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The Inherited Inequality: How Demographic Aging and Pension Reforms can Change the Intergenerational Transmission of Wealth*

Justina Klimaviciute,† Harun Onder,‡ Pierre Pestieau§

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Abstract

The role of inherited wealth in modern economies has increasingly come under scrutiny. This study presents one of the first attempts to shed light on how demographic aging could shape this role. It shows that, in the absence of retirement annuities, or for a given level of annuitization, both increasing longevity and decreasing fertility should reduce the inherited share of total wealth in a given economy. Thus, aging is not likely to explain a recent surge in this share in some advanced economies. Shrinking retirement annuities, however, could offset and potentially reverse these effects. The paper also shows that individual bequests will be more unequally distributed if aging is driven by a drop in fertility. In comparison, the effect of increasing longevity on their distribution is non-monotonic.

JEL Classification Numbers: D14, D31, D64, D91, E21, H55, J11
Keywords: Inherited wealth, Inheritance, Aging, Inequality, Social Security

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†CREPP, University of Liege, e-mail: justina.klimaviciute@ulg.ac.be.

‡Macroeconomics and Fiscal Management Global Practice, The World Bank, e-mail: honder@worldbank.org.

§CREPP, University of Liege, CORE, Université Catholique de Louvain and PSE, e-mail: p.pestieau@ulg.ac.be.

1 Introduction

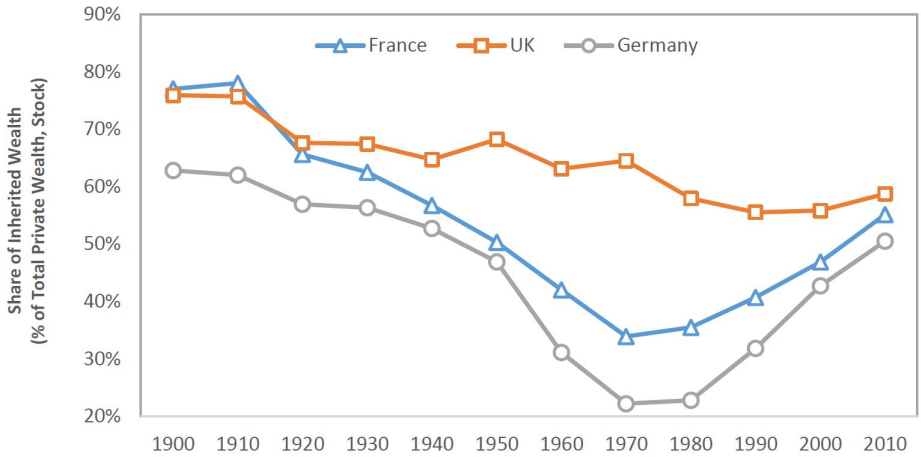
An important premise of modern capitalism is the idea that anyone, regardless of her parent's wealth, can become rich with the right entrepreneurial skills. A recent surge of self-made billionaires is often considered to be the proof of this. For instance, Kaplan and Rauh (2013) report that among the Americans who made it to the Forbes 400 list, which provides a list of the wealthiest people ranked by net worth, the share of those who grew up wealthy fell from 60 percent to 32 percent between 1982 and 2011. There seems to go the age of aristocracy, inherited wealth, and privilege. But, does it?

Notwithstanding the observation above, the role of inherited wealth is on the rise in a number of advanced economies. Piketty and Zucman (2014) show that although the inherited share of total wealth decreased steadily from the beginning of the 20th century until the 1970s in Europe (Figure 1 for France, UK and Germany), it began to increase again after that, a trend that has continued. Accordingly, the earlier reduction was driven by wars in the first half of the century, which impoverished the population across the board. Consequently, those who died between 1950-1960 were reported to be the least wealthy generation (measured at the time of death) in the 20th century. The increase in the inherited share of total wealth, on the other hand, stemmed from increasing *inter vivos* gifts. Although it is not clear why such gifts started increasing, Piketty and Zucman (2014) suggest that ever longer lives may have induced parents to transfer a portion of inheritance sooner to help their offspring. We can infer from this argument that the total bequeathed wealth (both in the form of inheritance and inter vivos gifts) has also increased as a response to longer lives, which is the main motivation for our discussion here.

How exactly does aging of a population affect the size and distribution of its inherited wealth? It is well known that decreasing mortality and fertility rates, both of which lead to an increase in average age in a society, have led to dramatic changes in the demographic structure of societies, especially in high income countries, in recent decades. Figure 2 shows the survival curves from 1950 to 2010. For instance, a 60 years old person could expect to live about 17 more years in 1950, and that has increased to 23 by now. In the meantime, the fertility rate fell from about 3 children per woman to 1.8 children. In this paper, we formalize the ideas summarized in a companion paper by Onder and Pestieau (2016a), to study the effects of these changes on the role of inheritance in modern societies in a somewhat systematic manner[1].

[1] See also Weil (1996).

Figure 1: The share of inherited wealth in total private wealth, 1900-2010



Source: Piketty and Zucman (2014a).

In particular, we are interested in shedding light on the effects of a decrease in fertility and of an increase in longevity on two indicators that define the size and distribution of inherited wealth, albeit in an imperfect manner. These are:

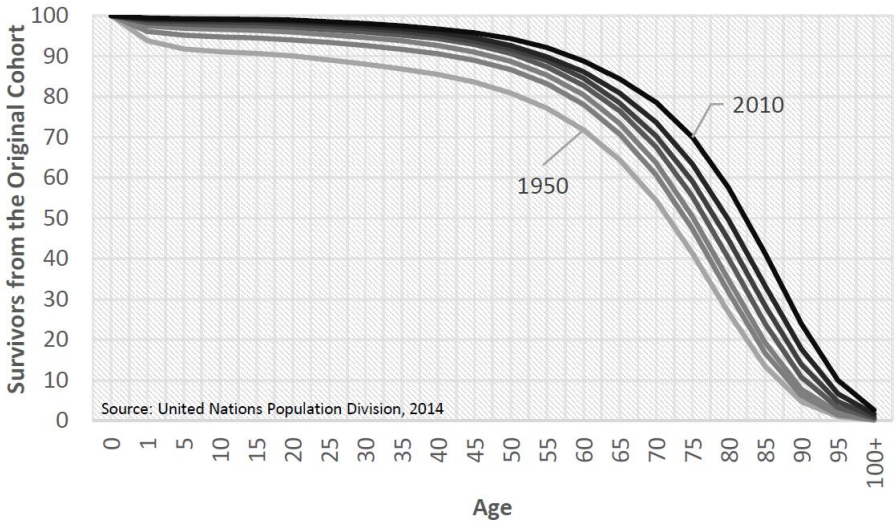
1. Inherited share of total wealth (ISW), which indicates the aggregate role of inheritance,

2. Inherited wealth inequality (IWI), which characterizes the distribution of inheritance.

In order to investigate the effects of aging on these two indicators, we adopt a simple two-period OLG model where individuals save for retirement and for the joy of bequeathing. As parts of these savings are not annuitized, the model also features accidental bequests in addition to altruistic bequests. Using this framework we find that, as opposed to the implication made by Piketty and Zucman (2014), aging in either form, a decrease in fertility or an increase in longevity, is not likely to explain the U-shaped pattern in the inherited share of total wealth in advanced economies. Both types of aging are expected to reduce the ISW. This is primarily because intentional bequests fall following a decrease in fertility, and although individual accidental bequests become larger with decreasing mortality, they also become less frequent, which dominates the effect on the size.

Our results also suggest an alternative mechanism that could generate such a U-shaped pattern of the ISW after the Second World War: the rise and fall of retirement annuities. In many high income countries, public and private defined benefits systems took up after the Second World War and the benefits provided by these systems increased steadily for several decades. Our findings show that, other things being equal, an increase in such annuitization could lead to a decrease in the ISW. Interestingly, however, the annuitization trend was reversed towards the end of the century following a decline in public pension benefits and a progressive shift from defined bene-

Figure 2: Evolution of survival in high income countries, 1950-2010



fit to defined contribution pensions. Thus, in theory, the rebound of the ISW could be driven by such a progressive abandonment of annuitized retirement savings.

Before proceeding two observations are in order. First, several authors have challenged the findings of Piketty and Zucman as to the recent upsurge of inherited wealth. Wolff (2015) notes that such upsurge does not apply to the US. Rognlie (2015), on the other hand, joins many other critiques to show that real estate wealth is not properly measured in Piketty's work. Second, the recent decline in the annuitization of retirement saving comes from two factors. There is the widespread decline in individual public pension benefits that is due to the increase in the number of retirees and to the governments' budgetary difficulties. For example, in most European countries one observes a sharp decline in the replacement ratio in public pensions. This is due to the fact that they apply indexation rules for pensions that do not fully reflect a 1:1 relationship with nominal wage increases. (See on this European Commission, 2015). Further, in private pensions, either mandatory or voluntary, there is a trend towards defined contribution formulas along with a payout in capital instead of in annuities. Such a trend can be explained by the desire of pension funds to reduce financial and longevity risks and by the preference of retirees towards cashing in assets accumulated in defined contribution pension plans (see Antonin, 2008 and Munnell et al., 2014).

This paper continues as follows. In Section 2, we present the general model and discuss what special cases we are going to use. Section 3 studies the effects of aging on the inherited share of total wealth, while Section 4 looks at the inequality of inherited wealth. In both sections, we separate the cases with and without annuitization. Finally, Section 5 focuses on some numerical simulations, whereas the last section provides some concluding remarks.

2 General Model

We use a simple two-period overlapping generations model to show our points. An individual who belongs to generation t can live for two periods: t and $t+1$. All individuals live a healthy life in the first period of their lifetimes; however, only a portion $\pi \in [0,1]$ of them can survive to live in the second period. We assume that the ex-ante probability of survival, π, is identical for all individuals regardless of their income and wealth.[2]

In the first period, each individual works and earns a wage w_t and receives a bequest from her parent b_t. A portion of these is used to finance the first-period consumption, c_t, and the rest is saved for two reasons: leaving bequests for own children (x_t) and financing a possible second period consumption, where the individual is assumed not to work for simplicity. The consumption oriented savings comprise a predetermined component in the form of annuities a_t and a voluntary component s_t, which is not annuitized. The amount of annuities is exogenous. It is supposed to comprise both public pensions and mandatory annuities.[3] The bequest is motivated as a "warm glow" giving that is based on some internal feeling of virtue arising form sacrifice in helping one's children or by the desire of controlling their life.[4]

If the individual lives over two periods successfully, then the second period consumption is given by $d_{t+1} = R_{t+1}(s_t + \frac{a_t}{\pi})$, where the second component in the brackets shows the annuities adjusted by the survival rate, and her bequest per child is given by $h_{t+1} = R_{t+1}x_t/n$, where R_{t+1} is the interest earnings on savings and n is the number of children. In comparison, if the individual dies at the end of the first period, all her savings will "accidentally" be inherited by her children, $h^*_{t+1} = R_{t+1}(s_t + x_t)/n$ and her claims to annuity will be reflected in other beneficiaries' annuities.

While everyone in the same generation is assumed to have the same wage, individuals differ according to the bequests they receive. Bequests depend on the individuals' family history and in particular, on their ancestors' longevity. In the general case, starting from generation $t = 0$, there are 2^t types of individuals in each generation t. These types include a dynasty where all ancestors happened to die prematurely, another dynasty where all ancestors enjoyed complete life spans, and all combinations in between these two. Denoting by j_t ($j_t \in [1, 2^t]$) an individual type in generation t, the utility maximization problem of type j_t individual is given by:

$$\max_{s,x} \; U_t^{j_t} = v(c_t^{j_t}) + \pi\delta\left[u(d_{t+1}^{j_t}) + n\gamma u(h_{t+1}^{j_t})\right] + (1-\pi)\delta n\gamma u(h_{t+1}^{j_t*}) \tag{1}$$

[2]See Lefebvre et al (2013) for an analysis on income-differentiated mortality.

[3]Implicitly we assume fully funded pensions. Introducing pay as you go pensions would not change the nature of results.

[4]See Andreoni (1990) for this.

$$s.t. \qquad c_t^{j_t} = w_t + b_t^{j_t} - s_t^{j_t} - x_t^{j_t} - a_t,$$

$$d_{t+1}^{j_t} = R_{t+1}(s_t^{j_t} + \frac{a_t}{\pi}),\ h_{t+1}^{j_t} = \frac{R_{t+1}x_t^{j_t}}{n},\ h_{t+1}^{j_t*} = \frac{R_{t+1}\left(s_t^{j_t} + x_t^{j_t}\right)}{n}$$

where δ is the discount factor, and $\gamma \in [0, 1]$ shows the relative utility weight of bequests in comparison to consumption. The first order conditions of this problem are given by:[5]

$$v'(c_t^{j_t}) = \pi\delta u'\left(d_{t+1}^{j_t}\right) R_{t+1} + (1-\pi)\,\delta\gamma u'\left(h_{t+1}^{j_t*}\right) R_{t+1} \tag{2}$$

$$v'(c_t^{j_t}) = \pi\delta\gamma u'\left(h_{t+1}^{j_t}\right) R_{t+1} + (1-\pi)\,\delta\gamma u'\left(h_{t+1}^{j_t*}\right) R_{t+1} \tag{3}$$

and the following equilibrium association between consumption and bequest motives:

$$u'\left(d_{t+1}^{j_t}\right) = \gamma u'\left(h_{t+1}^{j_t}\right) \tag{4}$$

To obtain more analytical insights, in what follows we are going to use specific sub-utility functions. In particular, we are going to assume $u(\cdot) = log(\cdot)$ and, when possible, also $v(c_t^{j_t}) = log(c_t^{j_t})$. However, for tractability reasons, we often need to assume that $v(c_t^{j_t})$ is linear and so the overall utility is of quasi-linear form. This assumption breaks the link between successive bequests, *i.e.* the optimal bequest chosen by an individual for her offspring is independent of the bequest she receives. Thus, at any given time, there are only two individual types in the economy: the offspring of those who die prematurely and leave a large bequest in intentional and accidental form, *i.e.* $b_t = h_t^*$, and the offspring of those who enjoy a long life span and leave only intentional bequests, *i.e.* $b_t = h_t$. This significantly simplifies the model and allows us to derive some analytical results when that is not possible with a more general specification. Nevertheless, in Section 5, we use the specification where all sub-utilities are logarithmic to provide some numerical simulations. Other assumptions we use are that annuities are actuarially fair and that, in spite of aging, the expected length of activity is constant.

We are primarily interested in showing how changes in fertility and survival rates, n and π, respectively, affect a commonly used indicator of wealth accumulation, namely the share of inherited wealth in total wealth (ISW). Furthermore, we look at the effect of aging on inherited wealth inequality (IWI).

[5]We assume that a_t is sufficiently small so that all individuals choose to save a positive amount.

3 Inherited Share of Total Wealth

In our simple model, the share of inherited wealth in total wealth can formally be defined as follows:

$$Inherited\ share\ of\ total\ wealth\ (ISW): \quad \Psi_{t+1} = \frac{(1-\pi)s_t + x_t}{s_t + a_t + x_t}, \tag{5}$$

where s_t is the average saving and x_t is the average intentional bequest in generation t.

We start by considering the logarithmic specification for all sub-utility functions in the individual utility. Thus, the utility of type j_t individual now writes as

$$U_t^{j_t} = \log(c_t^{j_t}) + \pi\delta\left[\log(d_{t+1}^{j_t}) + n\gamma\log(h_{t+1}^{j_t})\right] + (1-\pi)\delta n\gamma\log(h_{t+1}^{j_t*})$$

The first order conditions for $s_t^{j_t}$ and $x_t^{j_t}$, respectively, are as follows:

$$-\frac{1}{c_t^{j_t}} + \pi\delta\frac{R_{t+1}}{d_{t+1}^{j_t}} + (1-\pi)\delta\gamma\frac{R_{t+1}}{h_{t+1}^{j_t*}} = 0 \tag{6}$$

$$-\frac{1}{c_t^{j_t}} + \pi\delta\gamma\frac{R_{t+1}}{h_{t+1}^{j_t}} + (1-\pi)\delta\gamma\frac{R_{t+1}}{h_{t+1}^{j_t*}} = 0 \tag{7}$$

The first condition equalizes the marginal disutility from saving in the first period to the expected marginal utility of saving in the second period. Thus, the individual increases her savings until the expected sum of these utilities is equal to the opportunity cost of saving. The same idea applies to the voluntary bequests, $x_t^{j_t}$. Combining the two first order conditions above, we obtain the following relationship:

$$x_t^{j_t} = n\gamma\left[s_t^{j_t} + \frac{a_t}{\pi}\right] \tag{8}$$

which is a special case of (4). Intuitively, voluntary savings for future consumption and intentional bequests should yield the same marginal utilities after adjusting for the number of offspring and expected annuity benefits.

On the production side, we assume a Cobb-Douglas production function $y_t = k_t^\alpha$, where k_t is the average capital stock. From this it follows that

$$R_{t+1} = \alpha k_{t+1}^{\alpha-1} \tag{9}$$

and

$$w_t = (1-\alpha)k_t^\alpha \tag{10}$$

The motion of capital is described as:

$$nk_{t+1} = s_t + a_t + x_t \tag{11}$$

Note that capital is assumed to depreciate completely after each period. Although this assumption arises from convenience, it is not unrealistic considering the fact that a period denotes several decades in calendar.

The average saving and the average intentional bequest, respectively, write as

$$s_t = \sum_{j_t} s_t^{j_t} p_t^{j_t}$$

and

$$x_t = \sum_{j_t} x_t^{j_t} p_t^{j_t} = \sum_{j_t} n\gamma \left[s_t^{j_t} + \frac{a_t}{\pi} \right] p_t^{j_t} = n\gamma \left[s_t + \frac{a_t}{\pi} \right] \tag{12}$$

where $p_t^{j_t}$ is the probability of type j_t in the society.

The inherited share of total wealth thus writes as

$$\Psi_{t+1} = \frac{(1 - \pi + n\gamma)s_t + n\gamma \frac{a_t}{\pi}}{(1 + n\gamma)s_t + a_t + n\gamma \frac{a_t}{\pi}} \tag{13}$$

We will now discuss separately the case when there are no annuities ($a_t = 0$) and the case when annuities are present.

3.1 The Case without Annuities

Note that when $a_t = 0$, *i.e.* we shut down the annuity channel, all savings are bequeathable, and the optimal $x_t^{j_t}$ and $s_t^{j_t}$ become proportional in equation (8). Although this observation is not specific to logarithmic utility form, it proves to be an important property.[6]

When a_t is set to zero, equation (13) is reduced to

$$\Psi_{t+1} = \frac{(1 - \pi) + n\gamma}{1 + n\gamma}$$

from which $\partial\Psi/\partial\pi < 0$ and $-\partial\Psi/\partial n < 0$ follow through. We summarize this result in the following proposition.

[6] More generally, the proportionality between $x_t^{j_t}$ and $s_t^{j_t}$ can also be generated by any sub-utility function of the Constant Relative Risk Aversion (CRRA) form.

Proposition 1. *In the absence of annuities, the inherited share of aggregate wealth decreases with an increase in longevity (decrease in mortality) or a decrease in fertility.*

This is the most important and rather robust result of our analysis. A decrease in fertility or an increase in longevity has a depressive effect on the relative importance of inheritance in wealth accumulation. Thus, this finding does not support Piketty and Zucman's argument in this case.

3.2 The Case with Annuities

We have just seen that without annuities aging has a clear depressing effect on inherited wealth whether it arises from declining fertility or increasing longevity. We now explore the alternative idea that an increase in the share of inherited wealth could be driven by a decline in annuitized retirement savings. To investigate this, we now consider $a_t > 0$.

When a_t was equal to zero, we saw that the inherited share of total wealth reduced to a simple expression which does not depend on the model's endogenous variables. With $a_t > 0$, this is no longer the case and we have to consider the whole expression in equation (13). Thus, we now need to solve the model for s_t or at least to know how it depends on n, π and a_t. However, this becomes impossible when the utility function is non-linear in the first period consumption. Therefore, to analyze the case of $a_t > 0$, we use quasi-linear utility. In particular, individual utility now writes as

$$U_t = w_t + b_t - s_t - x_t - a_t + \pi\delta\left[\log(d_{t+1}) + n\gamma\log(h_{t+1})\right] + (1-\pi)\delta n\gamma\log(h^*_{t+1})$$

where $b_t \in \{h_t, h^*_t\}$. Note that we no longer use the superscript j_t since with quasi-linear utility, individual choices of savings and intentional bequests do not depend on the received bequests and are thus the same for all individuals.

The first order conditions for s_t and x_t are now as follows:

$$-1 + \frac{\pi\delta}{s_t + \frac{a_t}{\pi}} + \frac{(1-\pi)\delta n\gamma}{s_t + x_t} = 0 \tag{14}$$

$$-1 + \frac{\pi\delta n\gamma}{x_t} + \frac{(1-\pi)\delta n\gamma}{s_t + x_t} = 0 \tag{15}$$

which, as before, gives the relationship

$$x_t = n\gamma\left[s_t + \frac{a_t}{\pi}\right] \tag{16}$$

We are now going to explore the effects of annuities, fertility, and longevity on equilibrium

voluntary savings s_t and intentional bequests x_t and, consequently, on the inherited share of total wealth.

Fully differentiating (14) and (15) with respect to a_t and combining, we can obtain

$$\frac{\partial s_t}{\partial a_t} = \frac{-\left[\pi\left(s_t + x_t\right)^2 + (1-\pi)x_t^2\right]}{\pi\left[\pi\left(s_t + x_t\right)^2 + (1-\pi)n\gamma\left(s_t + \frac{a_t}{\pi}\right)^2 + (1-\pi)x_t^2\right]} < 0 \tag{17}$$

and

$$\frac{\partial x_t}{\partial a_t} = \frac{(1-\pi)x_t^2}{\pi\left[\pi\left(s_t + x_t\right)^2 + (1-\pi)n\gamma\left(s_t + \frac{a_t}{\pi}\right)^2 + (1-\pi)x_t^2\right]} > 0 \tag{18}$$

From an individual perspective, annuities and voluntary savings are substitutes in financing consumption in the second period of life. Thus, given the survival probability π, an increase in annuities decreases the expected marginal utility from saving an additional dollar voluntarily. However, the decrease in voluntary savings is smaller than the increase in annuity benefits, which allows to increase intentional bequests.

Using (17) and (18), we can derive the effect of a_t on the inherited share of total wealth, which gives

$$\frac{\partial \Psi_{t+1}}{\partial a_t} = \frac{\left[(1-\pi)\frac{\partial s_t}{\partial a_t} + \frac{\partial x_t}{\partial a_t}\right]\left[s_t + a_t + x_t\right] - \left[(1-\pi)s_t + x_t\right]\left[\frac{\partial s_t}{\partial a_t} + 1 + \frac{\partial x_t}{\partial a_t}\right]}{\left[s_t + a_t + x_t\right]^2} =$$

$$= \frac{-(1-\pi)\left[(s_t + 2x_t)(\pi s_t^2 + a_t s_t) + n\gamma(s_t + \frac{a_t}{\pi})^2\left(x_t + (1-\pi)s_t\right)\right]}{\left[\pi\left(s_t + x_t\right)^2 + (1-\pi)n\gamma\left(s_t + \frac{a_t}{\pi}\right)^2 + (1-\pi)x_t^2\right]\left[s_t + a_t + x_t\right]^2} < 0$$

Thus, an increase in annuities decreases the inherited share of total wealth. Such an increasing trend in annuities was indeed observed in the post-war era in advanced economies, where pensions coverage with defined benefits increased rapidly. Thus, this result suggests that an increase and a subsequent decrease in annuitization could theoretically explain the U-shaped pattern of inherited share of total wealth observed within the last half-century.

Fully differentiating (14) and (15) with respect to n and combining, we get

$$-\frac{\partial s_t}{\partial n} = \frac{-(1-\pi)\left(s_t + \frac{a_t}{\pi}\right)^2\gamma s_t}{\left[\pi\left(s_t + x_t\right)^2 + (1-\pi)n\gamma\left(s_t + \frac{a_t}{\pi}\right)^2 + (1-\pi)x_t^2\right]} < 0 \tag{19}$$

and

$$-\frac{\partial x_t}{\partial n} = \frac{-x_t\left[(s_t+x_t)(\pi s_t+x_t)+(1-\pi)n\gamma\left(s_t+\frac{a_t}{\pi}\right)^2\right]}{n\left[\pi\left(s_t+x_t\right)^2+(1-\pi)n\gamma\left(s_t+\frac{a_t}{\pi}\right)^2+(1-\pi)x_t^2\right]} < 0 \tag{20}$$

A decrease in fertility decreases both voluntary savings and intentional bequests. Indeed, it can be seen from the first order conditions (14) and (15) that an increase in n increases the marginal utility of voluntary savings from the accidental bequest channel and also increases the marginal utility of intentional bequests.

The effect of n on the inherited share of total wealth can be derived as

$$-\frac{\partial \Psi_{t+1}}{\partial n} = \frac{\left[(1-\pi)\frac{\partial s_t}{\partial n}+\frac{\partial x_t}{\partial n}\right]\left[s_t+a_t+x_t\right]-\left[(1-\pi)s_t+x_t\right]\left[\frac{\partial s_t}{\partial n}+\frac{\partial x_t}{\partial n}\right]}{-\left[s_t+a_t+x_t\right]^2} =$$

$$= \frac{\left[(1-\pi)\frac{\partial s_t}{\partial n}+\frac{\partial x_t}{\partial n}\right]a_t+\pi\left[\frac{\partial x_t}{\partial n}s_t-\frac{\partial s_t}{\partial n}x_t\right]}{-\left[s_t+a_t+x_t\right]^2} < 0$$

which follows from the fact that $\frac{\partial x_t}{\partial n}s_t - \frac{\partial s_t}{\partial n}x_t$ can be shown to be positive. Thus, as in the case without annuities, a decrease in fertility decreases the inherited share of total wealth.

Finally, fully differentiating (14) and (15) with respect to π and combining, we obtain

$$\frac{\partial s_t}{\partial \pi} = \frac{n\gamma\frac{a_t}{\pi}(\pi s_t+x_t)\left(s_t+\frac{a_t}{\pi}\right)+s_t(s_t+x_t)(2a_t+\pi s_t)}{\pi\left[\pi\left(s_t+x_t\right)^2+(1-\pi)n\gamma\left(s_t+\frac{a_t}{\pi}\right)^2+(1-\pi)x_t^2\right]} > 0 \tag{21}$$

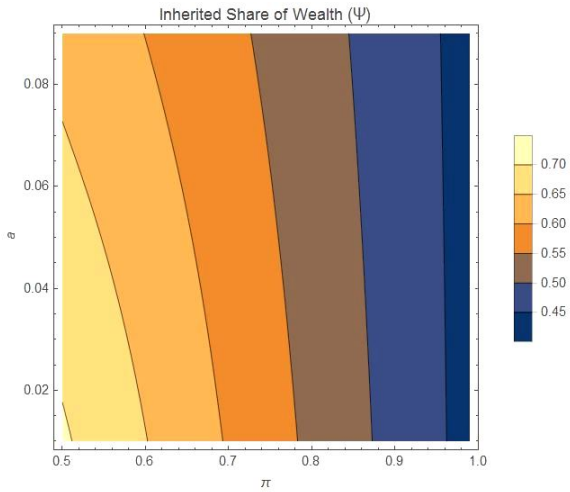
and

$$\frac{\partial x_t}{\partial \pi} = \frac{\pi^2 x_t s_t\left(s_t+x_t\right)^3+(1-\pi)x_t^4\left(\pi s_t-\frac{a_t}{\pi}\right)+\pi(1-\pi)s_t x_t^2\left[s_t x_t-\frac{a_t}{\pi}s_t-\frac{2a_t}{\pi}x_t\right]}{\pi\left[\pi\left(s_t+x_t\right)^2+(1-\pi)n\gamma\left(s_t+\frac{a_t}{\pi}\right)^2+(1-\pi)x_t^2\right]\left[\pi\left(s_t+x_t\right)^2+(1-\pi)x_t^2\right]} \gtrless 0 \tag{22}$$

An increase in longevity increases voluntary savings. This occurs as a combined result of three factors. First, consumption needs in the second period of life become more likely, which has a first order effect on savings. Second, for a given level of a_t, annuity benefits $\frac{a_t}{\pi}$ decrease with π. To compensate for this, and prevent a substantial loss in second period consumption, the voluntary savings need to be increased. Third, with higher π, accidental bequests become less likely, therefore the expected marginal utility from leaving an accidental bequest becomes smaller. The last negative effect counteracts the previous positive ones; however, in the end, the former effects dominate and voluntary savings increase in net terms.

On the other hand, the effect of longevity on intentional bequests is generally ambiguous. As it can be seen from (16), the increase in s_t pushes for increasing x_t, but the decrease in annuity

Figure 3: The impact of changes in longevity on inherited share of wealth (ISW)



Notes: The horizontal axis shows the survival probability (π), while the vertical axis shows the annuity values (a). The contour curves show the iso-ISW values, and a move from dark blue to light yellow denotes an increase in the ISW.

benefits $\frac{a_t}{\pi}$ has an opposite effect, and the overall impact is not clear.

The effect of π on the inherited share of total wealth is generally unclear as well. For this reason, we explore this effect by performing numerical simulations based on common parameter values from literature. In particular, taking $\gamma = 0.5$ and $n = 1.5$ and varying the level of annuities, the effect of longevity on the inherited share of total wealth is depicted in Figure 3. In this figure, the contours denote the iso-Ψ combinations, and as the shades shift from darker to lighter tones, the inherited share of total wealth increases. It can therefore be seen that an increase in longevity decreases the inherited share of total wealth, i.e. $\frac{\partial \Psi}{\partial \pi} < 0$. This is again the same effect as in the case without annuities.

The results of this sub-section can be summarized in the following proposition.

Proposition 2. *In the presence of annuities, the inherited share of aggregate wealth decreases with an increase in annuities and with a decrease in fertility. The effect of an increase in longevity (decrease in mortality) is generally not clear but is negative with reasonable parameter values.*

Therefore, we see that, as in the case without annuities, aging has a depressive effect on the inherited share of total wealth, which again goes against Piketty and Zucman's argument. At the same time, as discussed above, our results suggest that the U-shaped form of the inherited share of total wealth could rather be explained by trends in annuitization.

4 Inherited Wealth Inequality

In this section, we attempt to assess the impact of aging on the distribution of inherited wealth. For tractability reasons, to study the inherited wealth inequality we use the quasi-linear utility specification as in sub-section 3.2. As discussed before, in this case there are two levels of inherited wealth in the society: those of the children whose parent survives and those of the children whose parent did not survive. The indicator of the inherited wealth inequality can formally be defined as

$$Inherited\, wealth\, inequality\, (IWI): \quad \Phi_{t+1} = \frac{\sqrt{V(\omega_{t+1})}}{\bar{\omega}_{t+1}}, \tag{23}$$

where $\omega_{t+1} \in \{h_{t+1}, h^*_{t+1}\}$ denote different types of bequest, $V(\omega_{t+1})$ denotes the variance of bequests, and $\bar{\omega}_{t+1} = \pi h_{t+1} + (1-\pi)h^*_{t+1}$ shows the average size of bequest. Therefore, Φ_{t+1} is the coefficient of variation, defined for bequests.

4.1 The Case without Annuities

We first consider the case when $a_t = 0$. In this case, we know that $x_t = n\gamma s_t$, and we therefore obtain

$$\bar{\omega}_{t+1} = \frac{R_{t+1} s_t (1 + n\gamma - \pi)}{n}$$

and

$$V(\omega_{t+1}) = \frac{R^2_{t+1} s^2_t \pi (1-\pi)}{n^2}$$

From this it follows that the coefficient of variation is

$$\Phi_{t+1} = \frac{\sqrt{\pi(1-\pi)}}{n\gamma + (1-\pi)} \tag{24}$$

which lends itself to study the effect of n and π on it:

$$-\frac{\partial \Phi_{t+1}}{\partial n} > 0; \; \frac{\partial \Phi_{t+1}}{\partial \pi} \gtrless 0 \Leftrightarrow \pi \lesseqgtr 1/2$$

In words, we observe that aging has a dis-equalizing effect for any value of n and for $\pi < 1/2$. This result is summarized in the following proposition.

Proposition 3. *A decrease in fertility increases the inequality of inherited wealth in the absence of annuities. In comparison, starting from low levels ($\pi < 1/2$), an increase in longevity first increases this inequality and then, for relatively high levels of longevity ($\pi > 1/2$), decreases it.*

Note that this result depends on how we measure inequality. Using a variance based indicator

(coefficient of variation) to measure the dispersion, as it is known commonly, implies that the highest variance is reached when the probability of survival is equal to 0.5.

4.2 The Case with Annuities

When $a_t > 0$, the coefficient of variation writes as

$$\Phi_{t+1} = \frac{s_t \sqrt{\pi (1-\pi)}}{x_t + (1-\pi) s_t} \tag{25}$$

From this we can derive

$$\frac{\partial \Phi_{t+1}}{\partial a_t} = \frac{\sqrt{\pi (1-\pi)} \left[\frac{\partial s_t}{\partial a_t} x_t - \frac{\partial x_t}{\partial a_t} s_t \right]}{\left[x_t + (1-\pi) s_t \right]^2} < 0$$

and

$$-\frac{\partial \Phi_{t+1}}{\partial n} = \frac{-\sqrt{\pi (1-\pi)} \left[\frac{\partial s_t}{\partial n} x_t - \frac{\partial x_t}{\partial n} s_t \right]}{\left[x_t + (1-\pi) s_t \right]^2} > 0$$

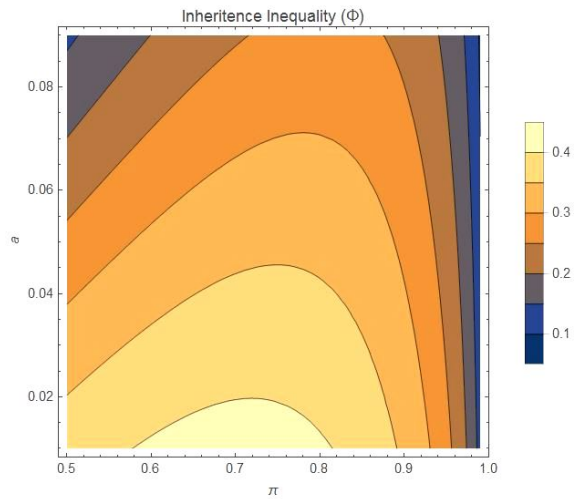
An increase in annuitization reduces inheritance inequality. Remember that there are two kinds of inheritance in our model, those that are received by children whose parents enjoyed a full life span and those that are received by children whose parents died prematurely. The difference between the two is accidental bequests. These bequests, in turn, are nothing but the voluntary savings that were not consumed by parents who died prematurely, which is depressed by an increase in annuities as we discussed above.

On the other hand, a decrease in fertility increases inheritance inequality. We have seen above that a decrease in fertility reduces both accidental bequests (through voluntary savings) and intentional bequests. However, it turns out that the decreases in the two types of bequests are such that the inequality between the two types of inheritances increases.

As it was the case with the inherited share of total wealth, the effect of longevity on inheritance inequality is generally not clear. We therefore again perform some numerical simulations which are shown in Figure 4. In this figure, the contours denote the iso-inequality combinations, and as the shades shift from darker to lighter tones, the inequality increases. As it can be seen from the figure, the effect of longevity on the inequality of inherited wealth is not monotonic: $\frac{d\Phi}{d\pi} > 0$ for low π and $\frac{d\Phi}{d\pi} < 0$ for high π. Starting from low levels, a rise in longevity initially increases the inequality; however, it eventually starts decreasing as the survival probability becomes high enough.

The above results are summarized in the following proposition.

Figure 4: The impact of changes in longevity on inherited wealth inequality (IWI)



Notes: The horizontal axis shows the survival probability (π), while the vertical axis shows the annuity values (a). The contour curves show the iso-IWI values, and a move from dark blue to light yellow denotes an increase in the IWI.

Proposition 4. *In the presence of annuities, a decrease in fertility increases and a rise in annuitization reduces the inequality of inherited wealth. With reasonable parameter values, an increase in longevity increases inheritance inequality for low levels of survival probability but decreases it for high probability levels.*

5 Numerical Simulations

In its general form, our model features bequests that are heterogeneous in size, and this heterogeneity is transmitted across generations by means of successive bequeathing. This follows from the fact that, for each generation, both accidental and planned bequests are functions of the transfers received from the previous generation. Thus, shocks to these transfers at any point in time are propagated across generations. As a result, each bequest depends on the receiving person's family history. More specifically, both the number of previous accidental bequests following early mortality cases and the exact sequence of those cases across periods matter in determining the size of inheritance received by an agent, or her type.

Therefore, with some abuse of biblical notation, suppose that an "Adam and Eve" economy constitutes an atomic generation in $t = 0$. Then there would be 2^t types of individuals in each generation $t > 0$. The lineages would include a dynasty where all ancestors happened to die prematurely, another dynasty where all ancestors enjoyed complete life spans, and all combinations in between these two extreme cases.

The problem with the exponentially increasing number of types is that the characteristics of

inherited wealth becomes intractable, analytically.[7] Thus, in this section, we consider a simulation approach to investigating the dynamic characteristics of the inherited share of total wealth and the inherited wealth inequality. We are primarily interested in analyzing the sensitivity of these measures to changes in demographic fundamentals, i.e. a decrease in fertility or an increase in life expectancy, and a reduction in retirement annuities. In doing so, we utilize the general model with annuities, which was developed earlier in this paper.

5.1 Simulation Strategy

In order to explore the genesis of the role played by inheritance in an open-small economy, we start our simulations with an identical group of individuals ($N = 10$), who receive no inheritance in period 0, i.e. $b^j_{t=0} = 0$ for all $j \in \{1, ..., 10\}$. This initial generation, nevertheless, leaves bequests like other generations.

In the baseline simulations, population doubles with each successive generation, i.e. the fertility rate is $n = 2$. The choice of fertility rate as an integer facilitates the numerical traction of expanding number of types, starting from a small group. Each individual survives to live in the second period of her lifetime with an ex-ante identical probability $\pi = 0.5$, as determined by a uniform distribution over the interval $[0, 1]$. Given the relatively small size of the population at the outset, however, the actual draws of survival outcomes in the early periods of the simulation exercise have sizable implications for the inherited wealth indicators in the subsequent periods: a coincidentally skewed realization of survival distribution in the first few periods could lead to arbitrary outcomes later. To address this problem, we run Monte Carlo simulations with 1000 draws, which ensures that the actual incidence of survival ratios is firmly centered around the chosen π.

With exponentially growing number of types and intergenerational linkages throughout the simulations, Monte Carlo simulations increase computational load significantly. In order to keep the simulations feasible, we conduct the exercise for a small, open economy, where wages and interest rates are fixed at pre-determined levels throughout the simulations. In addition, we restrict attention to 10 periods (after period 0), which allows sufficient time for the system to reach a stable level. We introduce permanent shocks to n, π, or a in period 3 to show impact on outcomes in both steady states and transition paths.

The results are displayed in Figure 5. In what follows, we will discuss important findings of this exercise with reference to our theoretical insights developed earlier in the paper.

[7] This is why we had to resort to quasi-linear utility function.

5.2 The Inherited Share of Total Wealth

The first panel in Figure 5 (panel a) shows the evolution of the ISW over time, as defined by Equation 13. The baseline case, where where the fertility, mortality and annuity variables are set at $n = 2$, $\pi = 0.5$ and $a = 0.1$, respectively, is shown by the solid-blue line in all three panels as benchmark. Because the initial generation by assumption receives no inheritance, the average bequests in the first few periods are also relatively small, but they increase over time despite the fixed wage and interest rate setting. This increase occurs at a decreasing rate, and the share of inherited wealth in the economy stabilizes towards the end of our simulations.

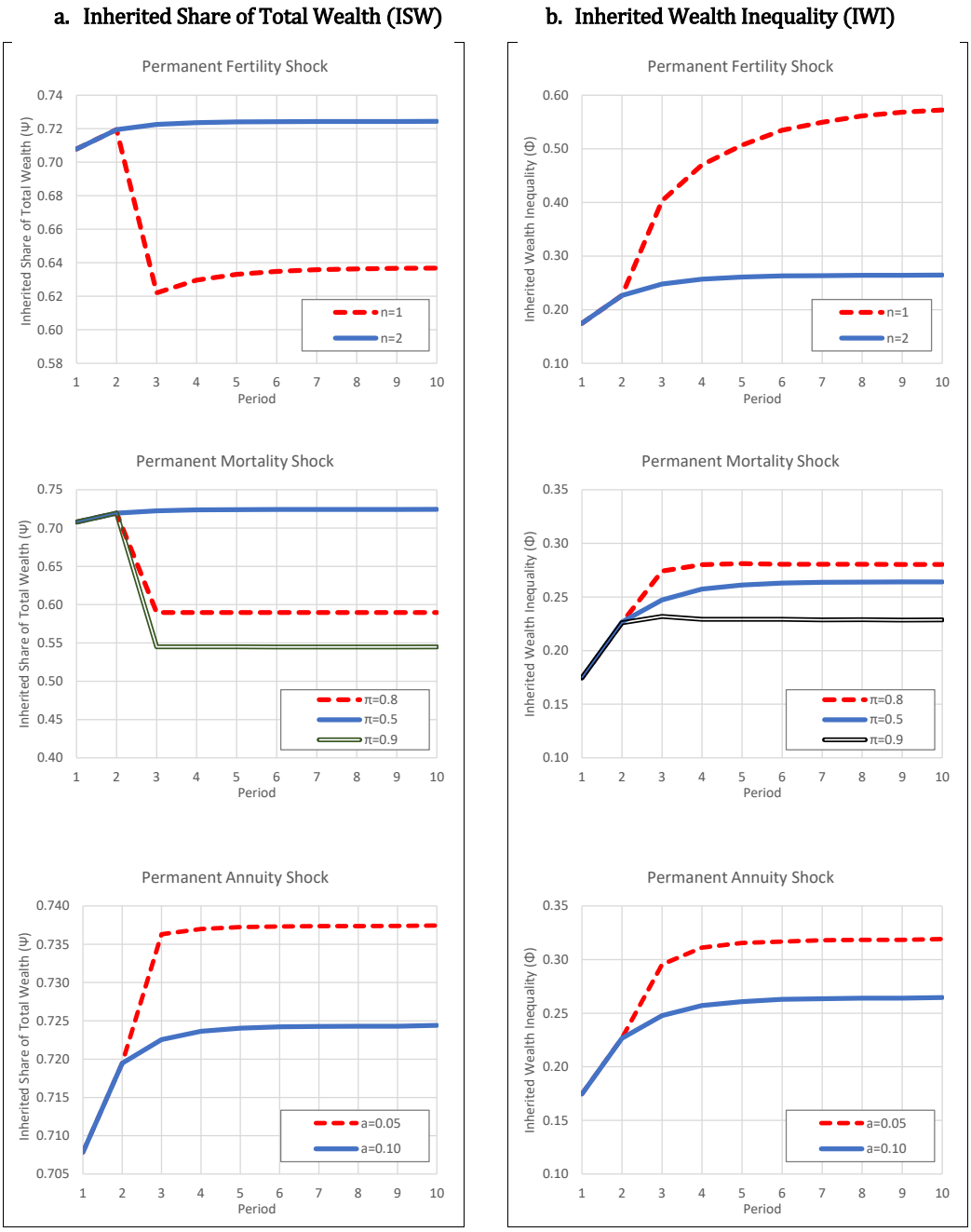
The simulations show the unanticipated and permanent changes in fertility, survival probability, and retirement annuities, all introduced in the second period, over three separate figures in panel a. The most important observation from this exercise is the confirmation that an increase in ISW is not likely to be driven by an aging demography, a result that is also suggested by our analytical solutions in the case of the general model without annuities (Proposition 1) and the quasi-linear model with annuities (Proposition 2). A decrease in fertility rate from $n = 2$ to $n = 1$ reduces the ISW from $\Psi = 0.72$ to $\Psi = 0.64$ in the steady state, as shown by the dashed-red line in the first figure. Similarly, an increase in survival probability from $\pi = 0.5$ to $\pi = 0.8$ or $\pi = 0.9$, reduces the ISW to $\Psi = 0.59$ and $\Psi = 0.55$, respectively. Our simulations also confirm the annuity related findings of Proposition 2. When annuities decrease from $a = 1$ to $a = 0.05$, the ISW increases from $\Psi = 0.72$ to $\Psi = 0.74$ in the long-term. Thus, overall, our analytical results hold when preferences are characterized by logarithmic utilities in both periods of life and retirement annuities are taken into consideration.

5.3 Inherited Wealth Inequality

The first observation to note from panel b is that, in the absence of shocks, the inherited wealth inequality has a concave and increasing shape over time. The dispersion of inheritances increases as the number of types grow over periods. This process, however, slows down in the outer years as the relative weight of extreme types (e.g. all or none of the ancestors had premature deaths) are reduced in the pool of types. In the baseline, where the fertility, mortality and annuity variables are set at $n = 2$, $\pi = 0.5$ and $a = 0.1$, respectively, the inherited wealth inequality stabilizes around 0.27 in the second half of the simulations.

Our second observation is about the non-linear effect of mortality rate changes on the inherited wealth inequality. Whereas a decrease in fertility from $n = 2$ to $n = 1$ increases the IWI from $\phi = 0.27$ to $\phi = 0.57$, the effect of mortality shock is non-monotonic. When π increases from 0.5 to 0.8, the long-term value of the inherited wealth inequality also increase from $\phi = 0.27$ to $\phi = 0.28$, as shown by the red dashed line. In comparison, a greater increase, from $\pi = 0.5$ to

Figure 5: Dynamics of inheritance with fertility, mortality, and annuity shocks



Notes: Simulations use baseline values of $n = 2$, $a = 0.1$, and $\pi = 0.5$ in cases that do involve shocks to these parameters. In all cases, the outcome paths admit these values until the unanticipated and permanent shocks are introduced in the second period. Common parameter values are as follows: $w = 1$, $R = 1.81$, $n = 2$, $\delta = 1$, and $\gamma = 0.5$. As no inheritance is received by the generation 0, both ISW and IWI take 0 values before period 1.

$\pi = 0.9$, reduces the inequality of wealth to about $\phi = 0.23$, as shown by the green hollow line. This result confirms the findings of the Propositions 3 and 4, where the former was specified for logarithmic preferences without annuities and the latter was specified with quasi-linear preferences with annuities. The simulations here suggest that these results are generalizable: starting from low levels, a rise in longevity initially increases the inequality; however, it eventually starts decreasing as the survival probability becomes high enough even when inheritances are linked across generations.

Finally, the third observation from our simulation exercise is about the ability of retirement annuities to explain the increase in the inequality of inheritances in a given generation. The last figure in panel b shows the scenario where the retirement annuity decreases from the baseline value of $a = 0.1$ to $a = 0.05$ from second period onward. As a result, the inequality of inherited wealth is stabilized around $\phi = 0.32$ (red dashed line) as opposed to its baseline value of $\phi = 0.27$ (blue solid line). Because the effect of annuity changes on inherited wealth inequality is monotonic, this exercise omits other possible shocks.

6 Conclusion

The purpose of this paper was to study the impact of aging, that is lower fertility and higher longevity, on the bequeathing decision and hence on the share of inheritance in capital accumulation. We also wanted to analyze the effect that the trends in annuitization could have on the level of inherited wealth. We show that aging has a depressive effect on the inherited share of total wealth, whereas declining annuitization, on the contrary, has a fostering impact.

On this point a caveat is in order. In this paper we have focused on two types of bequests, those relying on the absence of annuities and those arising from some joy of giving. In a companion paper, Onder and Pestieau (2016b) study other motives of bequeathing, such as pure altruism and preference for wealth. They show that declining fertility tends to foster altruistic bequests. As to the preference for wealth that characterizes the top wealthy individuals, it is likely that its relative importance increases with the increasing concentration of wealth.

Our findings rest on a quite simple model comprising a number of simplifying assumptions. The strongest of them is undoubtedly the quasi-linearity of the utility function, which we use to derive the most of our analytical results. On the other hand, our numerical simulations suggest that the analytical findings derived with quasi-linear utility are likely to hold with a more general specification as well. Finally, our paper is of relevance even in countries where the U-shaped evolution of bequests is not observed. In these countries, aging would indeed explain the observed decrease in the share of inherited wealth in total wealth if the shifts in annuities as discussed in this paper are absent.

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