Some Consequences of the Unknottedness of the Walras Correspondence

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Abstract

Two basic properties concerning the dynamic behavior of competitive equilibria of exchange economies with complete markets are derived essentially from the fact that the Walras correspondence has no knots.

I. One can view the work of Debreu, E. and H. Dierker, Balasko and others that introduced the notions of regular economy and regular equilibrium, and studied the local uniqueness and other more global properties of the Walras correspondence, as providing a formal mathematical framework for studying comparative statics properties of competitive equilibria. At the same time, P.A. Samuelson has pointed out repeatedly (see for instance Samuelson (1983)) that, in order to derive meaningful statements concerning the comparative statics properties of a given equilibrium, one also needs to take its stability properties into account. Samuelson also believed that, in many cases, perverse comparative statics properties are associated with unstable equilibria, and termed such a relationship, roughly, the correspondence principle. However, a basic problem with the correspondence principle and more importantly with the capability of doing meaningful comparative statics in Samuelson's sense, is that it is not clear what dynamics one should use when evaluating the stability properties of a given competitive equilibrium. In this paper, we show that important information concerning the stability properties of competitive equilibria can be obtained directly from

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the graph of the Walras correspondence and therefore independently of the particular dynamics chosen.

More precisely, we show that the graph of the Walras correspondence, embedded in the product space of prices and endowments, is topologically like the graph of a constant function of endowments, embedded in the same product space. This extends Balasko's (1975) celebrated structure theorem and in particular shows that the graph of the Walras correspondence is unknotted. This additional feature is then used to derive two fundamental consequences concerning possible dynamic behavior of competitive equilibria.

The first consequence shows that any two dynamics in the class of all price dynamics, whose zeros coincide with the competitive equilibria of underlying economies, are homotopic within the class, provided one takes the homotopy to be defined on the whole product space of prices and endowments. This gives a bound on how different any two dynamics can be from each other and, in particular, implies that all dynamics are homotopic to the tâtonnement dynamics. This provides in some sense a further, topological justification for the widespread use of the tâtonnement dynamics as a benchmark dynamics in much of the mathematical economics literature.

The second consequence shows that the indices of zeros of all dynamics from the mentioned class are completely determined and actually coincide with the local degrees of corresponding points on the graph of the Walras correspondence. In many cases, this allows to infer the instability of equilibria for arbitrary dynamics within the class, based only on certain geometric properties of the Walras correspondence, namely the local degrees of the equilibria, which are independent of the particular dynamics chosen. This establishes a basic link between comparative statics properties of competitive equilibria, which are embodied by the Walras correspondence, and dynamic properties of the equilibria under any of the dynamics from the above mentioned class.

The paper is organized as follows. Section II introduces basic notation, Section III contains the unknottedness result, and Section IV some consequences.

II. We consider exchange economies with n agents and l commodities, with endowments $\omega \in \Omega = I\!R^{nl}$ and preferences $\preceq \in \mathcal{P}^n$ where:

$$\mathcal{P} = \left\{ \preceq \subset I\!R^l \times I\!R^l \;\middle|\; \begin{array}{l} \preceq \textit{ smooth, strictly monotonic, strictly convex} \\ \textit{and indifference surfaces are bounded below} \end{array} \right\}.$$

Prices are taken from the positive part of the $(l-1)$ unit sphere:

$$S = \{p \in I\!R^l_{++} |\ \|p\| = 1\}.$$

Let $\preceq \in \mathcal{P}^n$ be a fixed preference profile, and let $f_i : S \times I\!R \to I\!R^l$, denote agent i's demand function. A Walrasian equilibrium for the exchange economy $\omega \in \Omega$ is a vector of prices $p \in S$ such that $\sum_{i=1}^n (f_i(p, p\omega^i) - \omega^i) = 0$, it is regular if the Jacobian matrix $D_p(\sum_{i=1}^n (f_i(p, p\omega^i) - \omega^i))$ has rank $l-1$. The Walras correspondence maps to each economy $\omega \in \Omega$ the set of Walrasian equilibria of ω. Denote by $\eta \subset S \times \Omega$ the graph of the Walras correspondence.

III. Balasko has shown that the graph of the Walras correspondence η has the global topological structure of the underlying space of economies Ω, i.e., η is diffeomorphic to the Euclidean space $I\!R^{nl}$, (see for example Balasko (1988), Theorem 3.2.2, p. 73). This leaves open the possibility that η may be knotted. The following theorem, which is central to our paper, extends Balasko's result by showing that this cannot happen. More precisely, we show that the pair $(S \times \Omega, \eta)$ is diffeomorphic to the pair $(S \times \Omega, \{p_0\} \times \Omega)$, by which we mean that there exists a diffeomorphism:

$$\Phi : (S \times \Omega, \eta) \xrightarrow{\approx} (S \times \Omega, \{p_0\} \times \Omega),$$

that maps $S \times \Omega$ diffeomorphically onto itself and maps the graph η diffeomorphically onto the trivial copy of the space of economies $\{p_0\} \times \Omega$. Furthermore, we show that the diffeomorphism can be connected to the identity on $(S \times \Omega, \eta)$ by a continuous family of diffeomorphisms, i.e., we show that the pairs $(S \times \Omega, \eta)$ and $(S \times \Omega, \{p_0\} \times \Omega)$ are also isotopic. This implies that topologically η, embedded in $S \times \Omega$, behaves like the graph of a constant function on Ω, embedded in $S \times \Omega$, i.e., as if equilibria were everywhere unique, and immediately implies that η is unknotted. It is this additional structure that allows us to derive Propositions 1 and 2 that follow below.

Unknottedness Theorem *There exists a diffeomorphism* $\Phi : (S \times \Omega, \eta) \xrightarrow{\approx} (S \times \Omega, \{p_0\} \times \Omega)$, *which takes* $S \times \Omega$ *diffeomorphically onto itself and takes the graph of the Walras correspondence* $\eta \subset S \times \Omega$ *diffeomorphically onto a trivial copy of the space of economies* $\{p_0\} \times \Omega \subset S \times \Omega$. *Moreover, the pairs* $(S \times \Omega, \eta)$ *and* $(S \times \Omega, \{p_0\} \times \Omega)$ *are isotopic.*[1]

[1] That is, there exists a family of diffeomorphisms $\Phi_t : S \times \Omega \xrightarrow{\approx} S \times \Omega$, continuous in $t \in [0, 1]$, such that Φ_0 is the identity on $S \times \Omega$, $\Phi_1 = \Phi$, and each Φ_t takes η diffeomorphically onto $\Phi_t(\eta)$, $t \in [0, 1]$.

Proof. The proof is in two steps. In the first step, we introduce the (equilibrium) graphs η_F and η_G of respectively the maps F and G, which are chosen such that η_F can be associated to η, and η_G can be associated to $\{p_0\} \times \Omega$. In the second step, using a vector field that essentially pushes η_F towards η_G, we show that η_F and η_G are isotopic within an appropriate ambient space, which with the first step readily implies that the pairs $(S \times \Omega, \eta)$ and $(S \times \Omega, \{p_0\} \times \Omega)$ are diffeomorphic and isotopic.

Step 1. Let $E_{-n} = S \times \Omega_{-n} \times I\!R$, $\partial E_{-n} = \partial S \times \Omega_{-n} \times I\!R$, where $\Omega_{-n} = I\!R^{(n-1)l}$ denotes the space of endowments of the first $n-1$ agents, $I\!R$ represents the space of the n-th agent's income, $\partial S = \{p \in S |\ p_k = 0,\ \text{some } k = 1, .., l\}$. Let:

$$
\begin{aligned}
F : E_{-n} \to \Omega_n \quad &, \quad (p, \omega^{-n}, y^n) \mapsto \sum_{i=1}^{n-1} (f_i(p, p\omega^i) - \omega^i) + f_n(p, y^n), \\
G : E_{-n} \to \Omega_n \quad &, \quad (p, \omega^{-n}, y^n) \mapsto (f_n(p, y^n)),
\end{aligned}
$$

where $f_i : S \times I\!R \to I\!R^l$ denotes agent i's demand function and $\Omega_n = I\!R^l$ is the space of endowments of the n-th agent. Let $E = E_{-n} \times \Omega_n$, $\partial E = \partial E_{-n} \times \Omega_n$, $\bar{E} = E \cup \partial E$, and let $\eta_F, \eta_G \subset E$ denote the mentioned graphs of respectively F and G. Given that for any $\omega \in \Omega$ the map f_n defines a diffeomorphism between $S \times I\!R$ and Ω_n, (see Balasko (1988), Theorem 2.4.1, p. 51), there exists a unique (equilibrium) point on η_G that moreover satisfies $y^n = p\omega^n$, and hence it is easy to see that a diffeomorphism exists between the pairs (E^*, η_G) and $(S \times \Omega, \{p_0\} \times \Omega)$, where:

$$
E^* = \{(p, \omega^{-n}, y^n, \omega^n) \in E |\ \ y^n = p\omega^n\} \subset E.
$$

(Notice that since $y^n = p\omega^n$ is satisfied everywhere on η_F, η_G, we have $\eta_F, \eta_G \subset E^*$.) Since we can always identify the space $S \times \Omega$ with the space E^*, and the graph of the Walras correspondence η with $\eta_F \subset E^*$, (this is similar to Schecter (1979), p. 2, or Balasko (1988), p. 73-74), it suffices to find a diffeomorphism $\Psi : (E^*, \eta_F) \overset{\approx}{\to} (E^*, \eta_G)$, that is connected to the identity via a continuous family of diffeomorphisms.

Step 2. Such a diffeomorphism Ψ can be seen to follow from the following two properties of E, η_F and η_G: (i) for any $x \in E$, there exists a unique $x' \in \eta_F$ and a unique $z' \in \eta_G$ such that x, x' and z' all coincide on the entries of E_{-n}; (in the figure, this means that any "horizontal" ray through x intersects η_F as well as η_G in exactly one point, which we label respectively x' and z'), and (ii) as x reaches the boundary ∂E, x' and z' both approach

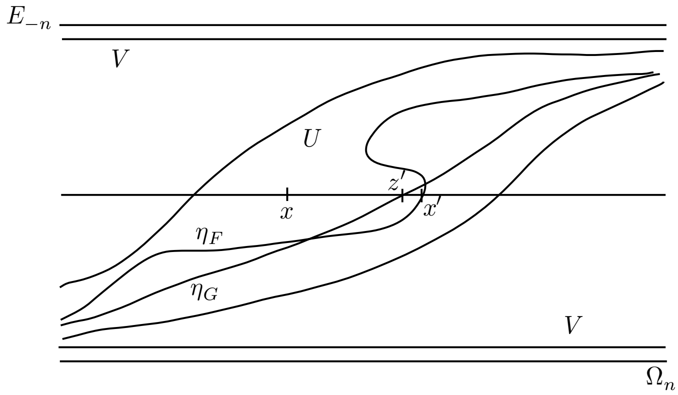

Figure 1: The graphs η_F, η_G in the space $E = E_{-n} \times \Omega_n$.

infinity in the sense that $\|x'\|$, $\|z'\| \to \infty$ and both are bounded below for given values of $(\omega^{-n}, y^n) \in \Omega_{-n} \times I\!R$ (see Mas-Colell (1985), Proposition 5.2.5, p. 170).

We now briefly sketch how to construct Ψ. Let $U \subset E$ be an open neighborhood containing η_F, η_G as well as all the "horizontal" segments between η_F, η_G. Notice that, by property (ii), U can be chosen so that the closure of U in $\bar{E}$ is disjoint from ∂E and so there exists an open set $V \subset E$, which satisfies $\partial E \subset \bar{V}$ and $\bar{U} \cap \bar{V} = \emptyset$, where $\bar{V} \subset \bar{E}$ denotes the closure of V. (In Proposition 1, we need the fact that the diffeomorphism Ψ is the identity on such a set V.) Given such U and V, there exists a smooth function $\varphi : E \to [0,1]$ such that $\varphi = 1$ on $\bar{U}$ and $\varphi = 0$ on $\bar{V}$. Now consider the (smooth) vector field on E defined by the differential equations:

$$\frac{dp}{dt} = 0, \frac{d\omega^{-n}}{dt} = 0, \frac{dy^n}{dt} = 0, \frac{d\omega^n}{dt} = \varphi \cdot \sum_{i=1}^{n-1} (\omega^i - f_i(p, p\omega^i)).$$

This vector field keeps the E_{-n} entries fixed and virtually pushes η_F "horizontally" (see the Figure 1) towards η_G. By Walras' law, this vector field is tangent to E^*, i.e., when restricted to E^* it maps into TE^*, therefore its flow gives a one parameter family $(\Psi_t)_{t\in[0,1]}$ of diffeomorphisms of E^* onto itself (see for example Hirsch (1976), Section 6.2, pp. 159-162), where all Ψ_t are the identity on V, Ψ_0 is the identity on all of E^*, and Ψ_1 takes η_F diffeomorphically onto η_G. Therefore, Ψ_1 is the desired diffeomorphism Ψ which is connected to the identity via a continuous family of diffeomorphisms, and which completes the proof. $\triangle$

IV. The rest of the paper is devoted to the derivation of some basic consequences of the preceding theorem. These consequences concern possible dynamic behavior of Walrasian equilibria for arbitrary dynamics from a certain class. In order to state the results, we introduce the relevant class of dynamics.

Definition 1 *A* **price dynamics** *is a map $F : S \times \Omega \to TS$ which belongs to the family $\mathcal{F}$ of maps F satisfying:*

(i) F is C^1,

(ii) $F^{-1}(0) = \eta$,

(iii) F points to the interior of S near $\partial S \times \Omega$, i.e., for every $\omega \in \Omega$, if $\{p^\nu\}_{\nu \in I\!N} \subset S$ is a sequence converging to $p \in \partial S$, then $\frac{F(p^\nu,\omega)}{\|F(p^\nu,\omega)\|} \to z \in I\!R^l$, where $z_k \geq 0$ whenever $p_k = 0$ with at least one such inequality strict.

The family $\mathcal{F}$ defines a large class of dynamics that basically contains all dynamics whose zeros coincide with the Walrasian equilibria and are inward pointing near $\partial S \times \Omega$. The tâtonnement dynamics, given by $F : S \times \Omega \to TS$, $(p, \omega) \mapsto \sum_{i=1}^n (f_i(p, p\omega^i) - \omega^i)$, is an element of $\mathcal{F}$, (see Mas-Colell (1985), Proposition 5.2.5, p. 170). An important property of the family $\mathcal{F}$ is the following.

Proposition 1 *The family of price dynamics in $\mathcal{F}$ is arcwise connected, i.e., any two price dynamics $F_0, F_1 : S \times \Omega \to TS$ in $\mathcal{F}$ are homotopic within $\mathcal{F}$.*

Proof. Given the preceding theorem, it suffices to carry out the proof for any two dynamics G_0, G_1 on the space $B \times \Omega$ within the class $\mathcal{G}$ of dynamics $G : B \times \Omega \to TB$ satisfying: (*i*) G is C^1, (*ii*) $G^{-1}(0) = \{0\} \times \Omega$, (*iii*) G is inward pointing near $\partial B \times \Omega$, where:

$$B = \{p \in I\!R^{l-1} |\, \|p\| < 1\},\ \partial B = \{p \in I\!R^{l-1} |\, \|p\| = 1\},\ \text{and } \bar{B} = B \cup \partial B.$$

It suffices to show that any dynamics $G_0 \in \mathcal{G}$ is homotopic to the dynamics:

$$G_1 : B \times \Omega \to TB, (p, \omega) \mapsto -p$$

within the class $\mathcal{G}$. By property (*ii*), we may assume without any loss that G_0 behaves like G_1 on an open neighborhood $V \subset B \times \Omega$ whose closure $\bar{V} \subset \bar{B} \times \Omega$ contains the boundary $\partial B \times \Omega$, (see also Mas-Colell (1985), Lemma 5.6.3, p. 203). Consider the homotopy:

$$H : [0,1] \times B \times \Omega \to TB,\ (t, p, \omega) \mapsto \begin{cases} G_1(p, \omega) \text{ if } \|p\| \geq 1 - t \\ \\ (1-t) \cdot G_0(\frac{p}{1-t}, \omega) \text{ else} \end{cases} .$$

It is easy to verify that $H_0 = G_0$, $H_1 = G_1$, and that any element $H_t \in \mathcal{G}$, for any $t \in [0,1]$. In particular $H_t^{-1}(0) = \{0\} \times \Omega$, $t \in [0,1]$. $\triangle$

This proposition shows that all price dynamics in $\mathcal{F}$ are related to each other, and to the tâtonnement dynamics in particular, via a homotopy on $S \times \Omega$ restricted to elements of $\mathcal{F}$. We interpret this as saying that there is a limit to how different dynamics in $\mathcal{F}$ can be from each other, and from the tâtonnement dynamics in particular. Notice, however, that while any two price dynamics in $\mathcal{F}$ are homotopic on the whole of $S \times \Omega$ within $\mathcal{F}$, any two arbitrarily given C^1 vector fields defined on say $S \times \{\omega\} \to TS$, $\omega \in \Omega$, that are inward pointing near $\partial S \times \{\omega\}$ and whose zeros coincide with the Walrasian equilibria of the underlying economy need not be homotopic on $S \times \{\omega\}$ within this class. So, in particular, a necessary condition to have a homotopy is that the dynamics be defined for all economies $\omega \in \Omega$.

Here are some obvious consequences of Proposition 1.

Corollary 1 *Any price dynamics in $\mathcal{F}$ is homotopic within $\mathcal{F}$ to the tâtonnement dynamics.*

Corollary 2 *Any price dynamics in $\mathcal{F}$ is homotopic within $\mathcal{F}$ to a dynamics that vanishes transversely on η.*

The first corollary follows immediately from the fact that the tâtonnement dynamics belongs to $\mathcal{F}$, and, for the second, notice that by Balasko (1988), Theorem 3.1.2, p. 68, the tâtonnement dynamics vanishes transversely on η, i.e., 0 is a regular value.

A more important consequence of Proposition 1 concerns the indices of of zeros of price dynamics. The next result establishes a basic link between the degrees and the indices of Walrasian equilibria and hence between comparative statics properties of Walrasian equilibria as embodied by the graph of the Walras correspondence and certain dynamic properties of the equilibria that are in principle dependent on the particular price dynamics chosen. Before we can state the result, we need to recall the definitions of index and degree.

Let F be a price dynamics in $\mathcal{F}$ and denote by F_ω the restriction of F to $S \times \{\omega\}$, for a given economy $\omega \in \Omega$. It is easy to see that this defines a C^1 vector field in the usual sense.

Definition 2 *Let $e = (p, \omega) \in S \times \Omega$ be such that p is a regular Walrasian equilibrium of the economy ω, and let F be a price dynamics in $\mathcal{F}$. The*

index *of F at e is defined as the integer,*

$$i(e, F) = \textit{ sign } | - D_p F_\omega(e)|,$$

where $|\cdot|$ denotes the determinant. Similarly, let $\pi : S \times \Omega \to \Omega, (p, \omega) \mapsto \omega$, denote the projection map. The **(local) degree** *of π at e is defined as the integer,*

$$d(e, \pi) = \textit{ sign } |D_{(\omega,p)}\pi(e)|.$$

Notice that the index is in following sense a much more local notion than that of the degree. To compute the index of a zero, one fixes an economy, say $\omega \in \Omega$, and a vector field, say F_ω, and looks at the dynamic behavior of F_ω around the zero, i.e., within $S \times \{\omega\}$. The degree, on the other hand, is computed by considering the projection map around a point $e \in S \times \Omega$, and therefore one necessarily considers how the equilibrium correspondence varies around e as one varies the underlying economy. The next proposition shows that for arbitrary dynamics in $\mathcal{F}$ the indices are completely determined and coincide with the corresponding (local) degrees.

Proposition 2 *Let $e = (p, \omega) \in S \times \Omega$ be such that p is a regular Walrasian equilibrium of ω, and let F be a price dynamics in $\mathcal{F}$, then, we have,*

$$i(e, F) = d(e, \pi).$$

Proof. Using the computations in Balasko (1988), Section 4.3, pp. 96-99, it follows that the indices computed with the tâtonnement dynamics coincide with the degrees of the projection map π. Therefore it remains to show that indices coincide for any pair of dynamics in $\mathcal{F}$. From Proposition 1, any two such dynamics are homotopic within $\mathcal{F}$, and therefore by a well known property of indices, which states that indices of zeros remain invariant under homotopies that do not alter the set of zeros, the indices of any two such dynamics in $\mathcal{F}$ must coincide. $\triangle$

The proposition asserts that the index of a given regular Walrasian equilibrium $e \in S \times \Omega$, computed with *any* dynamics in $\mathcal{F}$, coincides with the degree of the projection map π evaluated at e. This implies in particular that the indices for arbitrary dynamics in $\mathcal{F}$ are completely determined by the geometry of the Walrasian equilibrium correspondence. Given the more local nature of the index as compared to the (local) degree, it is somewhat surprising that the two notions agree for all dynamics in $\mathcal{F}$.

An immediate consequence of Proposition 2 is the following.

Corollary 3 *Let $p \in S$ be a regular equilibrium of degree -1 of the exchange economy $\omega \in \Omega$, then p cannot be locally stable under any dynamics in $\mathcal{F}$.*

In Dierker (1972), Theorem 2, it is shown that if all equilibria of an economy are locally stable under the tâtonnement dynamics, then there must be exactly one of them. Together with the index theorem, (see for example Mas-Colell (1985), Theorem 5.5.3, p. 189), our Proposition 2 implies that, if all equilibria are locally stable under some dynamics in $\mathcal{F}$, then there must be exactly one of them.

More importantly, with Corollary 3, we also provide a way of detecting whether certain regular equilibria may or may not be locally stable under any dynamics in $\mathcal{F}$, based directly on their degrees. If the degree is -1 then the corresponding equilibrium can never be locally stable. (Again we stress that the notion of degree is independent of the dynamics, while e.g. Dierker's index is strictly speaking the index of the tâtonnement dynamics.) This provides an additional argument in favor of equilibria of degree $+1$ (see Mas-Colell (1985), Section 5.6, pp. 201-215, for more on the discussion of the index theorem and the difference between equilibria of index $+1$ and -1). Together with the index theorem, Corollary 3 also implies that almost half of the equilibria (i.e., n out of $2n+1$) will be locally unstable in a regular economy under any dynamics in $\mathcal{F}$. Finally, with regards to Samuelson's correspondence principle mentioned in the introduction, Corollary 3 provides a partial validation, since, to the extent that equilibria with "perverse" comparative statics behavior are equilibria of degree -1, they will also always exhibit perverse, i.e., unstable dynamic behavior under all dynamics in $\mathcal{F}$.

Next we show that, although unknotted, the graph of the Walas correspondence may be almost arbitrarily twisted as one varies the underlying preference profiles of the agents. To see this, we need to extend the definitions of index and degree to components of equilibria that are not necessarily regular Walrasian equilibria. Let $C \subset S \times \Omega$ be an isolated, compact and connected component of Walrasian equilibria of the economy $\omega \in \Omega$, let $\{\omega^\nu\}_{\nu \in I\!N} \subset \Omega$ be a sequence of regular economies converging to ω, and let $N_C \subset \eta$ be a sufficiently small neighborhood of C in η. The **index** of a price dynamics $F \in \mathcal{F}$ at the component C is then defined as the integer:

$$i(C, F) \quad = \quad \lim_{\nu \to \infty} \sum_{e \in \pi^{-1}(\omega^\nu) \cap N_C} i(e, F).$$

Similarly, one defines the **(local) degree** of the component C. It can be verified that sequences of economies $\{\omega^\nu\}$ always exist and that the numbers

$i(C, F)$ and $d(C, \pi)$ do not depend on the sequence or on the (sufficiently small) neighborhood N_C chosen. Moreover, it follows immediately from Proposition 2 that the index and the degree will also agree for such components.

Proposition 2 should be compared with the following result.

Corollary 4 *Let $(C_i)_{i=1}^m \subset S$ be m disjoint, isolated, compact and connected subsets of S and let $(k_i)_{i=1}^m \in \mathbb{Z}$ be m arbitrary integers summing to one. Then there exists an economy described by endowments $\omega \in \Omega$ and preferences $\preceq \in \mathcal{P}^n$ such that $\cup_{i=1}^m C_i$ is the set of Walrasian equilibria of this economy and such that the degrees of $(C_i \times \{\omega\})_{i=1}^m$ are respectively $(k_i)_{i=1}^m$.*

Proof. Let $f : S \to TS$ be a smooth vector field vanishing precisely on $\cup_{i=1}^m C_i$ such that the indices of the components $(C_i)_{i=1}^m$ are respectively $(k_i)_{i=1}^m$, and such that f points strictly inwards on ∂S. Then it follows from a theorem of Mas-Colell (see for instance Mas-Colell (1985), Theorem 5.5.8, p. 192), that we can find an exchange economy $(\omega, \preceq) \in \mathbb{R}^{nl} \times \mathcal{P}^n$ with an excess demand function that coincides with f in a neighborhood of $\cup_{i=1}^m C_i$ and that has no further zeros. Using this excess demand function to define the tâtonnement dynamics, we have that the indices of the zeros are respectively $(k_i)_{i=1}^m$, and by the computations in Balasko (1988), Section 4.3, pp. 96-99, or more directly from our Proposition 2, which extend to components, these indices must coincide with the degrees of the corresponding components of Walrasian equilibria lying on the equilibrium manifold obtained for the preferences $\preceq \in \mathcal{P}^n$ with the endowments varying around $\omega \in \mathbb{R}^{nl}$. $\triangle$

Given that the degree of the projection map π in some sense measures how many times the graph η twists around an equilibrium point or a connected component of equilibria relative to the underlying space of parameters, this result, which combines a Debreu-Mantel-Sonneschein type result with our Proposition 2, shows that the Walras correspondence parameterized by endowments, for appropriate preference profiles in $\mathcal{P}^n$, may be almost arbitrarily twisted.

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