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Core-stable and equitable allocations of greenhouse gas emission permits[1]

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Abstract:

This paper deals with the issue of how to allocate greenhouse gas emission permits to nations in the long run. The so-called 'equitable' rules to allocate such permits under a global agreement (per capita or grandfathering allocation rules for instance) do not necessarily ensure stability in the sense of the core of a cooperative game: some nations and groups of nations may typically be worse off under the global agreement than in alternative non-cooperative situations. We present a way to compute allocations of permits satisfying core constraints *at each commitment period*, while being as close as possible to any given 'equitable' allocation. Then a simple world simulation model is used to analyze the long run welfare effects of these allocations.

Keywords:

Climate change negotiations, tradable permits, dynamic games, core, equity, Shapley value, nucleolus

1 Introduction

The issue of how to allocate greenhouse gas emission permits to nations has recently received a lot of attention. The first international agreement on the limitation of the emissions of such gases – the Kyoto Protocol— implicitly defines allocations of tradable permits to countries for a first commitment period (2008-2012). Moreover, discussions on the allocations of permits for subsequent commitment periods are taking place. Most of these discussions are actually driven by equity principles (such as allocating the same amount of permits per capita in all countries) and several authors have analyzed the welfare effects of these and other so-called 'equitable' allocation rules (see, among others, Reiner and Jacoby (1997), Rose, Stevens, Edmonds and Wise (1998) and Blanchard, Criqui, Trommetter and Viguier (2001)).

Although allocation rules based on equity principles seem very appealing, they may not necessarily be accepted by every nation or every group of nations. Indeed, such allocations might lead some of them to bear costs that largely exceed the benefits of the emissions limitation. In that case, no global international agreement can be reached since such agreements are signed on a voluntary basis. This issue of voluntary participation to international agreements, and the related problem of stability of these agreements, have been investigated by numerous authors using various approaches based on game theory concepts (for a survey of the early contributions on the issue, see Folmer, Hanley and Missfeldt (1998)).

In this paper, we link the literature on equity in the allocation of emission permits to the one on voluntary participation to a global agreement. Hence, this work is related to the analysis by Germain and van Steenberghe (2003) who look at a weak form of participation constraint, namely individual rationality. Here, we extend their analysis to a more subtle concept of stability of the global agreement: coalitional rationality. Our purpose is not to analyze the process of coalition formation. Using cooperative game theory, we rather consider the grand coalition (the global agreement) and analyze its stability. The notion of stability used is based on the concept of γ-core developed for cooperative games with externalities by Chander and Tulkens (1995, 1997).[1]

We present a method which allows to compute allocations of permits satisfying γ-core participation constraints while being as close as possible to an equitable allocation rule. Then, using a world simulation model, we evaluate the welfare effects of various allocation rules as well as the amount of permits allocated under these rules. Some of the rules analyzed are directly based on equity principles while others are based on game theoretic solution concepts (*nucleolus* and *Shapley value*).

Moreover, we employ a dynamic decision framework that ensures stability of the global agreement not only at the first decision period, but along the entire time path.[2] We use the dynamic –closed loop—approach introduced by Germain, Toint, Tulkens and De Zeeuw (2003) (GTTZ (2003) hereafter). The motivation for using such a dynamic framework is twofold. First, it is much more realistic than considering that an international agreement is signed once for all periods during decades. Indeed, negotiations are typically based on commitment periods and the obligations negotiated for each period are likely to be called into question from time to time. Second, a static decision setting would not allow one to compute such allocations of permits because, in that case, there would be a single (intertemporal) participation constraint instead of one constraint for each commitment period. In fact, an infinity of allocations satisfying participation constraints could then be defined at each period. The dynamic setting allows us to define an allocation of permits *at each period*.

[1] The γ-core is defined below in section 3. It differs from the concept of "stable coalitions" used by Carraro and Siniscalco (1993) and Barret (1994), derived from the literature on the stability of cartels. A comparison of these approaches is made in Tulkens (1998). For a discussion about the differences in cooperative and non-cooperative game theory approaches, see Bloch (1997) and Finus (2003).

[2] Stevens and Rose (2002) also present a dynamic analysis of tradable emission permits. In their paper, dynamics refer mainly to intertemporal trades of permits, not to the decision framework.

The structure of the paper is as follows. In Section 2, the dynamic decision model is presented and the international optimum is characterized. The global agreement with tradable permits is described in Section 3. Then various so-called 'equitable' rules to allocate permits are presented in Section 4. The method whereby these equitable allocations are constrained in order to ensure stability of the global agreement is also described and analyzed in the same section. Simulations results are presented in Section 5 where we analyze the welfare implications of the aforementioned 'equitable' allocation rules. Finally, Section 6 summarizes the results and concludes.

2 The model and international optimality

Consider a set N of countries indexed by $i \in N = \{1, 2, ..., n\}$ and a set Θ of commitment periods indexed by $t \in \Theta = \{1, 2, ..., T\}$.[3] At each period t, country i's emissions of greenhouse gases are a proportion ν_{it} of its output Y_{it}, which is considered as exogenous. However, this emissions-output ratio is affected by a domestic abatement policy represented by the choice of a positive value for the rate μ_{it} $(0 \leq \mu_{it} \leq 1)$. Emissions of country i at period t are therefore given by:

$$E_{it} = \nu_{it} [1 - \mu_{it}] Y_{it}. \tag{1}$$

The accumulation of these pollutants emitted by all countries leads to a change in the concentration of greenhouse gases. The level of concentration, M_{t+1} at time $t+1$, is expressed with respect to its preindustrial level, M_0:

$$M_{t+1} - M_0 = [1 - \delta] [M_t - M_0] + \beta \sum_{i=1}^{n} E_{it} \tag{2}$$

where δ $(0 < \delta < 1)$ is the rate of decay of the gases in the atmosphere and β $(0 < \beta < 1)$ is the marginal atmospheric retention ratio. The change in the concentration of

[3] We use here the economic-climatic model presented in GTTZ (2003) and Germain and van Steenberghe (2003). Our description of a global agreement (see section 3 below) is, however, different than theirs.

gases affects the radiative forcing which finally influences the atmospheric temperature. These effects are modelled by a simple equation:

$$\Delta T_t = \eta \ln \left(\frac{M_t}{M_0} \right) \tag{3}$$

where ΔT_t is the temperature change w.r.t. its preindustrial level and η ($\eta > 0$) is an exogenous parameter.

A positive temperature change –and the resulting sea level rise and precipitation changes– cause different kinds of damages on health, agriculture, forests, water supply, biodiversity, etc. The costs of these damages for country i at time t are expressed by:

$$D_{it}(\Delta T_t) = \left[b_{i1} \Delta T_t^{b_{i2}} \right] Y_{it} \tag{4}$$

where b_{i1} and b_{i2} are positive parameters with $b_{i2} > 1\ \forall i$. Hence, through equations (2) and (3), damage costs of country i (D_{it}) are a strictly increasing function of its own emissions (E_{it}) as well as of the emissions of all the other countries (E_{jt}, $\forall j \neq i$).

Controlling emissions, that is, abating, is also costly. The cost for country i of limiting its emissions up to the amount E_{it} at time t is taken to be a strictly decreasing function of its emissions E_{it} :

$$C_{it}\left(E_{it}\right) = a_{i1} \mu_{it}^{a_{i2}} Y_{it} \tag{5}$$

where a_{i1} and a_{i2} ($i \in N$) are positive parameters with $a_{i2} > 1\ \forall i$ and, by (1), $\mu_{it} = 1 - \frac{E_{it}}{\nu_{it} Y_{it}}$.

In the sequential dynamic decision framework that we use below, an international optimum is defined at each period t. It is the solution of the minimization of an unweighted sum of the abatement and damage costs borne by all countries from that period t until the last one, T. Thus, *at each period of time* t, an optimal policy is given by the solution of:

$$\min_{\{E_{i,u}\}_{i \in N, u \in \{t,\ldots,T\}}} \left\{ \sum_{u=t}^{T} \sum_{i=1}^{n} \alpha^s \left[C_{iu}\left(E_{iu}\right) + \alpha D_{i,u+1}\left(M_{u+1}\right) \right] \right\} \tag{6}$$

subject to (1)-(5),

$$E_{it} \leq \nu_{it} Y_{it}, \, \forall i, t \tag{7}$$

and

$$E_{it} > 0, \forall i, t \tag{8}$$

where α $(0 < \alpha \leq 1)$ is the discount factor.

Denoting by E_{it}^{*}, μ_{it}^{*} and M_t^{*} respectively the optimal level of emissions of country i, the optimal abatement rate of the same country and the optimal stock of gases at period t, the international optimum satisfies the following standard conditions:

$$-C_{i,t}' \left(E_{i,t}^{*} \right) = \alpha\beta \sum_{j=1}^{n} \sum_{u=t}^{T} \alpha^{u-t} \left[1 - \delta \right]^{u-t} D_{j,u+1}' \left(M_{u+1}^{*} \right) \, , \, \forall i, t \tag{9}$$

that is, the marginal abatement cost of every country is equal, at every point in time t, to the discounted sum, over all countries, of the marginal damage costs occurring from then on in the future.

3 Core-stable global agreements with tradable permits

In this section, we introduce tradable permits in the model just stated. We first show that, at any period t, the competitive equilibrium of the permits market induced by an appropriate initial allocation of permits is an international optimum. Then, we compute the costs borne by the countries at such permits market equilibrium. These costs allow us to define the coalitional rationality of any agreement that involves a competitive market equilibrium induced by some initial allocation of permits. The analysis is first performed for the last period, T, and its results are then used for computing costs and defining coalitional rationality at preceding periods. Finally, we present the cooperative game associated with the dynamic economic model.

3.1 Two properties of the competitive permits market equilibrium at any period t

At each period t, let $\widetilde{E}_t = \left(\widetilde{E}_{1,t}, ..., \widetilde{E}_{n,t}\right)$ be a vector of tradable permits allocated to the countries. This initial allocation is assumed to be such that the total amount of allocated permits is equal to the sum of the countries' optimal emissions, $E^*_{i,t}$, at the same period, i.e.

$$\sum_i \widetilde{E}_{i,t} = \sum_i E^*_{i,t}. \tag{10}$$

Countries are allowed to trade these permits. We assume that the market for tradable permits is perfectly competitive and that countries are allowed neither to borrow permits, nor to bank them. Then, we claim that, at every period t, a competitive market equilibrium is an international optimum; this property is used in the present section to establish coalitional rationality. Moreover, the equilibrium price is the same for any initial allocation satisfying (10); this result will be used later in section 4 to deal with coalitionally constrained equitable allocations of permits.

Formally, let $\left[E^{CE}_t\left(\widetilde{E}_t\right), \sigma^{CE}_t\left(\widetilde{E}_t\right)\right]$ denote the competitive equilibrium induced at any time t by the initial allocation $\widetilde{E}_t$, where $E^{CE}_t(\cdot) = \left\{E^{CE}_{1t}(\cdot), ..., E^{CE}_{nt}(\cdot)\right\}$ is a vector of emissions and $\sigma^{CE}_t(\cdot)$ is the market price of the permits. Then:

Claim 1 $E^{CE}_{it}\left(\widetilde{E}_t\right) = E^*_{it}$, $\forall i$ and $\forall \widetilde{E}_t$ satisfying (10).

Claim 2 $\sigma^{CE}_t\left(\widetilde{E}_t\right) = \sigma^{CE}_t\left(\widetilde{E}'_t\right)$, $\forall \widetilde{E}_t \neq \widetilde{E}'_t$ with $\widetilde{E}_t$ and $\widetilde{E}'_t$ satisfying (10).

Proofs See appendix 1.■

3.2 Coalitional rationality at the last period

Let $\widetilde{E}_T = \left(\widetilde{E}_{1,T}, ..., \widetilde{E}_{n,T}\right)$ be an allocation of the permits satisfying condition (10) in period T, i.e. $\sum_i \widetilde{E}_{i,T} = \sum_i E^*_{i,T}$. The total cost for a country i at the competitive permits market equilibrium[4] induced by the allocation $\widetilde{E}_T$ at period T is :

$$W_{i,T} = C_{i,T}\left(E_{i,T}^{CE}\right) + \alpha D_{i,T+1}\left(M_{T+1}^{CE}\right) + \sigma_T^{CE}\left[E_{i,T}^{CE} - \widetilde{E}_{i,T}\right] \tag{11}$$

with

$$M_{T+1}^{CE} - M_0 = [1-\delta]\,[M_T - M_0] + \beta \sum_{i=1}^{n} E_{iT}^{CE}$$

where M_T is the inherited stock at period T and where α is the discount factor. Thus, a country bears three types of costs: (i) the abatement costs corresponding to its effective emissions, (ii) the damage costs caused by the concentration and (iii) the costs (gains) of purchasing (selling) permits at the market equilibrium price. Recalling that $E_{i,T}^{CE}$ is a function of $\widetilde{E}_{i,T}$, we may write $W_{i,T}\left(\widetilde{E}_{i,T}\right)$.

Coalitional rationality of the competitive permits market equilibrium means that no group (coalition) of countries could benefit from refusing to sign the global agreement. Formally, the market equilibrium induced by the allocation $\widetilde{E}_T$ satisfying (10) guarantees coalitional rationality only if

$$(CR) \qquad \sum_{i\in S} W_{i,T}\left(\widetilde{E}_{i,T}\right) \leq V_{S,T}, \quad \forall S \tag{12}$$

where S is any non-empty subset (called a coalition) of N, the set of countries, and $V_{S,T}$ is the total cost for the members of S of the actions taken by them when S forms. The outcome of these actions –a vector of emissions $E_T = \{E_{1T}, ..., E_{nT}\}$ such that (i) the members of the coalition minimize the sum of their total costs, given what the other countries do and (ii) the non members of the coalition minimize their own total costs given what the coalition does and given what the other non members do– is called a Partial Agreement Nash Equilibrium (PANE) with respect to S and defined in Chander and Tulkens (1995, p. 284).

[4] To simplify notation, and only when no confusion arises, we shall henceforth drop the argument in $E_t^{CE}\left(\widetilde{E}_t\right)$ and in $\sigma_t^{CE}\left(\widetilde{E}_t\right)$ and write simply E_t^{CE} and σ_t^{CE} respectively.

In the context of the present dynamic model, the outcome $V_{S,T}$ of a PANE w.r.t. some S at time T reads as follows. Given an inherited stock M_T, the equilibrium vector of emissions E_T is such that:

(i) the level of emissions of the countries which are members of S, $E_{iT}, \forall i \in S$, is the solution of:

$$\min_{\{E_{iT}, \forall i \in S\}} \sum_{i \in S} \{C_{iT}(E_{iT}) + \alpha D_{i,T+1}(M_{T+1})\} \tag{13}$$

subject to (1)-(5), $E_{iT} \leq \nu_{iT} Y_{iT}$, $E_{iT} > 0$ and where E_{jT} $(j \notin S)$ is given by (14), while

(ii) the level of emissions of each country which is *not* a member of S, $E_{jT}, j \notin S$, is the solution of:

$$\min_{\{E_{jT}, j \notin S\}} \{C_{jT}(E_{iT}) + \alpha D_{j,T+1}(M_{T+1})\}, \ \forall j \notin S \tag{14}$$

subject to (1)-(5), $E_{jT} \leq \nu_{jT} Y_{jT}$, $E_{jT} > 0$, and where E_{iT} $(i \neq j)$ is given by (13). The value of (13) at the solution so-described defines $V_{S,T}$, a magnitude that will play a central role in the cooperative games defined below.

We will test the coalitional rationality of market equilibria induced by a series of 'equitable' initial allocations of permits. Let $\widetilde{E}_T$ satisfying (10) be such an equitable allocation. If $\widetilde{E}_T$ is such that condition (12) is satisfied, then that equitable allocation leads to coalitional rationality at period T.

Otherwise, we choose another initial allocation which necessarily leads to coalitional rationality at period T. Such an allocation will be defined in section 4.

For allocations of permits that guarantee that condition (12) is satisfied, it is thus rational for all countries, both individually and coalitionally, to sign the agreement in period T.

3.3 Coalitional rationality and rational expectations at preceding periods

The same reasoning is then applied in the preceding periods. At each such period, the countries face the same alternative of whether to sign or not the global agreement. However, countries expectations on the future have to be taken into account now. Following GTTZ (2003), we shall assume that countries have rational expectations in the sense that they *anticipate the signing of global agreements in the future thanks to allocations of permits that lead to coalitional rationality.*

Let $\widetilde{E}_t = \left(\widetilde{E}_{1,t}, ..., \widetilde{E}_{n,t}\right)$ be an allocation of the permits satisfying condition (10) in period t, i.e. $\sum_i \widetilde{E}_{i,t} = \sum_i E^*_{i,t}$. The total costs for a country i at the competitive permits market equilibrium induced by the allocation $\widetilde{E}_t$ at period t is

$$W_{i,t} = C_{i,t}\left(E^{CE}_{i,t}\right) + \alpha D_{i,t+1}\left(M^{CE}_{t+1}\right) + \sigma^{CE}_t\left[E^{CE}_{i,t} - \widetilde{E}_{i,t}\right] + \alpha W_{i,t+1} \qquad (15)$$

with

$$M^{CE}_{t+1} - M_0 = [1-\delta]\,[M_t - M_0] + \beta \sum_{i=1}^{n} E^{CE}_{it}$$

where M_t is the inherited stock at period t. This cost is composed of the current costs, including the costs of the net purchase of permits and of the future costs under global agreements in periods $t+1$ to T, $W_{i,t+1}$. Again, because $E^{CE}_{i,t}$ is a function of $\widetilde{E}_{i,t}$, we may write $W_{i,t}\left(\widetilde{E}_{i,t}\right)$.

As before, the coalitional rationality of the competitive market equilibrium induced by the allocation $\widetilde{E}_t$, is guaranteed only if

$$(CR) \qquad \sum_{i\in S} W_{i,t}\left(\widetilde{E}_{i,t}\right) \leq V_{S,t}, \quad \forall S, \forall t \qquad (16)$$

where $V_{S,t}$ is the total cost at time t for the members of S of the actions taken by them when S forms. The outcome of these actions is the Partial Agreement Nash Equilibrium (PANE) with respect to S (see above) at time t.

The outcome of a PANE w.r.t. some S at time t ($t \neq T$) reads as the outcome of a PANE w.r.t. some S at time T where subscripts T are replaced by subscripts t and where $\alpha W_{i,t+1}$ is added to (14) and in the summation term of (13) in order to account for future (discounted) costs.

Exactly as in the last period, we will test at each period t the coalitional rationality of the market equilibrium induced by alternative 'equitable' initial allocations of permits. If an equitable allocation $\widetilde{E}_t$ satisfying (10) does not lead to coalitional rationality at period t, we choose another initial allocation $\widehat{E}_t = \left(\widehat{E}_{1,t}, ..., \widehat{E}_{n,t}\right)$ obtained as the solution of program (17) defined below and which necessarily leads to coalitional rationality at period t. Therefore, all countries sign the global agreement in period t.

If we assume that this is perfectly anticipated in each of the preceding periods, we thus obtain, by backward induction, cooperation extending to all periods.

3.4 Cooperative games associated with the economic model

Implicit in the construction of the preceding paragraphs, which are in the spirit of GTTZ (2003), is a sequence of cooperative games in characteristic function form $[N, v_t(\,\cdot\,; M_t)]$ associated with the dynamic economic model at each period t. In each of these games, $N = \{1, ..., n\}$ is the set of players and $v_t(\,\cdot\,; M_t)$ is the characteristic function of the game that associates to every coalition $S \subseteq N$ a real number called the worth of the coalition, given the stock M_t prevailing at time t. In the present context, the worth of a coalition S is taken to be $v_t(S; M_t) \equiv V_{S,t}(M_t)$, that is, as defined by (13), the sum over all members of coalition S of the abatement and damage costs at the PANE w.r.t. S at time t.

For each such game, an *imputation* is an n-dimensional vector $W_t = [W_{1t}, ..., W_{nt}]$ of which all components sum up to $V_{N,t}(M_t)$, and a γ-*core* imputation is an imputation that satisfies the coalitional rationality constraints, i.e., such that $\sum_{i \in S} W_{i,t} \leq V_{S,t}(M_t)$, $\forall S$.

In our economic model, alternative allocations of permits $\widetilde{E}_t$ satisfying condition (10), i.e., $\sum_i \widetilde{E}_{i,t} = \sum_i E^*_{i,t}$, lead to alternative imputations $W_t\left(\widetilde{E}_t\right)$ which are not necessarily $\gamma-$core imputations. But in the following section, we shall identify an allocation of permits that does lead to a $\gamma-$core imputation. By choosing at every time t an allocation with that property, the sequence of such allocations will induce a sequence of $\gamma-$core imputations of games $[N, v_t(\,\cdot\,; M_t)]$.

The sequence of such permits allocations yields a sequence of equilibria on the permits market, as well as corresponding trajectories of emissions, abatement costs and damage costs. As such trajectories are core-stable at each time t, that is, at each commitement period, the solution of the model over the entire time horizon describes *a succession of core-stable global agreements*.

4 Core-stable and equitable allocations

Equitable allocations of permits are not necessarily core-stable. When they are not, it is unlikely that they ever be implemented since some of the countries can do better by themselves. However, as shown in Chander, Tulkens, van Ypersele and Willems (2002), allocations can be chosen so as to ensure core-stability. In this section, we present a method to modify equitable allocations in order to guarantee core-stability. Before doing so, some common allocation rules are briefly described. These are either directly based on equity principles or derived from well-known game-theoretic solution concepts.

4.1 Rules for the allocation

4.1.1 Rules based on equity principles

Various equity principles have been brought up in order to drive the discussions on how to share an amount of emission permits among nations, leading to the definition of various allocation rules. Among these rules (see Rose et al. (1998) for a review of them), we select those which are most often referred to, namely the *egalitarian*, *grandfathering* and *ability-to-pay* (ATP) rules. Under the *egalitarian* rule, each country receives the same amount of permits per head. Under the *grandfathering* rule, the permits are allocated in proportion to CO_2 emissions in 1990. Finally, under the *ability-to-pay* rule, the amount of permits allocated to a country is inversely related to its GDP per capita.[5]

4.1.2 Rules based on game-theoretic solution concepts

The issue of equity has received a lot of attention in the game theory literature. Two game-theoretic solution concepts have been regarded as particularly attractive equitable solutions: (i) the *nucleolus* (Schmeidler (1969)) and (ii) the *Shapley value* (Shapley (1953)). According to Maschler (1992, p. 634), they are *the most prominent game-theoretical solutions to revenue allocation problems*. These concepts share the following desirable feature: they both lead to unique imputations. Moreover, the *nucleolus* always lies in the γ-core –if it is not empty– (see Schmeidler (1969)), although this is not necessarily the case of the *Shapley value*.

The *nucleolus* and the *Shapley value* define imputations which for our games $[N, v_t(\cdot; M_t)]$, $t = 1, ..., T$, we shall denote respectively $NU_t = (NU_{1,t}, ..., NU_{i,t}, ..., NU_{n,t})$ and $SH_t = (SH_{1,t}, ..., SH_{i,t}, ..., SH_{n,t})$. From any imputation $\bar{W}_t$ we can infer the allocation of tradable permits $\bar{E}_t = \left(\bar{E}_{1,t}, ..., \bar{E}_{n,t}\right)$ inducing a competitive market equilibrium which results in the same imputation by computing

$$\bar{E}_{it} = E^*_{it} - \frac{C_{i,t}\left(E^*_{i,t}\right) + \alpha D_{i,t+1}\left(M^*_{t+1}\right) + \alpha \bar{W}_{i,t+1} - \bar{W}_{i,t}}{\sigma_t^{CE}}, \forall i, t,$$

[5] Formally, country i receives at time t the following share of the total amount of permits: $\lambda_{it} = POP_{it}(GDP_{it}/POP_{it})^{-\alpha} / \sum_j (POP_{jt}(GDP_{jt}/POP_{jt})^{-\alpha})$ with $\alpha < 1$ and POP denoting population. In the simulations presented below in section 5, we arbitrarily set $\alpha = 0.5$.

where $\bar{W}_{i,t} = NU_{i,t}$ or $\bar{W}_{i,t} = SH_{i,t}$.

Nucleolus

Consider $e_t(s, \bar{W}_t) = \sum_{i \in s} \bar{W}_{it} - V_{s,t}$ as the excess of a coalition s at time t. By definition, the nucleolus minimizes the greatest excess of any coalition. As suggested by Maschler, Peleg and Shapley (1979), *if one considers the excess of a coalition as a measure of its dissatisfaction, the nucleolus solution minimizes the highest dissatisfaction*, (which is equivalent to maximizing the lowest satisfaction). This *increases stability*, because the coalition with the highest dissatisfaction (or lowest satisfaction) is likely to have the greatest incentives to defect.[6]

The computation of the nucleolus is not an easy task and has given rise to many studies. Since, in our dynamic framework, the nucleolus solution must be computed more than 300 times[7], we implement in the simulations the particularly fast algorithm developed by Potters, Reijnierse and Ansing (1996).[8]

Shapley value

The usual interpretation of the Shapley value (see Roth (1988) for an introduction to the Shapley value) consists in considering that the players arrive in a random order to form the grand coalition. If a player i forms a coalition with the set of players who arrived before him –call it coalition s–, then it adds $V_{s \cup \{i\},t} - V_{s,t}$ to the coalition s. The probability that coalition s is present when i arrives is $\frac{|s|!(n-|s|-1)!}{n!}$. Hence, under the *Shapley value a country is rewarded according to an average of its contribution to every possible coalition.*[9] This value is easily computable.

[6] See Maschler (1992) for a further discussion.

[7] The solution must be computed $P(T-1)+T$ times, where $T = 31$ is the number of periods and $P = 10$ is the number of regression points on which the value functions are estimated (see the algorithm in GTTZ (2003)).

[8] They propose a prolonged simplex algorithm based on Mashler, Peleg and Shapley (1979).

[9] The Shapley value is often presented as the unique solution satisfying the four axioms of group rationality, symmetry, additivity and dummy player (see Shapley (1953)).

4.2 A method to compute core-stable and equitable allocations

Among the various 'equitable' allocation rules described above, only the one derived from the nucleolus solution concept is guaranteed to lead to an imputation belonging to the γ-core (if it is not empty). Hence, since this will typically not be the case under the other allocation rules, we propose now a method which allows to compute allocations of permits leading to γ-core imputations while taking as much as possible into account any initial allocation rule described above.[10]

The aim of the method is to minimize the deviation from the initial equitable allocation in order to satisfy the γ-core participation constraints. The notion of distance used here is a weighted Euclidean distance. This distance between the initial and the new allocations of permits is weighted (by the λ's, see below) in order to take the equitable allocation rule into account.

$\widetilde{E}_{it}$ denotes the amount of permits received by country i at time t under some rule and $W_{it}\left(\widetilde{E}_{it}\right)$ is the cost borne by the same country at the competitive market equilibrium induced by the initial allocation at the same period. Define λ_{it} ($\sum_i \lambda_{it} = 1$ $\forall t$) as country i's share of the total amount of permits allocated according to the same rule, i.e. $\lambda_{it} = \widetilde{E}_{it} / \sum_j \widetilde{E}_{jt}$.[11] Consider then $\widehat{E}_{it}$ as an alternative allocation of permits to country i at time t. If $W_{it}\left(\widetilde{E}_{it}\right)$ is not in the γ-core, the method consists in solving, at each period t:

$$\min_{\{\widehat{E}_{it}\}} \sum_i \frac{\left[\widehat{E}_{it} - \widetilde{E}_{it}\right]^2}{\lambda_{it}} \tag{17}$$

subject to the feasibility constraint

$$\sum_i \widehat{E}_{it} = \sum_i \widetilde{E}_{it} \tag{18}$$

[10] This method is an extension of the one proposed by Germain and van Steenberghe (2003) who deal with individual rationality but not with coalitional rationality.

[11] For instance, $\lambda_{it} = \frac{POP_{it}}{\sum_{j \in N} POP_{jt}}$ under the *egalitarian* rule.

and the γ-core constraints

$$\sum_{i\in S} W_{it}\left(\widehat{E}_{it}\right) \leq V_{St}, \quad \forall S \tag{19}$$

where $W_{it}\left(\widehat{E}_{it}\right)$ is given by

$$W_{it}\left(\widehat{E}_{it}\right) = W_{it}\left(\widetilde{E}_{it}\right) + \sigma_t^{CE}\left[\widetilde{E}_{it} - \widehat{E}_{it}\right] \tag{20}$$

and where σ_t^{CE} is the competitive market equilibrium price of the permits at time t. By claim 2 and relation (18), the price at the competitive market equilibrium induced by the new allocation $\widehat{E}_t$ is the same than the price at the equilibrium induced by the initial allocation $\widetilde{E}_t$. This allows us to write relation (20).

Interpretation

The langrangian of problem (17) under the γ-core constraints at period t is

$$L_t = \sum_i \frac{\left[\widehat{E}_{it} - \widetilde{E}_{it}\right]^2}{\lambda_{it}} + \varphi_t \sum_i \left[\widehat{E}_{it} - \widetilde{E}_{it}\right] + \sum_S \pi_{St} \sum_{i\in S}\left[W_{it}\left(\widetilde{E}_{it}\right) + \sigma_t^{CE}\left[\widetilde{E}_{it} - \widehat{E}_{it}\right]\right] - V_{St}$$

where φ_t is the multiplier associated to constraint (18) and π_{St} are the multipliers associated to constraints (19). The Kuhn-Tucker conditions of this problem are

$$\begin{aligned}
\frac{\partial L_t}{\partial \widehat{E}_{it}} &= \frac{2\left[\widehat{E}_{it} - \widetilde{E}_{it}\right]}{\lambda_{it}} + \varphi_t - \sigma_t^{CE} \sum_{S|i\in S} \pi_{St} = 0, \quad \forall i, && (21)\\
\frac{\partial L_t}{\partial \mu_t} &= \sum_i \widehat{E}_{it} - \sum_i \widetilde{E}_{it} = 0, && (22)\\
\frac{\partial L_t}{\partial \pi_{St}} &= \sum_{i\in S}\left[W_{it}\left(\widetilde{E}_{it}\right) + \sigma_t^{CE}\left[\widetilde{E}_{it} - \widehat{E}_{it}\right]\right] - V_{S,t} \leq 0, \quad \pi_{St} \geq 0 \text{ and} &&\\
& \pi_{St}\left\{\sum_{i\in S}\left[W_{it}\left(\widetilde{E}_{it}\right) + \sigma_t^{CE}\left[\widetilde{E}_{it} - \widehat{E}_{it}\right]\right] - V_{S,t}\right\} = 0, \ \forall S. && (23)
\end{aligned}$$

First order condition (22) is the feasibility constraint (18). It ensures that, at each period, the same total amount of permits are allocated under both the initial and the new allocations.

Conditions (23) imply that *any 'constrained coalition'* (that is, a coalition S for which $\pi_{St} > 0$) *receives exactly the same total costs as under the PANE*. Hence, the coalition is compensated in order to be induced to cooperate, but it does not receive more than what makes it indifferent between signing or not the global agreement. Accordingly, as few permits as possible are devoted to the satisfaction of the constraints.

Conditions (21), which determine the new allocation of the permits, are however more difficult to interpret. They can be written as

$$\frac{\widehat{E}_{it}}{\lambda_{it}} = \frac{\widetilde{E}_{it}}{\lambda_{it}} - \frac{\varphi_t}{2} + \frac{\sigma_t^{CE}}{2} \sum_{S|i \in S} \pi_{St}, \quad \forall i. \tag{24}$$

From these conditions, we derive five properties. Consider two countries $i \in N$ and $j \in N$. In order to save on notation, assume that only two non-empty coalitions, $K \subset N$ and $L \subset N$, are constrained at period t, i.e. $\pi_{Kt}, \pi_{Lt} > 0$. The reasoning easily extends to more than two constrained coalitions (CC).

Proposition. *Problem (17) leads to allocations of permits which satisfy the following properties:*

(a) Two countries which do not belong to any CC receive the same proportional allocation, i.e. if $i, j \notin K$ *and* $i, j \notin L$*, then* $\frac{\widehat{E}_{it}}{\lambda_{it}} = \frac{\widehat{E}_{jt}}{\lambda_{jt}}$.

(b) Countries belonging to the same CC receive the same proportional allocation, i.e. if $i, j \in K$ *and/or* $i, j \in L$*, then* $\frac{\widehat{E}_{it}}{\lambda_{it}} = \frac{\widehat{E}_{jt}}{\lambda_{jt}}$.

(c) Countries belonging to different CC receive different proportional allocations, i.e. if $i \in K$*,* $j \notin K$ *and* $j \in L$*,* $i \notin L$*, then* $\frac{\widehat{E}_{it}}{\lambda_{it}} \gtreqless \frac{\widehat{E}_{jt}}{\lambda_{jt}}$.

(d) A country belonging to at least one CC receives proportionately more permits than a country not belonging to any CC, i.e. if $i \in K$ *and/or* $i \in L$ *with* $j \notin K$*,* $j \notin L$*, then* $\frac{\widehat{E}_{it}}{\lambda_{it}} > \frac{\widehat{E}_{jt}}{\lambda_{jt}}$.

(e) A country belonging to the same CC as another country which also belongs to at least one more CC, receives a lower proportional allocation, i.e. if $i \notin K$*,* $j \in K$ *and* $i, j \in L$*, then* $\frac{\widehat{E}_{it}}{\lambda_{it}} \leq \frac{\widehat{E}_{jt}}{\lambda_{jt}}$.

Proof. *By combining (21) and (22), it is easily shown that* $\varphi_t \geq 0$. *Moreover, by definition,* $\frac{\widetilde{E}_{it}}{\lambda_{it}} = \frac{\widetilde{E}_{jt}}{\lambda_{jt}}$, $\forall i \neq j$. *Properties (a) to (e) derive then directly from (24).* ■

Attractive illustrations of these properties are provided in appendix (see Section 8.4). Let us now turn to the simulation of the model and show how these properties apply to alternative constrained equitable allocations of tradable CO_2 permits.

5 Simulations

The data set is based on the RICE model (see Nordhaus and Yang (1996) and Nordhaus and Boyer (1999)) as well as on Kverndokk (1994) (see the appendix).[12] Only CO_2 emissions are considered here. The world is divided into six regions: USA, Japan, European Union (EU), China, Former Soviet Union (FSU) and Rest of the World (ROW). Each period lasts for 10 years. Although the model assumes a time horizon of three centuries, results are shown only for the first twelve decades in order to avoid border effects.

5.1 The optimal path

Like in any other economic-climatic model, the optimal path is particularly sensitive to the following three elements. First, the levels of business-as-usual emissions in the far future are very uncertain.[13] Second, we know that the evaluation of damage costs is a particularly difficult task, as well as a disputable issue. Consequently, the range for the evaluation of the damage costs induced by a given temperature change goes from 1

[12] For comparison purposes, we use the same data and parameters as Germain and van Steenberghe (2003).

[13] In order to frame the debate on this issue, the Intergovernmental Panel on Climate Change (IPCC) has defined six families of economic scenarios leading alternative CO_2 emissions levels (see IPCC (2000)). When compared to these scenarios, reference emissions in our data set (23.8 GtC in 2090-2100) fall below those of the A1FI scenario but fit in these of the A2 one.

to 20. This is of crucial concern when an optimal path has to be computed. Third, the issue of discounting the far future is also very much debated.[14] In the absence of any consensus on the methodology to adopt and by simplicity, we use a constant discount rate that we set at a relatively low level, 1% per year, in order to still give some weight to the periods during which damages are likely to occur.

In fact, many global optimal emission paths may be justified. The one that is computed here is only one of them, from which we start analyzing the issue of γ-core equitable allocations of emission permits.

Given our parametrization, world optimal emissions reach 14.6 GtC per year in 2090-2100, leading to a temperature change of 2.5 C° in the same period compared to 1990-2000. Following condition (9), countries abate up to the point where their marginal abatement costs equalize. As shown in Table 1, all regions abate by a significant amount. The largest abatement rates are observed in CHI and ROW, due to their lower marginal abatement costs.

	USA		JPN		EU		CHI		FSU		ROW	
	Ref.	Opt.	Ref.	Opt.	Ref.	Opt.	Ref.	Opt.	Ref.	Opt.	Ref.	Opt.
2000-2010	5.77	4.27	2.40	1.95	2.29	1.80	0.85	0.42	3.63	2.23	1.33	0.82
2090-2100	6.72	4.85	2.41	1.97	1.73	1.36	1.66	1.03	3.13	2.23	2.56	1.48

Table 1. Reference and optimal emissions per head for selected periods (tC per year)

The sum of the countries optimal emissions determines the global emissions objective at each period. Under a global agreement with tradable permits, this emissions objective takes the form of tradable emission permits that are allocated to the regions according to any given equitable rule. We analyze now the allocations resulting from the use of the rules detailed above (section 4) as well as their welfare implications. For expositional convenience, we concentrate firstly on the egalitarian rule.

[14] See for instance the recent contributions by Weitzman (1998) and Newell and Pizer (2003).

5.2 Core-stable egalitarian allocations

Table 2 shows, for 4 selected periods, the *permits per head* allocated to each region and the *total discounted gains due to cooperation* for every coalition under both (i) the unconstrained egalitarian rule and (ii) the γ-core constrained egalitarian rule. Total discounted gains due to cooperation of a coalition S are defined as the difference between total discounted costs of S under the PANE w.r.t. S ($V_{S,t}$) and the sum of the members of S total discounted costs under the global agreement ($\sum_{i \in S} W_{i,t}$). Under the γ-core constrained allocations, total discounted costs under the global agreement are derived from the γ-core constrained allocations ($\widehat{E}_t$) computed at each period via the resolution of problem (17). Under the label '*un*constrained allocations', total costs under the global agreement are derived from allocations which are unconstrained in the period under investigation ($\widetilde{E}_t$) but γ-core constrained in the subsequent periods.[15] Moreover, a coalition S is said to be (γ-core) constrained if its total discounted gains due to cooperation equal zero.

Unconstrained egalitarian allocations

In each period, the unconstrained *egalitarian* allocations lead, by definition, to the same amount of permits per head in each region. We observe that two regions

[15] Formally, under the *un*constrained allocations, total gains of a coalition S at time t are given by $V_{S,t} - \sum_{i \in S} \bar{W}_{it}$ where

$$V_{S,t} = \min_{\{E_{it}, \forall i \in S\}} \sum_{i} \left\{ C_{it}\left(E_{it}\right) + \alpha D_{i,t+1}\left(M_{t+1}\right) + \alpha \widehat{W}_{i,t+1}(M_{t+1}) \right\} \tag{25}$$

and

$$\bar{W}_{i,t} = C_{i,t}\left(E^*_{i,t}\right) + \alpha D_{i,t+1}\left(M^*_{t+1}\right) + \sigma_t \left[E^*_{i,t} - \widetilde{E}_{i,t}\right] + \alpha \widehat{W}_{i,t+1} \tag{26}$$

with

$$\widehat{W}_{i,t} = C_{i,t}\left(E^*_{i,t}\right) + \alpha D_{i,t+1}\left(M^*_{t+1}\right) + \sigma_t \left[E^*_{i,t} - \widehat{E}_{i,t}\right] + \alpha \widehat{W}_{i,t+1}. \tag{27}$$

EGALITARIAN	Unconstrained allocations				Core constrained allocations			
	2000-2010	2030-2040	2060-2070	2090-2100	2000-2010	2030-2040	2060-2070	2090-2100
	Permits per head (1 quota = 1 ton of carbon)							
USA	1.05	1.11	1.30	1.54	3.92	4.03	4.19	4.34
JPN	1.05	1.11	1.30	1.54	1.35	1.13	1.17	1.41
EU	1.05	1.11	1.30	1.54	1.46	1.17	1.17	1.41
CHI	1.05	1.11	1.30	1.54	0.74	0.85	1.10	1.41
FSU	1.05	1.11	1.30	1.54	2.60	2.25	2.21	2.33
ROW	1.05	1.11	1.30	1.54	0.76	0.95	1.17	1.41
	Total discounted gains due to cooperation (in billion 1995 US$)							
	Coalitions of 1 country							
1 0 0 0 0 0	-522	-590	-583	-521	0	0	0	0
0 1 0 0 0 0	-10	12	42	72	16	13	31	62
0 0 1 0 0 0	-93	-2	97	189	17	13	59	155
0 0 0 1 0 0	465	527	529	474	197	247	314	343
0 0 0 0 1 0	-323	-271	-224	-180	5	4	3	2
0 0 0 0 0 1	1011	938	857	764	293	337	312	236
	Coalitions of 2 countries							
0 0 0 0 1 1	619	601	565	512	229	275	246	167
0 0 0 1 0 1	1398	1360	1242	1054	413	480	482	394
0 0 0 1 1 0	140	255	304	292	201	249	315	343
0 0 1 0 0 1	886	893	900	892	277	307	317	330
0 0 1 0 1 0	-421	-276	-129	8	17	13	59	156
0 0 1 1 0 0	367	521	621	659	209	255	369	494
0 1 0 0 0 1	989	932	873	802	297	332	317	263
0 1 0 0 1 0	-335	-260	-183	-109	19	15	33	63
0 1 0 1 0 0	454	537	569	544	212	258	343	403
0 1 1 0 0 0	-103	9	138	261	32	25	89	217
1 0 0 0 0 1	432	265	169	122	236	255	206	115
1 0 0 0 1 0	-851	-865	-809	-704	0	0	0	0
1 0 0 1 0 0	-63	-69	-60	-54	192	241	308	336
1 0 1 0 0 0	-619	-596	-489	-335	13	9	56	153
1 1 0 0 0 0	-533	-580	-542	-451	14	12	30	61
	Coalitions of 3 countries							
0 0 0 1 1 1	988	1005	931	784	331	399	397	307
0 0 1 0 1 1	464	534	591	629	184	223	234	249
0 0 1 1 0 1	1234	1271	1236	1132	358	405	438	438
0 0 1 1 1 0	32	241	389	471	203	250	363	488
0 1 0 0 1 1	582	584	573	546	218	259	243	190
0 1 0 1 0 1	1357	1331	1231	1063	397	451	460	393
0 1 0 1 1 0	124	261	340	358	210	256	341	399
0 1 1 0 0 1	850	871	902	919	267	287	308	347
0 1 1 0 1 0	-438	-269	-91	76	26	21	86	214
0 1 1 1 0 0	350	525	656	724	217	261	392	548
1 0 0 0 1 1	8	-99	-147	-148	140	165	117	28
1 0 0 1 0 1	775	632	488	341	311	342	310	203
1 0 0 1 1 0	-399	-350	-294	-243	185	234	301	329
1 0 1 0 0 1	266	174	167	211	179	179	166	170
1 0 1 0 1 0	-960	-880	-723	-523	0	0	49	147
1 0 1 1 0 0	-175	-90	18	118	189	235	348	474
1 1 0 0 0 1	391	235	161	139	220	226	187	122
1 1 0 0 1 0	-868	-859	-772	-637	8	7	26	57
1 1 0 1 0 0	-81	-66	-28	8	199	245	329	388
1 1 1 0 0 0	-635	-590	-453	-268	22	16	81	210

Table 2 (part I) Egalitarian allocations

	Coalitions of 4 countries							
0 0 1 1 1 1	792	890	902	841	245	299	330	329
0 1 0 1 1 1	930	961	906	780	299	356	361	292
0 1 1 0 1 1	411	498	579	643	157	188	211	253
0 1 1 1 0 1	1176	1223	1205	1121	325	358	396	417
0 1 1 1 1 0	7	239	418	531	203	249	381	538
1 0 0 1 1 1	330	246	147	43	195	230	196	87
1 0 1 0 1 1	-191	-216	-172	-80	51	63	54	61
1 0 1 1 0 1	564	491	429	368	210	216	213	195
1 0 1 1 1 0	-527	-384	-226	-81	165	216	331	458
1 1 0 0 1 1	-51	-143	-169	-145	108	122	84	21
1 1 0 1 0 1	711	575	446	319	273	286	258	170
1 1 0 1 1 0	-426	-354	-267	-187	183	231	316	375
1 1 1 0 0 1	208	126	139	208	147	132	127	157
1 1 1 0 1 0	-986	-881	-692	-461	0	0	68	199
1 1 1 1 0 0	-203	-96	43	173	187	231	362	518
	Coalitions of 5 countries							
0 1 1 1 1 1	716	826	856	815	194	236	273	294
1 0 1 1 1 1	85	77	62	46	59	75	74	56
1 1 0 1 1 1	248	173	90	6	138	158	128	40
1 1 1 0 1 1	-268	-280	-215	-97	0	0	0	35
1 1 1 1 0 1	482	414	366	324	153	139	139	142
1 1 1 1 1 0	-566	-398	-208	-33	152	203	337	496
	Grand coalition							
1 1 1 1 1 1	0	0	0	0	0	0	0	0

Table 2 (part II) Egalitarian allocations

never benefit from cooperation: total gains of USA (see key 100000) and FSU (key 000010) are strictly negative at all periods. Although their abatement costs are not particularly high, they do not benefit as much as the other countries from the decrease in the temperature (see parameters b_{1i} in appendix 3). Furthermore, as indicated in Table 1, their optimal level of emissions is relatively high due to a high level of GDP for USA and due to a low energy intensity for FSU. Accordingly, these regions –especially USA– import a large amount of permits which leads to a substantial increase in their total costs.[16]

Not surprisingly, coalition $\{USA, FSU\}$ (key 100010) bears very large costs due to cooperation. Many coalitions to which USA belongs also bear large costs due to cooperation. However, CHI (key 000100), ROW (key 000001) and coalition $\{CHI, ROW\}$ (key 000101) enjoy huge benefits due to cooperation.

The *egalitarian* allocation does not lead to coalitional rationality because some

[16] The net imports of quotas are given by the difference between the optimal amount of emissions (E^*_{it}) and the allocation ($\widetilde{E}_{it}$).

countries and group of countries do lose from global cooperation. Therefore, we constrain these allocations by using the method defined above (section 4.2) in order to find, at each period, a new allocation which leads to γ-core imputations and is as close as possible to the initial one.

γ-core constrained egalitarian allocations

The constrained allocations are very different from the unconstrained ones, particularly for USA and FSU. In 2090-2100, these regions receive, respectively, almost 4 times and 2.5 times more permits than under the initial rule. For CHI and ROW, the new allocations are about 25% lower than the initial ones.

By definition, no coalition bears costs due to cooperation anymore. The imputations belong to the γ-core at each period. Moreover, the new allocations stand well on a face of the γ-core, that is, $\sum_{i \in S} W_{it}\left(\widehat{E}_{it}\right) = V_{St}$ for at least one S and for all $t \in \Theta$.

In **period 2000-2010**, gains of coalitions $\{USA\}$, $\{USA, FSU\}$, $\{USA, EU, FSU\}$, $\{USA, JPN, EU, FSU\}$ and $\{USA, JPN, EU, FSU, ROW\}$ equal zero. They are constrained. According to properties (d) and (e) (see section 4.2), we observe that

$$\frac{\widehat{E}_{USA,t}}{\lambda_{USA,t}} > \frac{\widehat{E}_{FSU,t}}{\lambda_{FSU,t}} > \frac{\widehat{E}_{EU,t}}{\lambda_{EU,t}} > \frac{\widehat{E}_{JPN,t}}{\lambda_{JPN,t}} > \frac{\widehat{E}_{ROW,t}}{\lambda_{ROW,t}} > \frac{\widehat{E}_{CHI,t}}{\lambda_{CHI,t}}$$

for the same period t (2000-2010) and where $\lambda_{it} = \frac{POP_{it}}{\sum_{j \in N} POP_{jt}}$. Indeed, $\{USA\}$ is constrained and the country receives permits accordingly. The compensation of $\{USA\}$ is however not sufficient to compensate coalition $\{USA, FSU\}$. Hence, this coalition must receive even more permits. Since USA already receives more permits per head than FSU, the amount of permits needed to compensate the coalition is given to FSU rather than to USA. By this way, FSU receives enough permits in order to compensate $\{FSU\}$ which had to bear costs due to cooperation under the unconstrained allocation. Under the coalitionally constrained allocation, $\{FSU\}$ even enjoys positive –although small– gains due to cooperation while $\frac{\widehat{E}_{USA,t}}{\lambda_{USA,t}} > \frac{\widehat{E}_{FSU,t}}{\lambda_{FSU,t}}$.

A similar reasoning applies then to coalitions $\{USA, EU, FSU\}$, $\{USA, JPN, EU, FSU\}$ and $\{USA, JPN, EU, FSU, ROW\}$.

In **period 2090-2100**, fewer coalitions bear costs due to cooperation under the *un*constrained Egalitarian allocation. Then, only coalitions $\{USA\}$ and $\{USA, FSU\}$ are constrained and

$$\frac{\widehat{E}_{USA,t}}{\lambda_{USA,t}} > \frac{\widehat{E}_{FSU,t}}{\lambda_{FSU,t}} > \frac{\widehat{E}_{EU,t}}{\lambda_{EU,t}} = \frac{\widehat{E}_{JPN,t}}{\lambda_{JPN,t}} = \frac{\widehat{E}_{ROW,t}}{\lambda_{ROW,t}} = \frac{\widehat{E}_{CHI,t}}{\lambda_{CHI,t}}$$

in 2090-2100. Hence, the initial allocation rule –the *egalitarian* rule– is kept among the compensating regions, namely EU, JPN, CHI and ROW: each of them receives 1.41 permits per head in 2090-2100, instead of 1.54 under the initial (unconstrained) allocation.

5.3 Core-stable and equitable allocations: a comparison

We analyze now the allocations of permits resulting from the use of each other equitable allocation rule mentioned above in section 4.1. As a starting point, we note that, like the *egalitarian* allocations (see above), the *ability-to-pay* and *grandfathering* ones need to be γ-core constrained while the *Shapley value* based and the *nucleolus* based allocations lead to γ-core imputations. A comparison of these allocations follows their brief description.

Table 3 shows the annual amount of permits allocated per capita under each rule at the end of this century (2090-2100). The corresponding *individual* total discounted gains are also depicted.[17]

The *ability-to-pay* rule (A.T.P.) is defined in such a way that countries with low per capita GDP are favored. As a result, a country like CHI receives large gains due to cooperation. All the other regions are constrained, either individually or within

[17] Due to limited space, total gains due to cooperation by non-singleton coalitions are not reported here. These results are available upon request.

a coalition. Indeed, gains of coalitions $\{USA\}$, $\{USA, FSU\}$, $\{USA, JPN, FSU\}$, $\{USA, JPN, EU, FSU\}$ and $\{USA, JPN, EU, FSU, ROW\}$ equal zero (not shown in Table 3). This situation is therefore very similar to the one implied by the use of the (γ-core constrained) *egalitarian* rule, except that ROW receives much more permits, to the detriment of JPN and EU.

	Egal.	A.T.P.	Grandf.	Nucleo.	Shapley
Permits per head (1 quota = 1 ton of carbon) in 2090-2100					
USA	4.34	4.34	5.32	5.12	4.98
JPN	1.41	0.68	1.99	1.43	1.26
EU	1.41	0.86	1.57	1.2	1.12
CHI	1.41	1.44	1.25	1.26	1.23
FSU	2.33	2.33	2.82	2.64	2.59
ROW	1.41	1.45	1.36	1.41	1.44
Tot. disc. gains due to coop. (in billion 1995 US$) in 2090-2100					
USA	0	0	195	150	124
JPN	62	5	110	58	47
EU	155	13	197	96	77
CHI	343	382	181	191	162
FSU	2	2	122	73	64
ROW	236	395	0	231	323

Table 3. γ-core constrained allocations in 2090-2100 - Various rules

The *grandfathering* rule (Grandf.) draws a very different picture since constrained coalitions include $\{ROW\}$ and $\{CHI, ROW\}$. These regions receive much fewer permits than under the two other rules. Indeed, compared to the other regions, they are penalized by their lower level of historical emissions while their reference emissions are growing at a higher rate.

Before turning to the analysis of the rules based on the two game-theoretic solution concepts, we make a comparison with the results of Germain and van Steenberghe (2003) who perform the same kind of analysis but limit the participation constraint to individual rationality. Does the introduction of γ-core (coalitional rationality) constraints, instead of individual rationality constraints, change significantly the results? The answer depends on the rule under consideration. For instance, the amount of permits allocated to each region under the *egalitarian* rule are almost the same under

γ-core constraints than under individual rationality constraints. However, a significant difference is observed under the *ability-to-pay rule* (up to 28% for CHI in 2090-2100), as well as under the *grandfathering* one. Hence, the extension of the participation constraints from individual to coalitional (γ-core) rationality may significantly narrow the set of allocations leading to a global agreement.

The *nucleolus* based allocations lead to a high level of stability because the gain of the coalition with lowest gain is maximized. Since total gains are strictly positive, every coalition necessarily receives a positive gain and, consequently, the imputation necessarily belongs to the γ-core (since the γ-core exists at all periods). In the present model, the lowest gains are those of $\{JPN\}$ and its complementary coalition, that is all regions excluding JPN (not shown here). Both coalitions enjoy annual gains amounting to 58 billions US$. Note that giving more permits to JPN would increase its gains, but decrease those of the other countries in the complementary coalition. The lowest gains (those of the complementary coalition) would then not be maximized. Hence, the lowest gains need to be equal. The second lowest gains are those of $\{FSU\}$ and its complementary coalition, that is all regions excluding FSU (73 billions US$). And so on with the other countries and coalitions. Hence, the *nucleolus* imputations lie 'in the middle' of the γ-core at each period.

The *Shapley value* based allocations are such that countries are rewarded according to their average contribution to every possible coalition. In the present context, since the contribution of a country to a coalition is *a priori* an increase in costs, countries receive larger gains (and more permits) when they increase by a little amount the costs of the other members of the coalition that it joins. This increase may take place through two channels. First, each coalition adopts an optimal policy for its members. The optimal policy of the new coalition therefore changes. The change will depend on the level of the marginal abatement costs and marginal damage costs of the newcoming country relative to those of the other countries in the coalition. The relatively large gains enjoyed by CHI and ROW suggest that countries with low marginal abatement and damage costs tend to receive a large part of the gains. Second, the size of the countries, in terms of absolute abatement and damage costs, also matter since the costs are expressed in absolute value. This explains the large gains enjoyed by a country like USA.

A comparison of the amount of permits allocated and the gains obtained under the two game-theoretic rules shows that they are not very different form each other. In 2090-2100, the largest difference in the allocation of permits is observed for JPN, with 13% more permits under the *nucleolus* based allocation than under the Shapley based one.

A similar comparison among all rules shows that *the allocations based on equity principles allow for a relatively large flexibility in the selection of allocations leading to γ-core imputations*. For instance, USA receives 23% more permits under the *grandfathering* rule than under the *egalitarian* or the *ability-to-pay* one. This ratio increases up to 193% for JPN when the *grandfathering* rule is compared to the *ability-to-pay* one (83% for EU, -13% for CHI, 21% for FSU and -6% for ROW).

Moreover, the *nucleolus* based permits allocations (and thus also the gains), leading to imputations lying 'in the middle' of the γ-core, are between those resulting from the use of the *egalitarian* and the *ability-to-pay* rules. Indeed, the γ-core constrained allocations based on equity principles lead to imputations which are necessarily on the faces of the γ-core and the rules based on the equity principles that are considered here lead to imputations located on different faces of the γ-core.

6 Conclusion

Most so-called 'equitable' rules to allocate greenhouse gas emission permits among countries under a global agreement are likely not to be accepted by some nation or group of nations because they run too strongly against their self interest. Consequently, we have developed a way to compute allocations of permits which satisfy a participation constraint while keeping as much as possible any 'equitable' allocation rule into account. The members of any coalition of countries for which an 'equitable' allocation is not acceptable (their participation constraint is violated) receive more permits in order to compensate the coalition so as to make it indifferent for them to sign or not to sign the global agreement. Countries belonging to unconstrained coalitions (coalitions whose participation constraint is not violated) receive then fewer permits than the others. Since any given country may belong at the same time to a constrained coalition and to an unconstrained one, the way to modify the gains of such a country is not straightforward: this modification affects the gains –and therefore the incentives to participate in the agreement– of both coalitions. Our method deals with that problem. Moreover, the rule based on an equity principle is preserved as much as possible among the members of both constrained and unconstrained coalitions.

Simulations with the main permits allocation rules based on equity principles as well as on game-theoretic solution concepts, the *nucleolus* and the *Shapley value*, have been performed. These simulations highlight two results of particular interest. First, introducing γ-core constraints instead of individual rationality constraints only may significantly affect the amount of permits to be allocated under a constrained allocation rule. However, the magnitude of this effect differs very much from rule to rule. Second, the degree of freedom for allocating emission permits while satisfying at each period γ-core participation constraints is still significantly large. For instance, under a global agreement, the United States of America receive 23% more permits with the constrained *grandfathering* rule than with the *ability-to-pay* one at the end of this century. This figure goes up to 193% for Japan, 83% for the European Union, 21% for former Soviet Union and -13% for China.

The simplicity of the model used here is justified by the simultaneous introduction of three important features: (i) an international market for emission permits, (ii) core constraints and especially (iii) a dynamic negotiations framework. However, the usual shortcomings of economic-climatic models are present. Of particular concern is the evaluation of the damages caused in the different regions by a change in the atmospheric temperature and of the costs of these damages. This is certainly a crucial issue on which further research is needed.

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8 Appendix

8.1 Proof of claims 1 and 2

Let $\widetilde{E}_t = \left\{\widetilde{E}_{1,t}, ..., \widetilde{E}_{n,t}\right\}$ be a vector of tradable permits satisfying (10), i.e. $\sum_i \widetilde{E}_{i,t} = \sum_i E^*_{i,t}$, where $E^*_{i,t}$ is the optimal level of emissions for country i at period t and is given by (6) (section 3.2). Recall that $E^*_{i,t}$ satisfies (9), that is

$$-C'_{i,t}\left(E^*_{i,t}\right) = \Delta_t, \forall i \tag{28}$$

where $\Delta_t = \alpha\beta \sum_{j=1}^{n} \sum_{u=t}^{T} \alpha^{u-t} [1-\delta]^{s-t} D'_{j,u+1}\left(M^*_{u+1}\right)$.

Under competitive market conditions, given the allocation $\widetilde{E}_t$, each country minimizes its abatement costs and chooses its levels of emissions E_{it} and net sales of permits X_{it} accordingly. It solves

$$\min_{\{E_{it}, X_{it}\}} C_{it}\left(E_{it}\right) + \sigma_t X_{it} \text{ subject to } X_{it} \geq E_{it} - \widetilde{E}_{it} \tag{29}$$

where σ_t is the competitive market price of the permits. The FOC for an interior solution is

$$-C'_{i,t}\left(E_{i,t}\right) = \sigma_t \tag{30}$$

which, since $C'_{i,t}$ is strictly increasing, leads to a level of emissions $E_{i,t} = C'^{-1}_{i,t}\left(-\sigma_t\right)$.

The aggregate excess demand for permits is thus $\sum_i \left[C'^{-1}_{i,t}\left(-\sigma_t\right) - \widetilde{E}_{it}\right]$. The equilibrium market clearing price, σ^{CE}_t, satisfies thus

$$\sum_i C'^{-1}_{i,t}\left(-\sigma^{CE}_t\right) = \sum_i \widetilde{E}_{it}. \tag{31}$$

By $\sum_i \widetilde{E}_{i,t} = \sum_i E^*_{i,t}$, we have $\sum_i C'^{-1}_{i,t}\left(-\sigma^{CE}_t\right) = \sum_i E^*_{i,t}$ and, by (28), $E^*_{i,t} = C'^{-1}_{i,t}\left(-\Delta_t\right)$. Therefore, taking into account that $C'^{-1}_{i,t}$ is strictly increasing $\forall i$,

$$\sum_i C'^{-1}_{i,t}\left(-\sigma^{CE}_t\right) = \sum_i C'^{-1}_{i,t}\left(-\Delta_t\right) \tag{32}$$

and thus

$$\sigma_t^{CE} = \Delta_t. \tag{33}$$

By (28), we then have $-C'_{i,t}\left(E^*_{i,t}\right) = \sigma_t^{CE}$ and, by (30), $-C'_{i,t}\left(E^{CE}_{i,t}\right) = \sigma_t^{CE}$. Therefore $E^{CE}_{i,t} = E^*_{i,t}\ \forall i$. Furthermore, this reasoning is independent from $\widetilde{E}_t$ as long as it satisfies (10).

Moreover, (31) shows that the equilibrium price depends on the aggregate amount of permits allocated in a given period. ■

8.2 Detailed description of the two game theoretic solution concepts

nucleolus - In order to define the *nucleolus*[18], consider $e_t(s, \bar{W}_t) = \sum_{i \in s} \bar{W}_{it} - V_{s,t}$ as the excess of a coalition s at time t. Define a vector $\text{-}_t(\bar{W}_t) = (e_t(s_1, \bar{W}_t), e_t(s_2, \bar{W}_t), ...,$ $e_t(s_{2^n}, \bar{W}_t))$ where the various excesses of all coalitions are arranged in decreasing order. Then, $\text{-}_t(\bar{W}_t)$ is said to be lexicographically smaller than $\text{-}_t(\ddot{W}_t)$, i.e. $\text{-}_t(\bar{W}_t) \preceq \text{-}_t(\ddot{W}_t)$, if there exists a positive integer q such that $\text{-}_{jt}(\bar{W}_t) = \text{-}_{jt}(\ddot{W}_t)$ whenever $j < q$ and $\text{-}_{qt}(\bar{W}_t) < \text{-}_{qt}(\ddot{W}_t)$. Let Ψ_t be the set of imputations at period t, i.e. $\Psi_t = \{x \in R^n | \sum_{i=1}^n x_i = V_{s,t},\ s = N\}$. Then, the *nucleolus* in period t, NU_t, is defined as follows:

$$NU_t(\Psi_t) = \left\{\bar{W}_t \in \Psi_t : \text{-}_t(\bar{W}_t) \preceq \text{-}_t(\ddot{W}_t), \forall \ddot{W}_t \in \Psi_t\right\}.$$

Shapley value - The *Shapley value* is easier to define. The total costs (imputation) of a country i at time t are given by:

$$SH_{it} = \sum_{s | i \notin s} \frac{|s|!(n - |s| - 1)!}{n!} \left[V_{s \cup \{i\},t} - V_{s,t}\right].$$

[18] This definition is borrowed from Maschler (1992).

8.3 Data

M_0 (preindustrial level of the CO_2 atmospheric stock) : 590 billion tons of carbon equivalent

δ (rate of decay of CO_2 in the atmosphere): 0.0833 per decade

β (marginal atmospheric retention ratio of CO_2) : 0.64

η (parameter) : 2.5/ln(2)

Output and emissions growth rates are taken from a model developed at CORE by Germain, Tulkens, Tulkens and van Ypersele (2002) and which is based on the RICE'98 model (Nordhaus and Boyer, 1999). Population, output, energy intensity and parameters characterizing damage and abatement cost functions (4) and (5) are given below ([19]).

Population	USA	JPN	EU	CHI	FSU	ROW	World
1990-2000	0.250	0.124	0.367	1.134	0.289	3.103	5.266
2000-2010	0.264	0.124	0.382	1.255	0.307	3.685	6.019
2010-2020	0.274	0.125	0.394	1.354	0.321	4.227	6.694
2020-2030	0.281	0.125	0.403	1.432	0.333	4.714	7.286
2030-2040	0.285	0.125	0.410	1.492	0.341	5.141	7.795
2040-2050	0.288	0.125	0.415	1.539	0.348	5.510	8.225
2050-2060	0.291	0.125	0.419	1.574	0.354	5.821	8.583
2060-2070	0.292	0.125	0.422	1.601	0.358	6.082	8.879
2070-2080	0.293	0.125	0.424	1.621	0.361	6.298	9.121
2080-2090	0.294	0.125	0.425	1.636	0.363	6.475	9.318
2090-2100	0.294	0.125	0.426	1.647	0.365	6.620	9.477
2100-2110	0.294	0.125	0.427	1.656	0.366	6.738	9.606
Total	3.400	1.493	4.914	17.941	4.105	64.414	

GDP (/1000)	USA	JPN	EU	CHI	FSU	ROW	World
1990-2000	63.11	33.11	77.11	4.78	9.88	56.83	244.82
2000-2010	72.92	36.78	83.35	6.82	11.81	77.80	289.48
2010-2020	83.42	40.83	90.04	9.51	14.13	103.90	341.83
2020-2030	94.58	45.26	97.11	12.98	16.84	135.48	402.25
2030-2040	106.33	50.05	104.53	17.34	19.95	172.80	471.00
2040-2050	118.61	55.22	112.28	22.70	23.46	215.95	548.22
2050-2060	131.41	60.78	120.34	29.16	27.40	264.90	633.99
2060-2070	144.68	66.74	128.70	36.80	31.76	319.52	728.20
2070-2080	158.39	73.09	137.31	45.70	36.55	379.55	830.59
2080-2090	172.51	79.87	146.17	55.90	41.77	444.69	940.91
2090-2100	187.02	87.05	155.24	67.42	47.42	514.57	1058.72
2100-2110	201.89	94.67	164.50	80.26	53.47	588.76	1183.55
Total	1534.870	723.450	1416.680	389.370	334.440	3274.750	

[19] For more details, see Nordhaus and Yang (1996), pp. 744-745. Population is in billion and GDP is in billion US$ of 1990 per decade.

Energy intensity (*1000)

	USA	JPN	EU	CHI	FSU	ROW
1990-2000	0.231	0.092	0.118	2.069	1.199	0.693
2000-2010	0.209	0.081	0.105	1.573	0.944	0.632
2010-2020	0.190	0.072	0.094	1.239	0.760	0.578
2020-2030	0.174	0.064	0.085	1.006	0.624	0.531
2030-2040	0.160	0.058	0.077	0.838	0.522	0.490
2040-2050	0.148	0.053	0.070	0.714	0.444	0.454
2050-2060	0.137	0.048	0.064	0.620	0.384	0.422
2060-2070	0.128	0.044	0.059	0.547	0.336	0.395
2070-2080	0.120	0.040	0.055	0.490	0.298	0.370
2080-2090	0.112	0.037	0.051	0.443	0.267	0.348
2090-2100	0.106	0.035	0.048	0.405	0.241	0.329
2100-2110	0.100	0.032	0.045	0.374	0.220	0.312

Others

	USA	JPN	EU	CHI	FSU	ROW
a_{i1}	0.07	0.05	0.05	0.15	0.15	0.1
a_{i2}	2.2887	2.2887	2.2887	2.2887	2.2887	2.2887
b_{i1}	0.01102	0.01174	0.01174	0.015523	0.00857	0.02093
b_{i2}	1.5	1.5	1.5	1.5	1.5	1.5

8.4 Illustration of the method to compute γ-core-stable and equitable allocations (Section 4.2)

For the purpose of illustration, let us assume that there are only three countries named A, B and C.[20] The set of coalitions that we consider is therefore $\{\{A\},\{B\},\{C\},\{A,B\},\{A,C\},\{B,C\},\{A,B,C\}\}$, $\{A,B,C\}$ being the grand coalition.

Note that if no coalition is constrained, it is straightforward to show that the new allocation is the same as the initial one, i.e. $\widehat{E}_{it} = \widetilde{E}_{it}$, $i = A, B, C$. Consider the four following examples.

EXAMPLE I. (Only one singleton complains)

If only coalition $\{A\}$ is constrained under the initial allocation, i.e. $W_{At} > V_{\{A\}t}$, $W_{Bt} < V_{\{B\}t}$ and $W_{Ct} < V_{\{C\}t}$, then problem (17) subject to (18)-(20) leads to $\frac{\widehat{E}_{Bt}}{\lambda_{Bt}} = \frac{\widehat{E}_{Ct}}{\lambda_{Ct}} < \frac{\widetilde{E}_{At}}{\lambda_{At}} = \frac{\widetilde{E}_{Bt}}{\lambda_{Bt}} = \frac{\widetilde{E}_{Ct}}{\lambda_{Ct}} < \frac{\widehat{E}_{At}}{\lambda_{At}}$ (see properties (a) and (d)). This shows that the compensating regions, namely regions B and C, receive fewer permits than under the

[20] As mentionned in Section 4.2, the method applies to any number of countries.

initial rule because they must compensate region A in order to induce it to cooperate. Furthermore, *the allocation rule is preserved among these compensating regions.*

An illustration of this example –which will prove to be very useful for the discussion of the other, more complex, examples– is provided in figures 1a and 1b. The figures represent the hyperplane corresponding to the set of total discounted gains due to co-operation, $\sum_{S\in\{\{A\},\{B\},\{C\}\}} V_{St} - \sum_i \bar{W}_{it}\left(\bar{E}_{it}\right)$, for any allocation of permits satisfying relation (8), i.e. $\sum_i \bar{E}_{it} = \sum_i E^*_{it}$. Hence, it is the set of imputations which is rescaled so as to represent the individual gains due to cooperation.

Accordingly, the empty triangle is formed by the three individual rationality (IR) constraints: any point on the AB line corresponds to no gains for country C ($\bar{W}_{Ct} = V_{\{C\}t}$) and similarly for lines AC (no gains for B) and BC (no gains for A).

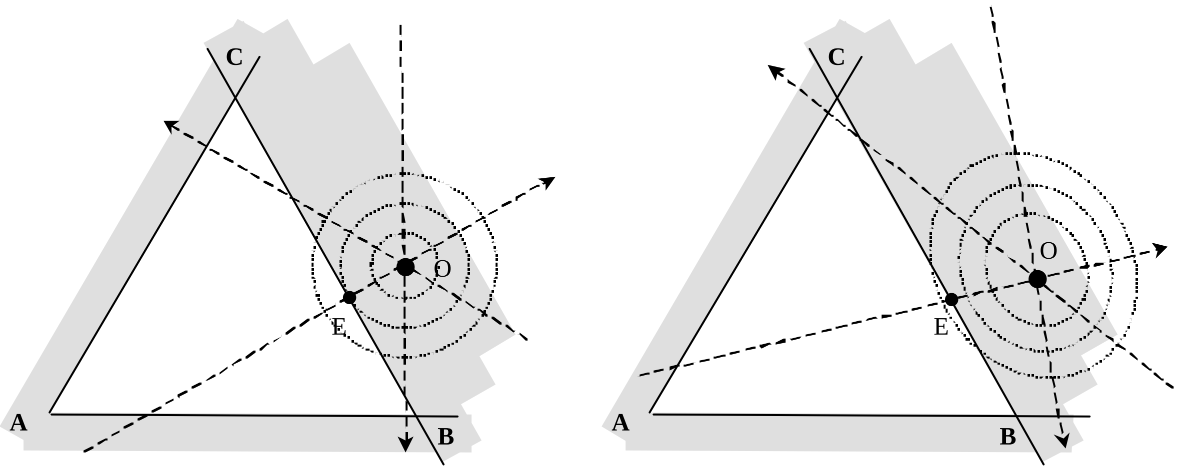

Fig. 1a and 1b. Illustration of example I

Now, in this example, the initial allocation of permits $\widetilde{E}_t$ is such that only one constraint is not satisfied: $W_{At} > V_{\{A\}t}$. In figure 1, this allocation of permits corresponds to a point which lies outside the triangle and which we call O. From this point, where we know that by definition $\frac{\widetilde{E}_{it}}{\lambda_{it}} = \frac{\widetilde{E}_{jt}}{\lambda_{jt}}$, $\forall i \neq j$, three dotted lines have been drawn. The dotted arrow starting from bottom-left (close to point A) represents the set of individual gains due to cooperation such that the initial allocation rule of permits is preserved among countries B and C ($\frac{\widehat{E}_{Bt}}{\lambda_{Bt}} = \frac{\widehat{E}_{Ct}}{\lambda_{Ct}}$). Any point on this arrow and to the left (respectively, to the right) of O corresponds to an allocation of permits such that, compared to the initial allocation, country A receives more (fewer) permits: $\frac{\widehat{E}_{At}}{\lambda_{At}} > \frac{\widehat{E}_{Bt}}{\lambda_{Bt}} = \frac{\widehat{E}_{Ct}}{\lambda_{Ct}}$ ($\frac{\widehat{E}_{At}}{\lambda_{At}} < \frac{\widehat{E}_{Bt}}{\lambda_{Bt}} = \frac{\widehat{E}_{Ct}}{\lambda_{Ct}}$). Similarly, the points on the top-down arrow correspond to allocations of permits such that $\frac{\widehat{E}_{At}}{\lambda_{At}} = \frac{\widehat{E}_{Bt}}{\lambda_{Bt}} \gtreqless \frac{\widehat{E}_{Ct}}{\lambda_{Ct}}$, and those on the bottom-right to top-left arrow correspond to allocations such that $\frac{\widehat{E}_{At}}{\lambda_{At}} = \frac{\widehat{E}_{Ct}}{\lambda_{Ct}} \gtreqless \frac{\widehat{E}_{Bt}}{\lambda_{Bt}}$.

Figure 1.a illustrates the situation in which $\lambda_{At} = \lambda_{Bt} = \lambda_{Ct}$. The dotted lines are then orthogonal to the IR constraints and the objective function is a sphere (a circle on the plane). However, in the more general case of $\lambda_{At} \neq \lambda_{Bt} \neq \lambda_{Ct}$, the dotted lines are not orthogonal to the IR constraints anymore and the objective function becomes an ellipsoid. This is illustrated in figure 1.b. Note that it can then be proven that the objective function is tangent to each IR constraint at the intersection of the IR constraint and the corresponding dotted arrow, e.g. the objective function is tangent to BC at point E.

Now, point E corresponds to the constrained allocation of permits as provided by the resolution of problem (17). In figures 1.a and 1.b, only country A needs to be compensated and countries B and C participate to this compensation in such a way that *the initial allocation rule is kept among them*, i.e. E is on the left-right dotted arrow.

EXAMPLE II. (One coalition complains)

If only coalition $\{A, B\}$ is constrained under the initial allocation, i.e. $W_{At} + W_{Bt} > V_{\{A,B\}t}$ and $\sum_{i \in S} W_{it} < V_{St}$ $\forall S \neq \{A, B\}$, then problem (17) subject to (18)-(20) leads to $\frac{\widehat{E}_{Ct}}{\lambda_{Ct}} < \frac{\widetilde{E}_{At}}{\lambda_{At}} = \frac{\widetilde{E}_{Bt}}{\lambda_{Bt}} = \frac{\widetilde{E}_{Ct}}{\lambda_{Ct}} < \frac{\widehat{E}_{At}}{\lambda_{At}} = \frac{\widehat{E}_{Bt}}{\lambda_{Bt}}$ (see properties (b) and (d)). Hence, country C receives fewer permits than the others in order to compensate coalition $\{A, B\}$ and, *within this coalition, each member receives permits according to the initial allocation rule*.

This example is illustrated in figure 2.a. The new horizontal line depicts the difference between (i) the sum of the total costs for A and B at the Nash equilibrium (any PANE w.r.t. a singleton) (e.g. $\sum_{S\in\{\{A\},\{B\}\},t} V_{S,t}$) and (ii) the total costs for coalition $\{A,B\}$ at the PANE w.r.t. $\{A,B\}$ ($V_{\{A,B\},t}$). Consequently, the set of coalitionally rational gains due to cooperation for each region lie therefore in the empty trapeze. Since the objective function is tangent to the horizontal line along the OE arrow, point E is the solution of problem (17) subject to (18)-(20).

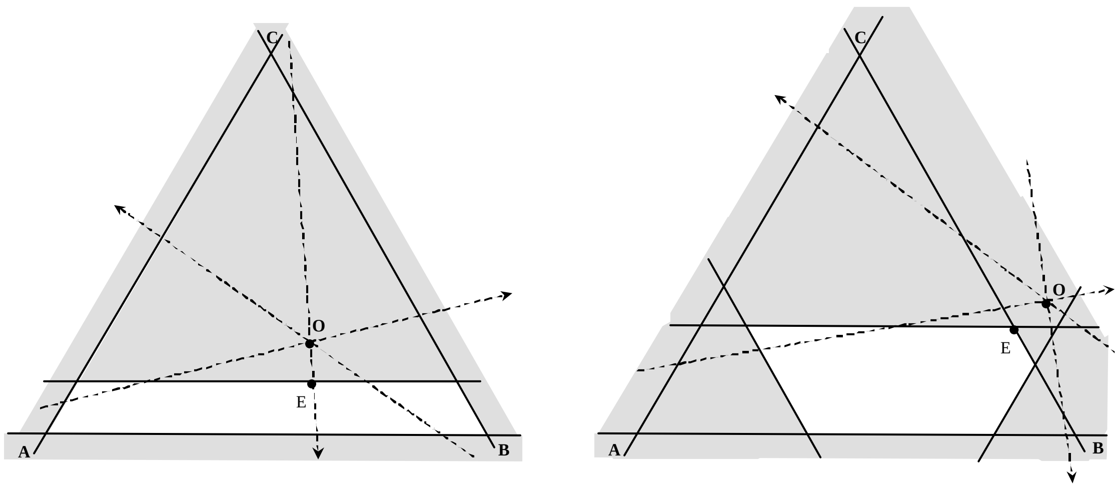

Fig. 2.a and 2.b. Illustration of examples II and III

EXAMPLE III. (One coalition and a subcoalition (singleton) belonging to that coalition complain)

If only coalition $\{A,B\}$ and singleton $\{A\}$ are constrained under the initial allocation, i.e. $W_{At}+W_{Bt}>V_{\{A,B\}t}$, $W_{At}>V_{\{A\}t}$ and $\sum_{i\in S} W_{it}<V_{St}$ $\forall S\neq\{A,B\},\{A\}$, then problem (17) subject to (18)-(20) leads to $\frac{\widehat{E}_{Ct}}{\lambda_{Ct}}\leq\frac{\widehat{E}_{Bt}}{\lambda_{Bt}}\leq\frac{\widehat{E}_{At}}{\lambda_{At}}$. Let's illustrate the reasoning. Country C has to receive necessarily fewer permits than under the initial allocation in order to compensate $\{A\}$ and $\{A,B\}$. Indeed, the payoff (gains due to cooperation) of $\{A\}$ has to be increased so as to give it an appropriate amount of permits

$\widehat{E}_{At}$ (with $\widehat{E}_{At} > \widetilde{E}_{At}$) in order to have $V_{\{A\}t} = W_{At}\left(\widehat{E}_{At}\right)$ ($\{A\}$ is just compensated). At this stage, $\frac{\widehat{E}_{Ct}}{\lambda_{Ct}} < \frac{\widetilde{E}_{Bt}}{\lambda_{Bt}} < \frac{\widehat{E}_{At}}{\lambda_{At}}$. By the same token, the payoff of $\{A, B\}$ has also been increased since A belongs to that coalition. Then, two situations emerge.

(i) if coalition $\{A, B\}$ is still constrained, the payoffs of A and B have to be increased. This increase will take place via an increase of country B's payoff as long as $\frac{\widehat{E}_{Bt}}{\lambda_{Bt}} < \frac{\widehat{E}_{At}}{\lambda_{At}}$ (i.e. as long as the payoffs stand to the left of the top-down dotted arrow which corresponds to $\frac{\widehat{E}_{Bt}}{\lambda_{Bt}} = \frac{\widehat{E}_{At}}{\lambda_{At}}$). This situation is illustrated in figure 2.b. We observe that both coalition $\{A, B\}$ and $\{A\}$ are constrained with $\frac{\widehat{E}_{Ct}}{\lambda_{Ct}} < \frac{\widehat{E}_{Bt}}{\lambda_{Bt}} < \frac{\widehat{E}_{At}}{\lambda_{At}}$ (see properties (d) and (e) together). However, if the payoff of B has increased in such a way that $\frac{\widehat{E}_{Bt}}{\lambda_{Bt}} = \frac{\widehat{E}_{At}}{\lambda_{At}}$ while coalition $\{A, B\}$ is not yet constrained, then the payoffs of B and A are increased according to the initial allocation rule (i.e. along the top-down dotted arrow). Then coalition $\{A, B\}$ is constrained but A is not, with $\frac{\widehat{E}_{Ct}}{\lambda_{Ct}} < \frac{\widehat{E}_{Bt}}{\lambda_{Bt}} = \frac{\widehat{E}_{At}}{\lambda_{At}}$ (see properties (b) and (d)).

(ii) if coalition $\{A, B\}$ is not constrained anymore after the compensation of $\{A\}$, then country B's payoff may be decreased as long as $\frac{\widehat{E}_{Bt}}{\lambda_{Bt}} > \frac{\widehat{E}_{Ct}}{\lambda_{Ct}}$. Hence, country B helps country C to compensate $\{A\}$ as long as $\{A, B\}$ is not constrained. Exactly as C, B will then receive fewer permits than under the initial allocation. We then end up with $\frac{\widehat{E}_{Ct}}{\lambda_{Ct}} \leq \frac{\widehat{E}_{Bt}}{\lambda_{Bt}} < \frac{\widehat{E}_{At}}{\lambda_{At}}$ with $\{A\}$ constrained and coalition $\{A, B\}$ not necessarily constrained (see property (d)).

EXAMPLE IV. (Two coalitions with a non-empty intersection complain)

For this example, consider a fourth country, denoted by D. If only coalitions $\{A, B\}$ and $\{A, C\}$ are constrained under the initial allocation, i.e. $W_{At} + W_{Bt} > V_{\{A,B\}t}$, $W_{At} + W_{Ct} > V_{\{A,C\}t}$ and $\sum_{i \in s} W_{it} < V_{St}\ \forall S \neq \{A, B\}, \{A, C\}$, then problem (17) subject to (18)-(20) leads to $\frac{\widehat{E}_{Bt}}{\lambda_{Bt}} < \frac{\widehat{E}_{At}}{\lambda_{At}}$ and $\frac{\widehat{E}_{Ct}}{\lambda_{Ct}} < \frac{\widehat{E}_{At}}{\lambda_{At}}$ with $\frac{\widehat{E}_{Bt}}{\lambda_{Bt}} \gtreqless \frac{\widehat{E}_{Ct}}{\lambda_{Ct}}$ (see properties (c) and (e)). $\square$