausible. The foregoing discussion also shows that, in our setting, history matters in a deeper way than in standard NEG models.

## 4 The impact of rising labor productivity

In this section, we turn our attention to the effect of a rising labor productivity and show that, regardless of its concrete form, a steadily increase in labor productivity brings about the agglomeration of the manufacturing sector. To avoid undue complexity, we assume that productivity gains stem from exogenous technological progress. Although both  $c$  and  $f$  are likely to be affected by technological progress, we will see that falling marginal and fixed requirements of labor do not have the same implications for consumers.

### 4.1 Marginal labor requirement

We consider a new thought experiment and show that a steady decrease in the marginal labor requirement  $c$  has an impact that greatly differs from that generated by falling transport costs, which is described in Proposition 2. Figure 2, very much like Figure 1, shows how the geographical distribution of economic activities and the marginal labor requirement are related. Observe that Figures 1 and 2 illustrate the difference in the impacts of a deeper market integration and a higher labor productivity:  $\bar{\lambda}$  decreases with  $\tau$  but increases with  $c$ .

Let  $\lambda_0$  be the initial distribution of economic activities. Since  $\Delta V(\lambda)$  decreases with  $c$ , the equation  $\Delta V(\lambda_0) = -\Delta V(1 - \lambda_0) = m$  has a unique solution  $c_0$ . The shaded domain describes the continuum of dispersed equilibria associated with any  $c$  exceeding  $c_0$ . Note that the vertical distance between these two curves now increases with the marginal requirement  $c$ . Indeed, since  $\partial\Delta V/\partial c < 0$ ,  $\bar{\lambda}$  decreases when  $c$  falls. Since the spatial equilibria arising under  $\lambda_0 \in [0, 1/2)$  are the mirror images of those arising under  $\lambda_0 \in (1/2, 1]$ , we assume without loss of generality that region 1 accommodates a priori a larger number of firms and consumers than region 2:  $\lambda_0 \in (1/2, 1]$ .

Insert Figure 2 about here

Suppose that the economy starts from a sufficiently high marginal production cost, which gradually decreases. Because  $c$  is arbitrarily large, it is readily verified that any distribution  $\lambda \in [1/2, \bar{\lambda}]$  is a stable equilibrium. In addition,  $\lambda^* = 1$  is always a stable equilibrium. From now on, we rule out the extreme cases where the initial distribution  $\lambda_0 = 1/2$  or 1 and assume that  $\lambda_0 \in (1/2, \bar{\lambda})$ . Then,  $\lambda^* = \lambda_0$  is a stable spatial equilibrium for all  $c \in (c_0, \infty)$ . Or, to put it differently, as long as  $c$  exceeds the threshold  $c_0$ , a rising labor productivity has no impact on the geographical distribution of the manufacturing sector.

However, as shown in subsection 3.1, once  $c$  is equal to  $c_0$  the equilibrium  $\lambda^* = \bar{\lambda}$  becomes unstable. Furthermore, the interval of partially dispersed equilibria  $[1 - \bar{\lambda}, \bar{\lambda}]$  shrinks as  $c$  decreases and is empty for  $c < c_0$ . Therefore,  $\lambda^* = \lambda_0$  ceases to be a spatial equilibrium for  $c$  smaller than  $c_0$ . In this case, the new stable equilibrium is given by  $\lambda^* = 1$  for all  $c \in (0, c_0)$ ; see the solid arrows in Figure 2. Evidently, lowering  $m$  implies a hike in  $c_0$ , and thus a faster concentration of firms and jobs in region 1.

The following proposition summarizes.

**Proposition 3** *Assume that the marginal labor requirement falls steadily. Then, for any initial distribution of activities  $\lambda_0 \in (1/2, \bar{\lambda})$ , there exists a threshold  $c_0$  such that (i)  $\lambda_0$  is a stable spatial equilibrium for all  $c > c_0$ ; and (ii)  $\lambda^* = 1$  is the unique stable spatial equilibrium for all  $c \leq c_0$ .*

This is reminiscent of Krugman's (1991) CP model: the evolutionary process involves, first, the (partial) dispersion of activities for a whole domain of values taken by the parameters  $\tau$  (Krugman) or  $c$  (here) and, then, the agglomeration of firms and workers in one region. Technological change is a slow process that displays strong inertia over time, and this inertia also characterizes the geographical dimension of change.

The reasons for Proposition 3 are easy to grasp. When  $c$  falls, the following three effects are at work. First, because  $\lambda_0$  exceeds  $1/2$ , it ensues from Proposition 1 that the nominal wage is higher in region 1 than in region 2. As long as  $\lambda = \lambda_0$ , (11) implies that a decreasing marginal labor requirement does not affect the equilibrium wage  $w^*$ . By contrast, when  $\lambda$  starts rising above  $\lambda_0$ , (11) shows that the nominal wage in region 1 also rises. Second, when  $\lambda = \lambda_0$ , (5) shows that a fall in  $c$  translates into a lower equilibrium price for the existing varieties, regardless of where they are produced. However, when  $\lambda$  starts rising above  $\lambda_0$ , the wage paid in region 1 also increases, which may result in a price hike in region 1. Since  $p_1^*/w^* = c\sigma/(\sigma - 1)$ , a falling  $c$  lowers  $p_1^*/w^*$ , thus implying that  $w^*$  rises faster than the equilibrium price  $p_1^*$ .

Third, and last, the productivity hike implies that fewer workers are needed to produce the existing varieties. Although the equilibrium output  $q_r^*$  increases with falling  $c$  from (7), every firm hires the same number of workers to produce a larger output because  $cq_r^* + f$  is independent of  $c$ . By implication, the total number of varieties remains the same (see also (9)). As a consequence, when  $c$  falls,  $1/P_1^* - 1/P_2^*$  rises, and thus the indirect utility differential  $\Delta V(\lambda_0)$  increases. As long as  $\Delta V(\lambda_0)$  remains smaller than the migration cost  $m$ , no region 2's consumer moves ( $\lambda = \lambda_0$ ), but all consumers are better off because of the price drop and the production hike. Note that the number of varieties remains the same.

Once  $c$  falls below the threshold  $c_0$ ,  $\Delta V(\lambda_0)$  exceeds the migration cost  $m$  and a few consumers living in the smaller region move to the larger one. As a consequence, more (fewer) varieties are produced in region 1 (2). Since  $w^*$  rises faster than  $p_1^*$ ,  $w_1^*/P_1^*$  increases at a higher rate than  $1/P_2^*$ . Therefore, the difference  $\Delta V(\lambda) - \Delta V(\lambda_0)$  gets bigger when  $c$  falls. Thus, as in Krugman (1991), the interplay between these various effects generates the cumulative causation that feeds the migration process until all consumers are agglomerated in region 1, and so even when  $c < c_0$  has stopped decreasing. Unlike the case of falling  $\tau$  in subsection 3.2, consumers migrate at most once because  $\Delta V(\lambda)$  is decreasing in  $c$  whereas it is increasing in  $\tau$ .

It is legitimate to ask what Proposition 3 becomes in Krugman's original CP model which, unlike ours, involves a farming sector. Because the CP model is not easy to handle analytically, we have undertaken this using the linear model of monopolistic competition (Ottaviano et al., 2002). If the number of farmers is not too high (otherwise there is always dispersion) and not too low (otherwise there is always agglomeration), the economy shifts from dispersion to agglomeration when labor productivity has reached a certain threshold. Therefore, disregarding the farming sector is *not* the reason for our main results. Similarly, accounting for housing and commuting costs, which both rise with  $\lambda$  in region 1 and fall in region 2, decreases the utility level in the larger region and increases it in the smaller one, thereby lowering the utility differential  $\Delta V(\lambda)$ . Nevertheless, a fall in  $c$  still drives the geographical concentration of the manufacturing sector because  $\partial\Delta V/\partial c > 0$  still holds. When that dispersion force is grafted onto our setting, it is natural to expect the economy to involve partial, rather than full, agglomeration of the manufacturing sector.

## 4.2 Fixed labor requirement

Consider now a fall in the fixed requirement of labor. As shown by (5), the price of existing varieties is unaffected. Even though a firm's output  $q_r^*$  increases with falling  $f$ , the number of firms and varieties in each region increases from (9). In other words, the productivity hike implies that some workers are freed from producing the existing varieties. Since their number is greater in region 1 than in region 2, a larger number of new varieties are launched in region 1 than in region 2, which implies that  $1/P_1^* - 1/P_2^*$  increases with falling  $f$ . In this case, the total number of varieties produced in the economy increases, but it does so more in region 1 than in region 2.

Because  $\Delta V(\lambda_0)$  is decreasing in  $f$ , the equation  $\Delta V(\lambda_0) = m$  has a single solution, which is denoted  $f_0$ . Applying the argument used to prove Proposition 3, we obtain the following result.

**Proposition 4** *Assume that the fixed labor requirement falls steadily. Then, for any initial distribution of activities  $\lambda_0 \in (1/2, \bar{\lambda})$ , there exists a threshold  $f_0$  such that (i)  $\lambda_0$*

is a stable spatial equilibrium for all  $f > f_0$ ; and (ii)  $\lambda^* = 1$  is the unique stable spatial equilibrium for all  $f \leq f_0$ .

A drop in  $c$  leads to a higher total output  $Q^* = n^*q_r^* = (\sigma - 1)/\sigma c$  through a bigger output per firm, whereas  $n^* = 1/\sigma f$  does not change. On the other hand, a fall in  $f$  increases the number of firms and varieties,  $n^* = 1/\sigma f$  but does not affect  $Q^*$ . Thus, although falling marginal and fixed labor requirements are not congruent in terms of their effects on the economy, the above two propositions have a clear implication: *a steady flow of labor-saving innovations brings about a transition from a (partially) dispersed configuration of activities to an agglomerated one.* Hence, when we disregard the problematic existence of immobile farmers whose role is to hold back industrial workers living in the less prosperous region in Krugman's setting, the effects of a rising labor productivity are in sharp contrast to those generated by falling transport costs. More precisely, a growing labor productivity widens the interregional utility differential, which eventually outweighs migration costs and generates interregional migration. In contrast, steady drops in transport costs reduce the interregional utility differential and keep the distribution of activities unaffected.

The following remarks are in order. First, very much like in NEG, the initial distribution of activities displays some sluggishness during the first phases of technological progress, but then abruptly takes the form of a large economic agglomeration of firms and consumers. Second, like in NEG again, a small initial inequality in the spatial distribution of activities can lead to a large and striking inequality at equilibrium.

Last, industrialization and urbanization are also fed by large rural-urban migrations. Although our model does not account for an agricultural sector, we may capture the impact of such migrations by studying how the space-economy changes when the labor force  $L$  rises. We have seen that an increase in  $L$  amounts to a decrease in  $f$ . Therefore, it follows from Proposition 4 that *the manufacturing sector gets more agglomerated when the population working in this sector grows.* In other words, stronger rural-urban migrations exacerbate the tendency toward the agglomeration of activities.

## 5 Spatial equilibrium with heterogeneous labor

So far, we have assumed that all workers bear the same migration cost. What is more, the assumption of equally productive workers is a very strong one. In this section, we assume that an  $e$ -type worker is endowed with  $e > 0$  efficiency units of labor and bears a migration cost  $m(e) > 0$ . Workers are thus heterogeneous along two dimensions, that is, migration cost and productivity. It is empirically well documented that skilled workers are more mobile than unskilled workers (OECD, 2005, Chart 2.10), which means that  $m$  is a decreasing function of  $e$ . Since the two types of heterogeneity are highly correlated, we may avoid the technicalities associated with two-dimensional heterogeneity by assuming that workers are heterogeneous along their productivity type only. Assuming that  $m$  decreases with the skill level strengthens the results obtained in this section. More generally, our analysis holds true when (i) consumers can be ranked according to a one-dimensional type  $\theta$  that embodies different forms of heterogeneity across individuals (e.g., productivity, mobility, taste for natural amenities) and (ii) the indirect utility of a type  $\theta$ -consumer residing in region  $r$  is given by  $V_r(\theta) = A(\theta)w_r/P_r$ , where  $A(\theta)$  is a monotone function of  $\theta$ .

Let the total number of efficiency units of labor available in the two regions be equal to 1 after normalization. When labor is heterogeneous, what determines the productive size of region  $r$  is no longer the number of workers  $\lambda_r$  residing in this region, but the number of efficiency units of labor  $E_r$  available therein. In other words,  $\lambda_r$  is to be replaced by  $E_r$  in the analysis developed above. Observe that what matters in our model is the value of  $E_r$ , not the composition of the group of workers residing in region  $r$ .

Individual types are initially distributed in region  $r = 1, 2$  according to the continuous density function  $g_r(e) > 0$  defined over  $(0, \infty)$ . The corresponding regional labor supply functions are then given by

$$E \equiv E_1 = \int_0^\infty eg_1(e) de \quad E_2 = 1 - E \equiv \int_0^\infty eg_2(e) de. \quad (17)$$

Since a region endowed with a given number of efficiency units of labor is equivalent to a region endowed with the same number of workers having the same productivity, region 1 is said to be more productive than region 2 if the total number of efficiency units of labor available in the former exceeds that in the latter, that is,  $E_1 > E_2$  or, equivalently,  $E > 1/2$ . Assuming  $E > 1/2$  does not imply that region 1 has a larger population than region 2 because a higher number of inefficient workers may be located in region 2 than in region 1. Since  $c$  and  $f$  are now expressed in efficiency units of labor, labor market clearing implies  $E_r = \sigma f n_r$  for  $r = 1, 2$ , so that region 1 accommodates a higher number of firms than region 2.

Denoting by  $w_r$  the price of one efficiency unit of labor in region  $r$ , the income of an  $e$ -type worker residing in region  $r$  is equal to  $ew_r$ . Therefore, her indirect utility is given by

$$V_r(e) = e \frac{w_r}{P_r}, \quad (18)$$

which increases linearly with  $e$ .

Both the equilibrium wages  $w_r$  and price indices  $P_r$  depend on  $E$  as they depend on  $\lambda$  in Section 2. Accordingly, for any initial distribution  $E > 1/2$ , we can call on Proposition 1 to assert that  $w_1^* > w_2^*$  and  $P_1^* < P_2^*$ . While  $e$  varies across types of labor, the variables  $w_r^*$  and  $P_r^*$  are common to all workers residing in region  $r$ . Thus, the interregional utility differential is given by

$$\Delta V(e, E) = V_1(e, E) - V_2(e, E) = e \left[ \frac{w_1^*(E)}{P_1^*(E)} - \frac{1}{P_2^*(E)} \right], \quad (19)$$

which is positive and increasing in  $e$ .

Since  $\Delta V(e, E)$  becomes arbitrarily large for sufficiently high values of  $e$ , the utility differential of the workers endowed with a large number of efficiency units of labor always exceeds their migration cost. As a consequence, *region 2's most productive workers choose to migrate to region 1*. But how many workers in region 2 want to migrate?

Let  $e^* \in (0, \infty)$  be the marginal  $e$ -type workers indifferent between moving to the more productive region and staying put in the less productive one. Thus, the equilibrium number of efficiency units of labor available in the larger region is given by

$$E(e^*) = \int_0^\infty eg_1(e) de + \int_{e^*}^\infty eg_2(e) de, \quad (20)$$

while the equilibrium number of workers residing in region 1 is given by

$$\int_0^\infty g_1(e) de + \int_{e^*}^\infty g_2(e) de,$$

where the first term is the initial number  $\lambda_0$  of workers and the second the number of migrants.

As in Section 2, we choose the efficiency unit of labor in region 2 as the numéraire, so that  $w_1 = w$  and  $w_2 = 1$ . The wage equation (11) then becomes

$$\frac{E(e^*)}{1 - E(e^*)} = \frac{w^{\sigma-1}(w^\sigma - \phi)}{1 - \phi w^\sigma}. \quad (21)$$

Clearly, the left-hand side of this expression decreases with  $e^*$ , whereas the right-hand side increases with  $w$ . The implicit function theorem thus implies that (21) has a unique solution  $w(e^*)$  while  $w'(e^*) < 0$  for all  $e^* \in (0, \infty)$ . As a consequence, when the number of migrants moving into region 1 increases, the price of one efficiency unit of labor in this region also increases.

The expressions (20) and (21) imply that there is a one-to-one correspondence between  $e^*$  and  $E$  as well as between  $e^*$  and  $w_r/P_r$ . As a consequence, the utility differential may be rewritten as a function of  $e$  only. An interior equilibrium  $e^*$  is then determined by the solution to the spatial equilibrium condition:

$$\Delta V(e) = e \left[ \frac{w_1^*(E(e))}{P_1^*(E(e))} - \frac{1}{P_2^*(E(e))} \right] = m. \quad (22)$$

Unlike (19), both the wages and price indices in (22) now depend upon  $e^*$  only. Set

$$h(e) \equiv \Delta V(e) - m. \quad (23)$$

We have  $h(0) = -m < 0$  and  $h(\infty) = \infty > 0$ . Hence, there exists an equilibrium  $e = e^*$  where  $h'(e^*) > 0$ . This inequality implies that  $e^*$  is stable because  $e^*$  decreases with  $E$ .

We can repeat the analysis of Section 4 and show that the equilibrium price  $w^*$  of one efficiency unit of labor rises when  $c$  decreases. Similarly, the inverse price index difference  $1/P_1^* - 1/P_2^*$  increases when  $c$  falls. As a consequence,  $\Delta V$  increases when  $c$  decreases. In other words,  $h(e)$  is shifted upward, which implies that  $e^*$  decreases when  $c$  falls. Note that the decrease in  $e^*$  is not necessarily continuous. Indeed, if there are multiple stable equilibria, some of them may disappear when  $c$  falls. In this case, the economy jumps to another stable equilibrium having a larger number of workers in region 1 because this region is more attractive. However, if there is a unique stable equilibrium,  $e^*$  gradually decreases when  $c$  steadily decreases.

Falling fixed requirements  $f$  yield the same qualitative result. Thus, we have the following result.

**Proposition 5** *Assume that  $E > 1/2$ . If the marginal or fixed labor requirement steadily decreases, the number of individuals residing in region 1 monotonically increases by attracting workers whose productive efficiency decreases.*

This proposition provides a rationale for the well-documented fact that the skilled workers ( $e > e^*$ ) living in a less efficient place tends to move toward a more efficient place.

Specifically, region 1 accommodates a more than proportionated share of skilled workers, which echoes Behrens et al. (2014) and Eeckhout et al. (2014). Through the migration of such workers, the economy ends up with one prosperous region, while the other is relatively poorer. It thus seems fair to say that Proposition 5 highlights a fundamental trend of the contemporary space-economy, which supplements other sorting mechanisms that have been studied in NEG and urban economics (Mori and Turrini, 2005; Eeckhout et al., 2014).

Empirical evidence also suggests that large and prosperous cities are characterized by a growing skill and income polarization of their population (Berry and Glaeser, 2005). In our setting, this can be explained by the fact that *the core region hosts both skilled and unskilled workers*, i.e., the skilled from regions 1 and 2 as well as the unskilled ( $e < e^*$ ) from region 1. Whether region 1 features a more than disproportionate share of unskilled workers ( $e < e^*$ ), as in Behrens et al. (2014) and Eeckhout et al. (2014), depends on the initial distribution of such workers between regions. That no sharp conclusion can be derived from our setting suggests that the initial conditions matter a lot for the equilibrium outcome. In addition, it should be kept in mind that migrants enjoy different welfare levels because they are heterogeneous, thereby making some comparisons especially difficult.

If the per capita income decreases in region 2, the impact of migration on the per capita income in region 1 depends on the position of the migrants on this region's skill ladder. Note also that the more productive region need not be the larger one since  $E > 1/2$  can be consistent with  $\lambda < 1/2$ . In this respect, different economic landscapes may emerge, depending on the densities  $g_r$ .

Observe that the problem studied above has the nature of an assignment problem with heterogeneous workers (Sattinger, 1993). However, it has two distinctive features. First, each worker is initially assigned to a specific location and, second, when moving away from her initial location, this worker incurs a positive dislocation cost. Since  $w^*(E)$  and  $P_2^*(E)$  increase with  $E$  while  $P_1^*(E)$  decreases with  $E$ , it follows from (19) that

$$\frac{\partial^2 \Delta V(e, E)}{\partial e \partial E} > 0, \quad (24)$$

which means that our setting satisfies the supermodularity property. Note that, unlike many authors, we do not assume that our setting is supermodular. Rather, we show that, owing to (admittedly simple) general equilibrium effects, our setting *is* supermodular.<sup>8</sup>

This property has two far-reaching implications. First, by raising the price of an efficient unit of labor, *the migration of the more productive workers pulls up the less productive workers residing in the core region*. Specifically, the unskilled born in region 1 enjoy higher nominal wages than their counterparts who live in region 2. For example, considering the 25 bottom percent of job earnings in 2013 Japan, we find that the corresponding workers living in the core regions of Japan (the 10 prefectures containing Tokyo, Osaka and Nagoya) earn 23 per cent more than their counterpart residing in the rest of the country. Thus, like in Moretti (2010), the unskilled benefit from the creation of skilled jobs in their region. However, in Moretti this effect is channelized through the creation of jobs for the unskilled, whereas it manifests here through higher wages. The reason for this difference lies in the assumption of full employment made in this paper.

To this income effect, we must add the standard effect that stems from a broader range of varieties produced in the core region, which increases even further the welfare of

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<sup>8</sup>Davis and Dingel (2013) derive the same property in a related, but different, context.

the unskilled residing in this region. On the other hand, this effect decreases the welfare of those who stay put in the poor region. Hence, when it is recognized that workers are heterogeneous, *migration affects all workers*, regardless of their productivity. Therefore, the wage hike paid to the unskilled is not a compensation for a higher cost-of-living as in Suedekum (2006) because the price index of region 1 decreases when more skilled workers settle down therein.

Second, (24) implies that there is perfect spatial sorting of workers across types belonging to  $(e^*, \infty)$ . In contrast, workers sharing the same types  $e < e^*$  are not located in the same region. Therefore, spatial sorting across all types is imperfect.

## 6 Conclusion

The central tenet that cuts across NEG is that falling transport costs explain the agglomeration of economic activities. In this paper, we propose a different explanation: the emergence of a core-periphery structure stems from technological progress in the manufacturing sector. Given the dramatic labor productivity growth observed ever since the beginning of the Industrial Revolution, we find this explanation both plausible and relevant. Therefore, the prime mover responsible for the emergence of a core-periphery structure would be *technological innovations in the manufacturing sector rather than technological innovations in the transportation sector*. In other words, rising labor productivity takes the place of falling transport costs as the main explanation for the emergence and persistence of an uneven distribution of activities across space. These results are proven by using a paper-and-pencil method that is disarmingly simple, whereas standard NEG models often appeal to numerical simulations. This has allowed us (i) to study in a detailed way the various effects at work and (ii) to take on board different types of asymmetry and/or heterogeneity, something which is not easy accomplish in models à la Krugman (Baldwin et al., 2003).

That said, we would be the last to claim that market integration does not play any role. Quite the opposite, we believe that market integration has been, and still is, one of the main drivers shaping the space-economy. For example, it is well documented that the commercial revolution in the 17th century, which has been facilitated by a large number of improvements in transportation techniques, went with the relocation of textile production. Likewise, larger and integrated markets make R&D more profitable and lead to more inventions. To a large extent, explaining the geographical pattern of production in various countries requires combining technological progress and market integration.

In contrast, we do not believe that the existence of the primary sector and other activities using immobile inputs is sufficient to explain the existence of (partially) dispersed patterns of activities in modern economies. Rather, we recognize that migration is governed by push and pull effects in which significant and continued migration costs plays the role of a dispersion force. Note that the existence of high commuting costs and land prices in the core region may hold back the agglomeration process, thereby implying that some manufacturing activities are located in the periphery.

We have shown that, once labor productivity has increased sufficiently, the interplay between the agglomeration and dispersion forces triggers the (partial) concentration of activities. However, there is no reason to expect the resulting pattern of activities to prevail forever. Indeed, we have assumed in the foregoing sections that technological progress

affected all regions equally. It is reasonable, however, to believe that labor requirement declines at different rates in various regions. In this case, even when region 1 is the core of the economy, a reversal of fortune becomes possible if region 2 experiences a stronger wave of innovations. In this event, the peripheral region or country is able to throw off its history. Such a redrawing of the map of economic activities is difficult to obtain in standard NEG models.

Our model, owing to its extreme flexibility, can be extended in several directions. First, it is well known that technological progress follows different trajectories across industries. Therefore, our approach allows to explain why different industries display contrasted location patterns. Second, for our main results to hold, we need only the following two intuitive conditions:  $d\Delta V/d\lambda > 0$  and  $\partial\Delta V/\partial c < 0$ , which hold in alternative preferences settings, such as those involving quadratic preferences or additive utilities.

Third, the model could also be extended to account for the internal functioning of regions, which do not often grow at the same pace. This could be done by introducing different microeconomic mechanisms that generate agglomeration (dis)economies, such as those analyzed by Duranton and Puga (2004). In such a context, it would be natural to focus on endogenous technological progress, which is often place-specific, and to add a housing sector to the model. Hopefully, such a microscopic extension of our macroscopic model would find out why some regions fare better than others.

Last, our setting could be used as a building block in a model of endogenous regional growth model. We expect such a model to predict a growing divergence between regions. This does not mean, however, that smaller regions or countries are doomed to lag behind forever. These regions may take leverage on their high degree of political homogeneity to react faster than larger regions or countries to new opportunities.

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## Appendix

We show that  $d\bar{\lambda}(\tau)/d\tau < 0$  or, equivalently,  $d\bar{\lambda}(\phi)/d\phi > 0$  over  $(0, 1)$  for all  $\sigma \geq \bar{\sigma}$ . The variable  $\bar{\lambda}(\tau) \in (1/2, 1]$  must satisfy the following two equilibrium conditions:

$$F_1(\lambda, w) \equiv w^\sigma - \phi - [w^\sigma - (w + 1)\phi + w^{1-\sigma}] \lambda = 0 \quad (25)$$

$$F_2(\lambda, w) \equiv \Delta V(\lambda) - m = 0. \quad (26)$$

It is readily verified from comparative statics that

$$\frac{d\bar{\lambda}}{d\phi} = \frac{-\frac{\partial F_1(\lambda, w)}{\partial \phi} \frac{\partial F_2(\lambda, w)}{\partial w} + \frac{\partial F_2(\lambda, w)}{\partial \phi} \frac{\partial F_1(\lambda, w)}{\partial w}}{\frac{\partial F_1(\lambda, w)}{\partial \lambda} \frac{\partial F_2(\lambda, w)}{\partial w} - \frac{\partial F_1(\lambda, w)}{\partial w} \frac{\partial F_2(\lambda, w)}{\partial \lambda}} \quad (27)$$

where  $\lambda$  and  $w$  solve (25) and (26).

The denominator of (27) is negative from (13). Plugging (11) into the numerator of (27), we get

$$G(W) \equiv G_1(W) [G_2(W) - G_3(W)]$$

where  $W \equiv w^\sigma \in (1, 1/\phi]$  while

$$\begin{aligned} G_1(W) &\equiv \frac{(1 - \phi^2)^{\frac{2-\sigma}{\sigma-1}} W^{\frac{\sigma^2-\sigma+1}{\sigma(\sigma-1)}}}{cf^{\frac{1}{\sigma-1}} \sigma^{\frac{\sigma}{\sigma-1}} \left[ W(W - \phi) + W^{\frac{1}{\sigma}} (1 - \phi W) \right]^{\frac{\sigma}{\sigma-1}}} > 0 \\ G_2(W) &\equiv W^{\frac{2\sigma-1}{\sigma(\sigma-1)}} (1 - \phi W) [2\sigma - 1 - \phi^2 - 2(\sigma - 1)\phi W] > 0 \\ G_3(W) &\equiv (W - \phi) [(2\sigma - 1 - \phi^2)W - 2(\sigma - 1)\phi] > 0. \end{aligned}$$

Thus,  $G(W)$  is positive if and only if  $G_2(W) - G_3(W) > 0$ . Since  $G_2$  and  $G_3$  are positive, the sign of  $G_2(W) - G_3(W)$  is the same as the sign of

$$G_4(W) \equiv \log G_2(W) - \log G_3(W).$$

Differentiating this expression yields

$$G'_4(W) = G_5(W) G_6(W)$$

where

$$G_5(W) \equiv \frac{2\sigma - 1}{\sigma(\sigma - 1) W^{\frac{\sigma^2-3\sigma+1}{\sigma(\sigma-1)}} G_2(W) G_3(W)}$$

is positive, while

$$\begin{aligned} G_6(W) &\equiv 2(\sigma - 1)\phi^2(2\sigma - 1 - \phi^2)W^4 + \phi(4\sigma - 3 - \phi^2) [\sigma^2 - 3\sigma + 1 - (\sigma^2 + \sigma - 3)\phi^2] W^3 \\ &\quad - [4\sigma^3 - 10\sigma^2 + 6\sigma - 1 - (2\sigma - 1)(10\sigma - 11)\phi^2 - (4\sigma^3 - 6\sigma^2 - 10\sigma + 11)\phi^4 - \phi^6] W^2 \\ &\quad + \phi(4\sigma - 3 - \phi^2) [\sigma^2 - 3\sigma + 1 - (\sigma^2 + \sigma - 3)\phi^2] W + 2\phi^2(\sigma - 1)(2\sigma - 1 - \phi^2) \end{aligned}$$

is negative as shown by studying the derivatives of this function.

- (i) Since  $G_6'''(W) \geq 0$ ,  $G_6'''(W)$  is increasing over  $(1, \phi^{-1}]$ .
- (ii) We have

$$\begin{aligned} G_6'''(\phi^{-1}) &= 6\phi(1 - \phi^2) [4\sigma^3 + \sigma^2 - 11\sigma + 5 - (\sigma^2 + \sigma - 3)\phi^2] \\ &\geq 6\phi(1 - \phi^2) [4\sigma^3 + \sigma^2 - 11\sigma + 5 - (\sigma^2 + \sigma - 3)] \\ &= 24\phi(1 - \phi^2)(\sigma - 1)^2(\sigma + 2) \\ &\geq 0 \end{aligned}$$

where the first inequality holds because  $\sigma^2 + \sigma - 3 > 0$  for all  $\sigma \geq \bar{\sigma}$ .

- (iii) We have

$$\begin{aligned} G_6''(\phi^{-1}) &= 2(1 - \phi^2) [8\sigma^3 - 11\sigma^2 - 3\sigma + 4 - (4\sigma^3 - 3\sigma^2 - 7\sigma + 3)\phi^2 - \phi^4] \\ &\geq 2(1 - \phi^2) [8\sigma^3 - 11\sigma^2 - 3\sigma + 4 - (4\sigma^3 - 3\sigma^2 - 7\sigma + 3) - 1] \\ &= 8\phi\sigma(1 - \phi^2)(\sigma - 1)^2 \\ &\geq 0 \end{aligned}$$

where the first inequality follows from  $4\sigma^3 - 3\sigma^2 - 7\sigma + 3 > 0$  for all  $\sigma \geq \bar{\sigma}$ .

(iv) The signs of  $G_6'''(1)$  and  $G_6''(1)$  are indeterminate. However, if  $G_6''(1) \geq 0$ , then  $G_6'''(1) \geq 0$  for all  $\sigma \geq \bar{\sigma}$ . Two subcases may arise.

(iv-a) If  $G_6''(1) \geq 0$ , then  $G_6'''(1) \geq 0$ . Since  $G_6'''(W)$  is increasing,  $G_6'''(W) \geq 0$  always holds. Since  $G_6''(1) \geq 0$ ,  $G_6''(W) \geq 0$  always holds too, i.e.,  $G_6'(W)$  is increasing.

(iv-b) If  $G_6''(1) < 0$ , then  $G_6'''(1)$  is indeterminate. However, since  $G_6'''(W)$  is increasing and  $G_6''(\phi^{-1}) \geq 0$ , it must be that  $G_6''(W) < 0$  for small  $W$ , and then  $G_6''(W) \geq 0$  for large  $W$ , i.e.,  $G_6'(W)$  is U-shaped.

(v) We have

$$\begin{aligned} G_6'(1) &= -2(1-\phi)^3(2\sigma-1+\phi) \left[ 2(\sigma-\bar{\sigma}+\sqrt{2})(\sigma-\bar{\sigma}) + 2(\sigma^2-1)\phi + \phi^2 \right] \\ &\leq -2(1-\phi)^3(2\sigma-1+\phi) 2(\sigma-\bar{\sigma}+\sqrt{2})(\sigma-\bar{\sigma}) \\ &\leq 0 \end{aligned}$$

where the second inequality holds if and only if  $\sigma \geq \bar{\sigma}$ . Since  $G_6'(W)$  is either increasing or U-shaped from (iv-a) and (iv-b), it must be that  $G_6(W)$  is either decreasing or U-shaped.

We have

$$\begin{aligned} G_6(1) &= \frac{1}{2}G_6'(1) \leq 0 \\ G_6(\phi^{-1}) &= -\frac{(1-\phi^2)^3\sigma(\sigma-1)}{\phi^2} < 0. \end{aligned}$$

Thus,  $G_6(W) < 0$  for all  $W \in (1, \phi^{-1}]$ .

(vi) Since  $\text{sgn}G_6(W) = \text{sgn}G_4'(W)$  and  $G_4(1) = 0$ , we get  $G_4(W) < 0$  for all  $W \in (1, \phi^{-1}]$ , which implies  $d\bar{\lambda}(\phi)/d\phi > 0$ .

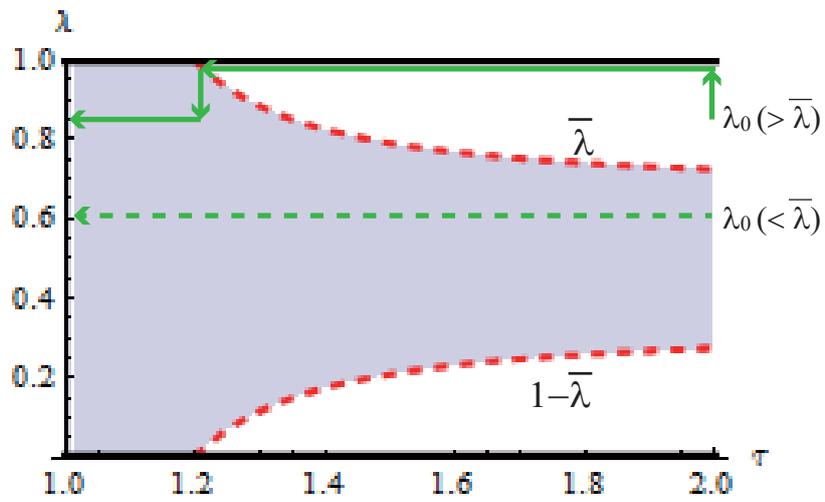


Figure 1: Stable equilibria for  $\tau$  with  $\sigma=3$ ,  $c=1$ ,  $m=1$ , and  $f=1/100$

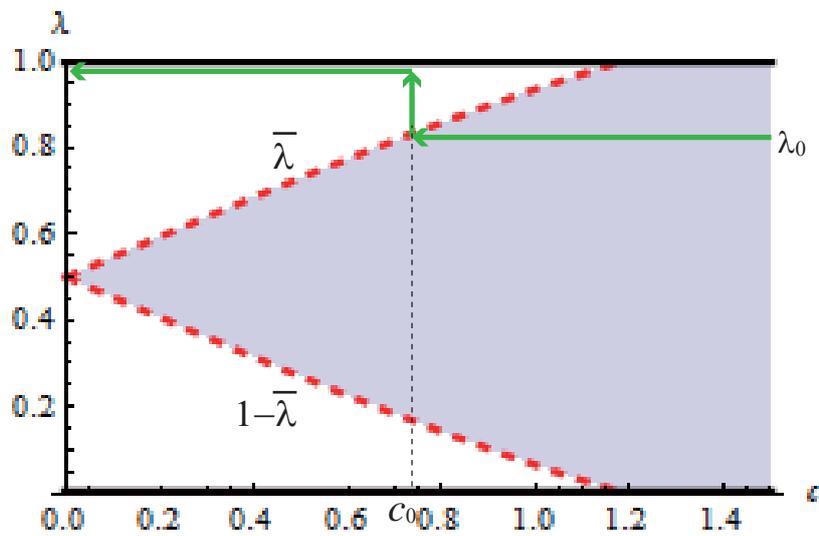


Figure 2: Stable equilibria for  $c$  with  $\sigma=3$ ,  $\phi=1/2$ ,  $m=1$ , and  $f=1/50$

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