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Justifying social discounting: the rank-discounted utilitarian approach

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Justifying social discounting:
the rank-discounted utilitarian approach

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Abstract

The popular discounted utilitarian criterion for infinite horizon social choice has been criticized on the ground that it treats successive generations unfairly. I propose to evaluate intergenerational welfare with a rank- discounted utilitarian (RDU) criterion instead. The criterion amounts to discounted utilitarianism on non-decreasing paths, but it treats all generations impartially: discounting becomes the mere expression of intergenerational inequality aversion. I show that more inequality averse RDU societies have higher social discount rates when future generations are better-off. I apply the RDU approach in two benchmark economic growth models and I prove that it promotes sustainable policies maximizing discounted utility.

Keywords: intergenerational equity, social discounting, discounted utilitarianism, sustainability.

JEL Classification: D63, H43, Q56

1 Introduction

The most popular objective function used to define the optimal policy in infinite horizon models is the discounted utilitarian (DU) criterion, $W(x) = \sum_{t=1}^{+\infty} \beta^t u(x_t)$, where $0 < \beta < 1$.

The criterion has been heavily criticized on the ground that it treats successive generations differently. Many economists in the utilitarian tradition have denounced this deviation from the ideal of equal regard for all individuals. For instance, Frank Ramsey famously described discounting as a "practice which is ethically indefensible and arises merely from the weakness of the imagination"(Ramsey , 1928, p.543). Among others, Pigou (1920) and Harrod (1948) have also stigmatized discounting.

Drawing on these criticisms, a prolific literature has studied whether it would be possible to combine the principle of procedural equity (equal regard for all generations) with the widely admitted Pareto principle. Although some positive results have been obtained, most of this literature stemming from Diamond (1965) has come to a negative conclusion. Even if equitable Paretian criteria do exist, they cannot be explicitly described (Basu and Mitra , 2003; Zame , 2007; Lauwers , 2010).

At the same time, several authors have pointed out the distributional consequences of not discounting future generations' welfare. Mirrlees (1967) computed optimal intertemporal consumption patterns in plausible economic models using the undiscounted utilitarian criterion (the so-called Ramsey criterion). He observed that present generations should save up to 50 % of their net income for the sake of future generations. The finding was best summarized by philosopher John Rawls who declared that "the utilitarian doctrine may direct us to demand heavy sacrifices of the poorer generations for the sake of greater advantages for the later ones that are far better-off"(Rawls, 1971, p. 253). He went on saying that "these consequences can be to some degree corrected by discounting the welfare of those living in the future"(Rawls, 1971, p. 262).

Although Rawls did not adopt discounted utilitarianism (for the very reason that it fails to comply with procedural equity), most of the economic literature

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has endorsed it, considering the solution as the lesser of two evils. Yet the conflict between procedural equity and distributional equity in a utilitarian context has remained unsolved.

The above distributional justification for discounted utilitarianism critically relies on the assumption that future generations can be made better-off. Asheim and Buchholz (2003) noticed that, in certain technological contexts, for instance in the Dasgupta-Heal-Solow model of growth with a non-renewable resource, future generations may not be better-off. Undiscounted utilitarianism may then yield more satisfactory recommendations than discounted utilitarianism. The key point is that the discount rate is only a way of preventing too much sacrifices for the sake of people who are already favored. For this to be true, it is critical that generations' rank in time corresponds to their rank in the income distribution.

If we retain the interpretation of the discount factor as preventing high sacrifices from the poor, it looks closely related to the social weights used in rank-dependent measures of social welfare. An example of a rank-dependent criterion is the Gini social welfare function. Generalizations thereof have been proposed by Weymark (1981) and Ebert (1988). The main feature of rank-depended social welfare functions is that they put more weight on the utility of the worse-offs. Rank-dependent weights simply represent the society's aversion to inequality.

In this paper, I propose to apply rank-dependent methods to intergenerational justice. More precisely, I put forward rank-discounted utilitarian social welfare functions:

$$W(\mathbf{x}) = (1 - \beta) \sum_{t \in \mathbb{N}} \beta^{t-1} u(x_{[t]})$$

In this expression, the consumption stream $\mathbf{x}_{[\ ]} = (x_{[1]}, x_{[2]}, \cdots, x_{[t]}, \cdots)$ is a reordering of the consumption stream $\mathbf{x} = (x_1, x_2, \cdots, x_t, \cdots)$ such that $x_{[1]} \leq x_{[2]} \leq \cdots \leq x_{[t]} \leq \cdots$.

The rank-discounted utilitarian approach coincides with discounted utilitarianism on the set of non-decreasing consumption streams. Utility discounting is then justified as an expression of inequality aversion when future generations are better-off. However, and contrary to the discounted utilitarian approach,

rank-discounted utilitarianism also satisfies procedural equity: two intergenerational consumption streams that are identical up to a permutation are deemed equally good.

Rank-discounted utilitarian social welfare functions respect both procedural equity and the weak Pareto principle on their domain of definition. They hence overcome the impossibility results in the tradition of Diamond (1965) on their domain. A natural question is then: under which conditions can rank-discounted utilitarian social welfare functions be defined? An evident necessary condition is that consumption streams can be re-ordered in non-decreasing sequences. It turns out that this is a non-trivial task. In Section 3, I characterize the set of infinite consumption streams that can be re-ordered. Restricting attention to this set permits to obtain equitable Paretian representations of social preferences.

In Section 4, I offer a complete characterization of rank-discounted utilitarian preferences. This characterization is clearly related to Koopmans (1960) characterization of discounted utilitarian preferences. The difference is that his separability and stationarity axioms are imposed on non-decreasing streams only. Independence axioms on ordered streams are common in the theory of decision under uncertainty (Gilboa , 1987; Wakker , 1993) and in the theory of inequality measurement (Weymark , 1981; Ebert , 1988). With the exception of Rébillé (2007), they have never been used in the theory of intertemporal decision making yet. They permit to weight utilities according to their rank in a distribution, which is exactly what rank-discounted utilitarian criteria do.

In Section 5, I provide conditions for a social observer using a rank-discounted utilitarian criterion to be inequality averse, in the sense that she always prefers a consumption stream obtained from another through a Pigou-Dalton redistributive transfer. I also provide conditions to compare two social observers in terms of inequality aversion. When the social observer has homothetic preferences, these conditions are very simple: she needs to discount ranks more and to use a more concave utility function.

The importance of distributive equity in the spirit of Atkinson (1970) has been addressed in many papers in the literature on intergenerational equity.[1]

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[1]For an early discussion, see Birchenhall and Grout (1979). Bossert, Sprumont and Suzu-

However, they have not investigated the consequences of different degrees of inequality aversion on society's choice. I will on the contrary uphold that inequality aversion is a central notion for intergenerational problems and that different degrees of inequality aversion have important policy implications.

A parallel can be drawn with optimal taxation problems. In optimal taxation problems, there is a trade-off between efficiency and distributional equity that usually arises from imperfect information issues (typically, work effort is not observable, so that the society needs to provide incentives preventing equality). In intertemporal problems, there is also an efficiency-equity trade-off, but it now arises from the technological asymmetry: it is possible today to accumulate capital in order to produce more tomorrow, but the reverse is impossible. Like in the optimal taxation problems, inequality aversion plays a key role in the definition of the optimal distribution. So in intergenerational problems inequality aversion should modify policy recommendations. I show that it is indeed the case.

In Section 6, I explore the implications of rank-discounted utilitarian social welfare functions for the social discount rate. The social discount rate has been one of the most debated economic parameter in recent years. The highly publicized debates surrounding the question of climate change have highlighted its importance for policy evaluation. An 'ethical' view has suggested low values for the social discount rate, on the ground that pure-time discounting violates procedural equity. Rank-discounted utilitarianism suggests an alternative 'ethical' view on discounting: discounting is only an expression of society's aversion to inequality.

In Section 6, I indeed prove that a more inequality averse social observer always discount more the future, *provided that future generations are better-off*. This has important policy implications. If future generations are expected to be better-off in spite of climate change, then a more inequality averse rank-discounted utilitarian decision maker will rather adopt the recommendations of Nordhaus (2008) to have gradual emission control policies rather than those of

mura (2007) and Hara, Shinotsuka, Suzumura and Xu (2008) are examples of recent discussions.

Stern (2006) who calls for immediate action. However, with the rank-dependent utilitarian model, discounting depends on a generation's rank in the intergenerational distribution rather than its rank in time. Then, if future generations are expected to be less well-off because of climate change, the social discount rate should on the contrary be negative, and strong action should be undertaken to mitigate climate change.

In Section 7, I show that inequality aversion also plays a role in the choice of the optimal growth policy. I first indicate how RDU preferences can be extended to obtain operational choice criteria. The extension is based on versions of the procedural equity and efficiency properties underlying RDU. I prove that, under a technological requirement of productivity, ERDU preferences promote sustainable discounted utilitarian policies: they chose the non-decreasing path maximizing discounted utility. The result can be applied to two benchmark models: the Ramsey growth model and the Dasgupta-Heal-Solow model of growth with a non-renewable resource. Inequality aversion plays a crucial role. A more inequality averse society indeed less often suggests any growth: if the initial stock of capital is high enough, the society prefers to maintain consumption for ever. More inequality aversion then yields lower long-run perspectives.

To reach these conclusions, I start in Section 2 by introducing the framework of our analysis.

2 The framework

Let $\mathbb{N}$ denote as usual the set of natural numbers $\{1, 2, 3, \cdots\}$. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}_+$ the set of nonnegative real numbers, and $\mathbb{R}_{++}$ the set of positive real numbers.

Denote $\mathbf{x} = (x_1, x_2, \cdots, x_t, \cdots)$ an infinite stream (or allocation), where $x_t \in \mathbb{R}_+$ is a one-dimensional indicator of the well-being of generation t. Contrary to most of the literature on the evaluation of infinite streams, x_t is not assimilated to a utility number, but to the aggregate consumption of generation t. Let restrict attention to intergenerational allocations consisting in bounded consumption

streams, and denote

$$\mathbf{X} = \left\{ \mathbf{x} = (x_1, \cdots, x_t, \cdots) \in \mathbb{R}_+^{\mathbb{N}} : \sup_t x_t < +\infty \right\}$$

the set of possible allocations.

For $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, write $\mathbf{x} \geq \mathbf{y}$ whenever $x_t \geq y_t$ for all $t \in \mathbb{N}$; write $\mathbf{x} > \mathbf{y}$ if $\mathbf{x} \geq \mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$; and write $\mathbf{x} \gg \mathbf{y}$ whenever $x_t > y_t$ for all $t \in \mathbb{N}$. For any $T \in \mathbb{N}$ and $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, denote $\mathbf{x_T y}$ the consumption stream $\mathbf{z}$ such that $z_t = x_t$ for all $t \leq T$ and $z_t = y_t$ for all $t > T$. For any $x \in \mathbb{R}_+$ and $\mathbf{y} \in \mathbf{X}$, $(x, \mathbf{y})$ denotes the stream $(x, y_1, y_2, \cdots)$.

Three subsets of $\mathbf{X}$ will be of particular interest. First, the set of stationary consumption streams, denoted $\mathbf{X^c}$. For any $x \in \mathbb{R}_+$, $\mathbf{x^c}$ denote the allocation $\mathbf{x^c} \in \mathbf{X}$ such that $x_t^c = x$ for all $t \in \mathbb{N}$. The set of stationary consumption streams is $\mathbf{X^c} = \{\mathbf{x^c}, x \in \mathbb{R}_+\}$.

A second subset of $\mathbf{X}$ is the set of non-decreasing streams in $\mathbf{X}$. This set is denoted $\mathbf{X^+} = \{\mathbf{x} \in \mathbf{X} : x_t \leq x_{t+1} \forall t \in \mathbb{N}\}$.

The last subset of $\mathbf{X}$ playing a key role in the remainder of the paper is the set of allocations whose elements can be permuted to obtain non-decreasing streams. This set is denoted $\bar{\mathbf{X}}$. To introduce it formally, some more notation is needed. Let Π be the set of all permutations on $\mathbb{N}$. For any $\pi \in \Pi$ and $\mathbf{x} \in X$, let $\mathbf{x_\pi} = (x_{\pi(1)}, x_{\pi(2)}, \cdots, x_{\pi(t)}, \cdots)$. The definition of the set $\bar{\mathbf{X}}$ is then as follows $\bar{\mathbf{X}} = \{\mathbf{x} \in \mathbf{X} : \exists \pi \in \Pi, \mathbf{x_\pi} \in \mathbf{X^+}\}$. For $\mathbf{x} \in \bar{\mathbf{X}}$, let $\mathbf{x_{[\ ]}}$ be an allocation such that there exist a permutation $\pi \in \Pi$ for which $x_{[t]} = x_{\pi(t)}$, whatever $t \in \mathbb{N}$, and $\mathbf{x_{[\ ]}} \in \mathbf{X^+}$. Hence $[t]$ is the generation whose rank is t in the intergenerational distribution. And $\mathbf{x_{[\ ]}}$ is a re-ordering of $\mathbf{x}$ in a non-decreasing sequence.[2] It will also be useful to denote $\{t\}$ the rank of generation t in the intergenerational distribution.

The following inclusions hold: $\mathbf{X^c} \subset \mathbf{X^+} \subset \bar{\mathbf{X}} \subset \mathbf{X}$. In a finite setting, $\bar{\mathbf{X}}$ would be exactly equal to $\mathbf{X}$. But, as will be discussed in the next section, this

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[2]There is not necessarily a unique $\mathbf{x_{[\ ]}}$ for a given $\mathbf{x} \in \bar{\mathbf{X}}$. For instance, if $x_t = x_\tau$ for some $t \neq \tau$, there are at least two re-orderings in a non-decreasing sequence. In the sequel, $\mathbf{x_{[\ ]}}$ denote any allocation satisfying the conditions. The Anonymity Axiom will guarantee that the choice of a particular allocation does not matter.

is not the case any more in an infinite setting, because some sequences may not be reordered.

A social welfare relation (SWR) on a subset $\widetilde{\mathbf{X}} \subset \mathbf{X}$ is a binary relation $\succeq$, where for any $\mathbf{x}, \mathbf{y} \in \widetilde{\mathbf{X}}$, $\mathbf{x} \succeq \mathbf{y}$ entails that the consumption stream $\mathbf{x}$ is deemed socially at least as good as $\mathbf{y}$. Let $\sim$ and $\succ$ denote the symmetric and asymmetric parts of $\succeq$. A social welfare function (SWF) representing $\succeq$ is a mapping $W : \widetilde{\mathbf{X}} \to \mathbb{R}$ with the property that for all $\mathbf{x}, \mathbf{y} \in \widetilde{\mathbf{X}}$, $W(\mathbf{x}) \geq W(\mathbf{y})$ if and only if $\mathbf{x} \succeq \mathbf{y}$.

3   The representation of equitable Paretian social welfare relations: the role of ordered streams

The difficulty of representing equitable and Paretian preferences over infinite utility streams has been the topic of a prolific literature since the seminal contribution by Diamond (1965). General possibility results exist (for instance Svensson , 1980), but most of the literature has reached negative a conclusion: although equitable efficient preferences exist, they cannot be explicitly described (Basu and Mitra , 2003; Zame , 2007; Lauwers , 2010).

In this section, I highlight the role that ordered streams can play to overcome the impossibility on part of the domain. Let $\succeq$ a SWR that is used to rank alternative intergenerational allocations in $\widetilde{\mathbf{X}}$. The set $\widetilde{\mathbf{X}}$ is a subset of $\mathbf{X}$ on which I seek to obtain an equitable Paretian (efficient) representation of $\succeq$. A first fundamental property that will be required of the SWR is the following:

Axiom 1 Order. *The relation $\succeq$ is complete, reflexive and transitive on $\widetilde{\mathbf{X}}$.*

A SWR satisfying Axiom 1 is named a social welfare order (SWO). The literature has often looked for SWOs on the whole set $\mathbf{X}$. However, many popular criteria for the evaluation of infinite streams are incomplete. Notable examples are the so-called von Weizsäcker criterion and Gale criterion. Recent literature has promoted appealing incomplete SWRs that can be completed by use of Szpilrajn's lemma (see, among others, Bossert, Sprumont and Suzumura , 2007; Basu and Mitra , 2007). In this paper, the analysis focuses on a subset $\widetilde{\mathbf{X}}$ only.

The SWR may be incomplete on the whole domain $\mathbf{X}$, but it is complete on the sub-domain $\widetilde{\mathbf{X}}$.

In addition to Axiom 1, the core axioms in the literature on the aggregation of infinite streams are the Pareto axioms and the anonymity axioms which represent the ideal of equal concern for all generations (procedural equity).

In this paper, I use an intermediate version of the widely admitted Pareto principle.

Axiom 2 Intermediate Pareto. *(i) For any $\mathbf{x}, \mathbf{y} \in \widetilde{\mathbf{X}}$, if $\mathbf{x} \geq \mathbf{y}$ then $\mathbf{x} \succeq \mathbf{y}$. (ii) For any $\mathbf{x}, \mathbf{y} \in \widetilde{\mathbf{X}}$, if there exists $T \in \mathbb{N}$ such that $x_t = y_t$ for all $t < T$ and $x_t > y_t$ for all $t \geq T$, then $\mathbf{x} \succ \mathbf{y}$.*

The Intermediate Pareto axiom is a requirement concerning the respect of generations' interests. It ensures that, whenever all generations experience an increase in their consumptions, the social welfare does not decrease. As indicated by its name, the axiom is weaker than the Strong Pareto axiom but stronger than the Weak Pareto axiom. The axiom is slightly different from the 'Intermediate Pareto' of Lauwers (2010): Lauwers postulates sensitivity in each infinite set of coordinates, while I assume sensitivity to a particular infinite set of coordinates, namely 'future' ones.

The reason why I use this new version of the axiom will appear in Section 7: it will make it possible to ensure that in the presence of a productive technology only non-decreasing paths are chosen. However, the Weak Pareto axiom would be sufficient to obtain the characterization result in Section 4. Remark that, contrary to the Weak Pareto axiom, the Intermediate Pareto axiom excludes a dictatorship of the present in the sense of Chichilnisky (1996).

Throughout the paper, the most general version of the anonymity requirement will be used.

Axiom 3 Anonymity. *For any $\pi \in \Pi$ and $\mathbf{x} \in \widetilde{\mathbf{X}}$, $\mathbf{x} \sim \mathbf{x}_\pi$.*

I use this strong version in order to draw conclusions in terms of inequality aversion. Inequality aversion deals with the distribution of resources, independently of whom receives the resource. It is hence necessary to deem equivalent

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consumption streams inducing the same intergenerational distribution, which is what the stronger Anonymity axiom does.

A weaker version of anonymity that is often considered in the literature involves only finite permutations. Finite permutations on $\mathbb{N}$ are permutations that differ from identity on a finite set. Let denote Π_F the set of finite permutations.

Axiom 3' (Weak Anonymity). *For any $\pi \in \Pi_F$ and $\mathbf{x} \in \widetilde{\mathbf{X}}$, $\mathbf{x} \sim \mathbf{x}_\pi$.*

A SWR satisfying Axiom 1-3 will be named an equitable Paretian SWO and a SWR satisfying axioms 1, 2 and 3' a weakly equitable Paretian SWO. A SWO can be 'explicitly described' if its graph is a definable set (i.e. there exists a set-theoretic formula that defines it; see Zame , 2007, p. 197, for a formal definition). A SWO is representable if there exists a SWF that represents it. A representable SWO can be explicitly described.

Focusing on the set $\bar{\mathbf{X}}$, it becomes possible to obtain representable equitable Paretian SWOs. This is exemplified by rank-discounted utilitarian SWOs (in short RDU SWOs).

Definition 1 Rank-Discounted Utilitarian SWO. *A SWR on a $\bar{\mathbf{X}}$ is a Rank-Discounted Utilitarian SWO (RDU SWO) if and only if it is represented by the social welfare function:*

$$W(\mathbf{x}) = (1 - \beta) \sum_{t \in \mathbb{N}} \beta^{t-1} u(x_{[t]}) \tag{1}$$

where $0 < \beta < 1$ is a real number and the function u is continuous and increasing.

An accepted response to impossibility results in social choice theory is to look for restricted domains on which possibility is restored. It turns out that $\bar{\mathbf{X}}$ is a domain which allows to overcome many negative results obtained in the literature on the aggregation of infinite streams. In particular, the next Proposition contrasts some possibilities allowed by $\bar{\mathbf{X}}$ with two important impossibility results found in the recent literature:

Proposition 1.

1. (a) *There exist no equitable Paretian SWOs on* $\mathbf{X}$.

   (b) *But there exist equitable Paretian SWOs on* $\bar{\mathbf{X}}$.

2. (a) *No weakly equitable Paretian SWOs on* $\mathbf{X}$ *can be 'explicitly described'.*

   (b) *But there exist representable weakly equitable Paretian SWOs on* $\bar{\mathbf{X}}$.

Proof. The proof of 1.(b) and 2.(b) is provided by RDU SWOs who satisfy Axioms 1 , 2 and 3 (and thus 3') on the set $\bar{\mathbf{X}}$.

The proof of 1.(a) is Theorem 1 in Fleurbaey and Michel (2003). Let us briefly recall the proof. Consider the intergenerational allocations $\mathbf{x} = (2, 3, 1, 4, 1/2, 5, 1/3, 6, \cdots, k, 1/k - 2 \cdots)$ and $\mathbf{y} = (1, 2, 1/2, 3, 1/3, 4, 1/4, \cdots, k, 1/k, \cdots)$. An equitable Paretian SWR $\succeq$ on $\mathbf{X}$ must rank $\mathbf{x} \succ \mathbf{y}$ by Axiom 2. But, by Axiom 3, it must also be indifferent between the two options, $\mathbf{x} \sim \mathbf{y}$, a contradiction.

The proof of 2.(a) is Theorem 4 in Zame (2007). $\qquad\square$

The set $\bar{\mathbf{X}}$ plays a key role in obtaining equitable Paretian SWFs in the intergenerational framework. It seems pertinent to characterize it in greater details, in order to evaluate its extend compared to the whole set $\mathbf{X}$.

To get an intuition of what is required of an infinite consumption stream to be re-ordered, consider the sequence $\mathbf{x} = (1, 0, 0, \cdots)$.[3] For any $\pi \in \Pi$, it is necessarily the case that[4] $\pi(1) < +\infty$ so that any re-ordered sequence has the form $(0, 0, \cdots, 0, 1, 0, \cdots)$. Hence $\mathbf{x}$ cannot be reordered to form a non-decreasing sequence.

The example illustrates what it takes for an infinite stream not to belong to $\bar{\mathbf{X}}$. This is all that is needed to completely characterize the set $\bar{\mathbf{X}}$. To do so, the following notation must be introduced: $\Lambda(\mathbf{x}, t) = \{\tau \in \mathbb{N}, \tau > t : x_\tau < x_t\}$. Denote $card\big[\Lambda(\mathbf{x}, t)\big]$ the cardinality of the set $\Lambda(\mathbf{x}, t)$.

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[3] I would like to thank Geir Asheim for mentioning this example, which led to the characterization of the set $\bar{X}$.

[4] The fact that $\pi(1) < +\infty$ for any $\pi \in \Pi$ does not mean that attention is restricted to finite length permutations. Finite length permutations entail that there exists a number $l < +\infty$ such that $|\pi(t) - t| < l$, which may not be the case if the distance between t and $\pi(t)$ increases with t (at least for some t). Still, $|\pi(1) - 1|$ must be finite for any infinite permutation.

Proposition 2. *An intergenerational allocation* $\mathbf{x}$ *belongs to* $\bar{\mathbf{X}}$ *if and only if for any* $t \in \mathbb{N}, card\big[\Lambda(\mathbf{x}, t)\big] < +\infty$.

Proof.

Necessity: Assume that $\mathbf{x} \in \mathbf{X}$ is such that, for some $t \in \mathbb{N}$, $card\big[\Lambda(\mathbf{x}, t)\big] = +\infty$. Then, for any $\pi \in \Pi$, $\pi(t) < +\infty$ and it is impossible that for all $\tau \in \Lambda(\mathbf{x}, t)$, $\pi(\tau) < \pi(t)$: $\mathbf{x}$ cannot be re-ordered to form a non-decreasing sequence.

Sufficency: Let $\mathbf{x} \in \bar{\mathbf{X}}$. The set $\Lambda(\mathbf{x}, 1)$ is finite and can be re-ordered in non-decreasing order. These coordinate will form the n_1 first elements of the ordered stream, with $n_1 = card\big[\Lambda(\mathbf{x}, 1)\big]$. And $\pi(1) = n_1 + 1$. Then let t_2 be the first period such that $x_{t_2} \geq x_1$. The set $\Lambda(\mathbf{x}, t_2) \setminus \Lambda(\mathbf{x}, 1)$ is finite and can be ordered in increasing order. These will form the n_2 next elements in the ordered stream, with $n_2 = card\big[\Lambda(\mathbf{x}, t_2) \setminus \Lambda(\mathbf{x}, 1)\big]$. And $\pi(t_2) = n_1 + n_2 + 2$. Pursuing this procedure, the stream $\mathbf{x}$ can be completely ordered. □

Proposition 2 delineates the set of sequences that can be reordered to form non-decreasing streams. It appears that sufficient 'increasingness' of the initial stream is required: at any point in time all future generations but a finite number of them must be better-off. This makes clear that many consumption streams, including decreasing ones, cannot be ranked by the RDU criterion. A question that one may naturally ask is whether it is possible to define equitable Paretian SWFs on other subsets of $\mathbf{X}$.

An obvious example is the set $\underline{\mathbf{X}}$ of infinite streams that can be re-ordered in *non-increasing sequences*. Using the same methods as in Proposition 2, this set can be characterized as the set of infinite streams such that at any period all future generations but a finite number must be less well-off. For any $\mathbf{x} \in \underline{\mathbf{X}}$, denoting $\mathbf{x}_{[\ ]_-}$ the re-ordering in a non-increasing sequence of $\mathbf{x}$, the following class of equitable Paretian SWFs exists:

$$W(\mathbf{x}) = \sum_{t \in \mathbb{N}} a_t u(x_{[t]_-})$$

where each a_t is positive scalar. The problem with these representations is that $\lim_{t \to +\infty} a_t = 0$, so that the welfare of the worst-off generation is not taken

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into account in the social welfare. This violates self-evident distributive equity principles.

Outside $\bar{\mathbf{X}} \cup \underline{\mathbf{X}}$, it is difficult to see whether one can represent equitable Paretian SWFs. It must be noticed however that outside $\bar{\mathbf{X}} \cup \underline{\mathbf{X}}$ there are sequences with both increasing and decreasing subsequences. Contradictions like the one in the proof of 1.a. of Proposition 1 are therefore likely to arise.

Despite their restricted domain of definition, I will show in Section 7 that RDU criteria can be operationalized by extending their range with properties underlying their definition.

4 Rank-discounted utilitarian social welfare functions

In this section, I offer a characterization of RDU SWFs.

Axioms 1-3 are usual ingredients in the characterization of SWFs. Another usual requirement is continuity. It guarantees that the social ranking does not change too much for small errors on the exact allocation.

Axiom 4 Continuity. *For all* $\mathbf{x}, \mathbf{y}$ *in* $\bar{\mathbf{X}}$*, if a sequence* $\mathbf{x}^1, \mathbf{x}^2, \cdots, \mathbf{x}^k, \cdots$ *of allocations in* $\bar{\mathbf{X}}$ *is such that* $\lim_{k \to \infty} \sup_{t \in \mathbb{N}} |x_t^k - x_t| = 0$ *and, for all* $k \in \mathbb{N}$*,* $\mathbf{x}^k \succeq \mathbf{y}$ *(resp.* $\mathbf{x}^k \preceq \mathbf{y}$*), then* $\mathbf{x} \succeq \mathbf{y}$ *(resp.* $\mathbf{x} \preceq \mathbf{y}$*).*

Continuity axioms have been carefully discussed in the literature on intergenerational equity because they may conflict with other ethical principles. In particular, some forms of continuity conflict with the anonymity axiom in infinite settings. In this respect, the sup norm continuity promoted by Axiom 4 seems appropriate, for it is compatible with the most general anonymity axiom (Efimov and Koshevoy , 1994, Theorem 4). The sup topology is the coarsest topology doing so under the monotonicity requirement (Lauwers , 1997, Theorem 1).

The fifth axiom used in the characterization is a dominance axiom that completes the efficiency requirements imposed by Axiom 2.

Axiom 5 Restricted Dominance. *For any positive real numbers* x *and* y*, if* $x > y$ *then* $(x, \mathbf{x^c}) \succ (y, \mathbf{x^c})$*.*

The Restricted Dominance axiom prevents a dictatorship of the future, in the sense of Chichilnisky (1996). Indeed, it entails that a strict preference by the current poor generation may be sufficient to impose a strict preference for the society: the situation of future generations is not the only thing that matters from a social point of view.

A key condition to obtain rank-dependent social welfare functions is an independence condition closely related to the comonotonic sure-thing principle that has been introduced in the theory of decision under uncertainty (see Gilboa , 1987; Wakker , 1993).

Axiom 6 Independence on Non-decreasing Streams. *For any* $\mathbf{x}, \mathbf{y}, \mathbf{x}', \mathbf{y}' \in \mathbf{X}^+$ *and for any* $\mathcal{T} \subset \mathbb{N}$, *if (i)* $x_t = x'_t$ *and* $y_t = y'_t$ *for all* $t \in \mathcal{T}$; *(ii)* $x_t = y_t$ *and* $x'_t = y'_t$ *for all* $t \in \mathbb{N} \setminus \mathcal{T}$; *then*

$$\mathbf{x} \succeq \mathbf{y} \iff \mathbf{x}' \succeq \mathbf{y}'$$

Axiom 6 states that the social evaluation is unaffected by unconcerned generations provided that the ranking of generations is left intact. Therefore, the only way the comparisons of the welfare of two generations may be affected by the welfare of other generations is through their ranks in the intergenerational distribution. Axiom 6 enables to retain part of the separability properties of independence axioms, while allowing the ranks to matter in the social evaluation. These are precisely the main features of RDU SWFs, which are additively separable on ordered streams, but have rank-dependent social weights.

Our last axiom is also a rank dependent axiom. It corresponds to the widely used stationarity axiom, but the application is restricted to non-decreasing streams. Stationarity implies constant utility trade-offs between consecutive period. Similarly, our axiom will imply constant utility trade-offs, but for generations with consecutive ranks in the intergenerational distribution.

Axiom 7 Stationarity on Non-decreasing Streams. *For any* $\mathbf{x}, \mathbf{y}, \in \mathbf{X}^+$ *and for any* $z \in \mathbb{R}_+$, *such that* $z \leq \min(x_1, y_1)$

$$(z, \mathbf{x}) \succeq (z, \mathbf{y}) \iff \mathbf{x} \succeq \mathbf{y}$$

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The main representation result of the paper can now be stated.

Proposition 3. *A SWR $\succeq$ on $\bar{\mathbf{X}}$ satisfies Axioms 1-7 if and only if it is a RDU SWO.*

Proof. The proof is in the Appendix. It relies on a simplified version of Koopmans (1960)'s proof, similar to the one in Bleichrodt, Rhode and Wakker (2008). The proof is applied to non-decreasing streams, so that it is necessary to use the techniques developed by Wakker (1993) for additive representation of preferences on rank-ordered sets. Continuity allows us to extend from a finite number of period to an infinite number of periods the representation on non-decreasing streams. Anonymity allows us to extend the representation to whole set $\bar{\mathbf{X}}$. □

Proposition 3 provides a rank-dependent extension of the DU model. Our axioms are close to Koopmans's ones, so that the comparison between discounted utility and rank-discounted utility is straightforward. Our anonymity axiom, which is intuitively appealing in the context of intergenerational justice, can be translated in terms of patience in the context of time preferences (see Rébillé , 2007). Proposition 3 therefore also provides an axiomatization of a family of patient intertemporal utilities on $\bar{\mathbf{X}}$.

One difficulty with RDU SWFs is that they are not recursive. In particular, they do not satisfy the property of separable future (see Fleurbaey and Michel , 2003). Indeed, the evaluation of a consumption stream from period t onward depends on the past through the rank of past generations (but not through their actual consumptions nor the distribution of consumption between them). Recursivity is restored on the set of non-decreasing sequences, so that traditional methods can be used if an additional sustainability requirement of non-decreasingness is imposed. Still, non-recursivity may be a source of analytical complexity. I will therefore discuss the applicability of the criterion in more detail in the last section of the paper.

It is convenient in applications to consider the more specific class of homothetic RDU SWFs (in short, HRDU SWFs). HRDU SWFs indeed yield clear-cuts

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results for comparisons of inequality aversions and for the expression of the discount rate.

Definition 2. *A SWR $\succeq$ on $\bar{\mathbf{X}}$ is a homothetic rank-discounted utilitarian SWR if and only if it can be represented by the SWF:*

$$W(\mathbf{x}) = \begin{cases} (1-\beta) \sum_{t\in\mathbb{N}} \beta^{t-1} \frac{x_{[t]}^{1-\eta}}{1-\eta} & \eta \neq 1 \\ or \\ (1-\beta) \sum_{t\in\mathbb{N}} \beta^{t-1} \ln\left(x_{[t]}\right) & \eta = 1 \end{cases} \tag{2}$$

HRDU SWFs satisfy a property of relative invariance. For any $\mathbf{x} \in \bar{\mathbf{X}}$ and $\lambda > 0$, let $\lambda\mathbf{x}$ be the consumption stream such that $\lambda\mathbf{x} = (\lambda x_1, \lambda x_2, \cdots, \lambda x_t, \cdots)$.

Axiom 8 Relative invariance. *For any $\mathbf{x}$ and $\mathbf{y}$ in $\bar{\mathbf{X}}$, and any $\lambda > 0$, $\mathbf{x} \sim \mathbf{y} \implies \lambda\mathbf{x} \sim \lambda\mathbf{y}$.*

Axiom 8 states that the equivalence between $\mathbf{x}$ and $\mathbf{y}$ is preserved by a simultaneous multiplication of the consumptions in $\mathbf{x}$ and $\mathbf{y}$ by the same positive factor. One alleged justification for the axiom is that change in the measurement unit of consumption (taking the form of a money conversion) should not alter social judgements. The axiom has actually broader consequences. It also means that the relative situations of generations (their consumption shares) is what matters from the social point of view.

The consequences of Axiom 8 are well-known in the literature on inequality measurement (Ebert , 1988). It can easily be showed that HRDU preferences are the only RDU preferences satisfying Axiom 8. The proof is therefore omitted.

5   Inequality aversion

Up to now, I have addressed the issue of procedural equity and its compatibility with efficiency. In this section, I am going to introduce concerns for distributional equity. I will show that inequality aversion can be properly measured and compared within the RDU class of preferences. The next two sections will show that inequality aversion has significant policy implications. I will henceforth restrict attention to RDU SWOs.

5.1 The Pigou-Dalton transfer principle and inequality aversion

It is a common practice to express distributional equity ideals by means of transfer axioms. In this paper a weak form of the Pigou-Dalton transfer principle is considered:

Axiom 9 Pigou-Dalton transfer principle. *For any* $\mathbf{x}, \mathbf{y} \in \bar{\mathbf{X}}$*, if there exists* $\varepsilon > 0$ *and a pair of positive integers* (τ, τ') *such that* $\varepsilon \leq y_\tau + \varepsilon = x_\tau \leq x_{\tau'} = y_{\tau'} - \varepsilon$ *and* $y_t = x_t$ *for all* $t \neq \tau, \tau'$ *then* $\mathbf{x} \succeq \mathbf{y}$.

In this section, I study the restrictions imposed by Axiom 9 on RDU criteria. These restrictions hold on the rank-discount factor β and on the utility function u in Equation (1). I shall henceforth refer to a particular RDU SWF as $W_{\beta,u}$.

The following index of non-concavity of the function u can be introduced:

$$\mathcal{C}_u = \sup_{0 < \varepsilon \leq x \leq x'} \frac{u(x' + \varepsilon) - u(x')}{u(x) - u(x - \varepsilon)}$$

This index has two interesting properties (Chateauneuf, Cohen and Meilijson , 2005): 1/ $\mathcal{C}_u \geq 1$, with $\mathcal{C}_u = 1$ corresponding to u concave; 2/ and, when u is differentiable, $\mathcal{C}_u = \sup_{y \leq x} \frac{u'(x)}{u'(y)}$.

The non-concavity index $\mathcal{C}_u$ and the discount factor β jointly characterize RDU SWFs satisfying the Pigou-Dalton transfer principle.

Proposition 4. *A RDU SWF* $W_{\beta,u}$ *on* $\bar{\mathbf{X}}$ *satisfies Axiom 9 if and only if*

$$\beta \times \mathcal{C}_u \leq 1$$

Proof. The proof of the Proposition is in the Appendix. □

Condition $\beta \times \mathcal{C}_u \leq 1$ means that the utility function u must not be 'too concave'. The concavity of u, though sufficient, is not necessary for a RDU SWO to satisfy the Pigou-Dalton transfer principle. The (weak) concavity of u is however necessary and sufficient if attention is restricted to HRDU SWFs. For a HRDU SWF it is indeed the case that $\mathcal{C}_u = 1$ whenever $\eta \geq 0$ and $\mathcal{C}_u = +\infty$ whenever $\eta < 0$. This is summarized in the following corollary of Proposition 4:

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Corollary 1. *A HRDU SWO on* $\bar{\mathbf{X}}$ *satisfies Axiom 9 if and only if* $\eta \geq 0$.

5.2 Comparative inequality aversion

Ranking different criteria according to the strength of their concerns for equality is an important prerequisite to study the policy implications of inequality aversion. The common way to do so is to compare the degree of inequality aversion of the underlying SWFs. The aim of this section is to perform such comparisons in the case of RDU SWFs.

I follow the procedure proposed in the literature on risk/uncertainty aversion to make such comparisons (Grant and Quiggin , 2005). It consists in: (i) defining an inequality relation $\succ_I$; (ii) declaring a SWO $\succeq$ at least as inequality averse as a SWO $\widehat{\succeq}$ if, for any allocation $\mathbf{y}$, whenever a less unequal allocation $\mathbf{x}$ (according to $\succ_I$) is preferred to $\mathbf{y}$ according to $\widehat{\succeq}$, then $\mathbf{x}$ is also preferred to $\mathbf{y}$ according to $\succeq$.

I use a simple definition of the relation 'more unequal than' based on the notion of local increase in inequality. is based on increases in the inequalities affecting only two generations and leaving generations' ranks unchanged. That is why I use the expression 'local increase in inequality'. For the definition, recall that $\{t\}$ denotes the rank of generation t in the intergenerational distribution.

Definition 3. *For any* $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\mathbf{y}$ *represents an elementary increase in inequality with respect to allocation* $\mathbf{x}$, *denoted* $\mathbf{y} \succ_I \mathbf{x}$, *if there exists a pair of positive real numbers* $(\varepsilon, \varepsilon')$ *and a pair of positive integers* (τ, τ') *such that* $y_\tau + \varepsilon = x_\tau \leq x_{\tau'} = y_{\tau'} - \varepsilon'$, $x_{\widehat{\tau}} \leq y_\tau$, $y_{\tau'} \leq x_{\widehat{\tau}'}$ *and* $y_t = x_t$ *for all* $t \neq \tau, \tau'$, *where* $\widehat{\tau}$ *and* $\widehat{\tau}'$ *are such that* $\{\widehat{\tau}\} = \{\tau\} - 1$ *and* $\{\widehat{\tau}'\} = \{\tau'\} + 1$.

The inequality relation $\succ_I$ is used to define comparative inequality aversion:

Definition 4. *A SWO* $\succeq$ *is at least as inequality averse as a SWO* $\widehat{\succeq}$ *if, for any* $\mathbf{x}$ *and any* $\mathbf{y} \succ_I \mathbf{x}$: *(i)* $\mathbf{x} \widehat{\succeq} \mathbf{y} \Longrightarrow \mathbf{x} \succeq \mathbf{y}$, *and (ii)* $\mathbf{x} \widehat{\succ} \mathbf{y} \Longrightarrow \mathbf{x} \succ \mathbf{y}$.

Consider now two RDU SWFs, $W_{\beta,u}$ and $W_{\hat{\beta},\hat{u}}$, representing two RDU SWOs $\succeq$ and $\widehat{\succeq}$. To assess relative inequality aversion, the discount factors β and $\hat{\beta}$

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and the relative concavity of the utility functions u and $\hat{u}$ must be compared. More precisely, the following two indices can be introduced:

$$\mathcal{D}_{(\beta,\hat{\beta})} = \inf_{t<t'} \frac{\beta^t/\hat{\beta}^t}{\beta^{t'}/\hat{\beta}^{t'}} = \begin{cases} \frac{\hat{\beta}}{\beta} & \text{if } \beta \le \hat{\beta} \\ 0 \text{ if} & \text{if } \beta > \hat{\beta} \end{cases}$$

and

$$\mathcal{C}_{(u,\hat{u})} = \sup_{0 \le x_1 < x_2 \le x_3 < x_4} \frac{\left[u(x_4) - u(x_3)\right]/\left[\hat{u}(x_4) - \hat{u}(x_3)\right]}{\left[u(x_2) - u(x_1)\right]/\left[\hat{u}(x_2) - \hat{u}(x_1)\right]}$$

The index $\mathcal{D}_{(\beta,\hat{\beta})}$ is an index of the relative decreasing speed of the social weights. The faster the social weights decrease, the less the society cares for better-off generations. The index $\mathcal{C}_{(u,\hat{u})}$ is an index of relative concavity of the utility functions u and $\hat{u}$. As noticed by Grant and Quiggin (2005), $\mathcal{C}_{(u,\hat{u})} \ge 1$, with $\mathcal{C}_{(u,\hat{u})} = 1$ corresponding to the case where u is an increasing concave transformation of $\hat{u}$. In addition, if u and $\hat{u}$ are differentiable, $\mathcal{C}_{(u,\hat{u})} = \sup_{y \le x} \frac{u'(x)\hat{u}'(y)}{u'(y)\hat{u}'(x)}$.

For two RDU SWFs, comparative inequality aversion can be characterized as follows:

Proposition 5. *Consider two RDU SWFs $W_{\beta,u}$ and $W_{\hat{\beta},\hat{u}}$ on $\bar{\mathbf{X}}$. $W_{\beta,u}$ is at least as inequality averse as $W_{\hat{\beta},\hat{u}}$ if and only if*

$$\mathcal{D}_{(\beta,\hat{\beta})} \ge \mathcal{C}_{(u,\hat{u})}$$

Proof. The proof is in the Appendix $\qquad\square$

It is clear from Proposition 5 that a necessary condition for $W_{\beta,u}$ to be at least as inequality averse as $W_{\hat{\beta},\hat{u}}$ is that $\beta \le \hat{\beta}$. A more inequality averse RDU social observer should have a lower rank-discount factor and thus discount more the utility of better-off generations. And in the case $\beta = \hat{\beta}$, u must be a concave transformation of $\hat{u}$.

Even clearer results can be obtained in the case of HRDU SWFs. Indeed, it is straightforward that, whenever $u(x) = \frac{x^{1-\eta}}{1-\eta}$ and $\hat{u}(x) = \frac{x^{1-\hat{\eta}}}{1-\hat{\eta}}$, $\mathcal{C}_{(u,\hat{u})} = 1$

if $\eta \geq \hat{\eta}$, and $\mathcal{C}_{(u,\hat{u})} = +\infty$ if $\eta < \hat{\eta}$. Denote $W_{\beta,\eta}$ a HRDU SWF with rank-discount factor β and utility function $u(x) = \frac{x^{1-\eta}}{1-\eta}$, there exist simple conditions for comparative inequality aversion of HRDU SWFs:

Corollary 2. *Consider two HRDU SWFs $W_{(\beta,\eta)}$ and $W_{\hat{\beta},\hat{\eta}}$ on $\bar{\mathbf{X}}$. $W_{\beta,\eta}$ is at least as inequality averse as $W_{(\hat{\beta},\hat{\eta})}$ if and only if $\beta \leq \hat{\beta}$ and $\eta \geq \hat{\eta}$ (with at least one strict inequality for the two SWFs not to be identical).*

As in the static case inequality aversion is a key policy parameter, so in intertemporal problems it is bound to play an important role in designing the optimal policy. In Section 6, I describe how it affects social discounting. In Section 7, I study optimal RDU policies, and I highlight the impact of inequality aversion.

6 Rank-discounted utilitarianism and social discounting

The social discount rate plays a key role in intertemporal social cost-benefit analysis. In recent years, few economic parameters have attracted as much attention.

Indeed, triggered by the Stern (2006) review of climate change, the social discount rate has been hotly debated, notably in contributions by Nordhaus (2007), Weitzman (2007), and Dasgupta (2008). The controversy has not held on the social welfare function used to assess different paths: all the authors have endorsed the DU approach. The controversy has held on the value of the parameters in the DU SWF, $W(\mathbf{x}) = \sum_{t=1}^{+\infty} \beta^{t-1} u(x_t)$. In particular, the time-discount factor β and the elasticity of marginal utility $-xu''(x)/u'(x)$ have a critical role in the determination of the social discount rate. But there has been no consensus on the interpretation and the value of these key parameters.

In this section, I will derive the social discount rate arising from RDU SWFs. Doing so I prove that key parameters of the social discount rate have interpretations in terms of inequality aversion.

Let assume that the function u in Equation (1) is twice continuously differentiable. In that case W is said to be a smooth RDU SWF. Let also consider

consumption paths such that no pairs of generations have the same consumption level. The set of such paths is denoted $\bar{\mathbf{X}}_{\neq}$. A smooth RDU SWF is differentiable on $\bar{\mathbf{X}}_{\neq}$ only.

The social discount rate evaluates how much an increase in marginal consumption in period t is 'worth' in terms of first period consumption. To obtain the formal expression, imagine that today (period 1) the society can make a marginal investment ε whose rate of return is r. The generation born in period t will therefore be able to consume $(1 + r)^{t-1}\varepsilon$ more units of aggregate good. The change in social welfare through this investment is:

$$dW(\mathbf{x}) = \frac{\partial W}{\partial x_t}(1 + r)^{t-1}\varepsilon - \frac{\partial W}{\partial x_1}\varepsilon$$

The social discount rate is the rate of return that makes the change in social welfare nil, so that the marginal welfare cost of differing consumption is exactly equal to its marginal welfare benefit. The formal definition of the social discount rate $\rho_t(\mathbf{x})$ is therefore:

Definition 5. *Let W be the SWF used to evaluate policies. The social discount rate at period t for a path $\mathbf{x}$ is:*

$$\rho_t(\mathbf{x}) = \left(\frac{\partial W/\partial x_1}{\partial W/\partial x_t}\right)^{\frac{1}{t-1}} - 1$$

Consider a smooth RDU SWF $W_{\beta,u}$. Denote $\delta = \frac{1}{\beta} - 1$ the rank discount rate. Also denote $\eta_u(x) = -xu''(x)/u'(x)$ the elasticity of marginal utility for the utility function u. In addition $g_t(\mathbf{x}) = \frac{x_t - x_1}{x_1}$ is the rate of growth in consumption between period 1 and t, and $\tilde{g}_t(\mathbf{x}) = \frac{g_t(\mathbf{x})}{t-1}$ is the rate growth in annualized terms (the 'average' per period growth between 1 and t).

A simple approximation of the social discount rate associated with a smooth RDU SWF can be derived:

Proposition 6. *Let $W_{\beta,u}$ be a smooth RDU SWF. The social discount rate at*

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period t for a path $\mathbf{x} \in \bar{\mathbf{X}}_{\neq}$ is approximatively equal to:

$$\rho_t(\mathbf{x}) \approx \frac{\{t\} - \{1\}}{t-1}\delta + \eta_u(x_1)\tilde{g}_t(\mathbf{x}) \tag{3}$$

Proof. Using the form of the RDU SWF, one obtains that

$$\rho_t(\mathbf{x}) = \left((1+\delta)^{\{t\}-\{1\}}\frac{u'(x_1)}{u'(x_t)}\right)^{\frac{1}{t-1}} - 1$$

But $u'(x_t) = u'(x_1 + x_t - x_1) \approx u'(x_1) + (x_t - x_1)u''(x_1)$ so that $\frac{u'(x_1)}{u'(x_t)} \approx \left(1 + \frac{x_1 u''(x_1)}{u'(x_1)}\frac{x_t - x_1}{x_1}\right)^{-1} \approx 1 + \eta_u(x_1)g_t(\mathbf{x})$. Besides $(1+\delta)^{\{t\}-\{1\}} \approx 1 + \left(\{t\} - \{1\}\right)\delta$.
Therefore, $(1+\delta)^{\{t\}-\{1\}}\frac{u'(x_1)}{u'(x_t)} \approx 1 + \left(\{t\} - \{1\}\right)\delta + \eta_u(x_1)g_t(\mathbf{x})$ and finally:

$$\begin{aligned}
\rho_t(\mathbf{x}) &\approx \left(1 + \left(\{t\} - \{1\}\right)\delta + \eta_u(x_1)g_t(\mathbf{x})\right)^{\frac{1}{t-1}} - 1 \\
&\approx 1 + \frac{\left(\{t\} - \{1\}\right)\delta + \eta_u(x_1)g_t(\mathbf{x})}{t-1} - 1
\end{aligned}$$

$\square$

Approximation (3) offers several insights concerning the social discount rate. First, it appears that the social discount rate is rank-dependent: it depends crucially on the distance between the welfare rank of generation t and the one of the first generation. The further generation t in the intergenerational distribution, the larger the social discount rate. On the contrary, the lower generation t in the intergenerational distribution, the lower the social discount rate.

This remark leads to a second insight. If generation t is less well-off than the first generation, the social discount rate will be negative, provided that $\eta_u(x_1) \geq 0$, which is always the case when u is concave. It has been pointed out in the literature using a DU approach that the social discount rate *may* be negative when future generations are sufficiently less well-off (see for instance Dasgupta , 2008, p. 150). With RDU, this should *always* be the case as soon as a future generations are less well-off and the function u is concave.

On the contrary, if one focuses on increasing consumption streams, the following familiar expression can be obtained: $\rho_t(\mathbf{x}) \approx \delta + \eta_u(x_1)\tilde{g}_t(\mathbf{x})$. For smooth HRDU SWFs, the expression becomes $\rho_t(\mathbf{x}) \approx \delta + \eta\tilde{g}_t(\mathbf{x})$. This expression emphasizes the crucial role played by the ethical parameters to determine the social discount rate. Indeed, δ and η conjointly characterize the attitude towards inequality: a more inequality averse social observer should have a higher δ (lower β) and/or a higher η. Therefore, a more inequality averse society should discount the future more whenever it believes that future generations will be better-off. This insight actually generalizes to all RDU SWFs.

Proposition 7. *Let $W_{\beta,u}$ and $W_{\hat{\beta},\hat{u}}$ be two smooth RDU SWF and $\rho_t(\mathbf{x})$, $\widehat{\rho}_t(\mathbf{x})$ the associated discount rate. If $W_{\beta,u}$ is at least as inequality averse than $W_{\hat{\beta},\hat{u}}$, then:*

1. *$\rho_t(\mathbf{x}) \geq \widehat{\rho}_t(\mathbf{x})$ for all $t \in \mathbb{N}$ and for all $\mathbf{x} \in \bar{\mathbf{X}}_{\neq}$ such that $x_t > x_1$.*

2. *$\rho_t(\mathbf{x}) \leq \widehat{\rho}_t(\mathbf{x})$ for all $t \in \mathbb{N}$ and for all $\mathbf{x} \in \bar{\mathbf{X}}_{\neq}$ such that $x_t < x_1$.*

Proof. For any $\mathbf{x} \in \bar{\mathbf{X}}_{\neq}$, and any $t \in \mathbb{N} \setminus \{1\}$, $\rho_t(\mathbf{x}) \geq \widehat{\rho}_t(\mathbf{x})$ if and only if $\frac{\partial W_{\beta,u}/\partial x_1}{\partial W_{\beta,u}/\partial x_t} - \frac{\partial W_{\hat{\beta},\hat{u}}/\partial x_1}{\partial W_{\hat{\beta},\hat{u}}/\partial x_t} \geq 0$. This expression can be computed:

$$
\frac{\partial W_{\beta,u}/\partial x_1}{\partial W_{\beta,u}/\partial x_t} - \frac{\partial W_{\hat{\beta},\hat{u}}/\partial x_1}{\partial W_{\hat{\beta},\hat{u}}/\partial x_t} = \frac{\beta^{\{1\}-1}u'(x_1)}{\beta^{\{t\}-1}u'(x_t)} - \frac{\widehat{\beta}^{\{1\}-1}\widehat{u}'(x_1)}{\widehat{\beta}^{\{t\}-1}\widehat{u}'(x_t)} \tag{4}
$$

$$
= \left(\frac{\beta^{\{1\}-1}/\widehat{\beta}^{\{1\}-1}}{\beta^{\{t\}-1}/\widehat{\beta}^{\{t\}-1}} - \frac{u'(x_t)\widehat{u}'(x_1)}{u'(x_1)\widehat{u}'(x_t)} \right) \frac{u'(x_1)/u'(x_t)}{\widehat{\beta}^{\{t\}-1}/\widehat{\beta}^{\{1\}-1}} \tag{5}
$$

$$
= \left(\frac{u'(x_1)\widehat{u}'(x_t)}{u'(x_t)\widehat{u}'(x_1)} - \frac{\beta^{\{t\}-1}/\widehat{\beta}^{\{t\}-1}}{\beta^{\{1\}-1}/\widehat{\beta}^{\{1\}-1}} \right) \frac{\widehat{u}'(x_1)/\widehat{u}'(x_t)}{\beta^{\{t\}-1}/\beta^{\{1\}-1}} \tag{6}
$$

We know by Proposition 5 that $\inf_{t<t'} \frac{\beta^{t-1}/\widehat{\beta}^{t-1}}{\beta^{t'-1}/\widehat{\beta}^{t'-1}} = \mathcal{D}_{\beta,\widehat{\beta}} \geq \mathcal{C}_{u,\widehat{u}} = \sup_{y<x} \frac{u'(x)\widehat{u}'(y)}{u'(y)\widehat{u}'(x)}$.

By Equation (5), for $x_t > x_1$, $\frac{\partial W_{\beta,u}/\partial x_1}{\partial W_{\beta,u}/\partial x_t} - \frac{\partial W_{\hat{\beta},\hat{u}}/\partial x_1}{\partial W_{\hat{\beta},\hat{u}}/\partial x_t} \geq \left(\inf_{t<t'} \frac{\beta^{t-1}/\widehat{\beta}^{t-1}}{\beta^{t'-1}/\widehat{\beta}^{t'-1}} - \right.$

$\left. \sup_{y<x} \frac{u'(x)\widehat{u}'(y)}{u'(y)\widehat{u}'(x)} \right) \frac{u'(x_1)/u'(x_t)}{\widehat{\beta}^{\{t\}-1}/\widehat{\beta}^{\{1\}-1}} \geq 0$. By Equation (6), for $x_t < x_1$, $\frac{\partial W_{\beta,u}/\partial x_1}{\partial W_{\beta,u}/\partial x_t} - $

$\frac{\partial W_{\hat{\beta},\hat{u}}/\partial x_1}{\partial W_{\hat{\beta},\hat{u}}/\partial x_t} \leq \left(\sup_{y<x} \frac{u'(x)\widehat{u}'(y)}{u'(y)\widehat{u}'(x)} - \inf_{t<t'} \frac{\beta^{t-1}/\widehat{\beta}^{t-1}}{\beta^{t'-1}/\widehat{\beta}^{t'-1}} \right) \frac{\widehat{u}'(x_1)/\widehat{u}'(x_t)}{\beta^{\{t\}-1}/\beta^{\{1\}-1}} \leq 0$. $\qquad \square$

Within the RDU model, the ethical parameters thus have a clear impact on social discounting. This contrasts with what occurs within the DU model. In the DU model, the two parameters δ and η represent different ethical notions. The time discounting parameter δ measures the intensity of intergenerational (procedural) inequity. A fair society should choose a lower δ. On the other hand, the elasticity of marginal utility η is often interpreted as a measure of *intra-temporal* inequality aversion.[5] A more egalitarian society should choose a higher η. As a consequence, it is not clear what the social discount rate of an 'equity-minded' society should be: on the one hand, it should discount less the future to avoid intergenerational inequity; on the other hand, it should discount more the future because it is more averse to intra-period inequalities.

One strength of the RDU model is that the two parameters have a consistent, common interpretation in terms of *intergenerational* inequality aversion. Within the RDU model, it is meaningless to simultaneously decrease δ and increase η (see Corollary 2). Therefore, a higher discount rate for a given increasing path has a clear ethical interpretation. Besides a more inequality averse society should discount less the consumption of a less well-off generation.

The result in Proposition 7 has important policy implications, in particular for the question of climate change. If one believes that future generations will be better-off in spite of climate change,[6] then a more inequality averse RDU decision maker will rather adopt the recommendation of Nordhaus (2008) to have a gradual emissions-control policy with increasing carbon price than those of Stern (2006) who calls for strong immediate action to mitigate climate change. Indeed, Nordhaus proposes to use $\delta = 0.015$ and $\eta = 2$ whereas Stern argues in favor of $\delta = 0.001$ and $\eta = 1$.

Of course, the policy recommendation would be totally different if one believed that climate change might strongly affect the economy so that declining consumption would occur for some generations in the future. This perspective

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[5]Non-ethical ('positive') interpretations of the discount rate consider that η instead represents risk aversion.

[6]The assumption is verified in the central scenario of most climate-economy integrated assessment models, such as the RICE model of Nordhaus (2008) and the PAGE model used in the Stern (2006) review.

may not be unrealistic for some poor developing countries particularly exposed to climate change. In that case, a RDU decision maker using $\eta = 1$ and $\delta > 0$ would always discount future consumption at a negative rate, lower than the one that would be promoted by Stern for decreasing consumption streams. This would prompt even stronger action than the one proposed in the Stern review.

To sum up, from a RDU perspective, climate change jeopardizes intergenerational equity only insofar as it may threaten the livelihood of future generations. If it only slows the growth rate, no harm is done, because future generations are still better-off. This remark extends to other dynamic problems: the RDU approach generally support sustainable policies.

7 Optimal rank-discounted utilitarian policies

The restricted domain of RDU criteria highlighted in Section 3 raises concerns about their applicability. Since there are many streams that they cannot order, they may not be able to suggest definite policies in specific economic environments. In particular, in traditional economic growth models, some decreasing streams are feasible and they cannot be ranked by RDU criteria.

In this section, I provide a method to extend RDU rankings in order to define optimal RDU policies. I show that, in two benchmark cases (the Ramsey growth model and the Dasgupta-Heal-Solow growth model), these policies are the same as the ones promoted by the sustainable discounted utilitarian (SDU) criteria recently studied by Asheim and Mitra (2010).

7.1 Extended Rank-Discounted Utilitarian preferences

In order to obtain effective SWRs capable of advocating specific policies, I propose to apply RDU on the set $\bar{\mathbf{X}}$ and to complete it on other paths with appealing principles. In this respect Axiom 2 (Intermediate Pareto) and Axiom 3' (Finite Anonymity) are particularly appropriate for two reasons. A first reason is that RDU SWOs already satisfy these axioms, so that imposing them on the whole set of bounded consumption streams $\mathbf{X}$ will not conflict with the RDU representation on $\bar{\mathbf{X}}$. A second reason is that efficiency and equity requirements are

known to recommend sustainable paths under certain technological assumptions (Asheim, Buchholz and Tungodden , 2001). Since RDU criteria are well-defined on sustainable paths, specific recommendations can be obtained.

We therefore propose the following definition of Extended RDU preferences (in short ERDU).

Definition 6 Extended RDU SWR. *A SWR $\succeq$ on $\mathbf{X}$ is an ERDU SWR with parameters β and u, denoted $\succeq_{\beta,u}$, if and only if it satisfies Axioms 2 and 3' on $\mathbf{X}$ and it is represented by the RDU SWF $W_{\beta,u}$ on $\bar{\mathbf{X}}$.*

In order to define a specific intertemporal economic environment, let introduce a technology F. The function F gives the maximum consumption attainable for generation t if k_t is inherited and k_{t+1} bequeathed, where k_t and k_{t+1} are vectors of capital stock. Hence a consumption path ${}_t\mathbf{x} = (x_t, x_{t+1}, \cdots)$ is *F-feasible* at t given k_t if and only if, for any $t' \geq t$, $0 < x_{t'} \leq F(k_{t'}, k_{t'} + 1)$ and $k_{t'} > 0$. Denote $\mathbf{X_F}(\mathbf{k_t})$ the set of F-feasible consumption path given k_t.

Definition 7 Productive technology. *A technology F is productive if for any $\mathbf{x} \in \mathbf{X_F}(\mathbf{k_1})$ and any $\tau < \tau'$, if $x_\tau > x_{\tau'}$, there exist scalars $\epsilon_t > 0$, $t > \tau$, such that the consumption stream $\mathbf{y}$ defined as follows is F-feasible given k_1: $y_t = x_t$ for all $t < \tau$, $y_\tau = x_{\tau'}$, $y_{\tau'} = x_\tau + \epsilon_{\tau'}$ and $y_t = x_t + \epsilon_t$ for all $t > \tau$, $t \neq \tau'$.*

The productivity assumption states that delaying consumption can improve the consumption of all future generations. It strengthens the immediate productivity assumption of Asheim, Buchholz and Tungodden (2001). As in the presence of the Strong Pareto axiom immediate productivity permits to show that only sustainable paths can be chosen, so does the productivity assumption in the presence of Intermediate Pareto. We will say that a F-feasible consumption stream $\mathbf{x}$ is optimal for $\succeq_{\beta,u}$ if and only if there exists no other F-feasible consumption stream $\mathbf{y}$ such that $\mathbf{y} \succ_{\beta,u} \mathbf{x}$.

Proposition 8. *Assume that the technology F is productive. A F-feasible path is optimal for $\succeq_{\beta,u}$ only if it is non-decreasing.*

Proof. Consider a F-feasible consumption stream $\mathbf{x}$ such that $x_1 > x_t$ for some $t > 1$. Since F is productive, there exist scalars $\epsilon_t > 0$, such that the following

consumption stream $\mathbf{y}$ is F-feasible: $y_1 = x_t$, $y_t = x_1 + \epsilon_t$ and $y_{t'} = x_{t'} + \epsilon_{t'}$ for all $t' > 1$, $t' \neq t$. Denote $\tilde{\mathbf{x}}$ the consumption stream such that $\tilde{x}_1 = x_t$, $\tilde{x}_t = x_1$ and $\tilde{x}_{t'} = x_{t'}$ for all $t' \notin \{1, t\}$. By Weak Anonymity (Axiom 3'), $\tilde{\mathbf{x}} \sim_{\beta,u} \mathbf{x}$. But by Intermediate Pareto (Axiom 2), $\mathbf{y} \succ_{\beta,u} \tilde{\mathbf{x}}$. Hence, $\mathbf{y} \succ_{\beta,u} \mathbf{x}$: the consumption stream $\mathbf{x}$ is not optimal for $\succeq_{\beta,u}$.

Now consider a F-feasible consumption stream $\mathbf{x}$ such that $x_1 \leq x_t$ for all $t > 1$ (otherwise it is not optimal). Assume that $x_2 > x_t$ for some $t > 2$. Since F is productive, the same reasoning as before implies that the consumption stream $\mathbf{x}$ cannot be optimal for $\succeq_{\beta,u}$. Repeating the method, we obtain that a F-feasible consumption stream $\mathbf{x}$ must be non-decreasing in order to be optimal for $\succeq_{\beta,u}$. $\qquad\square$

Proposition 8 can be contrasted with Proposition 4 of Asheim, Buchholz and Tungodden (2001). The result is similar but it is based on different assumptions: their productivity assumption has been strengthened but their Paretian requirement has been weakened (Intermediate Pareto is used instead of Strong Pareto). Arguably, the strengthening of the productivity assumption I propose is not too costly: I will show in the next Sections that it is satisfied by several models that also satisfy immediate productivity. On the other hand, the Intermediate Pareto axiom is exactly the strengthening of Weak Pareto that makes it possible to obtain the result in Proposition 8. The reason for proposing Axiom 2 was precisely to have the weakest axiomatics justifying sustainability in several technological contexts.

Proposition 8 is important to obtain explicit recommendations based on ERDU SWRs. Indeed, ERDU SWRs are well-defined and complete on the set of non-decreasing sequences. More precisely they correspond to DU preferences on this set.

For an ERDU SWR $\succeq_{\beta,u}$, let name *corresponding discounted utility* the SWO represented by $\sum_{t \in \mathbb{N}} \beta^{t-1} u(x_t)$. Denote $\mathbf{X}_{\mathbf{F}}^{+}(\mathbf{k_1})$ the set of non-decreasing F-feasible consumption streams. The following corollary of Proposition 8 completely characterizes optimal consumption streams for $\succeq_{\beta,u}$.

Corollary 3. *Assume that the technology F is productive. The set of optimal F-feasible consumption streams for $\succeq_{\beta,u}$ is the set of optimal consumption streams over $\mathbf{X}_\mathbf{F}^+(\mathbf{k_1})$ for the corresponding discounted utility.*

Corollary 3 highlights the similarities between the RDU approach and the DU approach. The RDU solution indeed corresponds to the DU solution constrained by a requirement of sustainability. As a consequence, the RDU solution will be the DU one whenever the latter corresponds to a non-decreasing consumption stream, a situation that typically arises in growth theory.

Corollary 3 also emphasizes the similarity with Asheim (1988) and Asheim (1991). Both papers have proposed criteria yielding to the maximization of discounted utility over the set of non-decreasing path. The SDU preferences of Asheim and Mitra (2010) also recommend this kind of policy in several models. The next Sections describe the common solutions of these different criteria in two benchmark growth models.

The particularity of the RDU approach is to highlight the role of inequality aversion. I will then also indicate the impact of inequality aversion on policy recommendations.

7.2 The Ramsey growth model

The canonical model of economic growth is the one-sector capital accumulation model also called Ramsey model. There is a stock of non-negative man-made physical capital k_t which is used to produce the single homogenous good which can either be consumed or accumulated as physical capital. Hence,

$$x_t + k_{t+1} \leq f(k_t) + k_t, \quad x_t \geq 0, \quad k_t > 0$$

for any $t \in \mathbb{N}$ along a feasible consumption stream.

In the Ramsey model, the production function f is assumed to be strictly increasing, concave, continuously differentiable on $\mathbb{R}_+$, with $\lim_{k \to +\infty} f'(k) = 0$ and $f(0) = 0$. Hence the Ramsey technology is $F_R(k_t, k_{t+1}) = f(k_t) + k_t - k_{t+1}$.

Lemma 1. *The Ramsey technology F_R is productive.*

Proof. Let $\mathbf{x} \in \mathbf{X_{F_R}}(\mathbf{k_1})$ be such that $x_\tau > x_{\tau'}$ for some $\tau < \tau'$. We construct an alternative feasible capital sequence $\tilde{\mathbf{k}}$ in the following way. For $t \leq \tau$, $\tilde{k}_t = k_t$. At period $\tau + 1$, $\tilde{k}_{\tau+1} = k_{\tau+1} + x_\tau - x_{\tau'} > k_{\tau+1}$. At period $\tau + 2 \leq t \leq \tau'$, $\tilde{k}$ is defined recursively by $\tilde{k}_t = k_t + \epsilon_{t-1} + \tilde{k}_{t-1} - k_{t-1}$, with $\epsilon_{t-1} = \frac{f(\tilde{k}_{t-1}) - f(k_{t-1})}{2}$. Since f is strictly increasing, $\epsilon_t > 0$ and $\tilde{k}_t > k_t$ for any $\tau + 1 \leq t \leq \tau' - 1$. Furthermore, $\tilde{k}_\tau = k_\tau + \sum_{t=\tau+1}^{\tau'-1} \epsilon_t + \tilde{k}_{\tau+1} - k_{\tau+1} = k_\tau + \sum_{t=\tau+1}^{\tau'-1} \epsilon_t + x_\tau - x_{\tau'}$. At period $\tau' + 1$, $\tilde{k}_{\tau'+1} = k_{\tau'+1} + \epsilon_{\tau'}$, where $\epsilon_{\tau'} = \frac{f(\tilde{k}_{\tau'}) - f(k_{\tau'} + \sum_{t=\tau+1}^{\tau'-1} \epsilon_t)}{2} > 0$. From period $\tau' + 2$, $\tilde{k}$ is defined recursively by $\tilde{k}_t = k_t + \epsilon_{t-1}$, with $\epsilon_{t-1} = \frac{f(\tilde{k}_{t-1}) + \tilde{k}_{t-1} - f(k_{t-1}) - k_{t-1}}{2} > 0$.

Now define the consumption stream $\mathbf{y}$ as follows: $y_t = x_t$ for all $t < \tau$, $y_\tau = x_{\tau'}$, $y_{\tau'} = x_\tau + \epsilon_{\tau'}$ and $y_t = x_t + \epsilon_t$ for all $t > \tau$, $t \neq \tau'$. By construction, $\mathbf{y}$ is F_R-feasible using the capital sequence $\tilde{\mathbf{k}}$. Hence the Ramsey technology F_R is productive. $\qquad\square$

Given that the Ramsey technology is productive, we know that ERDU criteria give explicit solutions, as described in Corollary 3. Actually, the solution is unique. To characterize it, additional notation is needed.

Write the gross output function as $g(k) = f(k) + k$. Denote $x(y)$ the unique solution to the equation $y = g(y - x(y))$ such that $0 \leq x(y) \leq y$. The function $x(y)$ is well-defined, continuous and differentiable (see Asheim and Mitra , 2010). It is the consumption level that leaves the capital stock intact. Finally, let $y_\infty(\beta) \equiv \min \left\{ y : \frac{1}{g'\left(y - x(y)\right)} \geq \beta \right\}$. The function y_∞ is strictly increasing in β (Asheim and Mitra , 2010).

Proposition 9 Asheim, 1991. *Consider an ERDU SWR $\succeq_{\beta,u}$, with $0 < \beta < 1$, a Ramsey technology, and an initial level of capital k_1. There exists a unique optimal F_R-feasible consumption streams for $\succeq_{\beta,u}$, denoted $\mathbf{x}^*$, which is characterized as follows:*

1. *If $y_1 = g(k_1) \geq y_\infty(\beta)$, then $\mathbf{x}^*$ is a stationary path with $x_t^* = x(y_1)$ for any $t \geq 1$.*

2. *If $y_1 = g(k_1) < y_\infty(\beta)$, then $\mathbf{x}^*$ is an increasing path, converging to $x(y_\infty(\beta))$ and maximizing the associated discounted utility over $\mathbf{X_{F_R}}(\mathbf{k_1})$.*

Proof. Lemma 1 and Proposition 8 imply that RDU optimal consumption streams must be the associated optimal DU streams among non-decreasing streams. Proposition 6 in Asheim (1991) then yields the result. □

Proposition 9 shows that RDU preferences can be operationalized in the basic Ramsey model. We are able to characterize a unique optimal solution, that we call the sustainable discounted utilitarian solution for it is the same as in Asheim (1991) and Asheim and Mitra (2010).

The advantage of RDU preferences over SDU preferences is that they emphasize the influence of inequality aversion on optimal policy. Indeed, we know that a necessary condition for a RDU SWF $W_{\beta,u}$ to more inequality averse than another RDU SWF $W_{\hat{\beta},\hat{u}}$ is that $\beta \leq \hat{\beta}$. From Proposition 9, it is clear that:

- A more inequality averse RDU society $\succeq_{\beta,u}$ will converge to a lower steady state than a less inequality averse society $\succeq_{\hat{\beta},\hat{u}}$ in many situations (actually, as soon as $g(k_1) < y_\infty(\hat{\beta})$).

- A more inequality averse RDU society $\succeq_{\beta,u}$ will prevent growth more often than a less inequality averse society $\succeq_{\hat{\beta},\hat{u}}$ (actually, whenever $y_\infty(\hat{\beta}) \geq g(k_1) \geq y_\infty(\beta)$).

Regarding the second point, one can notice that the maximin always prevents growth. The maximin is the special case of RDU preferences where $\beta \to 0$, an extreme aversion to inequality. For other values of β, growth is prevented only in certain circumstances. The lower β the more often this will happen.

Inequality aversion therefore modifies both the long-run perspectives of the society and the prospects of an egalitarian (stationary) distribution. Remark that only the parameter β determines the long-term impact of inequality aversion. The other dimension of inequality aversion, namely the concavity of the function u, would only have an impact on the speed of the convergence to the steady state in the case $g(k_1) < y_\infty(\beta)$.

7.3 The Dasgupta-Heal-Solow growth model

The Dasgupta-Heal Solow model (Dasgupta and Heal , 1974; Solow , 1974) is the standard model of growth with an exhaustible natural resource. Production depends on a man-made physical capital k_t^m (which is homogeneous to the consumption good), on the extraction d_t of a natural exhaustible resource k_t^n and of the labor supply l_t. The natural resource is depleted by the resource use, so that $k_{t+1}^n = k_t^n - d_t$. The production function $\hat{f}(k_t^m, d_t, l_t)$ is concave, non-decreasing, homogeneous of degree one, and twice continuously differentiable. It satisfies $(\hat{f}_{k^m}, \hat{f}_d, \hat{f}_l) \gg 0$ for all $(k^m, d, l) \gg 0$ and $\hat{f}(k^m, 0, l) = \hat{f}(0, d, l) = 0$ (both the physical capital and the natural resource are essential in the production). Besides, given $(\tilde{k}^m, \tilde{d}) \gg 0$, there exists a scalar $\tilde{\chi}$ such that for (k^m, d) such that $k^m \geq \tilde{k}^m$ and $0 \leq d \leq \tilde{d}$, $\frac{d\hat{f}_d(k^m, d, 1)}{\hat{f}_l(k^m, d, 1)} \geq \tilde{\chi}$.

Assume that the labor force is constant and normalized to 1. Denote $f(k^m, d) := \hat{f}(k^m, d, 1)$. Also assume that f is strictly concave and $f_{k^m, d}(k^m, d) \gg 0$ for all $(k^m, d) \gg 0$. Along a feasible plan for the Dasgupta-Heal-Solow technology, we have, for any $t \in \mathbb{N}$:

$$x_t + k_{t+1}^m \leq f(k_t^m, k_t^n - k_{t+1}^n) + k_t^m, \quad x_t \geq 0, \quad k_t^m \geq 0, \quad k_t^n \geq 0$$

Hence the Dasgupta-Heal-Solow technology is $F_{DHS}(k_t^m, f_t^n, k_{t+1}^m, f_{t+1}^n) = f(k_t^m, k_t^n - k_{t+1}^n) + k_t^m - k_{t+1}^m$.

Lemma 2. *The Dasgupta-Heal-Solow technology F_{DHS} is productive.*

Proof. Let $\mathbf{x} \in \mathbf{X_{F_R}}(\mathbf{k_1^m}, \mathbf{k_1^n})$ be such that $x_\tau > x_{\tau'}$ for some $\tau < \tau'$. For the natural capital, we keep the same path. For the physical capital, we compute an alternative path like in the proof of Lemma 1. Then the proof proceeds in a similar way. $\square$

One question has attracted particular attention in the literature on the Dasgupta-Heal-Solow model: is it possible maintain a constant consumption level for ever? Cass and Mitra (1991) have answered this issue by providing a necessary and sufficient condition for the existence of a sustainable consumption

level. To introduce their condition, let h be the resource requirement function defined as follows:

$$h(y, k^m) = \min d \quad \text{subject to} \quad y \leq f(k^m, d), \ d \geq 0, \ (y, k^m) \in \mathbf{H}$$

where $\mathbf{H} = \left\{ (y, k^m) : k^m \geq 0 \text{ and there exists } d \geq 0 \text{ such that } y \leq f(k^m, d) \right\}$. The function h describes the minimal level of resource required to yield a given output level from a given physical capital stock. Next define:

$$\underline{k}^m(y) = \inf \left\{ k^m : (y, k^m) \in \mathbf{H} \text{ and } \inf_{\tilde{k}^m \leq k^m, \ (y, \tilde{k}^m) \in \mathbf{H}} h(y, \tilde{k}^m) = 0 \right\}$$

The scalar $\underline{k}^m(y)$ is the lower bound on physical capital stock that permit to maintain a given level of output without resource depletion. We assume that the Dasgupta-Heal-Solow technology satisfies Cass-Mitra substitution condition:

$$\lim_{y \to 0^+} \frac{\int_{k^m}^{\underline{k}^m(y)} h(y, z) dz}{y} = 0, \qquad \text{for any } 0 < k^m < \underline{k}^m(y)$$

The condition roughly ensures that pure physical accumulation is feasible from any initial conditions, and permits to avoid resource exhaustion. In the Cobb-Douglas case, $f(k^m, d) = (k^m)^\alpha (d)^\gamma$, the condition amounts to $\alpha > \gamma$: the elasticity of production of man-made capital must exceed the elasticity of production of the natural resource use.

It is possible to show (see Cass and Mitra , 1991) that, under the other assumptions of the Dasgupta-Heal-Solow model and whenever the Cass-Mitra substitution condition holds, there exists a stationary, F_{DHS}-feasible and efficient consumption stream associated to any vector of initial conditions (k_1^m, k_1^n). Denote $x(k_1^m, k_1^n)$ this level of consumption that can be sustained for ever. It is possible to attach a sequence of (shadow)-prices $\mathbf{p(k_1^m, k_1^n)}$ to the corresponding stationary sequence (for a characterization of the prices, see Asheim and Mitra , 2010, Lemma 3). Denote $\beta_\infty(k_1^m, k_1^n) = \frac{\sum_{t=2}^{+\infty} p_t(k_1^m, k_1^n)}{\sum_{t=1}^{+\infty} p_t(k_1^m, k_1^n)}$. Optimal RDU consumption streams can be fully described:

Proposition 10 Asheim, 1988. *Consider an ERDU SWR $\succeq_{\beta, u}$, with $0 < \beta <$*

1, a Dasgupta-Heal-Solow technology, and initial levels of capital $(k_1^m, k_1^n) \gg$ 0. There exists a unique optimal F_{DHS}-feasible consumption streams for $\succeq_{\beta,u}$, denoted $\mathbf{x}^*$, which is characterized as follows:

1. If $\beta_\infty(k_1^m, k_1^n) \geq \beta$, then $\mathbf{x}^*$ is a stationary path with $x_t^* = x(k_1^m, k_1^n)$ for any $t \geq 1$.

2. If $\beta_\infty(k_1^m, k_1^n) < \beta$, then $\mathbf{x}^*$ is the efficient and non-decreasing consumption stream maximizing the associated discounted utility over $\mathbf{X}_{\mathbf{F_{DHS}}}^+(\mathbf{k_1^m}, \mathbf{k_1^n})$. Denoting $\tau = \min\left\{t \in \mathbb{N} : \beta_\infty\big((k_t^m)^*, (k_t^n)^*\big) < \beta\right\}$, this stream exhibits the following pattern:

   - For $t < \tau$, $x_t^* < x_{t+1}^*$.
   - For any $t \geq \tau$, $x_t^* = x\Big((k_\tau^m)^*, (k_\tau^n)^*\Big)$.

Proof. Lemma 2 and Proposition 8 imply that RDU optimal consumption streams must be the associated optimal DU streams among non-decreasing streams. Under the assumptions of the Dasgupta-Heal-Solow model (notably Cass-Mitra substitution condition), this optimal stream is unique and it is the one described in Lemma 4 in Asheim (1988). $\qquad\square$

Proposition 10 shows that the consequences of a higher level of inequality aversion exhibited in the Ramsey growth model still hold in the Dasgupta-Heal-Solow model. Indeed:

- A more inequality averse RDU society $\succeq_{\beta,u}$ will stop growing at a lower stationary level of consumption than a less inequality averse society $\succeq_{\hat\beta,\hat u}$ whenever $\beta_\infty(k_1^m, k_1^n) < \hat\beta$.

- A more inequality averse RDU society $\succeq_{\beta,u}$ will prevent growth more often than a less inequality averse society $\succeq_{\hat\beta,\hat u}$ (whenever $\beta \geq \beta_\infty(k_1^m, k_1^n) \geq \hat\beta$).

In particular, in the maximin case, growth will always be prevented. Once again, the maximin case represents an extreme form of inequality aversion, and less extreme cases allow for growth in some cases. Also note that, once again, only the parameter β determines the long-term impact of inequality aversion.

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The concavity of the function u would only have an influence on the transition to the stationary phase.

8 Conclusion

The RDU approach to intertemporal welfare has several appealing features. First, it reconciles intergenerational procedural equity and efficiency on its domain of definition. Second, it enables to express a range of distributive concerns for successive generations. A last appealing feature of the RDU approach is that it provides a consistent and intuitive interpretation of the ethical parameters determining the social discount rate. With the RDU interpretation, we have obtained the provocative statement that inequality aversion increases the social discount rate along increasing consumption streams.

The statement is at odds with the traditional ethical approach to social discounting. It comes from the fact that RDU criteria do satisfy procedural equity (the reason why people endorsing the traditional ethical approach have called for lower discount rates) while allowing for inequality-aversion-based discounting. I believe that RDU may spark off new debates on social discounting within the ethical approach to social discounting.

The RDU resulting policies are closely related to the one promoted by sustainable discounted utilitarian (SDU) SWFs that have been recently studied by Asheim and Mitra (2010). While their axiomatization is designed to obtain a complete ordering on $\mathbf{X}$, mine highlights the role of procedural equity and inequality aversion. But in practice SDU and RDU criteria would suggest similar recommendations (provided that RDU is completed by the sustainability Axioms 2 and 3'). One advantage of RDU is to provide an ethical interpretation of the parameters in the discounted sum of utilities in terms of inequality aversion.

In conclusion, the RDU model can be operationalized. While its recommendations may not be new, the RDU model offers an interesting new perspective that respects procedural equity and displays concerns for intergenerational redistribution. It shades some new lights on what we owe to future generations: we have to increase their resources if we can do so; we have to guarantee that they

won't be worse-off than we are; but we must not be unfair to present generations and we must ensure that intergenerational inequalities are not too large. This conception of intergenerational equity, more in line with the intuitive notion of distributive equity, may seem appealing to many.

Appendix

Proof of Proposition 3.

The sufficiency part of the Proposition is obvious.

For the necessity, first remark that axioms 1, 2, and 4 imply that there exists a monotonic SWF W representing $\succeq$ on $\bar{\mathbf{X}}$. By Axiom 3, we know that whatever $\mathbf{x} \in \bar{\mathbf{X}}$, $W(\mathbf{x}) = W(\mathbf{x}_{[\ ]})$. We can therefore restrict attention to the set $\mathbf{X}^+$.

Now, for each $T \in \mathbb{N}$, we introduce the following subset of $\mathbf{X}^+$: $\mathbf{X}_{\mathbf{T}}^+ = \{\mathbf{x} \in \mathbf{X}^+ : x_t = x_{T+1}, \forall t \geq T+1\}$. These are the nondecreasing intergenerational allocations with a constant tail from period $T+1$ onward. Let $\succeq_T$ be the restriction of $\succeq$ to the set $\mathbf{X}_{\mathbf{T}}^+$. It is a continuous monotonic weak order on $\{(x_1, \cdots, x_{T+1}) \in \mathbb{R}^{T+1} : x_1 \leq \cdots \leq x_{T+1}\}$, a rank-ordered set. Furthermore, it satisfies the usual independence condition (sure-thing principle). Hence, by Theorem 3.2. and Corollary 3.6 of Wakker (1993), there exists a cardinal additive representation of $\succeq_T$:[7]

$$W_T(\mathbf{x}) = \sum_{t=1}^{T} u_{t,T}(x_t) + V_T(x_{T+1}), \ \forall \mathbf{x} \in \mathbf{X}_{\mathbf{T}}^+ \tag{7}$$

The functions $u_{t,T}$ and V_T are all continuous and nondecreasing. In addition, by Axioms 2, 5 and 7, the functions $u_{1,T}$ and V_T must be increasing. By cardinality, we may set $u_{t,T}(0) = 0$ for all $t \leq T$ and $V_T(0) = 0$ (normalization condition).

Now, representation (7) exists for $\succeq_T$ whatever $T \in \mathbb{N}$. Furthermore, $\succeq_T$ and $\succeq_{T+1}$ represent the same ordering on $\mathbf{X}_{\mathbf{T}}^+$. By standard uniqueness results for additive functions on rank-ordered sets, we can take (after the appropriate

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[7] By Axioms 2, 5 and 7 and Gorman's theorem (Gorman , 1968), we know that all coordinates are essential.

normalization) $u_{t,T} \equiv u_{t,T+1}$ and $V_T \equiv u_{T,T+1} + V_{T+1}$. We can henceforth drop the subscript T in functions $u_{t,T}$.

By Axiom 7, we also know that $W_T(\mathbf{x}) = \sum_{t=1}^{T} u_t(x_t) + V_T(x_{T+1})$ and $W_T(\mathbf{x}) = \sum_{t=2}^{T+1} u_t(x_{t-1}) + V_{T+1}(x_{T+1})$ represent the same preferences $\forall \mathbf{x} \in \mathbf{X}_\mathbf{T}^+$. By the cardinality of the additive representation and the normalization condition, there must exists a $\beta > 0$ such that $u_{t+1}(x) = \beta u_t(x)$ and $V_{T+1}(x) = \beta V_T(x)$ for any $x \in X$. Remark that β does not depend on t. Denote $u \equiv u_1$ and $V \equiv V_1$, we have the following representation of $\succeq_T$:

$$W_T(\mathbf{x}) = \sum_{t=1}^{T} \beta^{t-1} u(x_t) + \beta^T V(x_{T+1}), \ \forall \mathbf{x} \in \mathbf{X}_\mathbf{T}^+$$

with u and V two increasing functions.

Now remark that we must also have $V(x) = u(x) + \beta V(x)$, so that $V(x) = \frac{u(x)}{1-\beta}$. This implies that $\beta < 1$ by Axioms 2. This also implies that $V(x) = \sum_{t=1}^{+\infty} \beta^{t-1} u(x)$. Hence we obtain the following representation of $\succeq$ on $\bigcup_{T \in \mathbb{N}} \mathbf{X}_\mathbf{T}^+$:

$$W(\mathbf{x}) = \sum_{t=1}^{+\infty} \beta^{t-1} u(x_t)$$

Now it remains to prove that the representation extends to the whole set $\mathbf{X}^+$. For any $\mathbf{x} \in \mathbf{X}^+$, we define the sequence $\mathbf{x}^1, \mathbf{x}^2, \cdots, \mathbf{x}^k, \cdots$ of allocations in $\mathbf{X}^+$ as follows: for any $k \in \mathbb{N}$, $x_t^k = x_t$ for all $t \leq k$ and $x_t^k = x_k$ for all $t > k$. Each allocation in the sequence belongs to $\bigcup_{T \in \mathbb{N}} \mathbf{X}_\mathbf{T}^+$. And $\lim_{k \to \infty} \sup_{t \in \mathbb{N}} |x_t^k - x_t| = 0$, since we consider bounded streams. By continuity (Axiom 4), we obtain that $W(\mathbf{x}) = \sum_{t=1}^{+\infty} \beta^{t-1} u(x_t)$ is a SWF representing $\succeq$ on $\mathbf{X}^+$.

Finally, by Axiom 3, we obtain that $\succeq$ can be represented on $\bar{\mathbf{X}}$ by:

$$W(\mathbf{x}) = W(\mathbf{x}_{[\ ]}) = \sum_{t=1}^{+\infty} \beta^{t-1} u(x_{[t]})$$

Proof of Proposition 4.

Necessity. Consider $\mathbf{x} \in \bar{\mathbf{X}}$ such that $x_t = 0$ for all $t \leq \tau$, $x_\tau \leq x_{\tau+1}$, and $x_t > x_\tau + x_{\tau+1}$ for $t > \tau + 1$. Now consider $\mathbf{y} \in \bar{\mathbf{X}}$ such that $y_\tau + \varepsilon = x_\tau \leq$

$x_{\tau+1} = y_{\tau+1} - \varepsilon$ and $y_t = x_t$ for all $t \neq \tau, \tau+1$, with $x_\tau \geq \varepsilon > 0$. According to representation (1) of RDU SWFs, for Axiom 9 to hold it must be the case that:

$$\beta^\tau u(x_\tau) + \beta^{\tau+1} u(x_{\tau+1}) \geq \beta^\tau u(y_\tau) + \beta^{\tau+1} u(y_{\tau+1}) = \beta^\tau u\big(x_\tau - \varepsilon\big) + \beta^{\tau+1} u\big(x_{\tau+1} + \varepsilon\big)$$

This inequation can be rewritten:

$$1 \geq \beta \frac{u\big(x_{\tau'} + \varepsilon\big) - u\big(x_{\tau'}\big)}{u\big(x_\tau\big) - u\big(x_\tau - \varepsilon\big)} \tag{8}$$

The construction of allocations $\mathbf{x}$ and $\mathbf{y}$ yielding this inequation can be done for any two integers $\tau < \tau'$, and for any real numbers $0 < \varepsilon \leq x_\tau \leq x_{\tau'}$. We therefore need

$$1 \geq \sup_{0 < \varepsilon \leq x \leq x'} \beta \times \frac{u\big(x' + \varepsilon\big) - u\big(x'\big)}{u\big(x\big) - u\big(x - \varepsilon\big)} = \beta \times \mathcal{C}_u$$

Sufficiency. Assume that the representation in Equation (1) holds and that $1 \geq \beta \times \mathcal{C}_u$.

Now consider $\mathbf{x}, \mathbf{y} \in \bar{\mathbf{X}}$ such that $\varepsilon \leq y_\tau + \varepsilon = x_\tau \leq x_{\tau'} = y_{\tau'} - \varepsilon$, and $y_t = x_t$ for all $t \neq \tau, \tau'$, where $x_\tau \geq \varepsilon > 0$ and (τ, τ') are positive integers. We want to show that $W(\mathbf{x}) - W(\mathbf{y}) \geq 0$.

Denote $\{\tau\}$ (resp. $\{\tau'\}$) the rank of generation τ (resp. τ') in the intergenerational distribution $\mathbf{x}$ and $\widehat{\{\tau\}}$ (resp. $\widehat{\{\tau'\}}$) the rank of generation τ (resp. τ') in the intergenerational distribution $\mathbf{y}$. Using the representation in Equation (1), straightforward algebra yields:

$$W(\mathbf{x}) - W(\mathbf{y}) = \sum_{\widehat{\{\tau\}} \leq t \leq \{\tau\}} \beta^{t-1}\Big(u(x_{[t]}) - u(y_{[t]})\Big) - \sum_{\{\tau'\} \leq t \leq \widehat{\{\tau'\}}} \beta^{t-1}\Big(u(y_{[t]}) - u(x_{[t]})\Big)$$

For $\widehat{\{\tau\}} \leq t \leq \{\tau\}$, we have $u(x_{[t]}) - u(y_{[t]}) \geq 0$, and for $\{\tau'\} \leq t \leq \widehat{\{\tau'\}}$, we

have $u(y_{[t]}) - u(x_{[t]}) \geq 0$. Hence:

$$\sum_{\widehat{\{\tau\}} \leq t \leq \{\tau\}} \beta^{t-1} \Big(u(x_{[t]}) - u(y_{[t]}) \Big) - \sum_{\{\tau'\} \leq t \leq \widehat{\{\tau'\}}} \beta^{t-1} \Big(u(y_{[t]}) - u(x_{[t]}) \Big) \geq$$

$$\beta^{\{\tau\}-1} \times \sum_{\widehat{\{\tau\}} \leq t \leq \{\tau\}} \Big(u(x_{[t]}) - u(y_{[t]}) \Big) - \beta^{\{\tau'\}-1} \times \sum_{\{\tau'\} \leq t \leq \widehat{\{\tau'\}}} \Big(u(y_{[t]}) - u(x_{[t]}) \Big)$$

By definition of the Pigou-Dalton transfer, $\sum_{\widehat{\{\tau\}} \leq t \leq \{\tau\}} \Big(u(x_{[t]}) - u(y_{[t]}) \Big) = u(x_\tau) - u(y_\tau)$ and $\sum_{\{\tau'\} \leq t \leq \widehat{\{\tau'\}}} \Big(u(y_{[t]}) - u(x_{[t]}) \Big) = u(y_{\tau'}) - u(x_{\tau'})$. Therefore:

$$
\begin{aligned}
W(\mathbf{x}) - W(\mathbf{y}) &\geq \beta^{\{\tau\}-1} \Big(u(x_\tau) - u(y_\tau) \Big) - \beta^{\{\tau'\}-1} \Big(u(y_{\tau'}) - u(x_{\tau'}) \Big) \\
&= \beta^{\{\tau\}-1} \Big(u(x_\tau) - u(y_\tau) \Big) \left(1 - \beta^{\{\tau'\}-\{\tau\}} \frac{u(y_{\tau'}) - u(x_{\tau'})}{u(x_\tau) - u(y_\tau)} \right) \\
&\geq \beta^{\{\tau\}-1} \Big(u(x_\tau) - u(y_\tau) \Big) \left(1 - \beta \times \frac{u(x_{\tau'} + \varepsilon) - u(x_{\tau'})}{u(x_\tau) - u(x_\tau - \varepsilon)} \right)
\end{aligned}
$$

When $1 \geq \beta \times \mathcal{C}_u$, it is clear that $W(\mathbf{x}) - W(\mathbf{y}) \geq 0$.

Proof of Proposition 5.

Denote $\succeq$ (resp. $\hat{\succeq}$) the SWO represented by $W_{(\beta,u)}$ (resp. $W_{(\hat{\beta},\hat{u})}$).

Necessity. Consider $\mathbf{x}$ and $\mathbf{y}$ in $\bar{\mathbf{X}}$ such that, for $\tau < \tau'$:

- $x_t = y_t = 0$ for all $t < \tau$;

- $0 \leq y_\tau < x_\tau \leq y_t = x_t \leq x_{\tau'}$ for all $\tau < t < \tau'$;

- $y_{\tau'} = \hat{u}^{-1} \Big(\hat{u}(x_{\tau'}) + \beta^{\tau'-\tau} \big(\hat{u}(x_\tau) - \hat{u}(y_\tau) \big) \Big)$ so that $y_{\tau'} \geq x_{\tau'}$;

- and $y_t = x_t > y_\tau$ for $t > \tau'$.

By definition, $\hat{\beta}^{\tau-\tau'} = \frac{\big(\hat{u}(y_{\tau'}) - \hat{u}(x_{\tau'}) \big)}{\big(\hat{u}(x_\tau) - \hat{u}(y_\tau) \big)}$ or, equivalently, $\hat{\beta}^{\tau'-1} \big(\hat{u}(y_{\tau'}) - \hat{u}(x_{\tau'}) \big) = \hat{\beta}^{\tau-1} \big(\hat{u}(x_\tau) - \hat{u}(y_\tau) \big)$, so that $\mathbf{x} \hat{\sim} \mathbf{y}$. Because $\mathbf{y} \succ_I \mathbf{x}$, and because $W_{(\beta,u)}$ is

at least as inequality averse as $W_{(\hat{\beta},\hat{u})}$, it must be the case that $\beta^{\tau-1}u(x_\tau) + \beta^{\tau'-1}u(x_{\tau'}) \geq \beta^{\tau-1}u(y_\tau) + \beta^{\tau'-1}u(y_{\tau'})$, or equivalently

$$\beta^{\tau-\tau'} \geq \frac{\left(u(y_{\tau'}) - u(x_{\tau'})\right)}{\left(u(x_\tau) - u(y_\tau)\right)}$$

Together with the equality $\hat{\beta}^{\tau-\tau'} = \frac{\left(\hat{u}(y_{\tau'}) - \hat{u}(x_{\tau'})\right)}{\left(\hat{u}(x_\tau) - \hat{u}(y_\tau)\right)}$, the above inequality yields

$$\frac{\beta^\tau/\hat{\beta}^\tau}{\beta^{\tau'}/\hat{\beta}^{\tau'}} \geq \frac{\left(u(y_{\tau'}) - u(x_{\tau'})\right)/\left(\hat{u}(y_{\tau'}) - \hat{u}(x_{\tau'})\right)}{\left(u(x_\tau) - u(y_\tau)\right)/\left(\hat{u}(x_\tau) - \hat{u}(y_\tau)\right)}$$

The construction of $\mathbf{x}$ and $\mathbf{y}$ yielding this inequality can be done for any two integers $\tau < \tau'$, and for any real numbers $0 \leq y_\tau < x_\tau \leq x_{\tau'} < y_{\tau'}$ such that $y_{\tau'} = \hat{u}^{-1}\left(\hat{u}(x_{\tau'}) + \beta^{\tau'-\tau}\left(\hat{u}(x_\tau) - \hat{u}(y_\tau)\right)\right)$. Denoting $\hat{\mathbf{X}} = \Big\{(x_1, x_2, x_3, x_4) \in \mathbb{R}_+^4 : 0 \leq x_1 < x_2 \leq x_3 < x_4, x_4 = \hat{u}^{-1}\left(\hat{u}(x_3) + \beta^{\tau'-\tau}\left(\hat{u}(x_2) - \hat{u}(x_1)\right)\right)$ for some $\tau < \tau'\Big\}$, we obtain that

$$\mathcal{D}_{(\beta,\hat{\beta})} = \inf_{t<t'} \frac{\beta^\tau/\hat{\beta}^\tau}{\beta^{\tau'}/\hat{\beta}^{\tau'}} \geq \sup_{(x_1,x_2,x_3,x_4)\in\hat{X}} \frac{\left[u(x_4) - u(x_3)\right]/\left[\hat{u}(x_4) - \hat{u}(x_3)\right]}{\left[u(x_2) - u(x_1)\right]/\left[\hat{u}(x_2) - \hat{u}(x_1)\right]} \equiv \widetilde{\mathcal{C}_{u,\hat{u}}}$$

The next lemma ends the necessity part of the proof:

Lemma 3. $\mathcal{C}_{u,\hat{u}} = \widetilde{\mathcal{C}_{u,\hat{u}}}$.

Proof. Denote $y_1 = \hat{u}(x_1)$, $y_2 = \hat{u}(x_2)$, $y_3 = \hat{u}(x_3)$ and $y_4 = \hat{u}(x_4)$. $\mathcal{C}_{u,\hat{u}}$ can be rewritten $\mathcal{C}_{u,\hat{u}} = \sup_{0\leq y_1 < y_2 \leq y_3 < y_4} \frac{u\circ\hat{u}^{-1}(y_4) - u\circ\hat{u}^{-1}(y_3)}{y_4 - y_3} / \frac{u\circ\hat{u}^{-1}(y_2) - u\circ\hat{u}^{-1}(y_1)}{y_2 - y_1} = G_{u\circ\hat{u}^{-1}}$, with $G_{u\circ\hat{u}^{-1}}$ the 'greediness' index of Chateauneuf, Cohen and Meilijson (2005) for function $u \circ \hat{u}^{-1}$.

Denote also $\hat{X}_\lambda = \Big\{(y_1, y_2, y_3, y_4) \in \mathbb{R}_+^4 : 0 \leq y_1 < y_2 \leq y_3 < y_4, \frac{y_4 - y_3}{y_2 - y_1} = \lambda\Big\}$ and $G_{u\circ\hat{u}^{-1}}(\lambda) = \sup_{(y_1,y_2,y_3,y_4)\in\hat{X}_\lambda} \frac{u\circ\hat{u}^{-1}(y_4) - u\circ\hat{u}^{-1}(y_3)}{y_4 - y_3} / \frac{u\circ\hat{u}^{-1}(y_2) - u\circ\hat{u}^{-1}(y_1)}{y_2 - y_1}$. We obtain that $\widetilde{\mathcal{C}_{u,\hat{u}}} = \sup_{\lambda=\beta^{\tau-\tau'},\tau<\tau'} G_{u\circ\hat{u}^{-1}}(\lambda)$. But by Lemma 1 in Chateauneuf, Cohen and Meilijson (2005) we know that $G_{u\circ\hat{u}^{-1}} = G_{u\circ\hat{u}^{-1}}(\lambda)$ for any $\lambda > 0$, so that $\mathcal{C}_{u,\hat{u}} = \widetilde{\mathcal{C}_{u,\hat{u}}}$. $\square$

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Sufficiency. Assume that $\mathbf{y} \succ_I \mathbf{x}$ and $\mathbf{x} \hat{\sim} \mathbf{y}$.[8] Equation (1) yields:

$$
\begin{aligned}
0 &= W_{(\hat{\beta},\hat{u})}(\mathbf{x}) - W_{(\hat{\beta},\hat{u})}(\mathbf{y}) \\
&= \hat{\beta}^{\{\tau\}-1}\Big(\hat{u}(x_\tau) - \hat{u}(y_\tau)\Big) - \hat{\beta}^{\{\tau'\}-1}\Big(\hat{u}(y_{\tau'}) - \hat{u}(x_{\tau'})\Big)
\end{aligned}
$$

Hence

$$
\hat{\beta}^{\{\tau\}-1}\Big(\hat{u}(x_\tau) - \hat{u}(y_\tau)\Big) = \hat{\beta}^{\{\tau'\}-1}\Big(\hat{u}(y_{\tau'}) - \hat{u}(x_{\tau'})\Big) \tag{9}
$$

Now, we want to compute the difference $W(\mathbf{x}) - W(\mathbf{y})$. Using Equality (9), we obtain:

$$
\begin{aligned}
W_{(\beta,u)}(\mathbf{x}) - W_{(\beta,u)}(\mathbf{y}) &= \beta^{\{\tau\}-1}\Big(u(x_\tau) - u(y_\tau)\Big) - \beta^{\{\tau'\}-1}\Big(u(y_{\tau'}) - u(x_{\tau'})\Big) \\
&= \hat{\beta}^{\{\tau'\}-1}\Big(\hat{u}(y_{\tau'}) - \hat{u}(x_{\tau'})\Big)\left(\frac{\beta^{\{\tau\}-1}\Big(u(x_\tau) - u(y_\tau)\Big)}{\hat{\beta}^{\{\tau\}-1}\Big(\hat{u}(x_\tau) - \hat{u}(y_\tau)\Big)} - \frac{\beta^{\{\tau'\}-1}\Big(u(y_{\tau'}) - u(x_{\tau'})\Big)}{\hat{\beta}^{\{\tau'\}-1}\Big(\hat{u}(y_{\tau'}) - \hat{u}(x_{\tau'})\Big)}\right) \\
&= \frac{\beta^{\{\tau'\}-1}\big(\hat{u}(y_{\tau'}) - \hat{u}(x_{\tau'})\big)}{\big(\hat{u}(x_\tau) - \hat{u}(y_\tau)\big)/\big(u(x_\tau) - u(y_\tau)\big)}\left(\Big(\beta/\hat{\beta}\Big)^{\{\tau\}-\{\tau'\}} - \frac{\big(u(y_{\tau'}) - u(x_{\tau'})\big)/\big(\hat{u}(y_{\tau'}) - \hat{u}(x_{\tau'})\big)}{\big(u(x_\tau) - u(y_\tau)\big)/\big(\hat{u}(x_\tau) - \hat{u}(y_\tau)\big)}\right) \\
&\geq \frac{\beta^{\{\tau'\}-1}\big(\hat{u}(y_{\tau'}) - \hat{u}(x_{\tau'})\big)}{\big(\hat{u}(x_\tau) - \hat{u}(y_\tau)\big)/\big(u(x_\tau) - u(y_\tau)\big)}\Big(\mathcal{D}_{\beta,\hat{\beta}} - \mathcal{C}_{u,\hat{u}}\Big)
\end{aligned}
$$

Consequently, whenever $\mathcal{D}_{\beta,\hat{\beta}} \geq \mathcal{C}_{u,\hat{u}}$, $W_{(\beta,u)}(\mathbf{x}) \geq W_{(\beta,u)}(\mathbf{y})$.

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[8]If $\mathbf{x} \hat{\succ} \mathbf{y}$ by monotonicity there exists $x_\tau - y_\tau > \varepsilon > 0$ such that $\mathbf{x}'$ defined by $x'_\tau = x_\tau - \varepsilon$ (where τ is as in Definition 2) and $x'_t = x_t$ for all $t \neq \tau$ satisfies $\mathbf{x}'\hat{\sim}\mathbf{y}$. By monotonicity, $\mathbf{x} \succ \mathbf{x}'$; by transitivity, $\mathbf{x} \succ \mathbf{y}$; it is also the case that $\mathbf{y} \succ_I \mathbf{x}'$. Hence, if $(\mathbf{y} \succ_I \mathbf{x}\ \&\ \mathbf{x} \hat{\sim} \mathbf{y}) \Longrightarrow (\mathbf{x} \succeq \mathbf{y})$ then $(\mathbf{y} \succ_I \mathbf{x}\ \&\ \mathbf{x} \hat{\succ} \mathbf{y}) \Longrightarrow (\mathbf{x} \succ \mathbf{y})$.

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