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R&D Network Formation with Myopic and Farsighted Firms

Ana Mauleon* Jose J. Sempere-Monerris† Vincent Vannetelbosch‡

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Abstract

We study the formation of R&D networks when each firm benefits from the research done by other firms it is connected to. Firms can be either myopic or farsighted when deciding about the links they want to form. We propose the notion of myopic-farsighted stable set to determine the R&D networks that emerge in the long run. When the majority of firms is myopic, stability leads to R&D networks consisting of either two asymmetric components with the largest component comprises three-quarters of firms or two symmetric components of nearly equal size with the largest component having only myopic firms. But, once the majority of firms becomes farsighted, only R&D networks with two asymmetric components remain stable. Firms in the largest component obtain greater profits, with farsighted firms having in average more collaborations than myopic firms that are either loose-ends or central for spreading the innovation within the component. Besides myopic and farsighted firms, we introduce yes-firms that always accept the formation of any link and never delete a link subject to the constraint of non-negative profits. We show that yes-firms can stabilize R&D networks consisting of a single component that maximize the social welfare. Finally, we look at the evolution of R&D networks and we find that R&D networks with two symmetric components will be rapidly dismantled, single component R&D networks will persist many periods, while R&D networks consisting of two asymmetric components will persist forever.

Key words: Networks; R&D collaborations; Oligopoly; Myopia; Farsightedness.
JEL classification: C70, L13, L20.

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1 Introduction

R&D alliances are coordinating devices among two or more partners, where members seek access to new knowledge that would be in mutual advantage, and at the same time they risk to disclose unintentionally some strategic information. Such collaborative agreements introduce external effects on competitors, who can reply by searching for their own alliances.[1] Given that the increase of innovation is acknowledged to enhance both growth and welfare, it is important to analyze the bilateral incentives competing firms have to form alliances and how these alliances add up to form R&D network alliances. The objective of the paper is to analyze the R&D networks that would arise in the long run in presence of both myopic or farsighted firms.

We consider a n-firm industry, where initially firms produce an homogeneous good at a given marginal cost. Each firm is able to reduce its cost by forming a link with another competitor. The marginal cost of production reduction for one firm is proportional to the number of firms it is connected to. When a new link is formed between two firms already linked with others, all connected firms benefit from that link.[2] The collection of all the bilateral links define the R&D network which in turn determines the marginal cost profile for the n oligopolists. Once the R&D network is formed, firms compete in quantities.[3]

Up to now, it has been assumed that all firms are either myopic or farsighted when they decide with which firms they want to form a partnership. Goyal and Moraga-Gonzalez (2001) among others adopt the notion of pairwise stability to predict the R&D networks that one might expect to emerge in the long run. A R&D network is pairwise stable if no firm benefits from cutting a collaboration and no two firms benefit from forming a collaboration between them. Forming a link requires the consent of both firms, while deleting a link can be done unilaterally. Pairwise stability presumes that firms are myopic: they do not anticipate that other firms may react to their changes. But farsighted firms are able to anticipate that once they add or delete some links, other firms could add or delete links afterwards. For instance, Mauleon, Sempere-Monerris and Vannetelbosch (2014)

[1]The number of alliances worldwide seems to be increasing, according to the CATI (Cooperative Agreements and Technology Indicators) database, in 2006, about 900 new worldwide business technology alliances were formed, approximately two-thirds of which involved at least one U.S.-owned company regardless of location.

[2]In Mauleon, Sempere-Monerris and Vannetelbosch (2008), the reduction in marginal costs also depends on the total number of connected firms, but the marginal effect of that reduction decreases with the distance. In Goyal and Joshi (2003), the reduction in marginal costs only depends on the number of direct links. As if each firm was able to isolate the knowledge coming from each firm to whom it is linked. In Goyal and Moraga-Gonzalez (2001), firms even benefit, although imperfectly, from the research done by firms to whom they are not connected. All these papers study the emergence of R&D networks among only myopic firms. See also König, Battiston, Napoletano and Schweitzer (2011, 2012) or Dawid and Hellmann (2014) among others.

[3]Firms collaborate in R&D but do not cooperate on R&D effort choices. For a general background on R&D cooperation in oligopoly the reader is directed to Amir (2000), d'Aspremont and Jacquemin (1988), Kamien, Muller, and Zang (1992) and Katz (1986), among others.

show that farsighted firms may not put an end to some R&D link that appears in deficit to them as this can induce the formation of other competing links, ultimately lowering their profits.[4] However, Kirchsteiger, Mantovani, Mauleon and Vannetelbosch (2016) find experimental evidence in favor of a mixed population consisting of both myopic and farsighted agents. Hence, we are interested in addressing the following questions. Which R&D networks are likely to emerge in the long run with myopic and farsighted firms? Which firms are more likely to occupy key positions in the R&D network? What is the relationship between the stable R&D networks and the social welfare?

We propose the notion of myopic-farsighted stable set to determine the R&D networks that emerge when some firms are myopic while others are farsighted. A myopic-farsighted stable set is the set of networks satisfying internal and external stability with respect to the notion of myopic-farsighted improving path. That is, a set of networks is a myopic-farsighted stable set if there is no myopic-farsighted improving path between networks within the set and there is a myopic-farsighted improving path from any network outside the set to some network within the set. A myopic-farsighted improving path is simply a sequence of networks that can emerge when farsighted firms form or delete links based on the improvement the end network offers relative to the current network while myopic firms form or delete links based on the improvement the resulting network offers relative to the current network.[5]

When the majority of firms are myopic, the myopic-farsighted stable set consists of R&D networks having either two "asymmetric" components of different sizes (close to $3n/4$ and $n/4$) with farsighted firms occupying key positions in the largest component and myopic firms as loose-ends or medians,[6] or two "symmetric" components of nearly equal size (close $n/2+1$ and $n/2-1$) with the largest component having only myopic firms (n is the total number of firms).[7] In the case all firms are myopic, the myopic-farsighted stable set consists only of the networks having two components of nearly equal size, namely the pairwise stable networks.

However, when the majority of firms becomes farsighted, networks having two components of nearly equal size are now unstable. The myopic-farsighted stable set consists only of the networks having two components of different sizes (close to $3n/4$ and $n/4$) with possibly myopic firms as loose-ends or medians in the largest component. Hence,

[4] Mauleon and Vannetelbosch (2016) provide a comprehensive overview of the (myopic and farsighted) solution concepts for solving network formation games.

[5] One could interpret myopia and farsightedness as a proxy for its past experience in running R&D collaborations. For instance, well-established firms that have gained enough experience to acquire a better understanding of R&D collaboration could be more likely farsighted.

[6] A firm is called a loose-end node when it only has a single link. In a minimally connected component, a firm is called a median node if (by cutting one of its links) it cannot split the component in two components with one having a size strictly greater than $n/2$. A myopic loose-end or median firm has no incentive to delete a link.

[7] In our model, firms belonging to the largest component of size close to $3n/4$ obtain their best payoff among all networks consisting of at most two components.

even if there is a large majority of farsighted firms, firms will not necessarily end up segregated: farsighted firms can be mixed with myopic firms in the largest component, with farsighted firms having more collaborations on average. In the largest component, myopic firms enjoy greater profits than the farsighted and myopic firms that end up in the smallest component. In addition, some myopic firms (median nodes) have a high (if not the highest) betweenness centrality.[8] Thus, even if myopic firms are less active in terms of R&D collaborations they may play a crucial role for spreading the innovation within the component.

Beside having myopic together with farsighted firms we next introduce another type of firms: the yes-firms. Yes-firms always accept the formation of any link and never delete a link subject to the constraint of non-negative profits. Yes-firms could be viewed as public sector firms or universities that usually agree to add a new collaboration, if their profits are positive in the resulting network.[9] Which R&D network structure is likely to emerge when the majority of firms are myopic and others are farsighted or yes-firms? In fact, three types of stable networks can emerge. We still obtain the networks having either two components of different sizes (close to $3n/4$ and $n/4$) with myopic firms as loose-ends or medians in the largest component and all yes-firms in the smallest component, or two components of nearly equal size (close $n/2 + 1$ and $n/2 - 1$) with only myopic firms in the largest component and so all yes-firms in the other component. But now, a third type of networks can arise consisting of a single component with yes-firms bridging all other firms and myopic firms as loose-ends. Hence, yes-firms may play a crucial role by bridging myopic and farsighted firms and helping the society to reach a socially optimal state: R&D networks consisting of a single component maximize both social welfare and consumer surplus.

One could argue that yes-firms are myopic too. What would happen if they were farsighted instead? Farsighted firms look at the ending network instead of the resulting one. Nothing would change since all firms always make non-negative profits in any network configuration at equilibrium.[10] Thus, yes-firms can be interpreted both as myopic or farsighted yes-firms. Statistics about the R&D networks in Japan are in line with our theoretical predictions. From Table 1 we have one component or two components of different sizes depending on the region. For instance, in Tohoku and Kanto, there are two

[8] The betweenness centrality is a measure that captures how important an agent is in terms of connecting other agents.

[9] An important feature of collaboration agreements is that they often engage both private and public firms or universities. See Zikos (2010) for a theoretical analysis of the formation of links between private and public firms and Roesler and Broekel (2017) for an empirical study of the role of universities in the subsidized network of R&D collaboration in the German biotechnology industry.

[10] Given that single component networks are the best networks in terms of social welfare, adding links subject to the constraint of non-negative profits is consistent with the objective of maximizing the social welfare. However, if yes-firms would maximize their number of collaborations, being myopic or farsighted could potentially matter.

components with the size of the largest component close to $3n/4$. In Okinawa, there are two components with the size of the largest one close to $n/2+1$. In addition, we observe a much higher degree centrality for universities and public sector, except in Okinawa where the major industry is tourism.

		Degree centrality			N° components	Size of largest
Region	N° firms	Industry	Academy	Public	with #firms ≥ 10	component
Hokkaido	245	5.92	12.47	9.44	1	238
Tohoku	261	6.93	10.72	14.24	2	215
Kanto	686	5.85	8.67	14.88	2	547
Chubu	369	5.89	9.43	14.50	1	336
Kinki	528	6.19	13.53	20.84	1	496
Chugoku	302	6.72	13.43	21.21	1	298
Shikoku	220	6.02	11.13	17.56	1	203
Kyushu	381	5.67	13.63	15.67	1	357
Okinawa	65	5.02	6.07	7.00	2	32

Table 1: Descriptive statistics of the R&D networks in Japan under the METI programme "Consortium R&D Project for Regional Revitalization" during the period 2001-2007. Source: Yokura, Matsubara and Sternberg (2013).

Finally, we look at the evolution and dynamics of R&D networks with a group of myopic firms that are initially unconnected to each other. Over time, pairs of firms meet and decide whether or not to form or sever links with each other. A link can be severed unilaterally but agreement by both firms is needed to form a link. Since all firms are initially myopic, they decide to form or sever links if doing so increases their current profits. The length of a period is sufficiently long so that the process can converge to some stable R&D network. At the beginning of each period, some myopic firms become farsighted. It can be interpreted as if some myopic firms have gained enough experience to acquire a better understanding of R&D collaborations. Depending on their positions in the network, the process either stays at the same R&D network or converges to another stable R&D network. We study this process and show that it can reproduce and predict most features that have been observed empirically.[11] Starting from the empty network, the process converges to either a network consisting of two components of nearly equal sizes or a single component network with the yes-firms bridging all other firms. In the case the process reaches first a network consisting of two components of nearly equal sizes, it suffices that next one firm belonging to the largest component becomes farsighted for dismantling the network and converging to a network consisting of two components of

[11] For the German biotechnology industry during the period 2007-2010, Roesler and Broekel (2017) find that universities dominate the network as partners in many subsidized R&D projects: knowledge links among universities form the core of the network and universities are central for facilitating knowledge diffusion by connecting local private firms to inter-regional knowledge networks.

different sizes.[12] However, in the case the process reaches first a single component network, one would need that a large number of myopic firms become farsighted to move away from it. Hence, this single component network that maximize social welfare will persist many periods before moving to a network consisting of two components of different sizes that will persist forever.

The formation of research collaborations is also studied using the group formation approach where collaborations are modeled in terms of a coalition structure which is a partition of the set of firms (i.e. each firm can only belong to one coalition). Bloch (1995) proposes a sequential game for forming associations of firms. In equilibrium, firms form two asymmetric associations, with the largest one comprising roughly three-quarters of industry members. So, the sizes of the two associations coincide with those we obtain when the majority of firms are farsighted. In fact, by assuming that all connected firms in a network fully benefit from a new link, we recover the assumption in Bloch (1995) where the benefits from cooperation increase linearly in the size of the association. The network approach differs from the group formation approach by focusing on bilateral relationships and allowing for a richer class of collaborations. It also differs in the decision making for establishing R&D collaborations. Mutual consent is needed for forming a new link between two firms, whereas the consent of all members of the association is required when a firm joins the association.[13] Both approaches lead to similar conclusions only if some firms are farsighted and anticipate the reactions of other firms to the decisions they take.[14] Farsightedness helps firms to better exploit all the collaborative opportunities they face.[15]

The paper is organized as follows. Section 2 describes the model. Section 3 introduces the notion of a myopic-farsighted improving path and the concept of a myopic-farsighted stable set. Section 3 also characterizes the stable set when myopic firms interact with farsighted firms. Section 4 provides a characterization of the myopic-farsighted stable sets when some firms are yes-firms. Section 5 studies the evolution and the dynamics of R&D networks. Section 6 concludes and discusses the robustness of our results with respect to costly link formation, product competition, spillovers and reduction in marginal costs decreasing with the distance.

[12] Along the transition from a network consisting of two components of nearly equal sizes to a network consisting of a two components of different sizes, single component networks are likely to be visited.

[13] An exception is the open membership game. Yi (1997) finds that only the grand coalition is stable, but this result is not always robust when firms are not identical (see Belleflamme, 2000; Yi and Shin, 2000). See Bloch (2005) for a survey on group and network formation in industrial organization.

[14] Mauleon, Sempere-Monerris and Vannetelbosch (2016) show that if firms are myopic (Δ-stability) there is no stable association structure for $n \geq 8$.

[15] Roketskiy (2018) studies collaboration between farsighted firms competing in a tournament and finds that stable networks consist of two asymmetric mutually disconnected complete components.

2 The Model

We consider a two-stage game in a setting with n competing firms that produce some homogenous good. In the first stage, firms decide the bilateral R&D collaborations (or links) they are going to establish in order to maximize their respective profits. Let $N = \{1, 2, ..., n\}$ be the set of firms.[16] A network g of R&D collaborations is a list of which pairs of firms are linked to each other and $ij \in g$ indicates that i and j are linked under g. The complete network on the set of firms $S \subseteq N$ is denoted by g^S and is equal to the set of all subsets of S of size 2. It follows in particular that the empty network is denoted by $g^{\emptyset}$. The set of all possible networks on N is denoted by $\mathcal{G}$ and consists of all subsets of g^N. The network obtained by adding link ij to an existing network g is denoted $g + ij$ and the network that results from deleting link ij from an existing network g is denoted $g - ij$. Let $N(g) = \{i \mid \text{there is } j \text{ such that } ij \in g\}$ be the set of firms who have at least one link in the network g. A path in a network g between i and j is a sequence of firms $i_1, \ldots, i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, \ldots, K-1\}$ with $i_1 = i$ and $i_K = j$. A network g is connected if for all $i \in N$ and $j \in N \setminus \{i\}$, there exists a path in g connecting i and j. A non-empty subnetwork $h \subseteq g$ is a component of g, if for all $i \in N(h)$ and $j \in N(h) \setminus \{i\}$, there exists a path in h connecting i and j, and for any $i \in N(h)$ and $j \in N(g)$, $ij \in g$ implies $ij \in h$. The set of components of g is denoted by $C(g)$. A component h of g is minimally connected if h has $\#N(h) - 1$ links (i.e. every pair of firms in the component are connected by exactly one path). Knowing the components of a network, we can partition the firms into groups within which firms are connected. Let $\Pi(g)$ denote the partition of N induced by the network g. That is, $S \in \Pi(g)$ if and only if either there exists $h \in C(g)$ such that $S = N(h)$ or there exists $i \notin N(g)$ such that $S = \{i\}$. We denote by $S(i)$ the coalition $S \in \Pi(g)$ such that $i \in S$.

R&D collaborations reduce marginal costs of production. Given a network g, the marginal cost for firm i is given by

$$c_i(g) = c_0 - (1 + \sum_{j \neq i} \delta^{t(ij)-1})$$

where c_0 is a firm's initial marginal cost, $\delta \in (0, 1]$ and $t(ij)$ is the number of links in the shortest path between i and j (setting $t(ij) = \infty$ if there is no path between i and j). That is, each firm benefits both from its own R&D (reducing its marginal cost by 1) and from the R&D done by the firms its is connected to (reducing its marginal cost by $\sum_{j \neq i} \delta^{t(ij)-1}$). Let $N_i^k(g) = \{j \mid t(ij) = k\}$ be the set of firms that are connected to firm i by a path of at least k links. Then,

$$c_i(g) = c_0 - 1 - \sum_{k=1}^{n-1} \#N_i^k(g) \delta^{k-1}.$$

[16] Throughout the paper we use the notation $\subseteq$ for weak inclusion, $\subset$ for strict inclusion, and $\text{int}(\cdot)$ for the integer part. Finally, $\#$ will refer to the notion of cardinality.

We mainly focus on the case where each firm fully benefits from the research done by the firms it is connected to, i.e. $\delta = 1$. Since the knowledge flows perfectly through the component, we assume that a firm bears an infinitesimally small costs for maintaining redundant (or superfluous) links. In a network g, a component $h \in C(g)$ has no redundant links if and only if h is minimally connected. It reflects the idea that firms avoid wasting resources. When a firm deletes a redundant or superfluous link, it remains connected to the same set of firms and so still benefits from the same reduction in marginal costs.[17]

In the second stage, firms compete in quantities in the oligopolistic market, taking as given the costs of production. Let $p = a - \sum_{i \in N} q_i$ with $a > 0$ be the linear inverse demand function. For any given R&D network g, one can easily show that there exists a unique Cournot equilibrium on the market, and that each firm's profit $u_i(g)$ is a monotonically increasing function of the following valuation or payoff function,[18]

$$U_i(g) = a - c_0 + (n+1)\#S(i) - \sum_{S \in \Pi(g)} (\#S)^2. \tag{1}$$

In fact, $U_i(g) = (n+1)\sqrt{u_i(g)} = (n+1)q_i(g)$ where $q_i(g)$ is the equilibrium output. We focus our analysis on the case where there are at least eight firms. Notice that R&D networks connecting all firms are the ones that maximize social welfare, i.e. the sum of the profits and the consumer surplus. For $n \geq 8$, this payoff function satisfies some general properties that are useful for characterizing the networks that will emerge in the long run. A first property is that linking two components decreases the payoffs of the firms that do not belong to those components: $U_i(g + jk) < U_i(g)$ if $S(i) \neq S(j) \neq S(k)$ and $S(i), S(j), S(k) \in \Pi(g)$. A second property is that, in any R&D network, firms belonging to bigger components obtain greater payoffs: $U_i(g) > U_j(g)$ if and only if $\#S(i) > \#S(j)$. A third property is that firms belonging to the two smallest components obtain greater payoffs by bridging the two components: in any g with $\#\Pi(g) \geq 3$, $U_i(g + ij) > U_i(g)$ and $U_j(g + ij) > U_j(g)$ if $S(i) \neq S(j)$, $S(i), S(j) \in \Pi(g)$, and $\#S \geq \max\{\#S(i), \#S(j)\}$ for all $S \in \Pi(g)$, $S \neq S(i), S(j)$. Throughout the paper we illustrate our main results by means of an example with eight firms. In Table 2 we give the equilibrium payoffs for $a - c_0 = 42$. We make a slight abuse of notation. For instance, $\{5, 2, 1\}$ should be interpreted as a network, composed of three "components" of size 5, 2 and 1, that can be formed by eight firms. Firms in the component of size 5 obtain a payoff of 57, firms in the component of size 2 obtain a payoff of 30, and the (isolated) firm in the "component" of size 1 obtains a payoff of 21.

[17] Assuming infinitesimally small costs for maintaining redundant links means that we focus on the strategic positioning within the network rather than on the payoff one.

[18] Excluding infinitesimally small costs for maintaining redundant links.

Networks:	$\{8\}$	$\{5,3\}$	$\{5,2,1\}$	$\{3,3,2\}$	$\{3,3,1,1\}$
Profits:	(50)	$(53,35)$	$(57,30,21)$	$(47,47,38)$	$(49,49,31,31)$
Networks:	$\{7,1\}$	$\{4,4\}$	$\{4,3,1\}$	$\{5,1,1,1\}$	$\{3,2,2,1\}$
Profits:	$(55,1)$	$(46,46)$	$(52,43,25)$	$(59,23,23,23)$	$(51,42,42,33)$
Networks:	$\{6,2\}$	$\{6,1,1\}$	$\{4,2,2\}$	$\{4,2,1,1\}$	$\{2,2,2,2\}$
Profits:	$(56,20)$	$(58,13,13)$	$(54,36,36)$	$(56,38,29,29)$	$(44,44,44,44)$

Table 2: Payoffs for the 8-firm case with $a - c_0 = 42$.

3 Myopic-Farsighted Stable Set of R&D Networks

3.1 Myopic-farsighted improving paths and stable sets

We propose the notion of myopic-farsighted stable set to determine the R&D networks that emerge in the long run when some firms are myopic while others are farsighted. A set of networks is a myopic-farsighted stable set if (internal stability) there is no myopic-farsighted improving path between networks within the set and (external stability) there is a myopic-farsighted improving path from any network outside the set to some network within the set.[19] A myopic-farsighted improving path is a sequence of networks that can emerge when farsighted firms form or delete links based on the improvement the end network offers relative to the current network while myopic firms form or delete links based on the improvement the resulting network offers relative to the current network. Since we only allow for pairwise deviations, each network in the sequence differs from the previous one in that either a new link is formed between two firms or an existing link is deleted. If a link is deleted, then it must be that either a myopic firm prefers the resulting network to the current network or a farsighted firm prefers the end network to the current network. If a link is added between some myopic firm i and some farsighted firm j, then the myopic firm i must prefer the resulting network to the current network and the farsighted firm j must prefer the end network to the current network.[20] Let N_m be the set of myopic firms and N_f be the set of farsighted firms, $N = N_m \cup N_f$.

Definition 1. A myopic-farsighted improving path from a network g to a network $g' \neq g$ is a finite sequence of networks $g_1, \ldots, g_K$ with $g_1 = g$ and $g_K = g'$ such that for any $k \in \{1, \ldots, K-1\}$ either

(i) $g_{k+1} = g_k - ij$ for some ij such that $U_i(g_{k+1}) > U_i(g_k)$ and $i \in N_m$ or $U_j(g_K) > U_j(g_k)$ and $j \in N_f$; or

[19] Herings, Mauleon and Vannetelbosch (2017b) define the myopic-farsighted stable set for two-sided matching problems.

[20] Along a myopic-farsighted improving path, myopic players do not care whether other players are myopic or farsighted, while farsighted players know exactly who is farsighted and who is myopic.

(ii) $g_{k+1} = g_k + ij$ for some ij such that $U_i(g_{k+1}) > U_i(g_k)$ and $U_j(g_{k+1}) \geq U_j(g_k)$ if $i, j \in N_m$, or $U_i(g_K) > U_i(g_k)$ and $U_j(g_K) \geq U_j(g_k)$ if $i, j \in N_f$, or $U_i(g_{k+1}) \geq U_i(g_k)$ and $U_j(g_K) \geq U_j(g_k)$ (with one inequality holding strictly) if $i \in N_m, j \in N_f$.

If there exists a myopic-farsighted improving path from a network g to a network g', then we write $g \to g'$. The set of all networks that can be reached from a network $g \in \mathcal{G}$ by a myopic-farsighted improving path is denoted by $\phi(g)$, $\phi(g) = \{g' \in \mathcal{G} \mid g \to g'\}$. A set of networks G is a myopic-farsighted stable set if the following two conditions hold. Internal stability: for any two networks g and g' in the myopic-farsighted stable set G there is no myopic-farsighted improving path from g to g' (and vice versa). External stability: for every network g outside the myopic-farsighted stable set G there is a myopic-farsighted improving path leading to some network g' in the myopic-farsighted stable set G (i.e. there is $g' \in G$ such that $g \to g'$).

Definition 2. A set of networks $G \subseteq \mathcal{G}$ is a myopic-farsighted stable set if: **(IS)** for every $g, g' \in G$, it holds that $g' \notin \phi(g)$; and **(ES)** for every $g \in \mathcal{G} \setminus G$, it holds that $\phi(g) \cap G \neq \emptyset$.

When all firms are farsighted, the notion of myopic-farsighted improving path reverts to Jackson (2008) or Herings, Mauleon and Vannetelbosch (2009) notion of farsighted improving path, and the myopic-farsighted stable set is simply the farsighted stable set as defined in Herings, Mauleon and Vannetelbosch (2009), Mauleon, Vannetelbosch and Vergote (2011) or Ray and Vohra (2015).[21] When all firms are myopic, the notion of myopic-farsighted improving path reverts to Jackson and Watts (2002) notion of improving path, and the myopic-farsighted stable set is simply the farsighted stable set as defined in Herings, Mauleon and Vannetelbosch (2017a) for two-sided matching problems.

3.2 More myopic firms than farsighted ones

Suppose first that the majority of firms are myopic. We say that firm i is a loose-end node in network g if it only has a single link, i.e. $\# \{j \mid ij \in g\} = 1$. If a loose-end firm deletes its single link, it becomes an isolated firm with no R&D collaboration. Since the profit of being involved in R&D collaborations is always greater than the profit of being isolated, a myopic loose-end firm will never delete its link. In a minimally connected component $h \in C(g)$, we say that firm $i \in N(h)$ is a median node if $\nexists\, ij \in h$ and $h' \in C(h - ij)$ such that $\#N(h') > n/2$. That is, a firm is called a median node if by cutting one of its link it cannot split the component in two components with one having a size strictly greater than $n/2$. In our model, a myopic firm only benefits from splitting its component if it belongs to a component of a size strictly greater than $n/2$ after the split. Thus, a myopic median node will never delete one of its links in any minimally connected network.

[21] Alternative notions of farsightedness are suggested by Chwe (1994), Dutta, Ghosal and Ray (2005), Dutta and Vohra (2017), Herings, Mauleon and Vannetelbosch (2004, 2018), Page, Wooders and Kamat (2005), Page and Wooders (2009) among others.

Mauleon, Sempere-Monerris and Vannetelbosch (2014) show that a network g is pairwise stable[22] if and only if g consists of two minimally connected components with the cardinality of the largest component equal to $\text{int}((n+3)/2)$ for n even and to $(n+1)/2$ for n odd. Formally, $G^{1/2} = \{g \mid C(g) = (h_1, h_2),\ h_1$ and h_2 are minimally connected, $N(h_1) \cup N(h_2) = N,\ \#N(h_1) = \text{int}((n+3)/2)$ if n even and $\#N(h_1) = (n+1)/2$ if n odd$\}$ is the set of pairwise stable networks.

Let $g \in G^{3/4}_{fm} = \{g \mid C(g) = (h_1, h_2),\ h_1$ and h_2 are minimally connected, $N(h_1) \cup N(h_2) = N,\ \#N(h_1) = \text{int}((3n+1)/4),\ N_f \cap N(h_1) \neq \emptyset$, and for any $i \in N(h_1) \cap N_m$ we have either $\#\{j \mid ij \in g\} = 1$ or i is a median node$\}$. That is, R&D networks belonging to $G^{3/4}_{fm}$ are such that they consist of two minimally connected components of different size close to $3n/4$ and $n/4$, respectively. In both components, there can be myopic and farsighted firms. Myopic firms in the largest component are either loose-end nodes or median nodes.

Let $G^{1/2}_m = \{g \mid C(g) = (h_1, h_2),\ h_1$ and h_2 are minimally connected, $N(h_1) \cup N(h_2) = N,\ N(h_1) \subseteq N_m,\ \#N(h_1) = \text{int}((n+3)/2)$ if n even and $\#N(h_1) = (n+1)/2$ if n odd$\}$. That is, R&D networks belonging to $G^{1/2}_m$ are such that they consist of two minimally connected components of nearly equal size $n/2+1$ and $n/2-1$, respectively. In the largest component, there are only myopic firms. In fact, $G^{1/2}_m \subseteq G^{1/2}$.

Proposition 1 shows that when the majority of firms are myopic, the myopic-farsighted stable set consists of R&D networks having either two components of different sizes (close to $3n/4$ and $n/4$) with farsighted firms occupying key positions in the largest component and myopic firms as loose-ends or medians, or two components of nearly equal size (close $n/2+1$ and $n/2-1$) with the largest component having only myopic firms. In Figure 1 we depict two networks belonging to $G^{3/4}_{fm}$ and one network belonging to $G^{1/2}_m$ when $n = 8$, firms 1 and 3 are farsighted, and all other firms are myopic. In $g' \in G^{3/4}_{fm}$, firms 2 and 4 are median nodes while firms 5 and 6 are loose-end nodes.

Proposition 1. *If $0 < \#N_f < n/2$ then the set of networks $G^{3/4}_{fm} \cup G^{1/2}_m$ is a myopic-farsighted stable set.*

All the proofs can be found in the appendix. We now provide the intuition behind the proof of Proposition 1. The set of networks $G^{3/4}_{fm} \cup G^{1/2}_m$ is a myopic-farsighted stable set if both internal and external stability conditions are satisfied.

(**IS**) Internal stability follows because from any network $g \in G^{3/4}_{fm} \cup G^{1/2}_m$ there is no myopic-farsighted improving path ending at $g' \in G^{3/4}_{fm} \cup G^{1/2}_m$, $g' \neq g$. Indeed, if $g \in G^{1/2}_m$, g is pairwise stable and then no myopic firm will delete or form a link from g. Only farsighted firms in the smallest component could form or delete a link from g. However, farsighted firms cannot modify g so that myopic firms in the largest component would like to modify the resulting network afterwards. Adding a link between farsighted firms does not change the profit of the myopic firms in the largest component, and deleting a

[22] A network g is pairwise stable if (i) for all $ij \in g$, $U_i(g) \geq U_i(g-ij)$ and $U_j(g) \geq U_j(g-ij)$, and (ii) for all $ij \notin g$, if $U_i(g) < U_i(g+ij)$ then $U_j(g) > U_j(g+ij)$ (Jackson and Wolinsky, 1996).

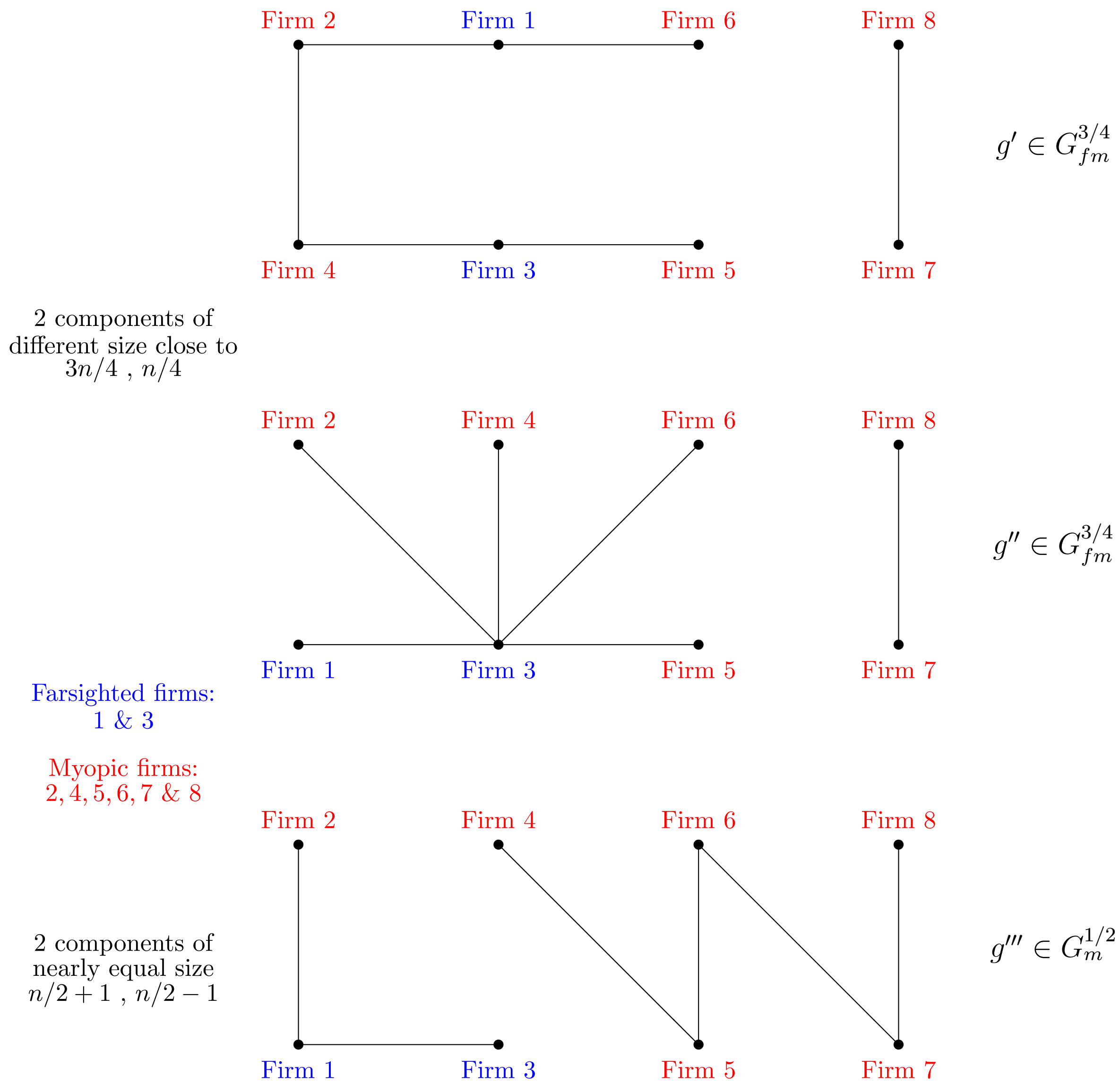

Figure 1: Stable R&D networks when $\#N_f < n/2$.

link would split the smallest component increasing even more the profits of the myopic firms in the largest component. Moreover, from any $g \in G^{3/4}_{fm}$, no myopic or farsighted firm in the largest component has an incentive to form or to delete a link (myopic firms because they are loose-end or median nodes in g, and farsighted firms because they are either equally well off or worse off at any $g' \in G^{3/4}_{fm} \cup G^{1/2}_{m}$ compared to $g \neq g'$). As before, firms in the smallest component cannot modify g so that firms in the largest component would like to modify the resulting network afterwards.

(**ES**) Given that profits only depend on the cardinality of the component and infinitesimally small costs for maintaining redundant links, we only need to check that there is a myopic-farsighted improving path from any minimally connected network $g \notin G^{3/4}_{fm} \cup G^{1/2}_{m}$ to some $\widetilde{g} \in G^{3/4}_{fm}$. Three cases have to be considered:

(a) First, we consider any minimally connected network g connecting the n firms of the industry. From g, looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, farsighted firms build links between them until we reach a network g' in which all the farsighted firms would still be connected once all links involving some myopic firm have been deleted from g'. From g', myopic and farsighted firms delete links until they reach a minimally connected network g'' such that all the farsighted firms remain connected once all links involving some myopic firm have been deleted from g''. Hence, at g'' there is no myopic firm in any path between two farsighted firms. From g'', some myopic firm linked to another myopic firm being a loose-end deletes its link to the loose-end node reaching a network with two components of sizes $n-1$ and 1, respectively. Next, some farsighted firm forms a link to the isolated myopic firm reaching again a minimally connected network with a single component. We repeat this two-step process until we reach a minimally connected network $\widehat{g}$ connecting the n firms of the industry in which the sum of all farsighted firms and the myopic firms that are loose-end nodes and are linked to a farsighted firm is equal to $\text{int}((3n+1)/4)$. From $\widehat{g}$, a farsighted firm looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, deletes its link to a myopic firm that is not loose-end obtaining a network $\widehat{g}'$ with two components and such that all farsighted firms belong to the largest component. From $\widehat{g}'$, a farsighted firm deletes its link to another myopic firm that is not loose-end obtaining a network $\widehat{g}''$ with three components and such that all farsighted firms belong to the largest component. From $\widehat{g}''$, two myopic firms in the two smallest components form a link to bridge the two smallest components obtaining a network $\widehat{g}'''$ with two components and such that all farsighted firms belong to the largest component. We then repeat this three-step process until we reach some $\widetilde{g} \in G^{3/4}_{fm}$ where all farsighted firms belong to the largest component of size $\text{int}((3n+1)/4)$ and myopic firms in the largest component are loose-ends.

(b) Second, we consider any minimally connected network $g \notin G^{3/4}_{fm} \cup G^{1/2}_{m}$ with two components, h_1, h_2, with $\#N(h_1) > \#N(h_2)$. Three types of networks have to be considered. In the first type, $\#N(h_1) \neq \text{int}((3n+1)/4)$ and there is at least a farsighted firm in the largest component h_1. Then, from g, looking forward to some $\widetilde{g} \in G_{fm}^{3/4}$, this farsighted firm will add a link to some (farsighted or myopic) firm in the smallest component obtaining a network g' with a unique component that connects the n firms of the industry. From g', we can then proceed as in **(a)**. In the second type, no farsighted firm belongs to the largest component h_1. Then, from g, we initiate a process that consists in isolating first a myopic firm from the largest component that next adds a link to some (farsighted or myopic) firm in the smallest component, until we reach some network $\widetilde{g} \in G_m^{1/2}$. In the third type, there is at least a farsighted firm in the largest component h_1, $\#N(h_1) = \text{int}((3n+1)/4)$ and some myopic firm j in h_1 is nor a loose-end node nor a median node. Then, from g, this myopic firm j has incentives to cut one of its links splitting h_1 in two components moving to a network g' with three components and such that the size of the largest component is smaller than $\text{int}((3n+1)/4)$ but larger than $n/2$. From g', we proceed as in **(c)** described below.

(c) Third, we consider any minimally connected network with three or more components. If the size of the largest component is smaller than $n/2$, two myopic firms belonging to two different components will successively form a link between them until we reach a network g where the size of the largest component h_1 is greater or equal to $n/2$. Three types of networks with three or more components have to be considered. In the first type, $\#N(h_1) \geq n/2$ and h_1 contains at least some farsighted firm. Then, from g, two (myopic or farsighted) firms belonging to the two smallest components form successively a link (with the farsighted firms looking forward to some $\widetilde{g} \in G_{fm}^{3/4}$) until we reach a network g' with two components h_1 (that has not changed along the process) and h_2 that has been formed at the end of the process. From g', we proceed as in **(b)** where h_1 does contain some farsighted firm. In the second type, $\#N(h_1) > n/2$ and there is no farsighted firm in h_1. From g, two (myopic or farsighted) firms belonging to the two smallest components form successively a link (with the farsighted firms looking forward to some $\widetilde{g} \in G_m^{1/2}$) until we reach a network g' with two components h_1 (that has not changed along the process) and h_2 that has been formed at the end of the process. From g', we proceed as in **(b)** where h_1 does not contain farsighted firms. In the third type, $\#N(h_1) = n/2$ and there is no farsighted firm in h_1. From g, two (myopic or farsighted) firms belonging to the two smallest components form successively a link (with the farsighted firms looking forward to some $\widetilde{g} \in G_{fm}^{3/4}$) until we reach a network g' with two components of size $n/2$. From g', two myopic firms belonging to different components have incentives to add a link between them to form a network g'' with a unique component. From g'', we proceed as in **(a)**.

When all firms are myopic (i.e. $N = N_m$, $N_f = \emptyset$), there is a unique myopic-farsighted stable set consisting of all pairwise stable networks. In fact, the set $G_m^{1/2}$ is equal to $G^{1/2}$

and the set $G^{3/4}_{fm}$ becomes empty (there is always a myopic firm that is nor a loose-end node nor a median node in the largest component).[23]

Proposition 2. *Suppose that all firms are myopic, $N = N_m$. The set of pairwise stable networks $G^{1/2}$ is the unique myopic-farsighted stable set.*

The idea behind Proposition 2 is the following. Mauleon, Sempere-Monerris and Vannetelbosch (2014) show that a network g is pairwise stable if and only if $g \in G^{1/2}$. It follows that from any network $g \in G^{1/2}$ there is no myopic-farsighted improving path leaving g. Hence, $G^{1/2}$ satisfies internal stability. For external stability, we only need to check that there is a myopic-farsighted improving path from any minimally connected network $g \notin G^{1/2}$ to some $\widetilde{g} \in G^{1/2}$. We proceed in four steps. First, from any minimally connected network g connecting the n firms of the industry, firms have incentives to isolate one firm. Second, from any minimally connected network with three or more components, two firms belonging to two different components of cardinality smaller than $n/2$ will successively form a link between them until we reach a network g with only two components. Third, from any minimally connected network with two components of different size, firms belonging to the largest component have incentives to isolate one firm until the cardinality of the component is equal to $\text{int}((n+3)/2)+1$ if n even or to $(n+3)/2$ if n is odd. Fourth, from any minimally connected network with two components of equal size, two firms belonging to two different components have incentives to add a link between them forming a network connecting the n firms of the industry. Then, from steps one to four, it follows that $G^{1/2}$ satisfies external stability. Finally, the fact that there is no myopic-farsighted improving path leaving g for any $g \in G^{1/2}$, guarantees that $G^{1/2}$ is the unique myopic-farsighted stable set when all firms are myopic.

3.3 More farsighted firms than myopic ones

Suppose now that the majority of firms are farsighted. Next Proposition shows that in this case the myopic-farsighted stable set consists of only R&D networks having two minimally connected components of different sizes close to $3n/4$ and $n/4$, respectively. In both components, there can be myopic and farsighted firms. Myopic firms in the largest component are still either loose-end nodes or median nodes. Despite there is a large majority of farsighted firms, firms will not necessarily end up segregated: farsighted and myopic firms can belong to the largest component, with farsighted firms having more R&D collaborations on average. Moreover, those myopic firms enjoy greater profits than the farsighted and myopic firms that end up in the smallest component. In Figure 2 we depict two networks belonging to $G^{3/4}_{fm}$ when $n = 8$, firms 2, 3, 4 and 8 are farsighted,

[23] Intuitively, a myopic firm that is not a median node nor a loose-end node in some network in $G^{3/4}$ has an incentive to split its component in two, leading to a network consisting of three components from which there is path towards some network in $G^{1/2}$. Hence, once there are only myopic firms, $G^{3/4} \cup G^{1/2}$ would violate internal stability.

and all other firms are myopic. In $g' \in G_{fm}^{3/4}$ firms 1, 5 and 6 are myopic loose-end nodes, while in $g'' \in G_{fm}^{3/4}$ firm 1 is a myopic median node and firms 5 and 6 are myopic loose-end nodes.

Proposition 3. *If $n > \#N_f \geq n/2$ then the set of networks $G_{fm}^{3/4}$ is a myopic-farsighted stable set.*

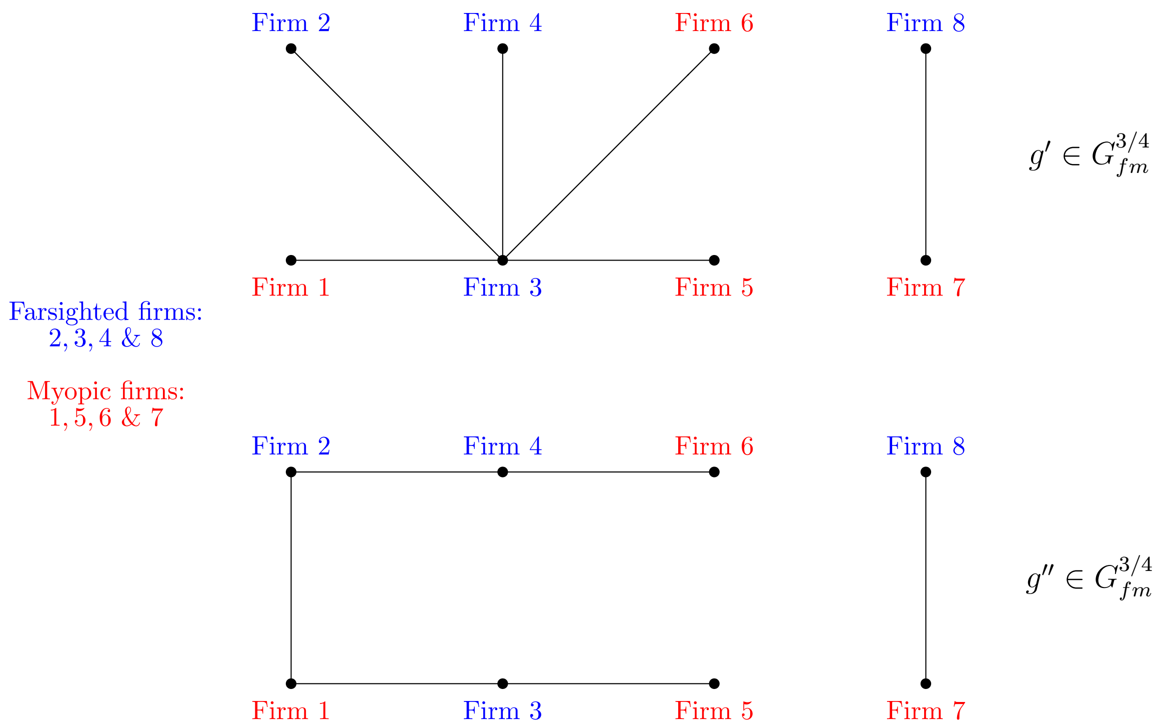

Figure 2: Stable R&D networks when $\#N_f \geq n/2$.

We now provide the intuition behind the proof of Proposition 3. The set of networks $G_{fm}^{3/4}$ is a myopic-farsighted stable set if both internal and external stability conditions are satisfied.

(**IS**) Internal stability follows because from any network $g \in G_{fm}^{3/4}$ there is no myopic-farsighted improving path ending at $g' \in G_{fm}^{3/4}$, $g' \neq g$. From any $g \in G_{fm}^{3/4}$, no myopic or farsighted firm in the largest component has an incentive to form or to delete a link (myopic firms because are they loose-end or median nodes in g while farsighted firms because they are either equally well off or worse off at any $g' \in G_{fm}^{3/4}$ compared to $g \neq g'$). Only myopic and farsighted firms in the smallest component could form or delete a link from g. However, firms in the smallest component cannot modify g so that firms in the largest component would like to modify the resulting network afterwards. Adding a link between firms of the smallest component does not change the profit of the firms in the largest component, and deleting a link would by splitting the smallest component increase the profits of the firms in the largest component.

(**ES**) Given that profits only depend on the cardinality of the component and infinitesimally small costs for maintaining redundant links, we only need to check that there is a myopic-farsighted improving path from any minimally connected network $g \notin G^{3/4}_{fm}$ to some $\widetilde{g} \in G^{3/4}_{fm}$. Three cases have to be considered:

(a) First, we consider any minimally connected network g connecting the n firms of the industry. From g, looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, farsighted firms build links between them until we reach a network g' in which a subset $N'_f \subseteq N_f$ of farsighted firms would be connected once all links involving some myopic firm and some farsighted firm outside N'_f have been deleted from g'. The cardinality of N'_f is equal to $\min\{\#N_f, \text{int}((3n+1)/4)\}$. From g', myopic and farsighted firms delete links until they reach a minimally connected network g'' such that the farsighted firms in N'_f remain connected once all links involving some myopic firm and some farsighted firm outside N'_f have been deleted from g''. At g'', if $\min\{\#N_f, \text{int}((3n+1)/4)\} = N_f$, we proceed as in **(a)** for Proposition 1. Otherwise, if $\min\{\#N_f, \text{int}((3n+1)/4)\} = \text{int}((3n+1)/4)$, we have that, at g'', there is no myopic firm or farsighted firm outside N'_f in any path between two farsighted firms belonging to N'_f. From g'', a farsighted firm belonging to N'_f and looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, deletes its link to a myopic firm or to a farsighted firm outside N'_f obtaining a network g''' with two components. From g''', a farsighted firm belonging to N'_f deletes its link to another myopic firm or to a farsighted firm outside N'_f obtaining a network g'''' with three components and such that all farsighted firms in N'_f belong to the largest component. From g'''', two firms in the two smallest components form a link to bridge the two smallest components obtaining a network with two components and such that all farsighted firms in N'_f belong to the largest component. We repeat this three-step process until we reach some $\widetilde{g} \in G^{3/4}_{fm}$ where all farsighted firms in N'_f belong to the largest component of size $\text{int}((3n+1)/4)$.

(b) Second, we consider any minimally connected network $g \notin G^{3/4}_{fm}$ with two components, h_1, h_2, with $\#N(h_1) > \#N(h_2)$. Two types of networks have to be considered. In the first type, $\#N(h_1) \neq \text{int}((3n+1)/4)$. Then, from g, looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, some farsighted firm in the largest component will add a link to some (farsighted or myopic) firm in the smallest component obtaining a network g' with a unique component that connects the n firms of the industry. From g', we can then proceed as in **(a)**. In the second type, $\#N(h_1) = \text{int}((3n+1)/4)$ and some myopic firm j in h_1 is nor a loose-end node nor a median node. Then, from g, this myopic firm j has incentives to cut one of its links splitting h_1 in two components moving to a network g' with three components and such that the size of the largest component is smaller than $\text{int}((3n+1)/4)$ but larger than $n/2$. From g', we proceed as in **(c)** described below.

(c) Third, we consider any minimally connected network with three or more components. Three types of networks with three or more components have to be considered. In the first type, $\#N(h_1) \geq n/2$. Then, from g, two (myopic or farsighted) firms belonging to the two smallest components form successively a link (with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$) until we reach a network g' with two components h_1 (that has not changed along the process) and h_2 that has been formed at the end of the process. From g', we proceed as in **(b)**. In the second type, $\#N(h_1) < n/2$ and there is no farsighted firm in h_1. From g, two (myopic or farsighted) firms belonging to the two largest components form successively a link (with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$) until we reach a network g' where the size of the largest component is greater or equal to $n/2$. Since the size of the largest two components is smaller than $n/2$, the two (myopic or farsighted) firms have incentives to link to each other. From g', two (myopic or farsighted) firms belonging to the two smallest components form successively a link (with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$) until we reach a network g'' with two components. From g'', we proceed as in **(b)**. In the third type, $\#N(h_1) < n/2$ and there is some farsighted firm in h_1. From g, one myopic firm of the largest component h_1 forms a link with some farsighted firm belonging to the largest component h_k that contains a farsighted firm (with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$). If $\#N(h_1) + \#N(h_k) < n/2$, from g', two (myopic or farsighted) firms belonging to the two largest components form successively a link (with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$) until we reach a network g'' such that the size of the largest component is greater or equal to $n/2$. From g'', two (myopic or farsighted) firms belonging to the two smallest components form successively a link (with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$) until we reach a network g''' with two components. From g''', we proceed as in **(b)**.

Let $G^{3/4}$ be the set of all networks g consisting of two minimally connected components h_1 and h_2 such that $\#N(h_1) = \text{int}((3n+1)/4)$ and $N(h_1) \cup N(h_2) = N$. Formally, $G^{3/4} = \{g \mid C(g) = (h_1, h_2),\ h_1 \text{ and } h_2 \text{ are minimally connected, } N(h_1) \cup N(h_2) = N \text{ and } \#N(h_1) = \text{int}((3n+1)/4)\}$ and $G^{3/4}_{fm} \subseteq G^{3/4}$. When all firms are farsighted (i.e. $N = N_f$, $N_m = \emptyset$), $G^{3/4}_{fm}$ is equal to $G^{3/4}$ and $G^{3/4}$ is a myopic-farsighted stable set. Hence, we obtain a collaboration architecture similar to the equilibrium structure of Bloch's (1995) sequential game for forming research associations of firms where firms form two asymmetric alliances, with the largest one comprising roughly three-quarters of industry members.

Corollary 1. *Suppose that all firms are farsighted, $N = N_f$. The set of networks $G^{3/4}$ is a myopic-farsighted stable set.*

Remember that the set of all pairwise stable networks $G^{1/2}$ is the unique myopic-farsighted stable set when all firms are myopic. However, $G^{1/2}$ is no more a myopic-farsighted stable set when the majority of firms become farsighted. Since $g \notin \phi(g')$ for all $g \in G^{1/2}$ and $g' \in G^{3/4}_{fm}$, the set $G^{1/2}$ violates the external stability condition when the majority of firms are farsighted.[24]

Once there is no majority of myopic firms, there is at least one farsighted firm in the largest component of any network belonging to $G^{1/2}$. This farsighted firm can form a link with a (myopic or farsighted) firm from the other component and by doing so, induce a myopic-farsighted improving path towards some network in $G^{3/4}_{fm}$ where it belongs to the largest component.

3.4 Social welfare

We now examine social welfare under the different networks that can emerge in the long run. The social welfare function $SW(g)$ is defined as the sum of consumer surplus plus aggregate profits. To compute social welfare under a network g we substitute equilibrium quantities and profits in the social welfare expression. Let $G^1 = \{g \mid \#C(g) = 1$ and $N(g) = N\}$ be the R&D networks that consist of a single component connecting all firms.

Proposition 4.

- **(i)** *Take any* $g \in G^1, g' \notin G^1$. *We have that* $SW(g) > SW(g')$.
- **(ii)** *Take any* $g \in G^{3/4}, g' \in G^{1/2}$. *We have that* $SW(g) > SW(g')$.

Proposition 4 tells us that a network g maximizes social welfare if and only if $g \in G^1$. In addition, R&D networks with two components of different sizes (close to $3n/4$ and $n/4$) dominate R&D networks with two components of nearly equal size (close $n/2 + 1$ and $n/2 - 1$). Hence, farsightedness helps in improving social welfare but fails to reach the societal optimum.

4 Stable R&D Networks with Yes-Firms

Suppose now that some firms are yes-firms. Yes-firms always accept the formation of any link and never delete a link subject to the constraint of non-negative profits. If they are myopic, yes-firms agree to add a new collaboration conditionally on making positive profits in the resulting network. What would happen if they were farsighted instead? Farsighted firms look at the ending network rather than at the resulting one. Nothing would change since at equilibrium all firms always make non-negative profits in any network configuration. Thus, yes-firms can be interpreted either as myopic or farsighted yes-firms.

Let N_y be the set of yes-firms, $N = N_m \cup N_f \cup N_y$. We adapt the definition of a myopic-farsighted improving path to allow for yes-firms. If a link is added between some myopic (farsighted) firm i and some yes-firm j, then only the myopic (farsighted) firm i must prefer the resulting (end) network to the current network.

[24] Since $\phi(g) \neq \emptyset$ for $g \in G^{3/4}_{fm}$ when the majority of firms are farsighted, the myopic-farsighted stable set may be not unique.

Definition 3. A y-myopic-farsighted improving path from a network g to a network $g' \neq g$ is a finite sequence of graphs $g_1, \ldots, g_K$ with $g_1 = g$ and $g_K = g'$ such that for any $k \in \{1, \ldots, K-1\}$ either

(i) $g_{k+1} = g_k - ij$ for some ij such that $U_i(g_{k+1}) > U_i(g_k)$ and $i \in N_m$ or $U_j(g_K) > U_j(g_k)$ and $j \in N_f$; or

(ii) $g_{k+1} = g_k + ij$ for some ij such that $i, j \in N_y$, or $U_i(g_{k+1}) \geq U_i(g_k)$ if $i \in N_m, j \in N_y$, or $U_i(g_K) \geq U_i(g_k)$ if $i \in N_f, j \in N_y$, or $U_i(g_{k+1}) > U_i(g_k)$ and $U_j(g_{k+1}) \geq U_j(g_k)$ if $i, j \in N_m$, or $U_i(g_K) > U_i(g_k)$ and $U_j(g_K) \geq U_j(g_k)$ if $i, j \in N_f$, or $U_i(g_{k+1}) \geq U_i(g_k)$ and $U_j(g_K) \geq U_j(g_k)$ (with one inequality holding strictly) if $i \in N_m, j \in N_f$.

If there exists a y-myopic-farsighted improving path from a network g to a network g', then we write $g \to g'$. The set of all networks that can be reached from a network $g \in \mathcal{G}$ by a y-myopic-farsighted improving path is denoted by $\phi^y(g)$, $\phi^y(g) = \{g' \in \mathcal{G} \mid g \to g'\}$.

Definition 4. A set of networks $G \subseteq \mathcal{G}$ is a myopic-farsighted stable set with yes-firms if: **(IS)** for every $g, g' \in G$, it holds that $g' \notin \phi^y(g)$; and **(ES)** for every $g \in \mathcal{G} \setminus G$, it holds that $\phi^y(g) \cap G \neq \emptyset$.

A component h of g is y-minimally connected if $N(h) \cap N_y \neq \emptyset$, $g^{N(h) \cap N_y} \subseteq h$ and h has $\#N(h) - \#(N(h) \cap N_y) + \#(N(h) \cap N_y)(\#(N(h) \cap N_y) - 1)/2$ links. That is, in a y-minimally connected component h, if we delete a link $ij \in h$ involving a myopic or a farsighted firm, then $\#N(h - ij) < \#N(h)$. But, if we delete any link $ij \in h$ involving only yes-firms, then $\#N(h - ij) = \#N(h)$. Indeed, all yes-firms in a y-minimally connected component are all linked to each other. In other words, in a y-minimally connected component, if we delete any link involving a myopic or a farsighted firm, the component would be split in two components. However, if we cut any link involving only yes-firms, the component would not be partitioned.

Suppose now that there is a majority of myopic firms. We show that the addition of yes-firms can stabilize a third type of R&D network that is socially optimal and that is never stable in the absence of yes-firms. Thus, yes-firms may play a crucial role by bridging myopic and farsighted firms in a socially optimal R&D network that connects the n firms of the industry in a unique component. In Figure 3 we depict the three types of stable R&D networks.

Let $G^{3/4,y}_{fm} = \{g \mid C(g) = (h_1, h_2)$, h_1 is minimally connected, h_2 is y-minimally connected, $\#N(h_1) = \text{int}((3n+1)/4)$, $N_f \cap N(h_1) \neq \emptyset$, $N_y \cap N(h_1) = \emptyset$, and for any $i \in N(h_1) \cap N_m$ we have either $\#\{j \mid ij \in g\} = 1$ or i is a median node$\}$. That is, R&D networks belonging to $G^{3/4,y}_{fm}$ are such that they consist of two components of different size close to $3n/4$ and $n/4$, respectively. There can be myopic and farsighted firms in both components, but all yes-firms belong to the smallest one. The largest component is minimally connected, while the smallest one is y-minimally connected. Myopic firms in the largest component are either loose-end nodes or median nodes.

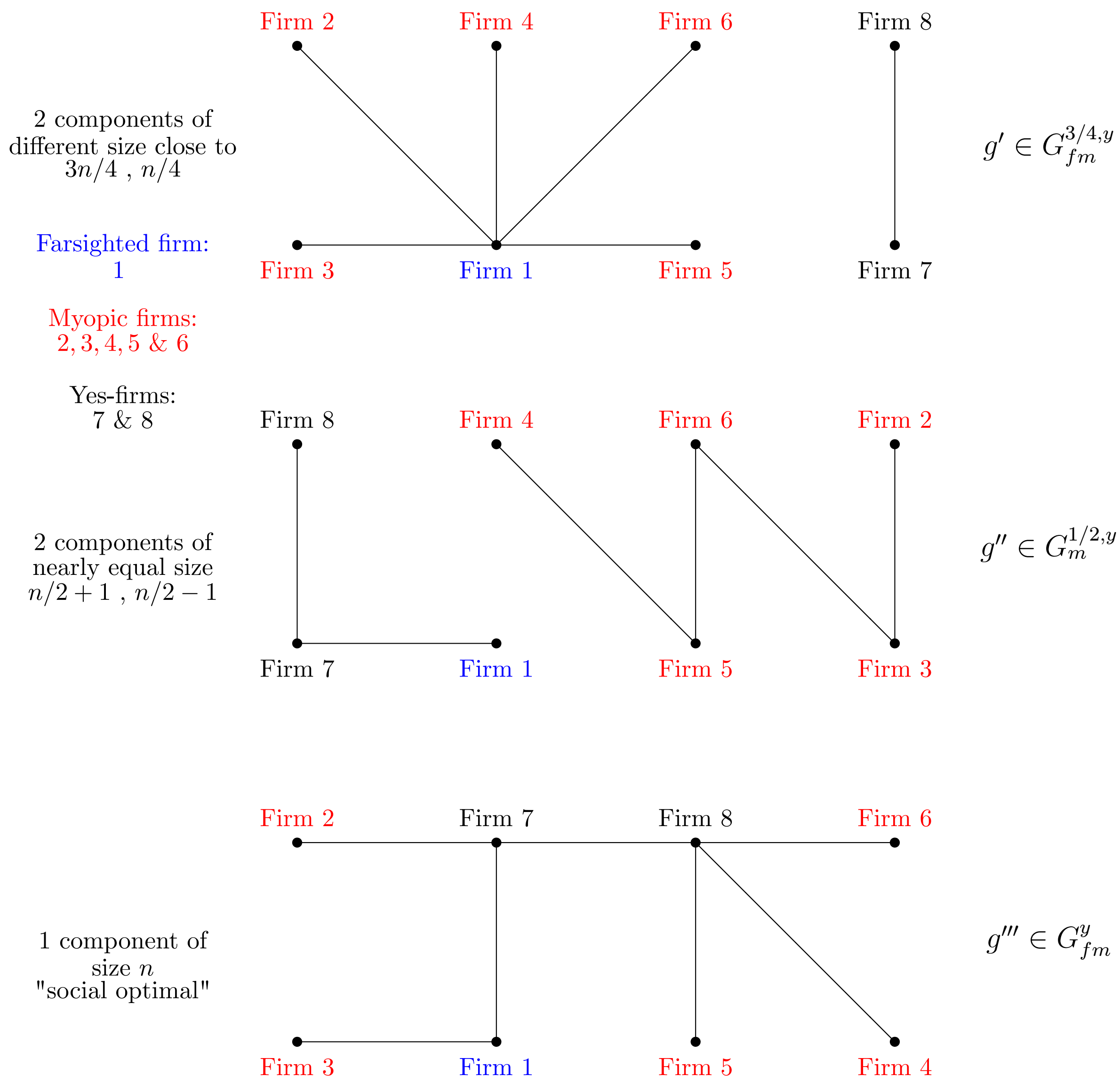

Figure 3: Stable R&D networks with yes-firms when the majority of firms is myopic.

Let $G_m^{1/2,y} = \{g \mid C(g) = (h_1, h_2),\ h_1$ is minimally connected, h_2 is y-minimally connected, $N(h_1) \cup N(h_2) = N,\ N(h_1) \subseteq N_m,\ \#N(h_1) = \text{int}((n+3)/2)$ if n even and $\#N(h_1) = (n+1)/2$ if n odd$\}$. That is, R&D networks belonging to $G_m^{1/2,y}$ are such that they consist of two components of nearly equal size $n/2+1$ and $n/2-1$, respectively. In the largest component, there are only myopic firms. All yes-firms and farsighted firms belong to the smallest component. The largest component is minimally connected, while the smallest one is y-minimally connected.

Let $G_{fm}^y = \{g \mid \#C(g) = 1,\ N(g) = N,\ g$ is y-minimally connected, $\#\{i \in N_m \mid ij \in g$ and $j \in N_f\} < n/2 - \#N_f$, and for any $i \in N_m$ we have $\#\{j \mid ij \in g\} = 1\}$. That is, R&D networks belonging to G_{fm}^y are such that they consist of a single y-minimally connected component that connects all firms. All myopic firms are loose-end nodes, while farsighted firms cannot have too many myopic firms as neighbors. Such R&D networks are socially optimal: they maximize the social welfare as well as the consumer surplus.

Proposition 5. *If $n-1 > \#N_m > n/2$, $n - \text{int}((3n+1)/4) > \#N_f > 0$ and $n - \text{int}((3n+1)/4) > \#N_y > 0$ then the set of networks $G_{fm}^{3/4,y} \cup G_m^{1/2,y} \cup G_{fm}^y$ is a myopic-farsighted stable set with yes-firms.*

We now provide the intuition behind the proof of Proposition 5. The set of networks $G_{fm}^{3/4,y} \cup G_m^{1/2,y} \cup G_{fm}^y$ is a myopic-farsighted stable set with yes-firms if both internal and external stability conditions are satisfied. (**IS**) Internal stability follows because $\phi^y(g) = \emptyset$ for all $g \in G_m^{1/2,y}$. From any network $g \in G_{fm}^{3/4,y}$, there is no myopic-farsighted improving path ending at $g' \in G_{fm}^{3/4,y} \cup G_m^{1/2,y} \cup G_{fm}^y$. The argument is similar to the one in Proposition 1. From any network $g \in G_{fm}^y$, only farsighted firms could want to add or to delete a link looking forward to some $g' \in G_{fm}^{3/4,y}$ where they would belong to the largest component. However, the number of loose-end myopic firms linked to farsighted firms is not enough for forming a component of size greater or equal than $n/2$, which is a necessary step for building a y-myopic-farsighted improving path towards some $g' \in G_{fm}^{3/4,y}$.

(**ES**) Given that profits only depend on the cardinality of the component and infinitesimally small costs for maintaining redundant links, we only need to check that there is a y-myopic-farsighted improving path from any network g consisting either of one y-minimally connected component h with $N_y \subset N(h)$ or of other minimally connected components to some $\widetilde{g} \in G_{fm}^{3/4,y} \cup G_m^{1/2,y} \cup G_{fm}^y$. First, we consider any minimally connected network g connecting the n firms of the industry, $g \notin G_{fm}^y$. From g, looking forward to some $\widetilde{g} \in G_{fm}^{3/4,y}$, farsighted firms build links between them until we reach a network g' in which all the farsighted firms would be connected once all links involving some myopic firm or some yes-firm have been deleted from g'. From g', myopic and farsighted firms delete links until they reach a minimally connected network g'' connecting the n firms of the industry and such that all the farsighted firms remain connected once all links involving some myopic firm or some yes-firm have been deleted from g''. Next, consider the two-step process (*). From g'', some myopic firm linked to a loose-end myopic firm deletes its link to the loose-end node reaching a network with two components of size $n-1$

and 1, respectively. Next some farsighted firm builds a link to the isolated myopic firm forming again a y-minimally connected network with a single component. We repeat this two-step process (*) until we reach a y-minimally connected network $\widehat{g} \notin G^{y}_{fm}$ connecting the n firms of the industry in which all farsighted firms remain connected once all links involving some myopic firm or some yes-firm have been deleted from $\widehat{g}$. In $\widehat{g}$, if the number of myopic firms that are loose-end nodes and linked to a farsighted firm is large enough, then farsighted firms can induce the move towards some $\widetilde{g} \in G^{3/4,y}_{fm}$. Otherwise, we show that there is a y-myopic-farsighted improving path from g leading to some $\widetilde{g} \in G^{y}_{fm}$. Second, we consider any minimally connected network $g \notin G^{3/4}_{fm} \cup G^{1/2}_{m}$ with two or more components and where all yes-firms belong to the same component. Using similar arguments as in the previous propositions, we can show the existence of a y-myopic-farsighted improving path from g to some $\widetilde{g} \in G^{3/4,y}_{fm} \cup G^{1/2,y}_{m} \cup G^{y}_{fm}$.

To measure the importance of a firm within the network, i.e. how central a firm is, some centrality measures have been proposed. The degree centrality of a firm is simply its number of collaborations (links) divided by $n-1$, so that it ranges from 0 to 1. It indicates how well a firm is connected in terms of direct connections. Another measure is betweenness centrality. It is based on how well located a firm is in terms of the paths that it lies on. It reflects how important a firm is in terms of connecting other firms. With yes-firms and a majority of myopic firms, three types R&D networks can emerge in the long run. In networks belonging to $G^{3/4,y}_{fm}$, myopic firms have in average a lower degree than farsighted firms. Myopic firms have either one link (loose-end node) or two links (median node). In addition, myopic firms that are loose-end nodes have the lowest betweenness centrality. However, myopic firms that are median nodes have a high (if not the highest) betweenness centrality. Thus, even if myopic firms are less active in terms of R&D collaborations they may play a crucial role for spreading the innovation within the component. In networks belonging to $G^{1/2,y}_{m}$, myopic and farsighted firms have in average a similar degree or betweenness centrality. Finally, in networks belonging to G^{y}_{fm} that consist of a single component (socially optimal) with yes-firms bridging other firms and myopic firms as loose-end nodes, myopic firms have in average a much lower degree and betweenness centrality than farsighted firms. Yes-firms have a high betweenness centrality, they play a crucial role for spreading the innovation throughout all the industry and they are the driving force for stabilizing the socially optimal structure.

Thus, our model predicts that R&D collaborations between yes-, myopic and farsighted firms often coexist in the long run. Thus, segregation (diversity) is unlikely (likely) to emerge in the long run in consistency with the data.[25]

Once there is a majority of farsighted firms, only networks belonging to $G^{3/4,y}_{fm}$ would

[25] Tomasello, Napoletano, Garas and Schweitzer (2017) find empirical evidence (using pooled data from 1984 to 2009 about R&D alliances among manufacturing firms) that new entrant firms are more likely to become part of the R&D network by attaching to the most central incumbents. Similarly, Roesler and Broekel (2017) find that entrants into the biotechnology industry tend to establish their first collaboration with organizations holding central positions in the subsidized R&D network.

emerge in the long run. The set $G_{fm}^{3/4,y} \cup G_{m}^{1/2,y} \cup G_{fm}^{y}$ is no more a myopic-farsighted stable set with yes-firms since internal stability would be violated. In any network consisting of two components of nearly equal size, there is now at least one farsighted firm in the largest component and this firm can induce a path towards some network in $G_{fm}^{3/4,y}$. In addition, there are now enough farsighted firms to give rise to a myopic-farsighted improving path from any network consisting of a single component leading to some network in $G_{fm}^{3/4,y}$. However, one can show that $G_{fm}^{3/4,y}$ is a myopic-farsighted stable set with yes-firms. Hence, if there is a majority of farsighted firms, the addition of yes-firms only slightly alters the structure of the stable R&D networks. The myopic-farsighted stable set still consists of R&D networks having two components of different sizes close to $3n/4$ and $n/4$ with farsighted firms and (loose-end or median) myopic firms in the largest component but now with all yes-firms in the smallest component (see Figure 4).

Proposition 6. *If $n > \#N_f \geq n/2$ and $n - \text{int}((3n+1)/4) \geq \#N_y \geq 1$ then the set of networks $G_{fm}^{3/4,y}$ is a myopic-farsighted stable set with yes-firms.*

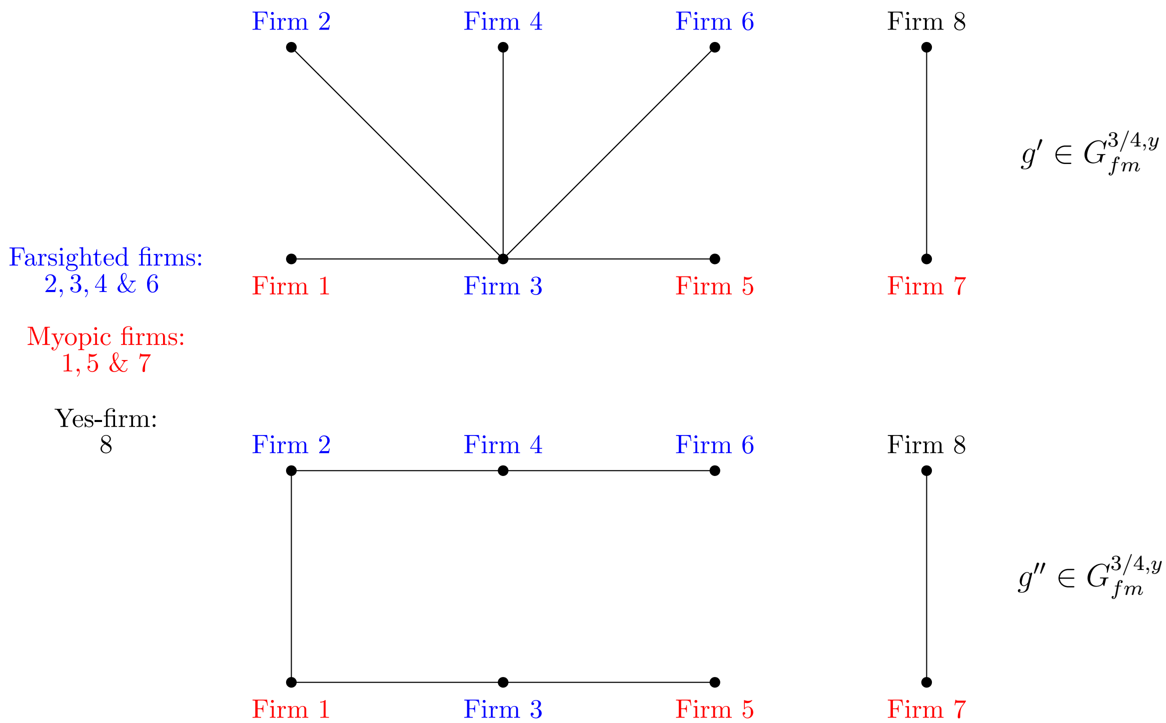

Figure 4: Stable R&D networks with yes-firms when the majority of firms is farsighted.

5 Evolution and Dynamics of R&D Networks

To study how networks evolve, we start with a group of firms who are initially unconnected to each other. Over time, pairs of firms decide whether or not to form or sever links with each other. A link can be severed unilaterally but agreement by both firms is needed to form a link. All firms are initially myopic, and thus decide to form or sever links if doing so increases their current profits. The length of a period is sufficiently long so that the process can converge to some stable R&D network. At the beginning of each period after the initial period, some myopic firms become farsighted. It can be interpreted as if some myopic firms have gained enough experience to acquire a better understanding of R&D collaboration agreements. Depending on their positions in the network, the process either stays at the same R&D network or evolves to another stable R&D network.

Time is divided into periods and is modeled as a countable and infinite set, $T = \{1, 2, ..., t, ...\}$. We denote by $g(t)$ the network that exists at the end of period $t \in T$ and by $g(0)$ the initial network. The process of forming links starts from the empty network. Hence, $g(0) = g^{\emptyset}$. We denote by $N_m(t)$ ($N_f(t)$) the set of myopic (farsighted) firms at the beginning of period $t \in T$. The population dynamics of firms is described by the following sequence $\{N_m(t), N_f(t)\}_{t=1}^{\infty}$ where $N_m(t) = N \setminus N_f(t)$, $N_m(1) = N$, $N_m(t) \subset N_m(t-1)$ for $2 \leq t < \bar{t}$ and $N_m(t) = \emptyset$ for $t \geq \bar{t}$. A myopic-farsighted improving path in period $t \in T$ from a network $g(t-1)$ to a network $g(t) \neq g(t-1)$ is a finite sequence of graphs $g_1, \ldots, g_K$ with $g_1 = g(t-1)$ and $g_K = g(t)$ such that for any $k \in \{1, \ldots, K-1\}$ either (i) $g_{k+1} = g_k - ij$ for some ij such that $U_i(g_{k+1}) > U_i(g_k)$ and $i \in N_m(t)$ or $U_j(g_K) > U_j(g_k)$ and $j \in N_f(t)$; or (ii) $g_{k+1} = g_k + ij$ for some ij such that $U_i(g_{k+1}) > U_i(g_k)$ and $U_j(g_{k+1}) \geq U_j(g_k)$ if $i, j \in N_m(t)$, or $U_i(g_K) > U_i(g_k)$ and $U_j(g_K) \geq U_j(g_k)$ if $i, j \in N_f(t)$, or $U_i(g_{k+1}) \geq U_i(g_k)$ and $U_j(g_K) \geq U_j(g_k)$ (with one inequality holding strictly) if $i \in N_m(t), j \in N_f(t)$. We denote by $\phi_t(g)$ the set of all networks that can be reached from g by a myopic-farsighted improving path in period t. We denote by G^{MF} the set of networks that belong to some myopic-farsighted stable set, $G^{MF} = \{g \in G \mid G \subseteq \mathcal{G}$ is a myopic-farsighted stable set$\}$.

Starting in period 1 from $g(0)$ with $N_m(1) = N$ and $N_f(1) = \emptyset$, the dynamic process will evolve to some $g(1)$ such that (i) there is a myopic-farsighted improving path from $g(0)$ to $g(1)$ and (ii) $g(1)$ belongs to some myopic-farsighted stable set, i.e. $g(1) \in G^{MF}(N_m(1), N_f(1))$. At the very beginning of period 2 some myopic firms become farsighted, $N_m(2) \subset N$ and $N_f(2) \neq \emptyset$. If $g(1)$ is no more stable (i.e. $g(1) \notin G^{MF}(N_m(2), N_f(2))$) then the dynamic process will evolve to some $g(2) \neq g(1)$ such that (i) $g(2) \in \phi_2(g(1))$ and (ii) $g(2) \in G^{MF}(N_m(2), N_f(2))$. Otherwise, it remains where it was, i.e. $g(2) = g(1)$. Given the population dynamics $\{N_m(t), N_f(t)\}_{t=1}^{\infty}$, we say that $\{g(t)\}_{t=1}^{\infty}$ is an evolution of stable R&D networks if and only if (i) $g(t) \in G^{MF}(N_m(t), N_f(t))$ and (ii) if $g(t) \neq g(t-1)$ then $g(t) \in \phi_t(g(t-1))$.

Proposition 7 shows that, starting from the empty network, the dynamic process first converges to a network consisting of two components of nearly equal sizes. Next, it suffices that one firm belonging to the largest component becomes farsighted for dismantling the network and converging to a network consisting of two components of different sizes close to $3n/4$ and $n/4$ with farsighted firms and loose-end or median myopic firms in the largest component. Along the transition, the dynamic process will visit single component networks that maximize social welfare.

Proposition 7. *Given the population dynamics* $\{N_m(t), N_f(t)\}_{t=1}^{\infty}$, *the sequence* $\{g(t)\}_{t=1}^{\infty}$ *is an evolution of stable networks if*

$$g(t) = \begin{cases} g(1) \in G^{1/2} & \text{for} \quad t < \widehat{t}_0(g(1)) \\ g(\widehat{t}_0(g(1))) \in G^{3/4}_{fm}(N_m(\widehat{t}_0(g(1)), N_f(\widehat{t}_0(g(1))) & \text{for} \quad t \geq \widehat{t}_0(g(1)) \end{cases}$$

where $\widehat{t}_0(g(1)) = \min\{t \in T \mid N_f(t) \cap \{i \in S \mid S \in \Pi(g(1)), \#S > n/2, g(1) \in G^{1/2}\} \neq \emptyset\}$.

Suppose now that there is a fixed number of yes-firms within the population of firms such that $n - \text{int}((3n+1)/4) \geq \#N_y \geq 1$. The population dynamics of firms is then described by the following sequence $\{N_m(t), N_f(t), N_y\}_{t=1}^{\infty}$ where $N_m(t) = N \setminus \{N_y \cup N_f(t)\}$, $N_m(1) = N \setminus N_y$, $N_m(t) \subset N_m(t-1)$ for $2 \leq t < \bar{t}$ and $N_m(t) = \emptyset$ for $t \geq \bar{t}$. Before looking at the population dynamics and the evolution of stable networks, we first characterize the myopic-farsighted stable set when the population consists only of myopic and yes-firms.

Let $G^y_m = \{g \mid \#C(g) = 1,\ N(g) = N,\ g$ is y-minimally connected, and for any $i \in N_m$ we have $\#\{j \mid ij \in g\} = 1\}$. That is, networks belonging G^y_m consists of a single y-minimally connected component that connects all firms and where all myopic firms are loose-end nodes. Remember that such R&D networks are socially optimal.

Lemma 1 shows that in the absence of farsighted firms, when the majority of firms are myopic and there are some yes-firms, the myopic-farsighted stable set consists of R&D networks having either two components of nearly equal size with only myopic firms in the largest component or a unique component connecting the n firms of the industry with myopic firms as loose-end nodes.

Lemma 1. *If* $n - 1 > \#N_m > n/2$, $\#N_f = 0$ *and* $\#N_y > 0$ *then the set of networks* $G^{1/2,y}_m \cup G^y_m$ *is the unique myopic-farsighted stable set with yes-firms.*

We now provide the intuition behind the proof of Lemma 1. (**IS**) Internal stability follows because $\phi^y(g) = \emptyset$ for all $g \in G^{1/2,y}_m$. From any $g \in G^y_m$, no myopic firm has an incentive to form or to delete a link (they are loose-end nodes). Thus, $\phi^y(g) = \emptyset$ for all $g \in G^y_m$, and $G^{1/2,y}_m \cup G^y_m$ satisfies internal stability. (**ES**) Given that profits only depend on the cardinality of the component and infinitesimally small costs for maintaining redundant links, we only need to check that there is a y-myopic-farsighted improving path from any minimally connected network $g \notin G^{1/2,y}_m \cup G^y_m$ to some $\widetilde{g} \in G^{1/2,y}_m \cup G^y_m$. Three cases have to be considered:

- **(a)** First, we consider any minimally connected network g connecting the n firms of the industry. From g, some myopic firm linked to another myopic firm being loose-end deletes its link to the loose-end reaching a network with two components of sizes $n-1$ and 1, respectively. Next, some yes-firm forms a link to the isolated myopic firm reaching again a minimally connected network with a single component. We repeat this two-step process until we reach a y-minimally connected network $\widetilde{g}$ connecting the n firms and such that $\widetilde{g} \in G^y_m$.

(b) Second, we consider any minimally connected network $g \notin G_m^{1/2,y}$ with two components, h_1, h_2, and such that h_2 is y-minimally connected. If $\#N(h_1) > n/2$, we initiate from g a process that consists in isolating first a myopic firm from the largest component that next adds a link to some firm in the smallest component, until we reach some network $\widetilde{g} \in G_m^{1/2}$. If $\#N(h_1) \leq n/2$, starting from g, some myopic firm in the largest component adds a link to some yes-firm in the smallest component obtaining a network g' with a unique component that connects the n firms of the industry. From g', we can then proceed as in **(a)**.

(c) Third, we consider any minimally connected network with three or more components and such that all yes-firms belong to the same component. If the size of the largest component $\#N(h_1) \geq n/2$, starting from g, two (myopic or yes-) firms belonging to the two smallest components form a link until we reach a network g' containing two components. From g', we proceed as in **(b)**. If $\#N(h_1) < n/2$, starting from g, two (myopic or yes-) firms belonging to the two largest components form a link until we reach a network g' such that the size of the largest component is greater or equal than $n/2$. From g', two (myopic or yes-) firms belonging to the two smallest components form a link until we reach a network g'' containing two components. From g'', we proceed as in **(b)**.

Finally, since there is no y-myopic-farsighted improving path leaving g for any $g \in G_m^{1/2,y} \cup G_m^y$, it guarantees that $G_m^{1/2,y} \cup G_m^y$ is the unique myopic-farsighted stable set.

From the empty network $g^{\emptyset}$, there are y-myopic-farsighted improving paths going to some $g \in G_m^y$. For instance, we first build a y-myopic-farsighted improving path leading to a network composed of two components with some yes-firms on both sides. Next, yes-firms will bridge both components.

Proposition 8. *Given the population dynamics $\{N_m(t), N_f(t), N_y\}_{t=1}^{\infty}$, the sequence of networks $\{g(t)\}_{t=1}^{\infty}$ is an evolution of stable networks if*

$$g(t) = \begin{cases} g(1) \in G_m^{1/2,y} & \text{for } \ t < \widehat{t}_1(g(1)) \\ g(\widehat{t}_1(g(1))) \in G_{fm}^{3/4,y}(N_m(\widehat{t}_1(g(1)), N_f(\widehat{t}_1(g(1))) & \text{for } \ t \geq \widehat{t}_1(g(1)) \end{cases}$$

where $\widehat{t}_1(g(1)) = \min\{t \in T \mid N_f(t) \cap \{i \in S \mid S \in \Pi(g(1)),\ \#S > n/2,\ g(1) \in G_m^{1/2,y}\} \neq \emptyset\}$*, or*

$$g(t) = \begin{cases} g(1) \in G_m^{y} & \text{for } \ t < \widehat{t}_2(g(1)) \\ g(\widehat{t}_2(g(1))) \in G_{fm}^{3/4,y}(N_m(\widehat{t}_2(g(1)), N_f(\widehat{t}_2(g(1))) & \text{for } \ t \geq \widehat{t}_2(g(1)) \end{cases}$$

where $\widehat{t}_2(g(1)) = \min\{t \in T \mid \#N_f(t) \geq n/2\}$.

When there are yes-firms together with a majority of myopic firms, Proposition 8 shows that, starting from the empty network, the process converges to either a network consisting of two components of nearly equal sizes or a single component network with the yes-firms bridging all other firms. If the process reaches first a network consisting of two components, it suffices that one firm belonging to the largest component becomes farsighted for dismantling the network and converging to a network consisting of two components of different sizes close to $3n/4$ and $n/4$ with all yes-firms in the smallest component. However, if the process reaches first a single component network, half of the firms need to become farsighted to move away from it. Hence, this single component network that maximize social welfare and consumer surplus will persist many periods before moving to a network that consists of two components with the largest component comprising three-quarters of firms. Figure 5 summarizes the persistence of stable R&D networks with yes-firms.

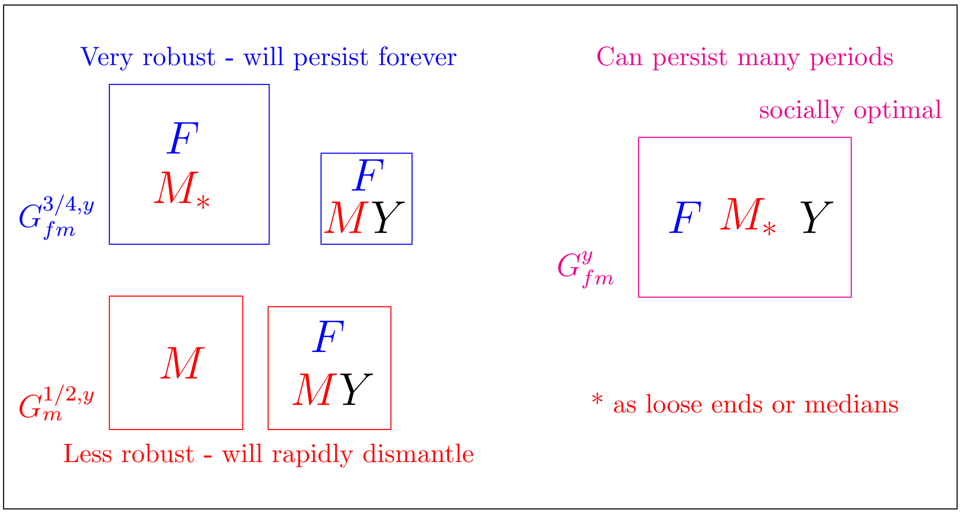

Figure 5: Evolution of stable R&D networks with yes-firms and a majority of myopic firms.

Firms generally come with relevant attributes that are related to the interaction pattern. For instance, a myopic firm (e.g. a start-up firm) will learn and evolve faster to become farsighted when interacting and collaborating in an environment composed mainly of farsighted firms (e.g. when it belongs to a component with a majority of farsighted or experienced private firms/public institutions). In the dynamic process, at the beginning of each period, some myopic firms become farsighted. It is interpreted as if some myopic firms have gained enough experience to acquire a better understanding of R&D collaborations. Hence, if the likelihood for a myopic firm of becoming farsighted now depends to which firms it is linked to, the R&D network consisting of two components of nearly equal sizes will tend to persist more periods before being dismantled. On the other hand, the socially optimal R&D network consisting of a single component will persist less periods than before. The idea is that a myopic firm belonging to a component with a majority of farsighted (myopic) will interact mostly with farsighted (myopic) firms, and so will acquire faster (slower) a better knowledge of the R&D process.

Hanaki, Nakajima and Ogura (2010) provide an empirical analysis of evolving networks of successful R&D collaborations in the IT industry in the U.S. between 1985 and 1995. In 1995 the largest component (i.e. the giant component) had a size of 52% of the firms.

It has some similarities with the pairwise stable structure, and in addition the largest component is still growing as predicted by our dynamic model. In 1995, the average degree was just above 2 links and the average geodesic distance between two connected firms was around 4.5. It became rather stable between 4.5 and 5.0 after that. Because many pairs of firms are connected by a few collaboration links, the spread of information or knowledge is fast in the R&D network. They also find that there are small numbers of "center or hub" firms and, at the same time, large numbers of peripheral firms with fewer links. In fact, the R&D network has evolved unevenly, and in the largest component, a core-periphery structure has emerged. This is in line with our model where experienced firms and/or public institutions have in average much more links than start-ups which are often loose-end nodes.

6 Conclusion

We studied the formation of R&D networks when each firm benefits from the research done by other firms it is connected to. We proposed the notion of myopic-farsighted stable set to determine the R&D networks that emerge when some firms are myopic while others are farsighted. When the majority of firms is myopic, stability leads to R&D networks consisting of either two asymmetric components with the largest component comprises three-quarters of firms or two symmetric components of nearly equal size with the largest component having only myopic firms. But, once the majority of firms becomes farsighted, only R&D networks with two asymmetric components remain stable. Firms in the largest component obtain greater profits, with farsighted firms having in average more collaborations than myopic firms that are either loose-ends or central for spreading the innovation within the component. Besides myopic and farsighted firms, we next introduced yes-firms that always accept the formation of any link and never delete a link subject to the constraint of non-negative profits. We showed that yes-firms may play a crucial role by bridging myopic and farsighted firms and stabilizing R&D networks consisting of a single component that maximize the social welfare. Finally, we looked at the evolution and dynamics of R&D networks and we found that R&D networks with two symmetric components will be rapidly dismantle, single component R&D networks that maximize social welfare will persist many periods, while R&D networks consisting of two asymmetric components will persist forever.

We focused on the case where each firm fully benefits from the research done by the firms it is connected to (δ is equal to one). If δ is close to one our results would not changed. What would happen if δ is close to zero as in Goyal and Joshi (2003)? As δ goes to zero, research spillovers disappear and each firm only benefits from the research done by the firms it is directly linked to. In other words, the reduction in marginal costs only depends on the number of direct links, as if each firm was able to isolate the knowledge coming from each firm to which it is linked. Then, it turns that, if there are only myopic firms or a large majority of myopic firms, the set $\{g^N\}$ is a myopic-farsighted stable set where g^N is the complete network. The complete network is the only pairwise stable network (Goyal and Joshi, 2003). Thus, contrary to δ close to one, one single farsighted firm cannot destabilize the pairwise stable network. One would need a majority of farsighted firms so that $\{g^N\}$ is no more a myopic-farsighted stable set. Indeed, from any network with two "complete" components where only farsighted firms belong to the largest one, there is no myopic-farsighted improving path to the complete network. Hence, external stability would be violated.

We assumed quantity competition on the goods market. With price competition and homogenous goods, all networks give zero profits for all firms. Once there is an infinitesimally small cost for forming links, stability only supports the empty network, which is also the unique pairwise stable network.

In terms of policy recommendations, our analysis suggests investments for turning myopic firms into farsighted and more precisely, focusing on the myopic firms who are linked to many other myopic ones. In addition, providing R&D subsidies to universities (or public institutions) for linking to private firms is likely to lead to the formation of R&D networks that are welfare improving. This theoretical recommendation is in line with recent empirical findings about the subsidized R&D network in the German biotechnology industry. Roesler and Broekel (2017) find that, by bridging local firms to geographically distant knowledge sources, universities play a central role for developing and stimulating local and inter-regional knowledge diffusion.

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Appendix

Proof of Proposition 1. We show that $G_{fm}^{3/4} \cup G_m^{1/2}$ satisfies both internal stability and external stability.

Internal stability: We have that $\phi(g) = \emptyset$ for all $g \in G_m^{1/2}$. The largest component of g consists of only myopic firms and those myopic firms do not want to add a link to g or to delete a link in g since g is pairwise stable. In addition, myopic firms in the smallest component do not want to delete a link. Only farsighted firms in the smallest component could start adding a link to g or deleting a link in g. But, adding a link between two farsighted firms in the smallest component would not change the profit of the myopic firms belonging to the largest component. In addition, deleting a link would split the smallest component in two and increase the profits of the firms belonging to the largest component. Hence, farsighted firms in the smallest component cannot modify the network g so that myopic firms in the largest component would have incentives to modify the network afterwards.

Take now any $g \in G_{fm}^{3/4}$. Myopic firms in the largest component do not want to add nor to delete a link since they are loose-end or median nodes in g. Farsighted firms in the largest component do not want to add a link nor to delete a link looking forward to any $g' \neq g$, $g' \in G_{fm}^{3/4} \cup G_m^{1/2}$, since they would obtain at most the same profit. As a result, myopic and farsighted firms in the smallest component can only add links among themselves. Of course, myopic firms will not do it because it's costly. Farsighted firms neither will add a link since it would not change the profit of the firms belonging to the largest component. Deleting a link would split the smallest component in two, decrease the profits of the firms that were in the smallest component, and increase the profits of the firms belonging to the largest component. Hence, myopic firms will not delete any link and farsighted firms in the smallest component cannot modify the network g so that farsighted firms in the largest component would have incentives to modify the network afterwards looking forward to some $g' \in G_{fm}^{3/4} \cup G_m^{1/2}$. Thus, $g \notin \phi(g')$ for all $g, g' \in G_{fm}^{3/4}$ and $g' \notin \phi(g)$ for all $g \in G_{fm}^{3/4}, g' \in G_m^{1/2}$. Hence, $G_{fm}^{3/4} \cup G_m^{1/2}$ satisfies internal stability.

External stability: Notice that we only need to show that there is a myopic-farsighted improving path from any network g consisting of minimally connected components to some $\widetilde{g} \in G_{fm}^{3/4}$. Indeed, if $\widetilde{g} \in \phi(g)$ then $\widetilde{g} \in \phi(g')$ for any $g' \supsetneq g$ with $\Pi(g') = \Pi(g)$ because profits only depend on the cardinality of the components and forming links is costly. We consider three cases.

(a) Take any network g with $\Pi(g) = \{N\}$. From g, looking forward to some $\widetilde{g} \in G_{fm}^{3/4}$, farsighted firms build links to other farsighted firms so that we reach a network g' such that $\Pi(g' - \{ij \mid i \vee j \in N_m\}) = \{N_f\}$. From g', myopic and farsighted firms delete links to reach a minimally connected network g'' such that $\Pi(g'') = \{N\}$ and $\Pi(g'' - \{ij \mid i \vee j \in N_m\}) = \{N_f\}$. Hence, at g'', there is no myopic firm in any path between two farsighted firms. Define process (*) as follows. Take in g'' any myopic firm i such that i is a loose-end node and linked to another myopic firm j in g''. Then, firm j deletes its link with firm i to form a network with two components of sizes $n-1$ and 1, respectively. Next some farsighted firm k builds a link to the isolated firm i to form again a minimally connected network with a single component. We now proceed repeatedly applying process (*) until we reach a network $\widehat{g}$ such that $\widehat{g}$ is minimally connected, $\Pi(\widehat{g}) = \{N\}$, $\Pi(\widehat{g} - \{ij \mid i \vee j \in N_m\}) = \{N_f\}$, and the number of myopic firms that are loose-end nodes and linked to a farsighted firm in $\widehat{g}$ is equal to $\text{int}((3n+1)/4) - \#N_f$, i.e. $\#\{i \in N_m \mid \#\{j \mid ij \in \widehat{g}\} = 1$ and $\{j \mid ij \in \widehat{g}\} \subseteq N_f\} = \text{int}((3n+1)/4) - \#N_f$. Next, a farsighted firm, looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, deletes its link in $\widehat{g}$ to a myopic firm that is not a loose-end node. We obtain a network $\widehat{g}'$ with two components, i.e. $\#C(\widehat{g}') = 2$. All farsighted firms belong to the largest component of $\widehat{g}'$. Next, a farsighted firm deletes its link in $\widehat{g}'$ to another myopic firm that is not a loose-end node. We obtain a network $\widehat{g}''$ with three components, i.e. $\#C(\widehat{g}'') = 3$. All farsighted firms belong to the largest component of $\widehat{g}''$. Next, two myopic firms, belonging to the two smallest components, form a link to bridge the two smallest components. We obtain a network $\widehat{g}'''$ with two components, i.e. $\#C(\widehat{g}''') = 2$. All farsighted firms belong to the largest component of $\widehat{g}'''$. We repeat this process until we reach some $\widetilde{g} \in G^{3/4}_{fm}$ where all farsighted firms belong to the largest component of size $\text{int}((3n+1)/4)$ and myopic firms in the largest component are loose-end nodes.

(b) Take any network $g \notin G^{3/4}_{fm} \cup G^{1/2}_{m}$ with two components such that $C(g) = (h_1, h_2)$, h_1 and h_2 are minimally connected, $N(h_1) \cup N(h_2) = N$. **(b1)** If there is a farsighted firm i in the largest component h_1 and $\#N(h_1) \neq \text{int}((3n+1)/4)$, then i adds a link to some (farsighted or myopic) firm j in h_2 with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$. From $g + ij$ we proceed as in **(a)** above since $\Pi(g + ij) = \{N\}$. **(b2)** If there is no farsighted firm in the largest component h_1, then some myopic firm $i \in N(h_1)$ deletes its link with another myopic firm $j \in N(h_1)$ to form $g - ij$ where j is isolated (i.e. j has no link). Next, firm j adds a link to some (farsighted or myopic) firm k in h_2, with the farsighted firms looking forward to some $\widetilde{g} \in G^{1/2}_{m}$, to form a network $g - ij + kj$ with two components $(h_1 - ij, h_2 + kj)$. We repeat the process of isolating a myopic firm in the largest component that next adds a link to some firm in the smallest component until we reach some $\widetilde{g} \in G^{1/2}_{m}$ where the size of the largest component is equal to $\text{int}((n+3)/2)$ if n even or $(n+1)/2$ if n odd. **(b3)** If there is a farsighted firm i in the largest component h_1, $\#N(h_1) = \text{int}((3n+1)/4)$ and some myopic firm j in h_1 is nor a loose-end node nor a median node, then this myopic firm j has incentives to cut one of its links to split h_1 in two components. From this network with three components, we proceed as in **(c1)** or **(c2)** below. Since the size of the largest of the three components is smaller than $\text{int}((3n+1)/4)$ but larger than $n/2$, the process will transit from **(c1)** or **(c2)** to either **(b1)** or **(b2)**.

(c) Take any network g with three or more minimally connected components. Let h_1 be the largest component of g, h_2 be the second largest component of g, and so forth. First, if $\#N(h_1) < n/2$ then one myopic firm i belonging to some h_k forms a link with some myopic firm j belonging to some h_l. If $\#N(h_k) + \#N(h_l) < n/2$, from $g' = g + ij$, at each step, two myopic firms belonging to some components form a link until we reach a network g'' such that the size of the largest component of g'' is greater or equal than $n/2$. **(c1)** Suppose $\#N(h_1) \geq n/2$ and $N(h_1) \cap N_f \neq \emptyset$. From g, at each step, two (myopic or farsighted) firms belonging to the two smallest components, with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, form a link until we reach a network g' consisting of only two components. From g' we proceed as in **(b)**. **(c2)** Suppose $\#N(h_1) > n/2$ and

$N(h_1) \cap N_f = \emptyset$. From g, at each step, two (myopic or farsighted) firms belonging to the two smallest components, with the farsighted firms looking forward to some $\widetilde{g} \in G_m^{1/2}$, form a link until we reach a network g' consisting of only two components with the largest component h_1 that did not change along the sequence. Hence, the largest component of g' still contains only myopic firms. From g' we proceed as in **(b)**. **(c3)** Suppose $\#N(h_1) = n/2$ and $N(h_1) \cap N_f = \emptyset$. From g, at each step, two (myopic or farsighted) firms belonging to the two smallest components, with the farsighted firms looking forward to some $\widetilde{g} \in G_{fm}^{3/4}$, form a link until we reach a network g' consisting of only two components of size $n/2$. From g' two myopic firms belonging to the two components have incentives to add a link between them to form a network g'' with a single component. From g'' we proceed as in **(a)**.

□

Proof of Proposition 2. We first show that $G^{1/2}$ satisfies both internal stability and external stability.

Internal stability: From Mauleon, Sempere-Monerris and Vannetelbosch (2014) Proposition 1 we have that g is pairwise stable if and only if $C(g) = (h_1, h_2)$, h_1 and h_2 are minimally connected, $N(h_1) \cup N(h_2) = N$, and

$$\#N(h_1) = \begin{cases} \text{int}((n+3)/2) & \text{if } n \text{ even} \\ (n+1)/2 & \text{if } n \text{ odd.} \end{cases}$$

It follows that $\phi(g) = \emptyset$ for all $g \in G^{1/2}$. Hence, $G^{1/2}$ satisfies internal stability.

External stability: We proceed in steps. **(i)** First, in any network g where all components are not minimally connected, some firms have incentives to delete one of their links without increasing the number of components. Second, in any minimally connected network g, no pair of firms belonging to the same component have incentives to add a link between them. In addition, in a minimally connected network g with $N(g) = N$, firms have incentives to isolate one of the firms. **(ii)** Take any g with $\Pi(g) = \{S_1, S_2, ..., S_m\}$ where all components are minimally connected and $m \geq 3$. Two minimally connected components of cardinality smaller than $n/2$ have incentives to add a link between them to form one component. Hence, from any g such that $\#\Pi(g) \geq 3$ there is some g' such that $\#\Pi(g) = 2$ and $g' \in \phi(g)$. **(iii)** Take any g with $\Pi(g) = \{S_1, S_2\}$ and $\#S_1 \geq \#S_2$. Firms belonging to the largest component have incentives to delete one link isolating one firm until the cardinality of the component is equal to $\text{int}((n+3)/2) + 1$ if n even or to $(n+3)/2$ if n odd. **(iv)** Take g with $\Pi(g) = \{S_1, S_2\}$ and $\#S_1 = \#S_2$. Then, firm $i \in S_1$ and firm $j \in S_2$ have incentives to add the link ij to form $g' = g + ij$ with $\Pi(g') = N$. From **(i)**-**(iv)**, we have that the set $G^{1/2}$ of pairwise stable networks satisfies external stability: for any g' that is not pairwise stable (i.e. $g' \notin G^{1/2}$), there is some pairwise stable network g such that $g \in \phi(g')$.

In addition, since $\phi(g) = \emptyset$ for all $g \in G^{1/2}$, the set $G^{1/2}$ is the unique myopic-farsighted stable set when all firms are myopic.

□

Proof of Proposition 3. We show that $G^{3/4}_{fm}$ satisfies both internal stability and external stability.

Internal stability: Take now any $g \in G^{3/4}_{fm}$. Myopic firms in the largest component do not want to add nor to delete a link since they are loose-end or median nodes in g. Farsighted firms in the largest component do not want to add a link nor to delete a link looking forward to any $g' \neq g$, $g' \in G^{3/4}_{fm}$, since they would obtain at most the same profit. As a result, myopic and farsighted firms in the smallest component can only add links among themselves. Of course, myopic firms will not do it. Farsighted firms neither will add a link since it would not change the profit of the firms belonging to the largest component. Deleting a link would split the smallest component in two, decrease the profits of the firms that were in the smallest component, and increase the profits of the firms belonging to the largest component. Hence, myopic firms will not delete any link and farsighted firms in the smallest component cannot modify the network g so that farsighted firms in the largest component would have incentives to modify the network afterwards looking forward to some $g' \in G^{3/4}_{fm}$. Thus, $G^{3/4}_{fm}$ satisfies internal stability since $g \notin \phi(g')$ for all $g, g' \in G^{3/4}_{fm}$.

External stability: We only need to show that there is a myopic-farsighted improving path from any network g consisting of minimally connected components to some $\widetilde{g} \in G^{3/4}_{fm}$. Indeed, if $\widetilde{g} \in \phi(g)$ then $\widetilde{g} \in \phi(g')$ for any $g' \supsetneq g$ with $\Pi(g') = \Pi(g)$ because profits only depend on the cardinality of the components and forming links is costly. We consider three cases.

(a) Take any network g with $\Pi(g) = \{N\}$. From g, looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, a set $N'_f \subseteq N_f$ of farsighted firms such that $\#N'_f = \min\{\#N_f, \text{int}((3n+1)/4)\}$ build links to other farsighted firms in N'_f so that we reach a network g' such that $\Pi(g' - \{ij \mid i \vee j \in N_m \cup (N_f \setminus N'_f)\}) = \{N'_f\}$. From g', firms delete links to reach a minimally connected network g'' such that $\Pi(g'') = \{N\}$ and $\Pi(g'' - \{ij \mid i \vee j \in N_m \cup (N_f \setminus N'_f)\}) = \{N'_f\}$. **(a1)** If $\min\{\#N_f, \text{int}((3n+1)/4)\} = \text{int}((3n+1)/4)$, a farsighted firm belonging to N'_f, looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, deletes its link in g'' to some firm belonging to $N_m \cup (N_f \setminus N'_f)$. We obtain a network g''' with two components. Next, a farsighted firm belonging to N'_f deletes its link in g''' to another firm belonging to $N_m \cup (N_f \setminus N'_f)$. We obtain a network g'''' with three components. Next, two firms, belonging to the two smallest components, form a link to bridge the two smallest components. We obtain a network with two components and we repeat this process until we reach some $\widetilde{g} \in G^{3/4}_{fm}$ where $\#N'_f$ farsighted firms belong to the largest component of size $\text{int}((3n+1)/4)$. **(a2)** If $\min\{\#N_f, \text{int}((3n+1)/4)\} = \#N_f$, define the process (*) as follows. Take in g'' any myopic firm i such that i is a loose-end node and linked to another myopic firm j in g''. Then, firm j deletes its link with firm i to form a network with two components of sizes $n-1$ and 1, respectively. Next some farsighted firm k builds a link to the isolated firm i to form again a minimally connected network with a single component. Next we proceed repeatedly applying process (*) until we reach a network $\widehat{g}$ such that $\widehat{g}$ is minimally connected, $\Pi(\widehat{g}) = \{N\}$, $\Pi(\widehat{g} - \{ij \mid i \vee j \in N_m \cup (N_f \setminus N'_f)\}) = \{N'_f\}$, and the number of myopic firms that are loose-end nodes and linked to a farsighted firm in $\widehat{g}$ is equal to $\text{int}((3n+1)/4) - \#N_f$, i.e. $\#\{i \in N_m \mid \#\{j \mid ij \in \widehat{g}\} = 1 \text{ and } \{j \mid ij \in \widehat{g}\} \subseteq N_f\} = \text{int}((3n+1)/4) - \#N_f$. Next, a farsighted firm, looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, deletes its link in $\widehat{g}$ to a myopic firm that is not a loose-end node. We obtain a network $\widehat{g}'$ with two components, i.e. $\#C(\widehat{g}') = 2$. All farsighted firms belong to the largest component of $\widehat{g}'$. Next, a farsighted firm deletes its link in $\widehat{g}'$ to another myopic firm that is not a loose-end node. We obtain a network $\widehat{g}''$ with three components, i.e. $\#C(\widehat{g}'') = 3$. All farsighted firms belong to the largest component of $\widehat{g}''$. Next, two myopic firms, belonging to the two smallest components, form a link to bridge the two smallest components. We obtain a network $\widehat{g}'''$ with two components, i.e. $\#C(\widehat{g}') = 2$. All farsighted firms belong to the largest component of $\widehat{g}'''$. We repeat this process until we reach some $\widetilde{g} \in G^{3/4}_{fm}$ where all farsighted firms belong to the largest component of size $\text{int}((3n+1)/4)$ and myopic firms in the largest component are loose-end nodes.

(b) Take any network $g \notin G^{3/4}_{fm}$ with two components such that $C(g) = (h_1, h_2)$, h_1 and h_2 are minimally connected, $N(h_1) \cup N(h_2) = N$. **(b1)** If $\#N(h_1) \neq \text{int}((3n+1)/4)$, then some farsighted firm i in h_1 adds a link to some (farsighted or myopic) firm j in h_2 with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$. From $g + ij$ we proceed as in **(a)** since $\Pi(g + ij) = \{N\}$. **(b2)** If $\#N(h_1) = \text{int}((3n+1)/4)$ and some myopic firm j in h_1 is nor a loose-end node nor a median node, then this myopic firm j has incentives to cut one of its link to split h_1 in two components. From this network with three components, we proceed as in **(c1)** or **(c2)** below. Since the size of the largest of the three components is smaller than $\text{int}((3n+1)/4)$ but larger than $n/2$, the process will transit from **(c1)** or **(c2)** to either **(b1)** or **(b2)**.

(c) Take any network g with three or more minimally connected components. Let h_1 be the largest component of g, h_2 be the second largest component of g, and so forth. **(c1)** Suppose $\#N(h_1) \geq n/2$. From g, at each step, two (myopic or farsighted) firms belonging to the two smallest components, with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, form a link until we reach a network g' consisting of only two components. From g' we proceed as in **(b)**. **(c2)** Suppose $\#N(h_1) < n/2$ and $N(h_1) \cap N_f \neq \emptyset$. From g, at each step, two (myopic or farsighted) firms belonging to the two largest components, with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, form a link until we reach a network g' such that the size of the largest component of g' is greater or equal than $n/2$. Since the size of the largest two components is smaller than $n/2$, the two (myopic or farsighted) firms have incentives to link to each other. From g', at each step, two (myopic or farsighted) firms belonging to the two smallest components, with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, form a link until we reach a network g'' consisting of only two components. From g'' we proceed as in **(b)**. **(c3)** Suppose $\#N(h_1) < n/2$ and $N(h_1) \cap N_f = \emptyset$. From g, first, one myopic i firm belonging to h_1 forms a link with some farsighted firm j belonging to the largest component h_k that contains a farsighted firm, with j looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$. If $\#N(h_1) + \#N(h_k) < n/2$, from $g' = g + ij$, at each step, two (myopic or farsighted) firms belonging to the two largest components, with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, form a link until we reach a network g'' such that the size of the largest component of g'' is greater or equal than $n/2$. From g'', at each step, two (myopic or farsighted) firms belonging to the two smallest components, with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, form a link until we reach a network g''' consisting of only two components. From g''' we proceed as in **(b)**.

□

Proof of Proposition 4. The social welfare function $SW(g)$ is defined as the sum of consumer surplus plus aggregate profits. The consumer surplus is obtained as the difference between the utility enjoyed by consumers minus total payments. Since the product is homogeneous and the demand function is given by $p = a - Q$ with $Q = \sum_{i \in N} q_i$, the utility of consumers is equal to $aQ - Q^2/2$. Take any network g consisting of two components h_1 and h_2 with $N(h_1) \cup N(h_2) = N$. Let $n_1 = \#N(h_1)$ and $n_2 = \#N(h_2) = n - n_1$ with $n_1 > n_2$. Given that all firms belonging to the same component obtain the same payoff, there are two types of firms. Those in the large component with marginal costs equal to $c_1 = c_0 - 1 - (n_1 - 1) = c_0 - n_1$, and those in the smaller component with marginal costs equal to $c_2 = c_0 - 1 - (n_2 - 1) = c_0 - n_2$. Denote by q_1 and q_2 the corresponding equilibrium levels of output for each firm, and denote by $Q_1 = n_1 q_1$ and $Q_2 = n_2 q_2$ the aggregate output per component, with $Q = Q_1 + Q_2$. Then, $SW(g)$ is given by

$$SW(g) = a(Q_1 + Q_2) - \frac{(Q_1 + Q_2)^2}{2} - p(Q_1 + Q_2) + (p - c_1)Q_1 + (p - c_2)Q_2,$$

which simplifies to

$$SW(g) = (a - c_0 + n_1)Q_1 + (a - c_0 + n_2)Q_2 - \frac{(Q_1 + Q_2)^2}{2}.$$

Noting that $Q_1 = (n_1(a - c_0 + n_1 + n_2(n_1 - n_2)))/(n + 1)$, $Q_2 = (n_2(a - c_0 + n_2 - n_1(n_1 - n_2)))/(n + 1)$, $Q = (n(a - c_0) + n_1^2 + n_2^2)/(n + 1)$ and $n_2 = n - n_1$, the expression for $SW(g)$ reduces to

$$\begin{aligned} SW(g) &= \frac{(a - c_0)^2 n(n + 2) + 2(a - c_0)(n + 2)\left(n^2 - 2nn_1 + 2n_1^2\right)}{2(n + 1)^2} \\ &\quad + \frac{\left((n + 2)n^2 + 2(2n + 3)n_1^2 - 2(2n + 3)nn_1\right)\left(2nn_1 + n - 2n_1^2\right)}{2(n + 1)^2}. \end{aligned}$$

Next we look for the extreme points of the above expression. Given that

$$\frac{\partial SW}{\partial n_1} = \frac{(n - 2n_1)\left(2(n + 2)(a - c_0) + n(3 - n^2) + 4(2n + 3)(n - n_1)n_1\right)}{(n + 1)^2},$$

there are three extreme points, a local minimum $n_1^{min} = n/2$ and two local maxima,

$$
\begin{aligned}
n_1^{max1} &= \frac{n}{2} - \frac{\sqrt{(2n+3)(2(n+2)(a-c_0)+n^3+3n^2+3n)}}{4n+6} \\
n_1^{max2} &= \frac{n}{2} + \frac{\sqrt{(2n+3)(2(n+2)(a-c_0)+n^3+3n^2+3n)}}{4n+6}.
\end{aligned}
$$

Since $n_1^{max1} < n_1^{min} = n/2 < n/2+1 < 3n/4 < n_1^{max2}$, we have that $SW(g) > SW(g')$ for any $g \in G^{3/4}, g' \in G^{1/2}$.

From Proposition 8 in Bloch (1995) we have that networks consisting of a single component that connects all firms maximize the social welfare: $SW(g) > SW(g')$ for any $g \in G^1, g' \notin G^1$.

□

Proof of Proposition 5. We show that $G_{fm}^{3/4,y} \cup G_m^{1/2,y} \cup G_{fm}^y$ satisfies both internal stability and external stability. Suppose $n-1 > \#N_m > n/2$, $n - \text{int}((3n+1)/4) > \#N_f > 0$ and $n - \text{int}((3n+1)/4) > \#N_y > 0$.

Internal stability: We have that $\phi^y(g) = \emptyset$ for all $g \in G_m^{1/2,y}$. The largest component of g consists of only myopic firms and those myopic firms do not want to add a link to g or to delete a link in g since g is pairwise stable. In addition, myopic and yes- firms in the smallest component do not want to delete a link. Only farsighted firms in the smallest component could start adding a link to g or deleting a link in g. But, adding a link would not change the profit of the myopic firms belonging to the largest component. In addition, deleting a link would split the smallest component in two and increase the profits of the firms belonging to the largest component. Hence, farsighted firms in the smallest component cannot modify the network g so that myopic firms in the largest component would have incentives to modify the network afterwards.

Take now any $g \in G_{fm}^{3/4,y}$. Myopic firms in the largest component do not want to add nor to delete a link since they are loose-end or median nodes in g. Farsighted firms in the largest component do not want to add a link nor to delete a link looking forward to any $g' \neq g$, $g' \in G_{fm}^{3/4,y} \cup G_m^{1/2,y} \cup G_{fm}^y$, since they would obtain at most the same profit. As a result, myopic, farsighted and yes- firms in the smallest component can only add links among themselves. Of course, myopic firms will not do it. Farsighted firms neither will add a link since it would not change the profit of the firms belonging to the largest component. Deleting a link would split the smallest component in two, decrease the profits of the firms that were in the smallest component, and increase the profits of the firms belonging to the largest component. Hence, myopic firms will not delete any link and farsighted firms in the smallest component cannot modify the network g so that farsighted firms in the largest component would have incentives to modify the network afterwards looking forward to some $g' \in G_{fm}^{3/4,y} \cup G_m^{1/2,y} \cup G_{fm}^y$.

Take now any $g \in G^y_{fm}$. Myopic firms do not want to add nor to delete a link since they are loose-end nodes in g. Farsighted firms do not want to add a link nor to delete a link looking forward to any $g' \neq g$, $g' \in G^{1/2,y}_m \cup G^y_{fm}$, since they would obtain at most the same profit. Thus, farsighted firms could only want to add a link or to delete a link looking forward to some $g' \neq g$, $g' \in G^{3/4,y}_{fm}$, where they would obtain a greater profit if they belong to the largest component of g'. However, the number of myopic firms that are both loose-end nodes and linked to farsighted firms is too small, $\# \{i \in N_m \mid ij \in g \text{ and } j \in N_f\} < n/2 - \#N_f$, for setting up a component of size greater or equal than $n/2$, which is a necessary step for building a y-myopic-farsighted improving path towards some $g' \in G^{3/4,y}_{fm}$.

Hence, $G^{3/4,y}_{fm} \cup G^{1/2,y}_m \cup G^y_{fm}$ satisfies internal stability.

External stability: We will show that there always exists a y-myopic-farsighted improving path from any network g consisting of one y-minimally connected component h with $N_y \subseteq N(h)$ (hence, $g^{N_y} \subseteq h$) and possibly other minimally connected components to some $\widetilde{g} \in G^{3/4,y}_{fm} \cup G^{1/2,y}_m \cup G^y_{fm}$. We consider three cases.

(a) Take any network $g \notin G^y_{fm}$ with $\Pi(g) = \{N\}$. Remember that $g^{N_y} \subseteq g$. From g, looking forward to some $\widetilde{g} \in G^{3/4,y}_{fm}$, farsighted firms build links to other farsighted firms so that we reach a network g' such that $\Pi(g' - \{ij \mid i \vee j \in N_m \cup N_y\}) = \{N_f\}$. From g', firms delete links to reach a y-minimally connected network g'' such that $\Pi(g'') = \{N\}$ and $\Pi(g'' - \{ij \mid i \vee j \in N_m \cup N_y\}) = \{N_f\}$. Notice that, under this process, myopic firms that are loose-end nodes and linked to a farsighted firm in g are still loose-end nodes and linked to the same farsighted firm in g'' (i.e. $g'' \notin G^y_{fm}$). Define process (*) as follows. Take in g'' any myopic firm i such that i is a loose-end node and linked to another myopic firm j. Then, firm j deletes its link with firm i to form a network with one component of size $n-1$ and one isolated firm. Next, some farsighted firm k builds a link to the isolated firm i. We proceed repeatedly applying process (*) until we reach a network $\widehat{g}$ such that $\widehat{g}$ is y-minimally connected, $\Pi(\widehat{g}) = \{N\}$, $\Pi(\widehat{g} - \{ij \mid i \vee j \in N_m \cup N_y\}) = \{N_f\}$, and all myopic firms that are loose-end nodes in $\widehat{g}$ are linked to either a farsighted firm or a yes-firm. Myopic firms that are not loose end nodes (if any) are on the path between some farsighted firm and some yes-firm. Obviously, $\widehat{g} \notin G^y_{fm}$.

(a1) Suppose first that the number of myopic firms that are loose-end nodes and linked to a farsighted firm in $\widehat{g}$ is greater or equal than $\text{int}((3n+1)/4) - \#N_f$, i.e. $\# \{i \in N_m \mid \# \{j \mid ij \in \widehat{g}\} = 1 \text{ and } \{j \mid ij \in \widehat{g}\} \subseteq N_f\} \geq \text{int}((3n+1)/4) - \#N_f$. Next, the only farsighted firm linked to a myopic firm that is not a loose end node or linked to a yes-firm (if there is no myopic firm that is not a loose end), looking forward to some $\widetilde{g} \in G^{3/4,y}_{fm}$, deletes its link in $\widehat{g}$ to this myopic firm or yes-firm. We obtain a network $\widehat{g}'$ with two components, $\#C(\widehat{g}') = 2$. All farsighted firms belong to the largest component of $\widehat{g}'$. If the size of the largest component is equal to $\text{int}((3n+1)/4)$, then we have reached $\widetilde{g} \in G^{3/4,y}_{fm}$. Otherwise, a farsighted firm, looking forward to $\widetilde{g} \in G^{3/4,y}_{fm}$, deletes its link to some myopic firm that is a loose-end node. Next, this isolated myopic firm adds a link to some firm belonging to the smallest component. We repeat this process until we reach

$\widetilde{g} \in G_{fm}^{3/4,y}$ where the largest component of size $\text{int}((3n+1)/4)$ consists of $\#N_f$ farsighted firms and $\text{int}((3n+1)/4) - \#N_f$ myopic firms that are loose-end nodes.

(a2) Suppose now that the number of myopic firms that are loose-end nodes and linked to a farsighted firm in $\widehat{g}$ is smaller than $\text{int}((3n+1)/4) - \#N_f$ but greater or equal than $n/2 - \#N_f$, i.e. $n/2 - \#N_f \leq \# \{i \in N_m \mid \# \{j \mid ij \in \widehat{g}\} = 1 \text{ and } \{j \mid ij \in \widehat{g}\} \subseteq N_f\} < \text{int}((3n+1)/4) - \#N_f$. Next, a farsighted firm, looking forward to some $\widetilde{g} \in G_{fm}^{3/4,y}$, deletes its link in $\widehat{g}$ to a myopic firm that is not a loose-end node or to a yes-firm (if there is no farsighted firm linked to a myopic firm that is not a loose-end node). We obtain a network $\widehat{g}'$ with two components, $\#C(\widehat{g}') = 2$. All farsighted firms belong to the same component. From $\widehat{g}'$, a farsighted firm i adds a link to a myopic firm j that belongs to the other component and is a loose-end node. We obtain a network $\widehat{g}''$ with a single component connecting all firms, $\#C(\widehat{g}'') = 1$ and $\Pi(\widehat{g}'') = \{N\}$. Next, this myopic firm j deletes its link it has with the non-farsighted firm and becomes a loose-end node in the largest component of the newly formed network $\widehat{g}'''$, $\#C(\widehat{g}''') = 2$. All farsighted firms belong to the largest component of $\widehat{g}'''$. We repeat this process until we reach some $\widetilde{g} \in G_{fm}^{3/4,y}$ where all farsighted firms belong to the largest component of size $\text{int}((3n+1)/4)$ and myopic firms in the largest component are loose-end nodes.

(a3) Suppose now that $\# \{i \in N_m \mid \# \{j \mid ij \in \widehat{g}\} = 1 \text{ and } \{j \mid ij \in \widehat{g}\} \subseteq N_f\} < n/2 - \#N_f$, i.e. the number of myopic firms that are loose-end nodes and linked to a farsighted firm in $\widehat{g}$ is smaller than $n/2 - \#N_f$. Then, we cannot build a myopic-farsighted improving path from $\widehat{g}$ to some $\widetilde{g} \in G_{fm}^{3/4,y}$. However, we can now build a myopic-farsighted improving path from g to some $\widetilde{g} \in G_{fm}^{y}$. Remember that $g \notin G_{fm}^{y}$ with $\Pi(g) = \{N\}$. Define process (**) as follows. From g take any (myopic or farsighted) firm i such that i is a loose-end node and linked to some myopic firm j. Then, firm j deletes its link with firm i to form a network with one component of size $n-1$ and one isolated firm. Next, some yes-firm builds a link to the isolated firm i. We proceed repeatedly applying process (**) until we reach a network g' such that g' is y-minimally connected and myopic firms that are loose-end nodes in g' are linked to either a farsighted firm or a yes-firm. Each myopic firm that is not a loose-end node in g' is such that if it deletes one of its links then the network g' is split into two components of unequal sizes. If there is no such myopic firm then we are already at $\widetilde{g} \in G_{fm}^{y}$ and the process stops. Otherwise, apply process (***): from g' some myopic firm i that is not a loose-end node has incentives to delete one of its link leading to $g' - ij$ where $C(g' - ij) = (h_1, h_2)$ with $N(h_1) \cup N(h_2) = N$, $\#N(h_1) \neq \#N(h_2)$, $\#N(h_1) > n/2$, $\#N(h_2) \geq 2$, and $N_y \cup \{i\} \subseteq N(h_1)$. If there is some myopic firm $k \in N(h_2)$ then firm k has incentives to bridge both components by adding a link to some yes-firm l. Otherwise, some farsighted firm $k \in N(h_2)$ adds a link to some yes-firm l. From $g' - ij + kl$ we proceed repeatedly as in process (**) until we reach a network g'' such that g'' is y-minimally connected and myopic firms that are loose-end nodes in g'' are linked to either a farsighted firm or a yes-firm. Each myopic firm that is not a loose-end in g'' is such that if it deletes one of its links then the network g'' is split into two components of unequal sizes. If there is no such myopic firm then we are already at $\widetilde{g} \in G^y_{fm}$ and the process stops. Otherwise, we repeat the process (***) until we reach some $\widetilde{g} \in G^y_{fm}$ (it is reached after a finite number of steps since the number of firms is finite).

(b) Take any network $g \notin G^{3/4,y}_{fm} \cup G^{1/2,y}_{m}$ with two components such that $C(g) = (h_1, h_2)$, h_1 is minimally connected, h_2 is y-minimally connected (hence, $g^{N_y} \subseteq h_2$), and $N(h_1) \cup N(h_2) = N$.

(b1) If $N(h_1) \cap N_f = \emptyset$ and $\#N(h_1) > n/2$, then some myopic firm $i \in N(h_1)$ deletes its link with another myopic firm $j \in N(h_1)$ to form $g - ij$ where j is isolated. Next, firm j adds a link to some (farsighted or myopic) firm k in h_2, with the farsighted firms looking forward to some $\widetilde{g} \in G^{1/2,y}_{m}$, to form a network $g - ij + kj$ with two components $(h_1 - ij, h_2 + kj)$. We repeat the process of isolating a myopic firm in the largest component that next adds a link to some firm in the smallest component until we reach some $\widetilde{g} \in G^{1/2,y}_{m}$ where the size of the largest component is equal to $\text{int}((n + 3)/2)$ if n even or $(n + 1)/2$ if n odd.

(b2) If $N(h_1) \cap N_f = \emptyset$ and $\#N(h_1) = n/2$, then a myopic firm $i \in N(h_1)$ adds a link to some myopic firm $j \in N(h_2)$ to form $g + ij$, and from $g + ij$ we can proceed as in **(a)**.

(b3) If $N(h_1) \cap N_m \neq \emptyset$ and $\#N(h_1) < n/2$, then a myopic firm $i \in N(h_1)$ adds a link to some yes-firm $j \in N(h_2)$ to form $g + ij$. From $g + ij$ we proceed as in **(a)** since $\Pi(g + ij) = \{N\}$.

(b4) If $N(h_1) \cap N = N_f$ (hence $\#N(h_1) < n/2$), then a farsighted firm $i \in N(h_1)$ adds a link to some yes-firm $j \in N(h_2)$ to form $g + ij$ looking forward to some $\widetilde{g} \in G^y_{fm}$, and from $g + ij$ we can reach such $\widetilde{g} \in G^y_{fm}$ (see **a3**).

(b5) If $N(h_1) \cap N_f \neq \emptyset$, $\#N(h_1) \geq n/2$ and $\#N(h_1) \neq \text{int}((3n + 1)/4)$, then looking forward to some $\widetilde{g} \in G^{3/4,y}_{fm}$, farsighted firms in h_1 build links to other farsighted firms in h_1 so that we reach a network g' such that $g^{N_f \cap N(h_1)} \subseteq g'$. From g' with $C(g') = (h'_1, h_2)$ and $\Pi(g') = \{N(h_1), N(h_2)\}$, firms in h'_1 delete links to reach a minimally connected network g'' such that $C(g'') = (h''_1, h_2)$, $\Pi(g'') = \{N(h_1), N(h_2)\}$ and $\Pi(h''_1 - \{ij \mid i \vee j \in \{N_m \cup N_y\} \cap N(h''_1)\}) = \{N_f \cap N(h_1)\}$. Next, apply process (*): take in h''_1 any myopic firm i such that i is a loose-end node and linked to another myopic firm j. Then, firm j deletes its link with firm i to split h''_1 into one component of size $\#N(h_1) - 1$ and one isolated firm. Next, some farsighted firm k belonging to the component of size $\#N(h_1) - 1$ builds a link to the isolated firm i. We proceed repeatedly as in process (*) until we reach a network $\widehat{g}$ such that $\widehat{g}$ is minimally connected, $C(\widehat{g}) = (\widehat{h}_1, h_2)$, $\Pi(\widehat{g}) = \{N(h_1), N(h_2)\}$ and $\Pi(\widehat{h}_1 - \{ij \mid i \vee j \in \{N_m \cup N_y\} \cap N(\widehat{h}_1)\}) = \{N_f \cap N(h_1)\}$. All myopic firms in $\widehat{h}_1$ are loose-end nodes linked to some farsighted firm. From $\widehat{g}$, some farsighted firm i in $\widehat{h}_1$ adds a link to some (farsighted or myopic) firm j in h_2 with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$. From $\widehat{g} + ij$ we proceed as in **(a)**.

(b6) If $N(h_1) \cap N_f \neq \emptyset$, $\#N(h_1) = \text{int}((3n+1)/4)$ and some myopic firm j in h_1 is nor a loose-end node nor a median node, then this myopic firm j has incentives to cut one of its link to split h_1 in two components (with one component having a size strictly greater than $n/2$). From this network with three components, we proceed as in **(c1)** below. Since the size of the largest of the three components is smaller than $\text{int}((3n+1)/4)$ but larger than $n/2$, the process will transit from **(c1)** to **(b5)**.

(c) Take any network g with three or more minimally connected components and such that $g^{N_y} \subseteq h$ for some $h \in C(g)$. Let h_1 be the largest component of g, h_2 be the second largest component of g, and so forth.

(c1) Suppose $\#N(h_1) \geq n/2$, $N(h_1) \cap N_y = \emptyset$ and $N(h_1) \cap N_f \neq \emptyset$. From g, at each step, two (myopic or farsighted or yes-) firms belonging to the two smallest components, with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4,y}_{fm}$, form a link until we reach a network g' consisting of only two components. From g' we proceed as in **(b5)** or **(b6)**.

(c1′) Suppose $\#N(h_1) \geq n/2$, $N(h_1) \cap N_y \neq \emptyset$ and $N(h_1) \cap N_f \neq \emptyset$. From g, at each step, two (myopic or farsighted) firms belonging to the two smallest components, with the farsighted firms looking forward either to some $\widetilde{g} \in G^{3/4,y}_{fm}$ or to some $\widetilde{g} \in G^{y}_{fm}$, form a link until we reach a network g' consisting of only two components. From g' we proceed as in **(b2)**, **(b3)** or **(b4)**.

(c2) Suppose $\#N(h_1) > n/2$, $N(h_1) \cap N_y = \emptyset$ and $N(h_1) \cap N_f = \emptyset$. From g, at each step, two (myopic or farsighted or yes-) firms belonging to the two smallest components, with the farsighted firms looking forward to some $\widetilde{g} \in G^{1/2,y}_{m}$, form a link until we reach a network g' consisting of only two components with the largest component h_1 that did not change along the sequence. Hence, the largest component of g' still contains only myopic firms. From g' we proceed as in **(b1)**.

(c2′) Suppose $\#N(h_1) > n/2$, $N(h_1) \cap N_y \neq \emptyset$ and $N(h_1) \cap N_f = \emptyset$. From g, at each step, two (myopic or farsighted) firms belonging to the two smallest components, with the farsighted firms looking forward either to some $\widetilde{g} \in G^{3/4,y}_{fm}$ or to some $\widetilde{g} \in G^{y}_{fm}$, form a link until we reach a network g' consisting of only two components with the largest component h_1 that did not change along the sequence. Hence, the largest component of g' still contains only myopic and yes-firms. From g' we proceed as in **(b3)** or **(b4)**.

(c3) Suppose $\#N(h_1) = n/2$, $N(h_1) \cap N_y = \emptyset$ and $N(h_1) \cap N_f = \emptyset$. From g, at each step, two (myopic or farsighted) firms belonging to the two smallest components, with the farsighted firms looking forward either to some $\widetilde{g} \in G^{3/4,y}_{fm}$ or to some $\widetilde{g} \in G^{y}_{fm}$, form a link until we reach a network g' consisting of only two components of size $n/2$. From g' two myopic firms belonging to the two components have incentives to add a link between them to form a network g'' with a single component. From g'' we proceed as in **(a)**.

(c3′) Suppose $\#N(h_1) = n/2$, $N(h_1) \cap N_y \neq \emptyset$ and $N(h_1) \cap N_f = \emptyset$. From g, at each step, two (myopic or farsighted) firms belonging to the two smallest components, with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4,y}_{fm}$, form a link until we reach a network g' consisting of only two components of size $n/2$. From g' we proceed as in **(b5)**.

(c4) Suppose $\#N(h_1) < n/2$, $N(h_1) \cap N_y = \emptyset$ and $N(h_1) \cap N_f \neq \emptyset$. From g, at each step, two (myopic or farsighted) firms belonging to the two largest components that do not include yes-firms, with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, form a link until we reach a network g' such that the size of the largest component of g' is greater or equal than $n/2$. From g', at each step, two (myopic or farsighted or yes-) firms belonging to the two smallest components, with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, form a link until we reach a network g'' consisting of only two components. From g'' we proceed as in **(b5)** or **(b6)**.

(c4′) Suppose $\#N(h_1) < n/2$, $N(h_1) \cap N_y \neq \emptyset$ and $N(h_1) \cap N_f \neq \emptyset$. From g, at each step, two (myopic or farsighted) firms belonging to the two largest components that do not include yes-firms, with the farsighted firms looking forward either to some $\widetilde{g} \in G^{3/4,y}_{fm}$ or to some $\widetilde{g} \in G^{1/2,y}_{m}$, form a link until we reach a network g' such that the size of the largest component of g' is greater or equal than $n/2$. From g', at each step, two (myopic or farsighted or yes-) firms belonging to the two smallest components, with the farsighted firms looking forward either to some $\widetilde{g} \in G^{3/4,y}_{fm}$ or to some $\widetilde{g} \in G^{1/2,y}_{m}$, form a link until we reach a network g'' consisting of only two components. From g'' we proceed as in **(b1)**, **(b5)** or **(b6)**.

(c5) Suppose $\#N(h_1) < n/2$, $N(h_1) \cap N_y = \emptyset$ and $N(h_1) \cap N_f = \emptyset$. From g, first, one myopic firm i belonging to h_1 forms a link with some farsighted firm j belonging to the largest component h_k that contains a farsighted firm but no yes-firm, with j looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$. If $\#N(h_1) + \#N(h_k) < n/2$, from $g' = g + ij$, at each step, two (myopic or farsighted) firms belonging to the two largest components that do not include yes-firms, with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, form a link until we reach a network g'' such that the size of the largest component of g'' is greater or equal than $n/2$. From g'', at each step, two (myopic or farsighted or yes-) firms belonging to the two smallest components, with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, form a link until we reach a network g''' consisting of only two components. From g''' we proceed as in **(b1)**, **(b5)** or **(b6)**.

(c5′) Suppose $\#N(h_1) < n/2$, $N(h_1) \cap N_y \neq \emptyset$ and $N(h_1) \cap N_f = \emptyset$. From g, first, one firm i belonging to h_2 forms a link with some farsighted firm j belonging to the largest component h_k that contains a farsighted firm, with j looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$. If $\#N(h_2) + \#N(h_k) < n/2$, from $g' = g + ij$, at each step, two (myopic or farsighted) firms belonging to the two largest components that do not include yes-firms, with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, form a link until we reach a network g'' such that the size of the largest component of g'' is greater or equal than $n/2$. From g'', at each step, two (myopic or farsighted or yes-) firms belonging to the two smallest components, with the farsighted firms looking forward to some $\widetilde{g} \in G^{3/4}_{fm}$, form a link until we reach a network g''' consisting of only two components. From g''' we proceed as in **(b1)**, **(b5)** or **(b6)**.

□

Proof of Proposition 6. We show that $G^{3/4,y}_{fm}$ satisfies both internal stability and external stability. Suppose $n > \#N_f \geq n/2$ and $n - \text{int}((3n+1)/4) \geq \#N_y \geq 1$.

Internal stability: Take now any $g \in G^{3/4,y}_{fm}$. Myopic firms in the largest component do not want to add nor to delete a link since they are loose-end or median nodes in g.

Farsighted firms in the largest component do not want to add a link nor to delete a link looking forward to any $g' \neq g$, $g' \in G_{fm}^{3/4,y}$, since they would obtain at most the same profit. As a result, myopic, farsighted and yes-firms in the smallest component can only add links among themselves. Of course, myopic firms will not do it. Farsighted firms neither will add a link since it would not change the profit of the firms belonging to the largest component. Deleting a link would split the smallest component in two, decrease the profits of the firms that were in the smallest component, and increase the profits of the firms belonging to the largest component. Yes-firms are already linked to any other yes-firms. Hence, myopic firms will not delete any link and farsighted firms in the smallest component cannot modify the network g so that farsighted firms in the largest component would have incentives to modify the network afterwards looking forward to some $g' \in G_{fm}^{3/4,y}$. Thus, $G_{fm}^{3/4,y}$ satisfies internal stability since $g \notin \phi^y(g')$ for all $g, g' \in G_{fm}^{3/4,y}$.

External stability: We will show that there always exists a y-myopic-farsighted improving path from any network g consisting of one y-minimally connected component h with $N_y \subseteq N(h)$ (hence, $g^{N_y} \subseteq h$) and possibly other minimally connected components to some $\widetilde{g} \in G_{fm}^{3/4,y}$. We consider three cases.

(a) Take any network g with $\Pi(g) = \{N\}$. Remember that $g^{N_y} \subseteq g$. From g, looking forward to some $\widetilde{g} \in G_{fm}^{3/4,y}$, a set $N'_f \subseteq N_f$ of farsighted firms such that $\#N'_f = \min\{\#N_f, \text{int}((3n+1)/4)\}$ build links to other farsighted firms in N'_f so that we reach a network g' such that $\Pi(g' - \{ij \mid i \vee j \in N_m \cup N_y \cup (N_f \setminus N'_f)\}) = \{N'_f\}$. From g', firms delete links to reach a y-minimally connected network g'' such that $\Pi(g'') = \{N\}$ and $\Pi(g'' - \{ij \mid i \vee j \in N_m \cup N_y \cup (N_f \setminus N'_f)\}) = \{N'_f\}$.

(a1) If $\min\{\#N_f, \text{int}((3n+1)/4)\} = \text{int}((3n+1)/4)$, a farsighted firm belonging to N'_f, looking forward to some $\widetilde{g} \in G_{fm}^{3/4,y}$, deletes its link in g'' to some firm belonging to $N_m \cup N_y \cup (N_f \setminus N'_f)$. We obtain a network g''' with two components, $\#C(g''') = 2$. Next, a farsighted firm belonging to N'_f deletes its link in g''' to another firm belonging to $N_m \cup N_y \cup (N_f \setminus N'_f)$. We obtain a network g'''' with three components, $\#C(g'''') = 3$. Next, two firms, belonging to the two smallest components, form a link to bridge the two smallest components. We obtain a network with two components and we repeat this process until we reach some $\widetilde{g} \in G_{fm}^{3/4,y}$ where $\#N'_f$ farsighted firms belong to the largest component of size $\text{int}((3n+1)/4)$.

(a2) If $\min\{\#N_f, \text{int}((3n+1)/4)\} = \#N_f$, define process (*) as follows. Take in g'' any myopic firm i such that i is a loose-end node and linked to another myopic firm j. Then, firm j deletes its link with firm i to form a network with one component of size $n-1$ and one isolated firm. Next, some farsighted firm k builds a link to the isolated firm i to form again a y-minimally connected network with a single component of size n. Next, we proceed repeatedly as in process (*) until we reach a network $\widehat{g}$ such that $\widehat{g}$ is y-minimally connected, $\Pi(\widehat{g}) = \{N\}$, $\Pi(\widehat{g} - \{ij \mid i \vee j \in N_m \cup N_y \cup (N_f \setminus N'_f)\}) = \{N'_f\}$, and all myopic firms that are loose-end nodes in $\widehat{g}$ are linked to either a farsighted firm or a yes-firm. **(a2i)** Suppose first that the number of myopic firms that are loose-end nodes and linked to a farsighted firm in $\widehat{g}$ is greater or equal than $\text{int}((3n+1)/4) - \#N_f$, i.e.

$\#\{i \in N_m \mid \#\{j \mid ij \in \widehat{g}\} = 1 \text{ and } \{j \mid ij \in \widehat{g}\} \subseteq N_f\} \geq \text{int}((3n+1)/4) - \#N_f$. Next, a farsighted firm, looking forward to some $\widetilde{g} \in G_{fm}^{3/4,y}$, deletes its link in $\widehat{g}$ to either a myopic firm that is not a loose-end node or a yes-firm. We obtain a network $\widehat{g}'$ with two components, $\#C(\widehat{g}') = 2$. All farsighted firms belong to the largest component of $\widehat{g}'$. Next, a farsighted firm deletes its link in $\widehat{g}'$ to either another myopic firm that is not a loose-end node or a yes-firm. We obtain a network $\widehat{g}''$ with three components, $\#C(\widehat{g}'') = 3$. All farsighted firms belong to the largest component of $\widehat{g}''$. Next, two myopic firms (or one myopic and one yes-firm), belonging to the two smallest components, form a link to bridge the two smallest components. We obtain a network $\widehat{g}'''$ with two components, $\#C(\widehat{g}') = 2$. All farsighted firms belong to the largest component of $\widehat{g}'''$. We repeat this process until we reach some network with two components where all farsighted firms belong to the largest component of size greater or equal than $\text{int}((3n+1)/4)$ and myopic firms in the largest component are loose-end nodes, and all yes-firms belong to the smallest component. If the size of the largest component is equal to $\text{int}((3n+1)/4)$, then we have reached $\widetilde{g} \in G_{fm}^{3/4,y}$. Otherwise, a farsighted firm, looking forward to $\widetilde{g} \in G_{fm}^{3/4,y}$, deletes its link to some myopic firm that is a loose-end node. Next, this isolated myopic firm adds a link to some firm belonging to the smallest component. We repeat this process until we reach $\widetilde{g} \in G_{fm}^{3/4,y}$ where the largest component of size $\text{int}((3n+1)/4)$ consists of $\#N_f$ farsighted firms and $\text{int}((3n+1)/4) - \#N_f$ myopic firms that are loose-end nodes. **(a2ii)** Suppose now that the number of myopic firms that are loose-end nodes and linked to a farsighted firm in $\widehat{g}$ is smaller than $\text{int}((3n+1)/4) - \#N_f$, i.e. $\#\{i \in N_m \mid \#\{j \mid ij \in \widehat{g}\} = 1 \text{ and } \{j \mid ij \in \widehat{g}\} \subseteq N_f\} < \text{int}((3n+1)/4) - \#N_f$. Next, a farsighted firm, looking forward to some $\widetilde{g} \in G_{fm}^{3/4,y}$, deletes its link in $\widehat{g}$ to a myopic firm that is not a loose-end node or to a yes-firm (if there is no farsighted firm linked to a myopic firm that is not a loose-end node). We obtain a network $\widehat{g}'$ with two components, $\#C(\widehat{g}') = 2$. All farsighted firms belong to the same component. Next, a farsighted firm deletes its link in $\widehat{g}'$ to a myopic firm that is not a loose-end node or to a yes-firm (if there is no farsighted firm linked to a myopic firm that is not a loose-end node). We obtain a network $\widehat{g}''$ with three components, $\#C(\widehat{g}'') = 3$. All farsighted firms belong to the largest component. Next, two myopic firms (or one myopic and one yes-firm), belonging to the smallest components, form a link to bridge the two smallest components. We obtain a network $\widehat{g}'''$ with two components, $\#C(\widehat{g}''') = 2$. We repeat this process until we reach some network $\overline{g}$ with two components, $\#C(\overline{g}) = 2$, such that all farsighted firms belong to the largest component and myopic firms in the largest component are loose-end nodes. From $\overline{g}$, a farsighted firm i adds a link to a myopic firm j that belongs to the other component and is a loose-end node. We obtain a network $\overline{g}'$ with a single component connecting all firms, $\#C(\overline{g}') = 1$ and $\Pi(\overline{g}') = \{N\}$. Next, this myopic firm j deletes its link it has with the non-farsighted firm and becomes a loose-end node in the largest component of the newly formed network $\overline{g}''$, $\#C(\overline{g}'') = 2$. All farsighted firms belong to the largest component of $\overline{g}''$. We repeat this process until we reach some $\widetilde{g} \in G_{fm}^{3/4,y}$ where all farsighted firms belong to the largest component of size $\text{int}((3n+1)/4)$ and myopic firms in the largest component are loose-end nodes.

(b) Take any network $g \notin G_{fm}^{3/4,y}$ with two components such that $C(g) = (h_1, h_2)$, h_1 is minimally connected and h_2 is y-minimally connected, $N(h_1) \cup N(h_2) = N$ and $N_y \subseteq N(h)$ (hence, $g^{N_y} \subseteq h$).

(b1) If $\#N(h_1) \neq \text{int}((3n+1)/4)$ and $\#N(h_1) \geq n/2$, then some farsighted firm i in h_1 adds a link to some (farsighted or myopic or yes-) firm j in h_2 with the farsighted firms looking forward to some $\widetilde{g} \in G_{fm}^{3/4,f}$. From $g+ij$ we proceed as in **(a)** since $\Pi(g+ij) = \{N\}$.

(b2) If $\#N(h_1) = \text{int}((3n+1)/4)$ and some myopic firm j in h_1 is nor a loose-end node nor a median node, then this myopic firm j has incentives to cut one of its links to split h_1 in two components. From this network with three components, we proceed as in **(c1)** below. Since the size of the largest of the three components is smaller than $\text{int}((3n+1)/4)$ but larger than $n/2$, the process will transit from **(c1)** to **(b1)**.

(b3) If $\#N(h_1) < n/2$, then some (farsighted or myopic) firm i in h_1 adds a link to some yes-firm j in h_2 with the farsighted firm looking forward to some $\widetilde{g} \in G_{fm}^{3/4,f}$. From $g + ij$ we proceed as in **(a)** since $\Pi(g + ij) = \{N\}$.

(c) Take any network g with three or more minimally connected components and such that $g^{N_y} \subseteq h$ for some $h \in C(g)$. Let h_1 be the largest component of g, h_2 be the second largest component of g, and so forth.

(c1) Suppose $\#N(h_1) \geq n/2$. From g, at each step, two (myopic or farsighted or yes-) firms belonging to the two smallest components, with the farsighted firms looking forward to some $\widetilde{g} \in G_{fm}^{3/4,y}$, form a link until we reach a network g' consisting of only two components, one minimally connected component and one y-minimally connected component. From g' we proceed as in **(b)**.

(c2) Suppose $\#N(h_1) < n/2$ and $N(h_1) \cap N_f \neq \emptyset$. From g, at each step, two (myopic or farsighted or yes-) firms belonging to the two largest components, with the farsighted firms looking forward to some $\widetilde{g} \in G_{fm}^{3/4,y}$, form a link until we reach a network g' such that the size of the largest component of g' is greater or equal than $n/2$. Since the size of the largest two components is smaller than $n/2$, the two (myopic or farsighted or yes-) firms have incentives to link to each other. From g', at each step, two (myopic or farsighted or yes-) firms belonging to the two smallest components, with the farsighted firms looking forward to some $\widetilde{g} \in G_{fm}^{3/4,y}$, form a link until we reach a network g'' consisting of only two components, one minimally connected component and one y-minimally connected component. From g'' we proceed as in **(b)**.

(c3) Suppose $\#N(h_1) < n/2$ and $N(h_1) \cap N_f = \emptyset$. From g, first, one myopic (or yes-) i firm belonging to h_1 forms a link with some farsighted firm j belonging to the largest component h_k that contains a farsighted firm, with j looking forward to some $\widetilde{g} \in G_{fm}^{3/4,y}$. If $\#N(h_1) + \#N(h_k) < n/2$, from $g' = g + ij$, at each step, two (myopic or farsighted or yes-) firms belonging to the two largest components, with the farsighted firms looking forward to some $\widetilde{g} \in G_{fm}^{3/4,y}$, form a link until we reach a network g'' such that the size of the largest component of g'' is greater or equal than $n/2$. From g'', at each step, two (myopic or farsighted or yes-) firms belonging to the two smallest components, with the farsighted firms looking forward to some $\widetilde{g} \in G_{fm}^{3/4,y}$, form a link until we reach a network g''' consisting of only two components, one minimally connected component and one y-minimally connected component. From g''' we proceed as in **(b)**.

□

Proof of Lemma 1.

We show that $G_m^{1/2,y} \cup G_m^y$ satisfies both internal stability and external stability.

Internal stability: We have that $\phi^y(g) = \emptyset$ for all $g \in G_m^{1/2,y}$. The largest component of g consists of only myopic firms and those myopic firms do not want to add a link to g or to delete a link in g since g is pairwise stable. In addition, myopic firms in the smallest component do not want to delete a link. Only yes-firms in the smallest component could start adding a link to g. But, adding a link to a myopic firm in the smallest component would decrease the profit of this myopic firm and would not change the profit of the myopic firms belonging to the largest component. Take now any $g \in G_m^y$. Myopic firms in the largest component do not want to add (it would not increase their profits) nor to delete a link since they are loose-end nodes in g. Hence, $\phi^y(g) = \emptyset$ for all $g \in G_m^y$, and $G_m^{1/2,y} \cup G_m^y$ satisfies internal stability.

External stability: We will show that there always exists a y-myopic-farsighted improving path from any network g consisting of one y-minimally connected component h with $N_y \subseteq N(h)$ (hence, $g^{N_y} \subseteq h$) and possibly other minimally connected components to some $\widetilde{g} \in G_m^{1/2,y} \cup G_m^y$. We consider three cases.

(a) Take any network $g \notin G_m^y$ with $\Pi(g) = \{N\}$. Remember that $g^{N_y} \subseteq g$. Define process (*) as follows. Take in g any myopic firm i such that i is a loose-end node and linked to another myopic firm j. Then, firm j deletes its link with firm i to form a network with one component of size $n-1$ and one isolated firm. Next, some yes-firm k builds a link to the isolated firm i. We proceed repeatedly applying process (*) until we reach a network g' such that $\#C(g') = 1$, $N(g') = N$, g' is y-minimally connected, and for any $i \in N_m$ we have $\#\{j \mid ij \in g'\} = 1$. Hence, $g' \in G_m^y$ and $g' \in \phi^y(g)$.

(b) Take any network $g \notin G_m^{1/2,y}$ with two components such that $C(g) = (h_1, h_2)$, h_1 is minimally connected, h_2 is y-minimally connected (hence, $g^{N_y} \subseteq h_2$), and $N(h_1) \cup N(h_2) = N$. **(b1)** If $\#N(h_1) > n/2$, then some myopic firm $i \in N(h_1)$ deletes its link with another myopic firm $j \in N(h_1)$ to form $g - ij$ where j is isolated (i.e. j has no link). Next, firm j adds a link to some (myopic or yes-) firm k in h_2 to form a network $g - ij + kj$ with two components $(h_1 - ij, h_2 + kj)$. We repeat the process of isolating a myopic firm in the largest component that next adds a link to some firm in the smallest component until we reach some $\widetilde{g} \in G_m^{1/2,y}$ where the size of the largest component is equal to $\text{int}((n+3)/2)$ if n even or $(n+1)/2$ if n odd. **(b2)** If $\#N(h_1) \leq n/2$, then a myopic firm $i \in N(h_1)$ adds a link to some yes- firm $j \in N(h_2)$ to form $g + ij$, and from $g + ij$ we can proceed as in **(a)**.

(c) Take any network g with three or more minimally connected components and such that $g^{N_y} \subseteq h$ for some $h \in C(g)$. Let h_1 be the largest component of g, h_2 be the second largest component of g, and so forth. **(c1)** Suppose $\#N(h_1) \geq n/2$. From g, at each step, two (myopic or yes-) firms belonging to the two smallest components form a link until we reach a network g' consisting of only two components. From g' we proceed as in **(b)**. **(c2)** Suppose $\#N(h_1) < n/2$. From g, at each step, two (myopic or yes-) firms belonging to the two largest components form a link until we reach a network g' such that the size of the largest component of g' is greater or equal than $n/2$. From g', at each step, two (myopic or yes-) firms belonging to the two smallest components form a link until we reach a network g'' consisting of only two components. From g'' we proceed as in **(b)**.

Since $\phi^y(g) = \emptyset$ for all $g \in G_m^{1/2,y} \cup G_m^y$, the union $G_m^{1/2,y} \cup G_m^y$ is the unique myopic-farsighted stable set.

□

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