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CORE

DISCUSSION PAPER

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Optimal Fertility under Age-dependent Labor Productivity*

Pierre Pestieau† and Gregory Ponthiere‡

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Abstract

In the so-called *Rapport Sauvy* (1962), the French demographer Alfred Sauvy argued that Wallonia's fertility rate was socially suboptimal, and recommended a 20 % rise of fertility, on the grounds that a society with too low a fertility leads to a low-productive economy composed of old workers having old ideas. This paper examines how Sauvy's intuition can be incorporated in the seminal Samuelsonian optimal fertility model (Samuelson 1975). For that purpose, we build a 4-period OLG model with physical capital and with two generations of workers (young and old), the skills of the latter being subject to some form of decay. We characterize the optimal fertility rate, and show that this equalizes, at the margin, the sum of the capital dilution effect (Solow effect) and the labor age-composition effect (Sauvy effect) with the intergenerational redistribution effect (Samuelson effect). Finally, we develop a numerical example, and Examine how Sauvy's recommendation can be reconcilied with facts.

Keywords: optimal fertility, age structure, overlapping generations, social optimum

JEL-Classification: E13, E21, J13, J24.

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1 Introduction

Published in 1962, the official report written by the French demographer Alfred Sauvy concluded that the level of fertility in Wallonia was too low in comparison with what would be the socially optimal fertility level. According to Sauvy, the too low fertility level in Wallonia at that time led to a too low proportion of young workers with respect to older workers. As a consequence, Wallonia's economy was, in Sauvy's own words, peopled of old workers having old thoughts in old houses. In the light of this observation, Sauvy recommended a 20 % rise in the fertility level of Wallonia, in such a way as to maintain the dynamism of the Walloon economy.

Undoubtedly, Sauvy's views on what constitutes the "optimal fertility" are deep, and cannot be summarized in one single formula. According to his *Theorie Générale de la Population* (1956, chapter 4), there exist not one, but several possible social objectives to be used to define what one means by an optimal fertility. One could take, for instance, not only total or average welfare (as economists often do), but, also, the total quantity of life, the total population size, the development of culture and knowledge, the promotion of longevity and health, and also power (available means to be affected to some goal, military or not). Thus there exist, according to Sauvy, lots of ways to define the goal with respect to which some fertility can be regarded as "optimal".

The objective of this paper is to reconsider Sauvy's view on optimal fertility within a formal model. Our purpose is to try to incorporate Sauvy's view as expressed in his 1962 report in standard optimal fertility models in economics, taking social welfare as a goal.

The economic theory of optimal fertility dates back to Samuelson's (1975) seminal paper, where he characterized the optimal fertility rate in a Diamond-type 2-period OLG economy with physical capital accumulation. Samuelson showed there that the interior optimal fertility rate, when it exists, equalizes, at the margin, two effects: on the one hand, the capital dilution effect (or Solow effect), according to which a higher fertility rate makes it more difficult to sustain a high steady-state capital to labor ratio; on the other hand, the intergenerational redistribution effect (also called Samuelson effect), according to which a higher fertility, by reducing the demographic weight of the elderly, contributes to relax the economy's resource constraint by reducing the weight of the elderly's consumption. Following Samuelson, several papers analyzed the conditions under which an interior optimal fertility rate exists in the standard Samuelsonian economy (see Deardorff 1976, Michel and Pestieau 1993), or in other economies, with, for example, endogenous fertility choices (Abio 2003), risky lifetime (de la Croix et al 2012) and several fertility periods (Pestieau and Ponthiere 2014).

Our goal is to extend the standard Samuelsonian economy to examine the impact of old worker's skills decay on the socially optimal fertility. Our questions are the following. Is it possible, within a simple Samuelsonian model, to do justice to Sauvy's intuition, according to which an older labor force justifies a higher optimal fertility? Was Sauvy right in his diagnosis on the suboptimality of fertility in Wallonia? In order to answer those questions, we proceed in two stages. First, we develop a simple 4-period OLG model with capital accumulation. A specificity of our model with respect to Samuelson's framework is that there exists not one, but two generations of workers (young and old), the skills of the latter being subject to some form of decay. We then use that model to characterize the optimal fertility rate, and study then the impact of skills decay on its level. Second, we calibrate that model using general functional form for utility functions and production functions, and compare the optimal fertility levels with Sauvy's recommendations.

Anticipating our results, we show, when characterizing the optimal fertility level, that this equalizes, at the margin, the sum of the capital dilution effect (Solow effect) and the labor age-composition effect (Sauvy effect) with the intergenerational redistribution effect (Samuelson effect). Hence, it is, in theory, possible to reconcile Sauvy's views with standard economic views on optimal fertility. Note, however, that, in theory, the impact of the decay in skills on optimal fertility is not monotonous, and depends strongly on the economic fundamentals. Thus, although Sauvy's views that a higher decay justifies a higher fertility can be rationalized in some cases, there exist also cases in theory where a higher degree of decay does not lead to a higher optimal fertility. On the empirical side, our numerical simulations suggest that, for standard values of the structural parameters of the economy, it is extremely difficult to reconcile our framework with Sauvy's views. Our analysis suggests that the socially optimal fertility rate is lower than the one observed in Wallonia at the time of Sauvy's report, and thus we cannot find support for his recommendation.

In sum, although Sauvy's intuitions can be incorporated in the theoretical analysis of optimal fertility, and although a higher decay of old workers' skills can, in theory, lead to a higher optimal fertility rate, our numerical analysis does not seem to support Sauvy's views on the suboptimal level of fertility in Wallonia in the 1950s and 1960s.

The rest of the paper is organized as follows. Section 2 presents the model. The laissez-faire is derived in Section 3. The optimal fertility rate is characterized in Section 4. Section 5 provides some numerical simulations. Section 6 concludes.

2 The model

We consider a four-period OLG model with physical capital accumulation. Time goes from 0 to $+\infty$. The duration of each period is normalized to one. The first period is childhood. The second period is young adulthood, during which each individual works, saves, consumes and has n children. The third period is a period during which individuals work, save and consume. Finally, the fourth period is the old age, during which individuals enjoy their savings. That period is lived through a probability π.

2.1 Demography and labour force

The population size follows the dynamic law:

$$N_{t+1} = nN_t \tag{1}$$

where N_t denotes the number of individuals born at period t, while n is the fertility rate.

The total labour force at time t, denoted by L_t, is equal to:

$$L_t = N_{t-1} + \alpha N_{t-2} \tag{2}$$

where $\alpha \in [0, 1]$ captures the extent of decay in the skills of old workers. When $\alpha \to 0$, old workers have a very low productivity in comparison to young workers. On the contrary, when $\alpha \to 1$, old workers are almost as productive as young workers. This gap in productivity between the young and the old may be due to lots of different causes. One obvious cause lies in the fact, emphasized by Boucekkine et al (2002), that the education of old workers dates back to a more distant epoch, which can make their skills relatively out of date. Thus our modelling of labor can be regarded as a reduced form of the continuous time vintage human capital economy considered in Boucekkine et al (2002), which is here simplified by the fact that we do not take education choices into account, but, rather, suppose that time naturally depreciates human skills and productivity. One could reply to this argument that, under standard learning by doing, we should have $\alpha > 1$ instead of $\alpha \leq 1$, since workers are learning how to better produce over time, and thus would become more productive with the age. We will further discuss the link between age and productivity when we will calibrate the model in Section 5.

Using the law for population dynamics, total labour can be rewritten as:

$$\begin{aligned} L_t &= nN_{t-2} + \alpha N_{t-2} \\ &= N_{t-2}(n + \alpha) \end{aligned} \tag{3}$$

2.2 Production

The production of an output Y_t involves capital K_t and labor L_t, according to the function:

$$Y_t = F(K_t, L_t) = \bar{F}(K_t, L_t) + (1 - \delta)K_t \tag{4}$$

where δ is the depreciation rate of capital. The production function $\bar{F}(K_t, L_t)$ is supposed to be homogeneous of degree 1. Thus the total production function $F(K_t, L_t)$ is also homogeneous of degree 1. The production process can be rewritten in intensive terms as:

$$\begin{aligned} y_t &= F\left(k_t, \frac{N_{t-2}(n + \alpha)}{N_{t-2}n}\right) \\ &= F\left(k_t, 1 + \frac{\alpha}{n}\right) \end{aligned} \tag{5}$$

where $y_t \equiv \frac{Y_t}{N_{t-1}}$ denotes the output per young worker, whereas $k_t \equiv \frac{K_t}{N_{t-1}}$ denotes the capital stock per young worker.

The resource constraint of the economy is:

$$F(K_t, L_t) = c_t N_{t-1} + d_t N_{t-2} + b_t \pi N_{t-3} + K_{t+1} \tag{6}$$

where c_t, d_t and b_t denote consumption at period 2, 3 and 4 of life. Dividing by N_{t-1} allows us to rewrite the resource constraint in intensive terms:

$$F\left(k_t, 1 + \frac{\alpha}{n}\right) = c_t + \frac{d_t}{n} + \frac{b_t \pi}{n^2} + n k_{t+1} \tag{7}$$

2.3 Markets

We suppose that the economy is perfectly competitive, so that production factors are paid at their marginal productivity. Given that $L_t - \alpha N_{t-2} = N_{t-1}$ and that $k_t \equiv \frac{K_t}{N_{t-1}} = \frac{K_t}{L_t - \alpha N_{t-2}}$, we have:

$$\begin{aligned} w_t &= \frac{\partial F(K_t, L_t)}{\partial L_t} = \frac{\partial (L_t - \alpha N_{t-2}) F\left(\frac{K_t}{L_t - \alpha N_{t-2}}, \frac{L_t}{L_t - \alpha N_{t-2}}\right)}{\partial L_t} \\ &= \left[F\left(k_t, 1 + \frac{\alpha}{n}\right) - k_t F_k\left(k_t, 1 + \frac{\alpha}{n}\right)\right]\left[\frac{n}{n + \alpha}\right] \end{aligned} \tag{8}$$

$$R_t = \frac{\partial F(K_t, L_t)}{\partial K_t} = \frac{\partial N_{t-1} F\left(k_t, 1 + \frac{\alpha}{n}\right)}{\partial N_{t-1} k_t} = F_k\left(k_t, 1 + \frac{\alpha}{n}\right) \tag{9}$$

where w_t denotes the wage rate and R_t is the return on savings at period t.

We also suppose that there exists a perfect annuity market with actuarially fair returns, so that the gross return on savings $\hat{R}_t$ equals:

$$\hat{R}_t = \frac{R_t}{\pi} \tag{10}$$

3 The laissez-faire

The second-period budget constraint is:

$$c_t + s_t = w_t \tag{11}$$

where s_t is savings in the first period of labor.

The third-period budget constraint is:

$$d_{t+1} + z_{t+1} = w_{t+1} + s_t R_{t+1} \tag{12}$$

The fourth-period budget constraint is:

$$b_{t+2} = \frac{z_{t+1} R_{t+2}}{\pi} \tag{13}$$

where z_{t+1} is savings in second period of labor.

Hence the intertemporal budget constraint is:

$$c_t + \frac{d_{t+1}}{R_{t+1}} + \frac{\pi b_{t+2}}{R_{t+1}R_{t+2}} = w_t + \frac{w_{t+1}}{R_{t+1}} \tag{14}$$

Conditionally on beliefs on future factor prices w^e_{t+1}, R^e_{t+1} and R^e_{t+2}, the problem of individuals can be written as:

$$\begin{aligned} &\max_{c_t, d_{t+1}, b_{t+2}} u(c_t) + \beta u(d_{t+1}) + \pi\beta^2 u(b_{t+2}) \\ &\text{s.t. } w_t + \frac{w^e_{t+1}}{R^e_{t+1}} = c_t + \frac{d_{t+1}}{R^e_{t+1}} + \frac{\pi b_{t+2}}{R^e_{t+1}R^e_{t+2}} \end{aligned}$$

The first-order conditions yield:

$$\frac{u'(c_t)}{\beta u'(d_{t+1})} = R^e_{t+1} \tag{15}$$

$$\frac{u'(d_{t+1})}{\beta u'(b_{t+2})} = R^e_{t+2} \tag{16}$$

Individuals save in such a way as to equalize the marginal rate of substitution of two successive periods with the expected rate of return on savings. Higher impatience (i.e. a lower β) pushes, for a given interest factor, towards more consumption early in life.

4 The long-run social optimum

Let us now focus on the long-run social optimum in our economy. We will follow here Samuelson's approach (Samuelson 1975), which consists in studying the problem of a social planner choosing consumptions, capital and fertility, in such a way as to maximize the expected lifetime welfare of an agent living at the stationary equilibrium.[1]

4.1 The social planner's problem

Let us assume that there exists a unique stable stationary equilibrium in our economy. The social planner's problem can be written as follows:

$$\begin{aligned} &\max_{c,d,b,k,n} u(c) + \beta u(d) + \pi\beta^2 u(b) \\ &\text{s.t. } F\left(k, 1 + \frac{\alpha}{n}\right) = c + \frac{d}{n} + \frac{b\pi}{n^2} + nk \end{aligned}$$

[1] Note that this objective is here similar to maximizing the average *ex post* (i.e. realized) lifetime well-being among agents living at the steady-state (some agents being short-lived, whereas others are long-lived).

An interior optimum $(c^*, d^*, b^*, k^*, n^*)$ satisfies the following FOCs:

$$\frac{u'(c^*)}{\beta u'(d^*)} = \frac{u'(d^*)}{\beta u'(b^*)} = n^* \tag{17}$$

$$F_k\left(k^*, 1+\frac{\alpha}{n^*}\right) = n^* \tag{18}$$

$$k^* + \frac{\alpha F_L\left(k^*, 1+\frac{\alpha}{n^*}\right)}{(n^*)^2} = \frac{d^*}{(n^*)^2} + \frac{2b^*\pi}{(n^*)^3} \tag{19}$$

The first condition states that, at the long-run social optimum, the marginal rate of substitution between consumption at two successive periods is equal to the optimal fertility rate. The second condition is the standard Golden Rule of capital accumulation (Phelps 1961), stating that the optimal capital equalizes the marginal productivity of capital net of depreciation and the fertility rate. The third condition characterizes the (interior) socially optimal fertility rate. It depends on three forces. When α is equal to zero, so that only young adults are productive, the optimal fertility rate equalizes, at the margin, the capital dilution effect or Solow effect (first term of LHS) with the intergenerational distribution effect or Samuelson effect (RHS). The Solow effect states that, under a higher fertility, it is more difficult to sustain a large capital to labor ratio at the stationary equilibrium. This effect pushes towards less fertility. The Samuelson effect states that a higher fertility relaxes the economy's resource constraint, by making the elderly's consumption relatively less sizeable. This second effect pushes towards a higher fertility. Note that the size of this effect depends on the age-structure, through the parameter π.[2]

But besides those two effects, there is another, third determinant of optimal fertility, represented by the second term of the LHS. Clearly, if the productivity of old workers is zero (i.e. $\alpha = 0$), this third effect is absent, since all productive workers are young, and thus changing fertility has no effect on the composition of the labor force. In that case, the optimal fertility equalizes, at the margin, the capital dilution effect and the Samuelson effect, as in Samuelson's pioneer work (Samuelson 1975). However, under $\alpha > 0$, a third effect is at work. Given that this effect depends on the composition of the labor force by age, let us call this effect the "labor age-composition effect".

To understand this labor age-composition effect, note first that an increase in n reduces the total amount of labor per young worker, and, hence, reduces the output per young worker. Thus the immediate effect of a rise in n is a fall of the ratio total labor force / young workers (i.e. $\frac{N_{t-1}+\alpha N_{t-2}}{N_{t-1}} = \frac{n+\alpha}{n}$), which leads to a fall of productivity per young worker. Note, however, that the size of this negative productivity effect depends itself on the extent of decay in old workers' skills. Obviously, as already mentioned, when $\alpha = 0$, a rise in the fertility rate has no effect on productivity per young worker. However, when $\alpha > 0$, a rise in n will affect the ratio total labor force / young workers, leading to a variation in productivity per young worker. Given that the total effect is equal to

[2] The necessity to raise fertility when longevity increases raise the number of inactive persons is also emphasized in Sauvy (1959).

$\frac{\alpha F_L\left(k^*,1+\frac{\alpha}{n^*}\right)}{(n^*)^2}$, an important question that arises is whether this term is increasing or decreasing in α. We know that it is equal to 0 when α equals 0. However, a rise in α tends to increase the first term of the product $\alpha F_L\left(k^*, 1+\frac{\alpha}{n^*}\right)$, and to reduce the second term of the product $\alpha F_L\left(k^*, 1+\frac{\alpha}{n^*}\right)$. We thus have two opposite influences of decay on the size of the labor composition effect. When the first influence dominates the second, a larger decay of old workers' skills (i.e. a lower α) contributes to reduce the negative productivity effect induced by a higher n, leading, *ceteris paribus*, to a higher optimal fertility n^*. But when the second influence dominates the first, a larger decay raises the negative productivity effect induced by a higher n, leading, *ceteris paribus*, to a lower optimal fertility n^*.

When Sauvy (1962) argued that the ageing of the workforce could lead to an ageing of production techniques, and, hence, to a reduction of total labor productivity, he probably had in mind that an increase in the degree of decay in old worker's skills would necessarily make a quicker renewal of the workforce more desirable, leading to a larger socially desirable fertility. This way of thinking is quite intuitive: when old workers'skill become more old-fashioned or depreciated, it makes sense to desire a higher fertility rate. This kind of rationale is possible in our framework, through the labor age-composition effect, but provided the fundamentals of the economy are such that the product $\alpha F_L\left(k^*, 1+\frac{\alpha}{n^*}\right)$ is indeed *increasing* with α. Otherwise, when the product $\alpha F_L\left(k^*, 1+\frac{\alpha}{n^*}\right)$ is decreasing with α, a higher decay justifies a lower fertility, unlike Sauvy's intuition.

In the light of all this, the interior optimal fertility rate equalizes at the margin, on the one hand, the sum of the capital dilution effect and the labor composition effect, and, on the other hand, the intergenerational redistribution effect. Those three effects can also be called, respectively, the Solow effect, the Sauvy effect and the Samuelson effect. It is thus possible, by merely introducing two generations of workers and the possibility of decay of old worker's skills, to incorporate Sauvy's intuitions into the neoclassical model of optimal fertility.

Finally, it should be stressed that our discussion assumed implicitly that the optimal fertility rate is an interior optimum. As this is well-known in the literature (see Deardorff 1976, Michel and Pestieau 1993), the optimum fertility rate in a Samuelsonian economy is not necessary an interior one. As shown by Michel and Pestieau (1993), interiority holds only when there is enough complementarity between consumption at the different ages of life, and when there is enough complementarity between capital and labor in the production process.

Two kinds of corner solutions can arise. First, it can be the case that:

$$k^* + \frac{\alpha F_L\left(k^*, 1+\frac{\alpha}{n^*}\right)}{(n^*)^2} > \frac{d^*}{(n^*)^2} + \frac{2b^*\pi}{(n^*)^3} \tag{20}$$

for any value of n^*, in which case the marginal utility gain from increasing fertility associated to the Samuelson effect is always lower than the marginal utility loss due to the capital dilution effect and the labor age-composition effect, implying that the optimal fertility rate is equal to 0. This case arises when consumptions at the old age do not really matter for individual well-being (because of a high elasticity of intertemporal substitution in consumption), which pushes the intergenerational redistribution effect down.

On the contrary, when we have

$$k^* + \frac{\alpha F_L\left(k^*, 1+\frac{\alpha}{n^*}\right)}{\left(n^*\right)^2} < \frac{d^*}{\left(n^*\right)^2} + \frac{2b^*\pi}{\left(n^*\right)^3} \tag{21}$$

for any value of n^*, the solution is the other corner: the optimal fertility is infinite. Indeed, in that case, the welfare loss from increasing fertility associated to the capital dilution effect is, whatever the level of fertility, lower than the marginal utility gain from increasing fertility due to the intergenerational redistribution effect. In that case, the low impact of capital dilution can arise from the high degree of substitutability between capital and labor, which makes further rises in fertility always beneficial.

Those two corner solutions look pathological, but, as we shall see when calibrating the model, corner solutions arise quite often when considering optimal fertility in a Samuelsonian economy, in line with what Michel and Pestieau (1993) proved analytically.

4.2 The Serendipity Theorem

First stated by Samuelson (1975), the Serendipity Theorem states that, if there exists a unique stable stationary equilibrium in a two-period OLG model with physical capital, then the perfectly competitive economy will converge towards the long-run social optimum provided the optimal fertility rate is imposed on individuals.

Does this result still hold in our four-period OLG setting? To check this, let us rewrite the problem of individuals living at the steady-state. That problem is:

$$\max_{c,d,b} u(c) + \beta u(d) + \pi\beta^2 u(b)$$
$$\text{s.t. } w + \frac{w}{R} = c + \frac{d}{R} + \frac{\pi b}{R^2}$$

The first-order conditions yield:

$$\frac{u'(c)}{\beta u'(d)} = \frac{u'(d)}{\beta u'(b)} = R \tag{22}$$

Hence, if the social planner fixes n such that $F_k\left(k^*, 1+\frac{\alpha}{n^*}\right) = n^*$ where k takes its socially optimal level, then individuals, being price-takers, will choose their savings optimally, since the above FOC will then coïncide with the FOC of the social optimum.

5 Numerical illustrations

In his report, Sauvy argued that fertility in Wallonia was suboptimal, and that this should be raised by 20 %. This section aims at providing answers to the following questions. On the basis of the model developed above, was Sauvy right when claiming that the actual fertility rate in Wallonia in 1950s and 1960s was below the socially optimum one? What about the recommendation of a 20 % rise in fertility?

In order to answer that question, this section calibrates the model presented above, and find the optimal fertility rate. It is then compared with the one prevailing in Wallonia at that time, and with the one recommended by Sauvy.

5.1 Data

At this stage, it is worth looking at the dynamics of fertility in Belgian regions since the 19th century. As shown by Capron et al (1998), the total fertility rate (TFR) has strongly declined in Wallonia since 1800. At that time, the TFR was about 4.5 children per women. The decline was first smooth, with about 4 children per women in 1860, but then strongly declined, to reach 2 children per women in 1930. Then, the TFR fluctuated between 2.5 in 1960 to reach 1.5 in the early 1980s.

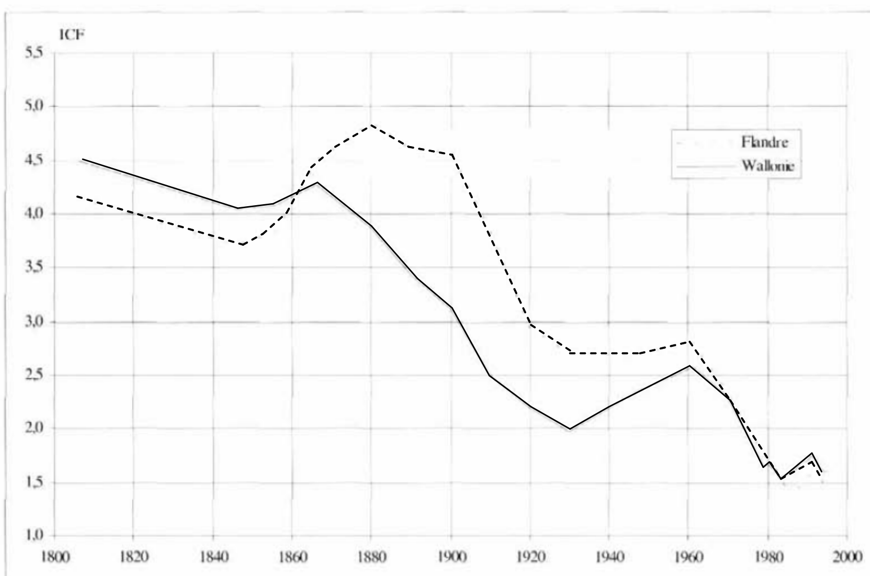

Figure 1: TFR in Flanders and Wallonia (Source: Capron et al 1998 p. 264).

Flanders's TFR followed a quite different dynamics: it declined slightly in the first part of the 19th century, but, then, it grew to reach about 4.7 children per women in 1880. In 1900, the fertility gap between Flanders and Wallonia was equal to about 1.5 children per women. That gap has reduced progressively after 1900, when Flanders' TFR has progressively converged towards the one prevailing in Wallonia. Note that, in 1960, the TFR in Flanders was still superior to the one prevailing in Wallonia (2.8 against 2.5 children per women).

5.2 Functional forms

In order to check to what extent fertility in Wallonia around 1960 can be regarded as suboptimal from the perspective of the model developed above, we need first to impose some particular functional forms on the utility function and the production function.

Regarding the temporal utility function, we use the standard CIES form:

$$u\left(c_t\right) = \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \tag{23}$$

where $\sigma > 0$ and $\sigma \neq 0$.

From the FOCs for optimal consumption profiles, we have $\frac{c_t^{-\frac{1}{\sigma}}}{\beta d_t^{-\frac{1}{\sigma}}} = n$ and $\frac{d_t^{-\frac{1}{\sigma}}}{\beta b_t^{-\frac{1}{\sigma}}} = n$, from which we obtain:

$$c_t = \left[n\beta d_t^{-\frac{1}{\sigma}}\right]^{-\sigma} \text{ and } b_t = \left[\frac{d_t^{-\frac{1}{\sigma}}}{\beta n}\right]^{-\sigma}$$

Substituting for those expressions in the economy's resource constraint yields:

$$F\left(k_t, 1+\frac{\alpha}{n}\right) = \left[n\beta d_t^{-\frac{1}{\sigma}}\right]^{-\sigma} + \frac{d_t}{n} + \frac{\left[\frac{d_t^{-\frac{1}{\sigma}}}{\beta n}\right]^{-\sigma}\pi}{n^2} + nk_t \tag{24}$$

The production function takes a CES form:

$$\bar{F}\left(K_t, L_t\right) = A\left[\gamma K_t^{-\rho} + (1-\gamma)L_t^{-\rho}\right]^{-1/\rho} \tag{25}$$

where $A > 0$, $0 < \gamma < 1$, $\rho > -1$, $\rho \neq 0$. Assuming a full depreciation of capital after one period, output per young worker is here:

$$\begin{aligned} y_t \quad &\equiv \quad \frac{Y_t}{N_{t-1}} = \frac{A\left[\gamma\left(N_{t-1}k_t\right)^{-\rho} + (1-\gamma)\left(N_{t-1}+\frac{\alpha N_{t-1}}{n}\right)^{-\rho}\right]^{-1/\rho}}{N_{t-1}} \\ &= \quad A\left[\gamma\left(k_t\right)^{-\rho} + (1-\gamma)\left(1+\frac{\alpha}{n}\right)^{-\rho}\right]^{-1/\rho} \end{aligned} \tag{26}$$

Hence we have

$$F_k\left(k_t, 1+\frac{\alpha}{n}\right) = A\left[\gamma\left(k_t\right)^{-\rho}+(1-\gamma)\left(1+\frac{\alpha}{n}\right)^{-\rho}\right]^{-(1/\rho)-1}\gamma\left(k_t\right)^{-\rho-1} \tag{27}$$

$$\begin{aligned}
F_L\left(k_t, 1+\frac{\alpha}{n}\right) &= \left[F\left(k_t, 1+\frac{\alpha}{n}\right)-k_t F_k\left(k, 1+\frac{\alpha}{n}\right)\right]\left[\frac{n}{n+\alpha}\right] \\
&= \left[\frac{n}{n+\alpha}\right]\left[\begin{array}{c} A\left[\gamma\left(k_t\right)^{-\rho}+(1-\gamma)\left(1+\frac{\alpha}{n}\right)^{-\rho}\right]^{-1/\rho} \\ -A\left[\gamma\left(k_t\right)^{-\rho}+(1-\gamma)\left(1+\frac{\alpha}{n}\right)^{-\rho}\right]^{-(1/\rho)-1}\gamma\left(k_t\right)^{-\rho} \end{array}\right]
\end{aligned} \tag{28}$$

Hence, from the Golden Rule, we have:

$$\begin{aligned}
n &= A\left[\gamma\left(k_t\right)^{-\rho}+(1-\gamma)\left(1+\frac{\alpha}{n}\right)^{-\rho}\right]^{-(1/\rho)-1}\gamma\left(k_t\right)^{-\rho-1} \\
\Longleftrightarrow \quad & k_t = \frac{\left(\frac{n}{A\gamma}\right)^{\frac{-1}{2}}-\gamma}{(1-\gamma)\left(1+\frac{\alpha}{n}\right)^{-1}}
\end{aligned} \tag{29}$$

Thus, for any level of fertility n, we can deduce the optimal level of k by means of the Golden Rule expression. Then, given the levels of n and k, the new formulation of the resource constraint yields a unique level of second-period consumption d, from which we can find the remaining variables c and b.[3]

5.3 Calibration

The model is a 4-period model. Each period lasts 20 years. Regarding the survival probability π, we use life expectancy estimates around 1960. Life expectancy in 1960 is equal, for both men and women, to 69.63 years. This implies that π is equal to $9.63/20 = 0.48$.

Let us now calibrate preference parameters. Regarding the time preference parameter β, we use the standard approach in the literature: given a quarterly discount factor equal to 0.99, we obtain $\beta = (0.99)^{80} = 0.45$. Regarding σ, we use the empirical estimates of the elasticity of intertemporal substitution. According to Browning at al (1999), the elasticity of intertemporal substitution is slightly above unity. We take here the estimate of Blundell et al (1994) and fix $\sigma = 1.25$.

[3]Indeed, from the resource constraint, we have:

$$\begin{aligned}
F\left(k, 1+\frac{\alpha}{n}\right) &= \left[n\beta d^{-\frac{1}{\sigma}}\right]^{-\sigma}+\frac{d}{n}+\frac{\left[\frac{d_t^{-\frac{1}{\sigma}}}{\beta n}\right]^{-\sigma}\pi}{n^2}+nk \\
\Longleftrightarrow \quad & d = \frac{F\left(k, 1+\frac{\alpha}{n}\right)-nk}{\left[[n\beta]^{-\sigma}+\frac{1}{n}+\frac{\left[\frac{1}{\beta n}\right]^{-\sigma}\pi}{n^2}\right]}
\end{aligned}$$

As far as the production process is concerned, three parameters must be calibrated: A, γ and ρ. In order to calibrate those parameters, we follow de la Croix and Michel (2002, p. 340), who calibrate jointly those three parameters. Those authors fix $A = 20$ and $\rho = 1$, which implies an elasticity if substitution between capital and labor equal to 0.50. Finally, in order to obtain a labor share in production equal to 2/3, those authors fix $\gamma = 0.49$.

We need also to calibrate the depreciation of physical capital δ and the depreciation of human skills α. Regarding physical capital depreciation, we assume, given the duration of periods (20 years), that there is full depreciation of capital after one period of use (i.e. $\delta = 1$). Regarding the parameter of old workers skills decay α, empirical studies on the age/productivity relation are far from unanimous, as discussed in the recent study by van Ours and Stoeldraijer (2010). Those authors highlight that the existing literature presents quite contradictory results on the age/productivity relationship. According to Johnson (1993), most employers and employees believe that average labor productivity declines after some age between ages 40 and 50. Skirbekk (2003) argues that job performance declines after age 50. But Aubert and Crépon (2007) found that productivity increases until age 40-45 and then remains stable after that age. Finally, Göbel and Zwick (2009) find that productivity grows until age 40-45, and then stabilizes until age 60. Given those contradicting results, we will take $\alpha = 1$ as a benchmark case, and consider alternative values for α as a way to show how Sauvy's intuition can affect optimal fertility in our model.

The following table summarizes the calibration of parameters taken as a benchmark.

parameters	α	β	σ	π	A	γ	ρ	δ
values	1.00	0.45	1.25	0.48	20	0.49	1.00	1.00

5.4 Results

This subsection computes the socially optimal fertility rate in our economy, and compares it with the actual fertility rate for Wallonia in 1960, equal to 2.6 children per women, which, in normalized terms, is equal to $n = 1.3$. For that purpose, we will proceed as follows. For each possible fertility rate within a biologically feasible interval $n \in [0, 6]$, we derive the associated optimal level of k, and, then, the optimal levels of c, d and b, as well as the associated average lifetime welfare. We then naturally find the level of fertility which leads to the maximum average lifetime well-being, and we compare it with the actual fertility. We then repeat that procedure for different sets of structural parameters, to assess the robustness of our results.

A first important numerical result is the fact that, under the benchmark calibration proposed in the previous subsection, the optimal fertility rate is a corner solution at $n = 0$, whatever the postulated level of old skills'decay (α) is. This first result is shown on Figure 2.

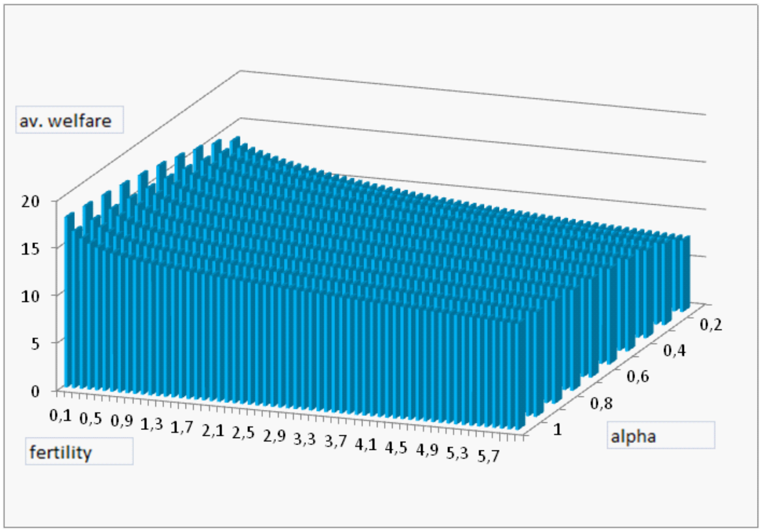

Figure 2: Average lifetime welfare as a function of fertility n and decay α (benchmark values).

This result, which may appear surprising at first glance, is not surprising for economists studying optimal fertility in an OLG setting. Indeed, since the study by Michel and Pestieau (1993), economists know that the existence of an interior optimal fertility rate within a Diamond-type economy requires that the elasticity of intertemporal substitution is sufficiently low and that complementarity between capital and labor is sufficiently high. However, given the postulated parameters' values, there remains too much substitutability, leading to a corner solution for fertility.

Let us now consider alternative calibrations, involving a lower value of the elasticity of intertemporal substitution σ. As shown on Figure 3 (where axes are reversed given the negative utility), assuming $\sigma = 0.5$ allows us to obtain an interior optimal fertility rate. Under $\alpha = 1$ (no decay), the optimal fertility is 0.5. The optimal n becomes slightly larger, equal to 0.6, when α decreases to 0.6. Note that the impact of α on the optimal fertility is non-monotonic, since for $\alpha = 0.1$ the optimal fertility goes back to 0.5. This result is in line with the theoretical discussion we had above. Since the product $\alpha F_L\left(k^*, 1 + \frac{\alpha}{n^*}\right)$ is non-monotonic in α, there is no reason to expect that the optimal fertility rate is necessarily increasing with the extent of decay in old worker's skills. Having stressed this, it remains true, in any case, that the optimal fertility is far lower than the one prevailing in Wallonia at that time (i.e. $n = 1.3$).

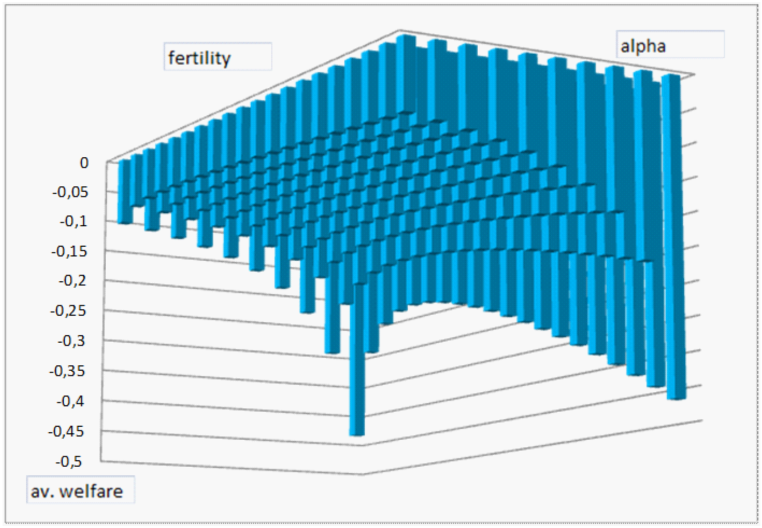

Figure 3: Average lifetime welfare as a function of fertility (n) and decay (α), under $\sigma = 0.5$.

Note that taking even lower values for σ does not substantially affect our results. Under $\sigma = 0.2$, the optimal fertility rate equals 0.8 when $\alpha = 1$, rises to 0.9 when $\alpha = 0.9$, but then goes down to 0.7 when $\alpha = 0.1$. Hence it appears, here again, that taking into account the decay of old worker's skills does not suffice, on its own, to lead to a large optimal fertility rate. This result is robust, and prevails even when we reduce strongly the elasticity of intertemporal substitution. One reason for this low level of optimal fertility lies in the relatively high level of impatience (i.e. $\beta = 0.45$), which, by construction, has the effect to weaken the strength of the Samuelson effect, and, hence, pushes optimal fertility down.

In the light of this, it seems hard to reconcile Sauvy's recommendation with the predictions of the model. True, it is possible, in theory, to account for the intuition that a higher decay of old workers skills may lead, in some cases, to justify a higher fertility rate. However, our numerical exercise suggests that, for reasonable values of the structural parameters, we obtain optimal fertility rates that are far below the one prevailing in Wallonia in 1950s and 1960s. Thus our numerical findings to not seem to support Sauvy's recommendation.

Finally, let us notice that, on the basis of the actual life expectancy prevailing in Wallonia, i.e. 79 years, implying π equal to $19/20 = 0.95$), the optimal fertility still remains, under $\sigma = 0.8$, quite low, and between 0.2 (when $\alpha = 1$) and 0.5 (when $\alpha = 0.3$). Thus better survival conditions push, *ceteris paribus*, towards a higher optimal fertility, by reinforcing Samuelson's effect. However, even under a high decay of old workers' skills, it is hard to regard fertility encouragement as welfare-improving in the long-run in our first-best setting.

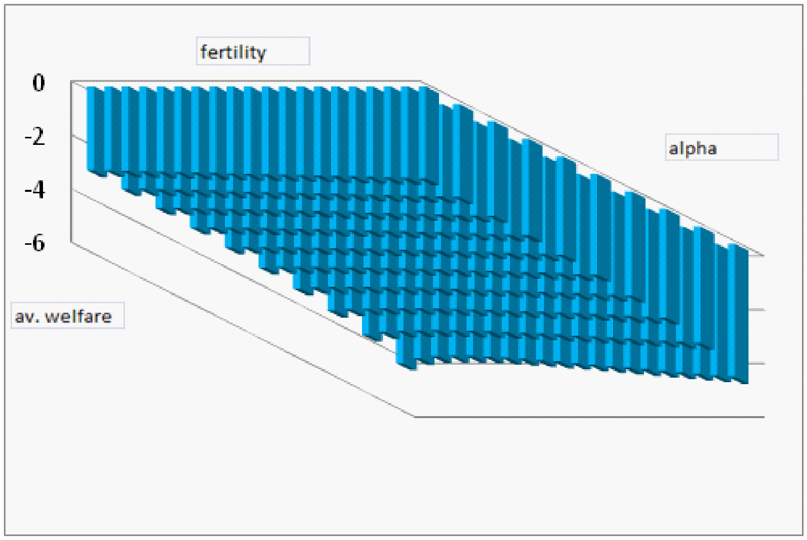

Figure 4: Average lifetime welfare as a function of fertility (α) and decay (n), under $\sigma = 0.8$ and $\pi = 0.95$ (contemporary survival conditions).

6 Conclusions

In sum, this paper provides a contrasted view on Sauvy's diagnosis on Walloon fertility in the early 1960s. True, it is possible to incorporate Sauvy's view on ageing of workers and productivity within a simple OLG model *à la* Samuelson. Once taking the decay of human skills into account, the optimal fertility rate equalizes, at the margin, the sum of the capital dilution effect (Solow effect) and the labor age-composition effect (Sauvy effect) with the intergenerational redistribution effect (Samuelson effect). There is thus some simple way to account for issues of old workers productivity when discussing optimal fertility in a Samuelsonian economy.

However, when turning to numbers, it appears that even large levels of decay in old workers' skills do not suffice to support Sauvy's views on Walloon fertility. Our numerical simulations suggest that, even though a higher extent of decay leads, in general (but not always), to a larger optimal fertility, the levels of optimal fertility rates remain far below the observed ones, and also below the ones recommended by Sauvy in his report.

One should not take these results as a criticism of Sauvy's thought on population. Quite the contrary, our calculations suggest that the rationale used by Sauvy to define the optimal fertility could not take, as a unique social objective, the maximization of average social welfare in Wallonia at that time. Obviously, some other considerations, either for numbers, or for culture and knowledge, or for what he called power, were playing a role in his recommendations. Hence, before saying that he was right or wrong, we need first to consider how those other concerns could be properly included in an exhaustive economic analysis of optimal fertility. This remains to be done.

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DISCUSSION PAPER

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CORE DISCUSSION PAPER
2015/43

Optimal Fertility under Age-dependent Labor Productivity*

Pierre Pestieau† and Gregory Ponthiere‡

October 19, 2015

Abstract

In the so-called *Rapport Sauvy* (1962), the French demographer Alfred Sauvy argued that Wallonia's fertility rate was socially suboptimal, and recommended a 20 % rise of fertility, on the grounds that a society with too low a fertility leads to a low-productive economy composed of old workers having old ideas. This paper examines how Sauvy's intuition can be incorporated in the seminal Samuelsonian optimal fertility model (Samuelson 1975). For that purpose, we build a 4-period OLG model with physical capital and with two generations of workers (young and old), the skills of the latter being subject to some form of decay. We characterize the optimal fertility rate, and show that this equalizes, at the margin, the sum of the capital dilution effect (Solow effect) and the labor age-composition effect (Sauvy effect) with the intergenerational redistribution effect (Samuelson effect). Finally, we develop a numerical example, and Examine how Sauvy's recommendation can be reconciled with facts.

Keywords: optimal fertility, age structure, overlapping generations, social optimum

JEL-Classification: E13, E21, J13, J24.

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1 Introduction

Published in 1962, the official report written by the French demographer Alfred Sauvy concluded that the level of fertility in Wallonia was too low in comparison with what would be the socially optimal fertility level. According to Sauvy, the too low fertility level in Wallonia at that time led to a too low proportion of young workers with respect to older workers. As a consequence, Wallonia's economy was, in Sauvy's own words, peopled of old workers having old thoughts in old houses. In the light of this observation, Sauvy recommended a 20 % rise in the fertility level of Wallonia, in such a way as to maintain the dynamism of the Walloon economy.

Undoubtedly, Sauvy's views on what constitutes the "optimal fertility" are deep, and cannot be summarized in one single formula. According to his *Theorie Générale de la Population* (1956, chapter 4), there exist not one, but several possible social objectives to be used to define what one means by an optimal fertility. One could take, for instance, not only total or average welfare (as economists often do), but, also, the total quantity of life, the total population size, the development of culture and knowledge, the promotion of longevity and health, and also power (available means to be affected to some goal, military or not). Thus there exist, according to Sauvy, lots of ways to define the goal with respect to which some fertility can be regarded as "optimal".

The objective of this paper is to reconsider Sauvy's view on optimal fertility within a formal model. Our purpose is to try to incorporate Sauvy's view as expressed in his 1962 report in standard optimal fertility models in economics, taking social welfare as a goal.

The economic theory of optimal fertility dates back to Samuelson's (1975) seminal paper, where he characterized the optimal fertility rate in a Diamond-type 2-period OLG economy with physical capital accumulation. Samuelson showed there that the interior optimal fertility rate, when it exists, equalizes, at the margin, two effects: on the one hand, the capital dilution effect (or Solow effect), according to which a higher fertility rate makes it more difficult to sustain a high steady-state capital to labor ratio; on the other hand, the intergenerational redistribution effect (also called Samuelson effect), according to which a higher fertility, by reducing the demographic weight of the elderly, contributes to relax the economy's resource constraint by reducing the weight of the elderly's consumption. Following Samuelson, several papers analyzed the conditions under which an interior optimal fertility rate exists in the standard Samuelsonian economy (see Deardorff 1976, Michel and Pestieau 1993), or in other economies, with, for example, endogenous fertility choices (Abio 2003), risky lifetime (de la Croix et al 2012) and several fertility periods (Pestieau and Ponthiere 2014).

Our goal is to extend the standard Samuelsonian economy to examine the impact of old worker's skills decay on the socially optimal fertility. Our questions are the following. Is it possible, within a simple Samuelsonian model, to do justice to Sauvy's intuition, according to which an older labor force justifies a higher optimal fertility? Was Sauvy right in his diagnosis on the suboptimality of fertility in Wallonia? In order to answer those questions, we proceed in two stages. First, we develop a simple 4-period OLG model with capital accumulation. A specificity of our model with respect to Samuelson's framework is that there exists not one, but two generations of workers (young and old), the skills of the latter being subject to some form of decay. We then use that model to characterize the optimal fertility rate, and study then the impact of skills decay on its level. Second, we calibrate that model using general functional form for utility functions and production functions, and compare the optimal fertility levels with Sauvy's recommendations.

Anticipating our results, we show, when characterizing the optimal fertility level, that this equalizes, at the margin, the sum of the capital dilution effect (Solow effect) and the labor age-composition effect (Sauvy effect) with the intergenerational redistribution effect (Samuelson effect). Hence, it is, in theory, possible to reconcile Sauvy's views with standard economic views on optimal fertility. Note, however, that, in theory, the impact of the decay in skills on optimal fertility is not monotonous, and depends strongly on the economic fundamentals. Thus, although Sauvy's views that a higher decay justifies a higher fertility can be rationalized in some cases, there exist also cases in theory where a higher degree of decay does not lead to a higher optimal fertility. On the empirical side, our numerical simulations suggest that, for standard values of the structural parameters of the economy, it is extremely difficult to reconcile our framework with Sauvy's views. Our analysis suggests that the socially optimal fertility rate is lower than the one observed in Wallonia at the time of Sauvy's report, and thus we cannot find support for his recommendation.

In sum, although Sauvy's intuitions can be incorporated in the theoretical analysis of optimal fertility, and although a higher decay of old workers' skills can, in theory, lead to a higher optimal fertility rate, our numerical analysis does not seem to support Sauvy's views on the suboptimal level of fertility in Wallonia in the 1950s and 1960s.

The rest of the paper is organized as follows. Section 2 presents the model. The laissez-faire is derived in Section 3. The optimal fertility rate is characterized in Section 4. Section 5 provides some numerical simulations. Section 6 concludes.

2 The model

We consider a four-period OLG model with physical capital accumulation. Time goes from 0 to $+\infty$. The duration of each period is normalized to one. The first period is childhood. The second period is young adulthood, during which each individual works, saves, consumes and has n children. The third period is a period during which individuals work, save and consume. Finally, the fourth period is the old age, during which individuals enjoy their savings. That period is lived through a probability π.

2.1 Demography and labour force

The population size follows the dynamic law:

$$N_{t+1} = nN_t \tag{1}$$

where N_t denotes the number of individuals born at period t, while n is the fertility rate.

The total labour force at time t, denoted by L_t, is equal to:

$$L_t = N_{t-1} + \alpha N_{t-2} \tag{2}$$

where $\alpha \in [0, 1]$ captures the extent of decay in the skills of old workers. When $\alpha \to 0$, old workers have a very low productivity in comparison to young workers. On the contrary, when $\alpha \to 1$, old workers are almost as productive as young workers. This gap in productivity between the young and the old may be due to lots of different causes. One obvious cause lies in the fact, emphasized by Boucekkine et al (2002), that the education of old workers dates back to a more distant epoch, which can make their skills relatively out of date. Thus our modelling of labor can be regarded as a reduced form of the continuous time vintage human capital economy considered in Boucekkine et al (2002), which is here simplified by the fact that we do not take education choices into account, but, rather, suppose that time naturally depreciates human skills and productivity. One could reply to this argument that, under standard learning by doing, we should have $\alpha > 1$ instead of $\alpha \leq 1$, since workers are learning how to better produce over time, and thus would become more productive with the age. We will further discuss the link between age and productivity when we will calibrate the model in Section 5.

Using the law for population dynamics, total labour can be rewritten as:

$$\begin{aligned} L_t &= nN_{t-2} + \alpha N_{t-2} \\ &= N_{t-2}(n + \alpha) \end{aligned} \tag{3}$$

2.2 Production

The production of an output Y_t involves capital K_t and labor L_t, according to the function:

$$Y_t = F(K_t, L_t) = \bar{F}(K_t, L_t) + (1 - \delta)K_t \tag{4}$$

where δ is the depreciation rate of capital. The production function $\bar{F}(K_t, L_t)$ is supposed to be homogeneous of degree 1. Thus the total production function $F(K_t, L_t)$ is also homogeneous of degree 1. The production process can be rewritten in intensive terms as:

$$\begin{aligned} y_t &= F\left(k_t, \frac{N_{t-2}(n + \alpha)}{N_{t-2}n}\right) \\ &= F\left(k_t, 1 + \frac{\alpha}{n}\right) \end{aligned} \tag{5}$$

where $y_t \equiv \frac{Y_t}{N_{t-1}}$ denotes the output per young worker, whereas $k_t \equiv \frac{K_t}{N_{t-1}}$ denotes the capital stock per young worker.

The resource constraint of the economy is:

$$F\left(K_t, L_t\right) = c_t N_{t-1} + d_t N_{t-2} + b_t \pi N_{t-3} + K_{t+1} \tag{6}$$

where c_t, d_t and b_t denote consumption at period 2, 3 and 4 of life. Dividing by N_{t-1} allows us to rewrite the resource constraint in intensive terms:

$$F\left(k_t, 1 + \frac{\alpha}{n}\right) = c_t + \frac{d_t}{n} + \frac{b_t \pi}{n^2} + n k_{t+1} \tag{7}$$

2.3 Markets

We suppose that the economy is perfectly competitive, so that production factors are paid at their marginal productivity. Given that $L_t - \alpha N_{t-2} = N_{t-1}$ and that $k_t \equiv \frac{K_t}{N_{t-1}} = \frac{K_t}{L_t - \alpha N_{t-2}}$, we have:

$$\begin{aligned} w_t &= \frac{\partial F\left(K_t, L_t\right)}{\partial L_t} = \frac{\partial \left(L_t - \alpha N_{t-2}\right) F\left(\frac{K_t}{L_t - \alpha N_{t-2}}, \frac{L_t}{L_t - \alpha N_{t-2}}\right)}{\partial L_t} \\ &= \left[F\left(k_t, 1 + \frac{\alpha}{n}\right) - k_t F_k\left(k_t, 1 + \frac{\alpha}{n}\right)\right]\left[\frac{n}{n + \alpha}\right] \end{aligned} \tag{8}$$

$$R_t = \frac{\partial F\left(K_t, L_t\right)}{\partial K_t} = \frac{\partial N_{t-1} F\left(k_t, 1 + \frac{\alpha}{n}\right)}{\partial N_{t-1} k_t} = F_k\left(k_t, 1 + \frac{\alpha}{n}\right) \tag{9}$$

where w_t denotes the wage rate and R_t is the return on savings at period t.

We also suppose that there exists a perfect annuity market with actuarially fair returns, so that the gross return on savings $\hat{R}_t$ equals:

$$\hat{R}_t = \frac{R_t}{\pi} \tag{10}$$

3 The laissez-faire

The second-period budget constraint is:

$$c_t + s_t = w_t \tag{11}$$

where s_t is savings in the first period of labor.

The third-period budget constraint is:

$$d_{t+1} + z_{t+1} = w_{t+1} + s_t R_{t+1} \tag{12}$$

The fourth-period budget constraint is:

$$b_{t+2} = \frac{z_{t+1} R_{t+2}}{\pi} \tag{13}$$

where z_{t+1} is savings in second period of labor.

Hence the intertemporal budget constraint is:

$$c_t + \frac{d_{t+1}}{R_{t+1}} + \frac{\pi b_{t+2}}{R_{t+1}R_{t+2}} = w_t + \frac{w_{t+1}}{R_{t+1}} \tag{14}$$

Conditionally on beliefs on future factor prices w^e_{t+1}, R^e_{t+1} and R^e_{t+2}, the problem of individuals can be written as:

$$\begin{aligned} &\max_{c_t,d_{t+1},b_{t+2}} u\left(c_t\right) + \beta u(d_{t+1}) + \pi\beta^2 u\left(b_{t+2}\right) \\ &\text{s.t. } w_t + \frac{w^e_{t+1}}{R^e_{t+1}} = c_t + \frac{d_{t+1}}{R^e_{t+1}} + \frac{\pi b_{t+2}}{R^e_{t+1}R^e_{t+2}} \end{aligned}$$

The first-order conditions yield:

$$\frac{u'(c_t)}{\beta u'(d_{t+1})} = R^e_{t+1} \tag{15}$$

$$\frac{u'(d_{t+1})}{\beta u'\left(b_{t+2}\right)} = R^e_{t+2} \tag{16}$$

Individuals save in such a way as to equalize the marginal rate of substitution of two successive periods with the expected rate of return on savings. Higher impatience (i.e. a lower β) pushes, for a given interest factor, towards more consumption early in life.

4 The long-run social optimum

Let us now focus on the long-run social optimum in our economy. We will follow here Samuelson's approach (Samuelson 1975), which consists in studying the problem of a social planner choosing consumptions, capital and fertility, in such a way as to maximize the expected lifetime welfare of an agent living at the stationary equilibrium.[1]

4.1 The social planner's problem

Let us assume that there exists a unique stable stationary equilibrium in our economy. The social planner's problem can be written as follows:

$$\begin{aligned} &\max_{c,d,b,k,n} u(c) + \beta u(d) + \pi\beta^2 u\left(b\right) \\ &\text{s.t. } F\left(k, 1+\frac{\alpha}{n}\right) = c + \frac{d}{n} + \frac{b\pi}{n^2} + nk \end{aligned}$$

[1] Note that this objective is here similar to maximizing the average *ex post* (i.e. realized) lifetime well-being among agents living at the steady-state (some agents being short-lived, whereas others are long-lived).

An interior optimum $(c^*, d^*, b^*, k^*, n^*)$ satisfies the following FOCs:

$$\frac{u'(c^*)}{\beta u'(d^*)} = \frac{u'(d^*)}{\beta u'(b^*)} = n^* \tag{17}$$

$$F_k\left(k^*, 1+\frac{\alpha}{n^*}\right) = n^* \tag{18}$$

$$k^* + \frac{\alpha F_L\left(k^*, 1+\frac{\alpha}{n^*}\right)}{(n^*)^2} = \frac{d^*}{(n^*)^2} + \frac{2b^*\pi}{(n^*)^3} \tag{19}$$

The first condition states that, at the long-run social optimum, the marginal rate of substitution between consumption at two successive periods is equal to the optimal fertility rate. The second condition is the standard Golden Rule of capital accumulation (Phelps 1961), stating that the optimal capital equalizes the marginal productivity of capital net of depreciation and the fertility rate. The third condition characterizes the (interior) socially optimal fertility rate. It depends on three forces. When α is equal to zero, so that only young adults are productive, the optimal fertility rate equalizes, at the margin, the capital dilution effect or Solow effect (first term of LHS) with the intergenerational distribution effect or Samuelson effect (RHS). The Solow effect states that, under a higher fertility, it is more difficult to sustain a large capital to labor ratio at the stationary equilibrium. This effect pushes towards less fertility. The Samuelson effect states that a higher fertility relaxes the economy's resource constraint, by making the elderly's consumption relatively less sizeable. This second effect pushes towards a higher fertility. Note that the size of this effect depends on the age-structure, through the parameter π.[2]

But besides those two effects, there is another, third determinant of optimal fertility, represented by the second term of the LHS. Clearly, if the productivity of old workers is zero (i.e. $\alpha = 0$), this third effect is absent, since all productive workers are young, and thus changing fertility has no effect on the composition of the labor force. In that case, the optimal fertility equalizes, at the margin, the capital dilution effect and the Samuelson effect, as in Samuelson's pioneer work (Samuelson 1975). However, under $\alpha > 0$, a third effect is at work. Given that this effect depends on the composition of the labor force by age, let us call this effect the "labor age-composition effect".

To understand this labor age-composition effect, note first that an increase in n reduces the total amount of labor per young worker, and, hence, reduces the output per young worker. Thus the immediate effect of a rise in n is a fall of the ratio total labor force / young workers (i.e. $\frac{N_{t-1}+\alpha N_{t-2}}{N_{t-1}} = \frac{n+\alpha}{n}$), which leads to a fall of productivity per young worker. Note, however, that the size of this negative productivity effect depends itself on the extent of decay in old workers' skills. Obviously, as already mentioned, when $\alpha = 0$, a rise in the fertility rate has no effect on productivity per young worker. However, when $\alpha > 0$, a rise in n will affect the ratio total labor force / young workers, leading to a variation in productivity per young worker. Given that the total effect is equal to

[2] The necessity to raise fertility when longevity increases raise the number of inactive persons is also emphasized in Sauvy (1959).

$\frac{\alpha F_L\left(k^*,1+\frac{\alpha}{n^*}\right)}{(n^*)^2}$, an important question that arises is whether this term is increasing or decreasing in α. We know that it is equal to 0 when α equals 0. However, a rise in α tends to increase the first term of the product $\alpha F_L\left(k^*, 1+\frac{\alpha}{n^*}\right)$, and to reduce the second term of the product $\alpha F_L\left(k^*, 1+\frac{\alpha}{n^*}\right)$. We thus have two opposite influences of decay on the size of the labor composition effect. When the first influence dominates the second, a larger decay of old workers' skills (i.e. a lower α) contributes to reduce the negative productivity effect induced by a higher n, leading, *ceteris paribus*, to a higher optimal fertility n^*. But when the second influence dominates the first, a larger decay raises the negative productivity effect induced by a higher n, leading, *ceteris paribus*, to a lower optimal fertility n^*.

When Sauvy (1962) argued that the ageing of the workforce could lead to an ageing of production techniques, and, hence, to a reduction of total labor productivity, he probably had in mind that an increase in the degree of decay in old worker's skills would necessarily make a quicker renewal of the workforce more desirable, leading to a larger socially desirable fertility. This way of thinking is quite intuitive: when old workers'skill become more old-fashioned or depreciated, it makes sense to desire a higher fertility rate. This kind of rationale is possible in our framework, through the labor age-composition effect, but provided the fundamentals of the economy are such that the product $\alpha F_L\left(k^*, 1+\frac{\alpha}{n^*}\right)$ is indeed *increasing* with α. Otherwise, when the product $\alpha F_L\left(k^*, 1+\frac{\alpha}{n^*}\right)$ is decreasing with α, a higher decay justifies a lower fertility, unlike Sauvy's intuition.

In the light of all this, the interior optimal fertility rate equalizes at the margin, on the one hand, the sum of the capital dilution effect and the labor composition effect, and, on the other hand, the intergenerational redistribution effect. Those three effects can also be called, respectively, the Solow effect, the Sauvy effect and the Samuelson effect. It is thus possible, by merely introducing two generations of workers and the possibility of decay of old worker's skills, to incorporate Sauvy's intuitions into the neoclassical model of optimal fertility.

Finally, it should be stressed that our discussion assumed implicitly that the optimal fertility rate is an interior optimum. As this is well-known in the literature (see Deardorff 1976, Michel and Pestieau 1993), the optimum fertility rate in a Samuelsonian economy is not necessary an interior one. As shown by Michel and Pestieau (1993), interiority holds only when there is enough complementarity between consumption at the different ages of life, and when there is enough complementarity between capital and labor in the production process.

Two kinds of corner solutions can arise. First, it can be the case that:

$$k^* + \frac{\alpha F_L\left(k^*, 1+\frac{\alpha}{n^*}\right)}{(n^*)^2} > \frac{d^*}{(n^*)^2} + \frac{2b^*\pi}{(n^*)^3} \tag{20}$$

for any value of n^*, in which case the marginal utility gain from increasing fertility associated to the Samuelson effect is always lower than the marginal utility loss due to the capital dilution effect and the labor age-composition effect, implying that the optimal fertility rate is equal to 0. This case arises when consumptions at the old age do not really matter for individual well-being (because of a high elasticity of intertemporal substitution in consumption), which pushes the intergenerational redistribution effect down.

On the contrary, when we have

$$k^* + \frac{\alpha F_L\left(k^*, 1 + \frac{\alpha}{n^*}\right)}{\left(n^*\right)^2} < \frac{d^*}{\left(n^*\right)^2} + \frac{2b^*\pi}{\left(n^*\right)^3} \tag{21}$$

for any value of n^*, the solution is the other corner: the optimal fertility is infinite. Indeed, in that case, the welfare loss from increasing fertility associated to the capital dilution effect is, whatever the level of fertility, lower than the marginal utility gain from increasing fertility due to the intergenerational redistribution effect. In that case, the low impact of capital dilution can arise from the high degree of substitutability between capital and labor, which makes further rises in fertility always beneficial.

Those two corner solutions look pathological, but, as we shall see when calibrating the model, corner solutions arise quite often when considering optimal fertility in a Samuelsonian economy, in line with what Michel and Pestieau (1993) proved analytically.

4.2 The Serendipity Theorem

First stated by Samuelson (1975), the Serendipity Theorem states that, if there exists a unique stable stationary equilibrium in a two-period OLG model with physical capital, then the perfectly competitive economy will converge towards the long-run social optimum provided the optimal fertility rate is imposed on individuals.

Does this result still hold in our four-period OLG setting? To check this, let us rewrite the problem of individuals living at the steady-state. That problem is:

$$\max_{c,d,b} u\left(c\right) + \beta u(d) + \pi\beta^2 u\left(b\right)$$
$$\text{s.t. } w + \frac{w}{R} = c + \frac{d}{R} + \frac{\pi b}{R^2}$$

The first-order conditions yield:

$$\frac{u'(c)}{\beta u'(d)} = \frac{u'(d)}{\beta u'\left(b\right)} = R \tag{22}$$

Hence, if the social planner fixes n such that $F_k\left(k^*, 1 + \frac{\alpha}{n^*}\right) = n^*$ where k takes its socially optimal level, then individuals, being price-takers, will choose their savings optimally, since the above FOC will then coïncide with the FOC of the social optimum.

5 Numerical illustrations

In his report, Sauvy argued that fertility in Wallonia was suboptimal, and that this should be raised by 20 %. This section aims at providing answers to the following questions. On the basis of the model developed above, was Sauvy right when claiming that the actual fertility rate in Wallonia in 1950s and 1960s was below the socially optimum one? What about the recommendation of a 20 % rise in fertility?

In order to answer that question, this section calibrates the model presented above, and find the optimal fertility rate. It is then compared with the one prevailing in Wallonia at that time, and with the one recommended by Sauvy.

5.1 Data

At this stage, it is worth looking at the dynamics of fertility in Belgian regions since the 19th century. As shown by Capron et al (1998), the total fertility rate (TFR) has strongly declined in Wallonia since 1800. At that time, the TFR was about 4.5 children per women. The decline was first smooth, with about 4 children per women in 1860, but then strongly declined, to reach 2 children per women in 1930. Then, the TFR fluctuated between 2.5 in 1960 to reach 1.5 in the early 1980s.

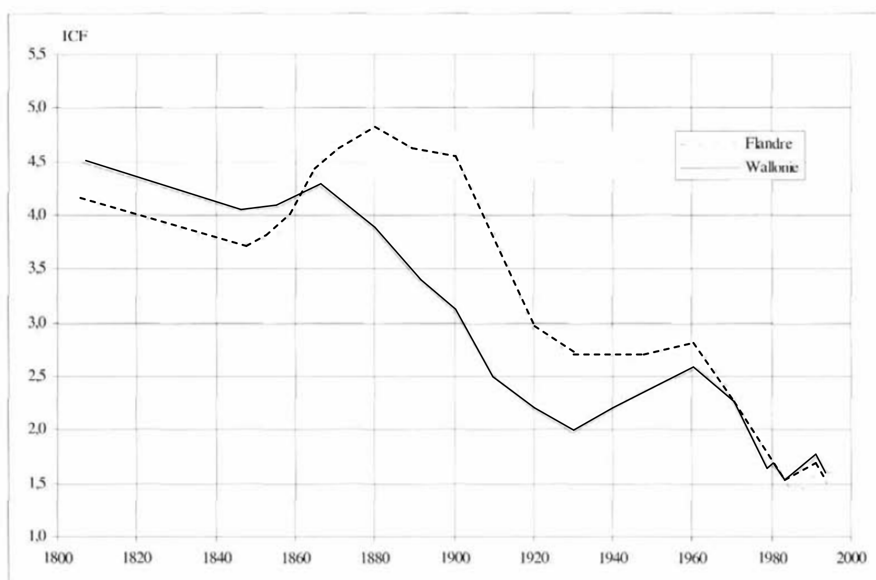

Figure 1: TFR in Flanders and Wallonia (Source: Capron et al 1998 p. 264).

Flanders's TFR followed a quite different dynamics: it declined slightly in the first part of the 19th century, but, then, it grew to reach about 4.7 children per women in 1880. In 1900, the fertility gap between Flanders and Wallonia was equal to about 1.5 children per women. That gap has reduced progressively after 1900, when Flanders' TFR has progressively converged towards the one prevailing in Wallonia. Note that, in 1960, the TFR in Flanders was still superior to the one prevailing in Wallonia (2.8 against 2.5 children per women).

5.2 Functional forms

In order to check to what extent fertility in Wallonia around 1960 can be regarded as suboptimal from the perspective of the model developed above, we need first to impose some particular functional forms on the utility function and the production function.

Regarding the temporal utility function, we use the standard CIES form:

$$u\left(c_t\right) = \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \tag{23}$$

where $\sigma > 0$ and $\sigma \neq 0$.

From the FOCs for optimal consumption profiles, we have $\frac{c_t^{-\frac{1}{\sigma}}}{\beta d_t^{-\frac{1}{\sigma}}} = n$ and $\frac{d_t^{-\frac{1}{\sigma}}}{\beta b_t^{-\frac{1}{\sigma}}} = n$, from which we obtain:

$$c_t = \left[n\beta d_t^{-\frac{1}{\sigma}}\right]^{-\sigma} \text{ and } b_t = \left[\frac{d_t^{-\frac{1}{\sigma}}}{\beta n}\right]^{-\sigma}$$

Substituting for those expressions in the economy's resource constraint yields:

$$F\left(k_t, 1+\frac{\alpha}{n}\right) = \left[n\beta d_t^{-\frac{1}{\sigma}}\right]^{-\sigma} + \frac{d_t}{n} + \frac{\left[\frac{d_t^{-\frac{1}{\sigma}}}{\beta n}\right]^{-\sigma}\pi}{n^2} + nk_t \tag{24}$$

The production function takes a CES form:

$$\bar{F}\left(K_t, L_t\right) = A\left[\gamma K_t^{-\rho} + (1-\gamma)L_t^{-\rho}\right]^{-1/\rho} \tag{25}$$

where $A > 0$, $0 < \gamma < 1$, $\rho > -1$, $\rho \neq 0$. Assuming a full depreciation of capital after one period, output per young worker is here:

$$\begin{aligned} y_t \quad &\equiv \quad \frac{Y_t}{N_{t-1}} = \frac{A\left[\gamma\left(N_{t-1}k_t\right)^{-\rho} + (1-\gamma)\left(N_{t-1}+\frac{\alpha N_{t-1}}{n}\right)^{-\rho}\right]^{-1/\rho}}{N_{t-1}} \\ &= \quad A\left[\gamma\left(k_t\right)^{-\rho} + (1-\gamma)\left(1+\frac{\alpha}{n}\right)^{-\rho}\right]^{-1/\rho} \end{aligned} \tag{26}$$

Hence we have

$$F_k\left(k_t,1+\frac{\alpha}{n}\right) = A\left[\gamma\left(k_t\right)^{-\rho}+(1-\gamma)\left(1+\frac{\alpha}{n}\right)^{-\rho}\right]^{-(1/\rho)-1}\gamma\left(k_t\right)^{-\rho-1} \tag{27}$$

$$\begin{aligned} F_L\left(k_t,1+\frac{\alpha}{n}\right) &= \left[F\left(k_t,1+\frac{\alpha}{n}\right)-k_tF_k\left(k,1+\frac{\alpha}{n}\right)\right]\left[\frac{n}{n+\alpha}\right] \\ &= \left[\frac{n}{n+\alpha}\right]\left[\begin{array}{l} A\left[\gamma\left(k_t\right)^{-\rho}+(1-\gamma)\left(1+\frac{\alpha}{n}\right)^{-\rho}\right]^{-1/\rho} \\ -A\left[\gamma\left(k_t\right)^{-\rho}+(1-\gamma)\left(1+\frac{\alpha}{n}\right)^{-\rho}\right]^{-(1/\rho)-1}\gamma\left(k_t\right)^{-\rho}\end{array}\right] \end{aligned} \tag{28}$$

Hence, from the Golden Rule, we have:

$$\begin{aligned} n &= A\left[\gamma\left(k_t\right)^{-\rho}+(1-\gamma)\left(1+\frac{\alpha}{n}\right)^{-\rho}\right]^{-(1/\rho)-1}\gamma\left(k_t\right)^{-\rho-1} \\ \Longleftrightarrow &\quad k_t=\frac{\left(\frac{n}{A\gamma}\right)^{\frac{-1}{2}}-\gamma}{(1-\gamma)\left(1+\frac{\alpha}{n}\right)^{-1}} \end{aligned} \tag{29}$$

Thus, for any level of fertility n, we can deduce the optimal level of k by means of the Golden Rule expression. Then, given the levels of n and k, the new formulation of the resource constraint yields a unique level of second-period consumption d, from which we can find the remaining variables c and b.[3]

5.3 Calibration

The model is a 4-period model. Each period lasts 20 years. Regarding the survival probability π, we use life expectancy estimates around 1960. Life expectancy in 1960 is equal, for both men and women, to 69.63 years. This implies that π is equal to $9.63/20 = 0.48$.

Let us now calibrate preference parameters. Regarding the time preference parameter β, we use the standard approach in the literature: given a quarterly discount factor equal to 0.99, we obtain $\beta = (0.99)^{80} = 0.45$. Regarding σ, we use the empirical estimates of the elasticity of intertemporal substitution. According to Browning at al (1999), the elasticity of intertemporal substitution is slightly above unity. We take here the estimate of Blundell et al (1994) and fix $\sigma = 1.25$.

[3] Indeed, from the resource constraint, we have:

$$\begin{aligned} F\left(k,1+\frac{\alpha}{n}\right) &= \left[n\beta d^{-\frac{1}{\sigma}}\right]^{-\sigma}+\frac{d}{n}+\frac{\left[\frac{d_t^{-\frac{1}{\sigma}}}{\beta n}\right]^{-\sigma}\pi}{n^2}+nk \\ \Longleftrightarrow &\quad d=\frac{F\left(k,1+\frac{\alpha}{n}\right)-nk}{\left[[n\beta]^{-\sigma}+\frac{1}{n}+\frac{\left[\frac{1}{\beta n}\right]^{-\sigma}\pi}{n^2}\right]} \end{aligned}$$

As far as the production process is concerned, three parameters must be calibrated: A, γ and ρ. In order to calibrate those parameters, we follow de la Croix and Michel (2002, p. 340), who calibrate jointly those three parameters. Those authors fix $A = 20$ and $\rho = 1$, which implies an elasticity if substitution between capital and labor equal to 0.50. Finally, in order to obtain a labor share in production equal to 2/3, those authors fix $\gamma = 0.49$.

We need also to calibrate the depreciation of physical capital δ and the depreciation of human skills α. Regarding physical capital depreciation, we assume, given the duration of periods (20 years), that there is full depreciation of capital after one period of use (i.e. $\delta = 1$). Regarding the parameter of old workers skills decay α, empirical studies on the age/productivity relation are far from unanimous, as discussed in the recent study by van Ours and Stoeldraijer (2010). Those authors highlight that the existing literature presents quite contradictory results on the age/productivity relationship. According to Johnson (1993), most employers and employees believe that average labor productivity declines after some age between ages 40 and 50. Skirbekk (2003) argues that job performance declines after age 50. But Aubert and Crépon (2007) found that productivity increases until age 40-45 and then remains stable after that age. Finally, Göbel and Zwick (2009) find that productivity grows until age 40-45, and then stabilizes until age 60. Given those contradicting results, we will take $\alpha = 1$ as a benchmark case, and consider alternative values for α as a way to show how Sauvy's intuition can affect optimal fertility in our model.

The following table summarizes the calibration of parameters taken as a benchmark.

parameters	α	β	σ	π	A	γ	ρ	δ
values	1.00	0.45	1.25	0.48	20	0.49	1.00	1.00

5.4 Results

This subsection computes the socially optimal fertility rate in our economy, and compares it with the actual fertility rate for Wallonia in 1960, equal to 2.6 children per women, which, in normalized terms, is equal to $n = 1.3$. For that purpose, we will proceed as follows. For each possible fertility rate within a biologically feasible interval $n \in [0, 6]$, we derive the associated optimal level of k, and, then, the optimal levels of c, d and b, as well as the associated average lifetime welfare. We then naturally find the level of fertility which leads to the maximum average lifetime well-being, and we compare it with the actual fertility. We then repeat that procedure for different sets of structural parameters, to assess the robustness of our results.

A first important numerical result is the fact that, under the benchmark calibration proposed in the previous subsection, the optimal fertility rate is a corner solution at $n = 0$, whatever the postulated level of old skills'decay (α) is. This first result is shown on Figure 2.

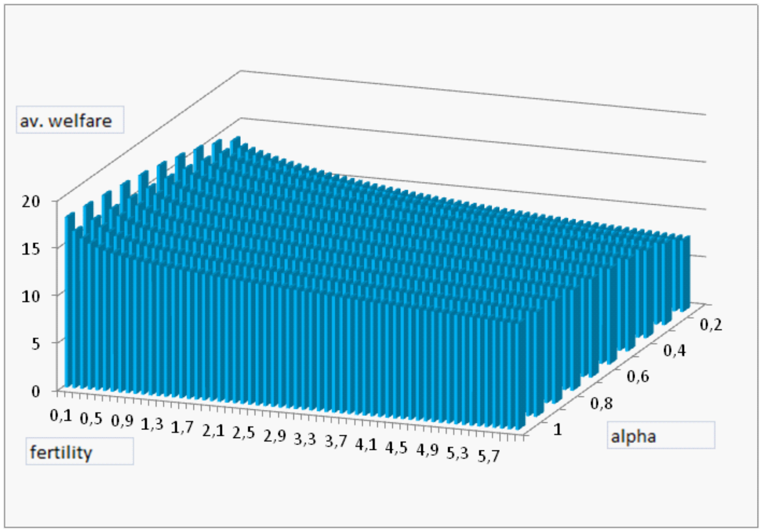

Figure 2: Average lifetime welfare as a function of fertility n and decay α (benchmark values).

This result, which may appear surprising at first glance, is not surprising for economists studying optimal fertility in an OLG setting. Indeed, since the study by Michel and Pestieau (1993), economists know that the existence of an interior optimal fertility rate within a Diamond-type economy requires that the elasticity of intertemporal substitution is sufficiently low and that complementarity between capital and labor is sufficiently high. However, given the postulated parameters' values, there remains too much substitutability, leading to a corner solution for fertility.

Let us now consider alternative calibrations, involving a lower value of the elasticity of intertemporal substitution σ. As shown on Figure 3 (where axes are reversed given the negative utility), assuming $\sigma = 0.5$ allows us to obtain an interior optimal fertility rate. Under $\alpha = 1$ (no decay), the optimal fertility is 0.5. The optimal n becomes slightly larger, equal to 0.6, when α decreases to 0.6. Note that the impact of α on the optimal fertility is non-monotonic, since for $\alpha = 0.1$ the optimal fertility goes back to 0.5. This result is in line with the theoretical discussion we had above. Since the product $\alpha F_L\left(k^*, 1 + \frac{\alpha}{n^*}\right)$ is non-monotonic in α, there is no reason to expect that the optimal fertility rate is necessarily increasing with the extent of decay in old worker's skills. Having stressed this, it remains true, in any case, that the optimal fertility is far lower than the one prevailing in Wallonia at that time (i.e. $n = 1.3$).

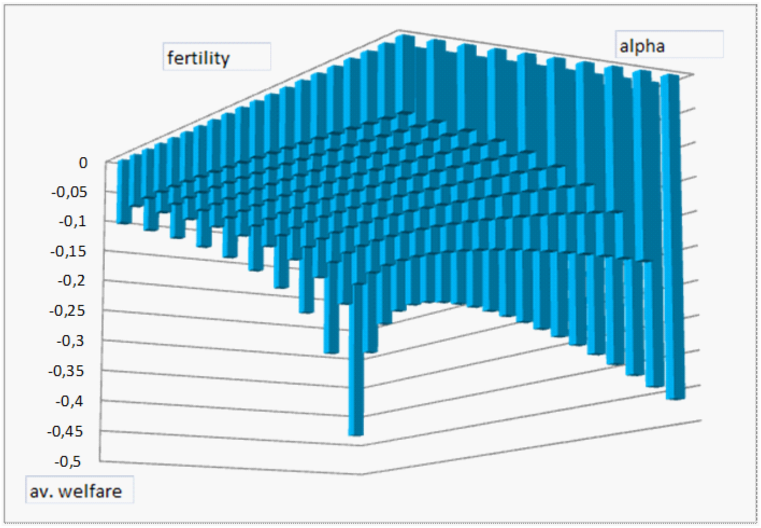

Figure 3: Average lifetime welfare as a function of fertility (n) and decay (α), under $\sigma = 0.5$.

Note that taking even lower values for σ does not substantially affect our results. Under $\sigma = 0.2$, the optimal fertility rate equals 0.8 when $\alpha = 1$, rises to 0.9 when $\alpha = 0.9$, but then goes down to 0.7 when $\alpha = 0.1$. Hence it appears, here again, that taking into account the decay of old worker's skills does not suffice, on its own, to lead to a large optimal fertility rate. This result is robust, and prevails even when we reduce strongly the elasticity of intertemporal substitution. One reason for this low level of optimal fertility lies in the relatively high level of impatience (i.e. $\beta = 0.45$), which, by construction, has the effect to weaken the strength of the Samuelson effect, and, hence, pushes optimal fertility down.

In the light of this, it seems hard to reconcile Sauvy's recommendation with the predictions of the model. True, it is possible, in theory, to account for the intuition that a higher decay of old workers skills may lead, in some cases, to justify a higher fertility rate. However, our numerical exercise suggests that, for reasonable values of the structural parameters, we obtain optimal fertility rates that are far below the one prevailing in Wallonia in 1950s and 1960s. Thus our numerical findings to not seem to support Sauvy's recommendation.

Finally, let us notice that, on the basis of the actual life expectancy prevailing in Wallonia, i.e. 79 years, implying π equal to $19/20 = 0.95$), the optimal fertility still remains, under $\sigma = 0.8$, quite low, and between 0.2 (when $\alpha = 1$) and 0.5 (when $\alpha = 0.3$). Thus better survival conditions push, *ceteris paribus*, towards a higher optimal fertility, by reinforcing Samuelson's effect. However, even under a high decay of old workers' skills, it is hard to regard fertility encouragement as welfare-improving in the long-run in our first-best setting.

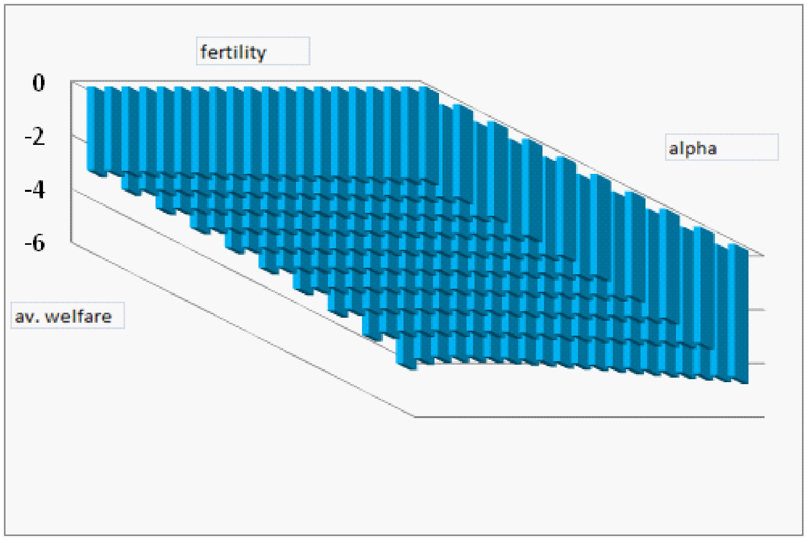

Figure 4: Average lifetime welfare as a function of fertility (α) and decay (n), under $\sigma = 0.8$ and $\pi = 0.95$ (contemporary survival conditions).

6 Conclusions

In sum, this paper provides a contrasted view on Sauvy's diagnosis on Walloon fertility in the early 1960s. True, it is possible to incorporate Sauvy's view on ageing of workers and productivity within a simple OLG model *à la* Samuelson. Once taking the decay of human skills into account, the optimal fertility rate equalizes, at the margin, the sum of the capital dilution effect (Solow effect) and the labor age-composition effect (Sauvy effect) with the intergenerational redistribution effect (Samuelson effect). There is thus some simple way to account for issues of old workers productivity when discussing optimal fertility in a Samuelsonian economy.

However, when turning to numbers, it appears that even large levels of decay in old workers' skills do not suffice to support Sauvy's views on Walloon fertility. Our numerical simulations suggest that, even though a higher extent of decay leads, in general (but not always), to a larger optimal fertility, the levels of optimal fertility rates remain far below the observed ones, and also below the ones recommended by Sauvy in his report.

One should not take these results as a criticism of Sauvy's thought on population. Quite the contrary, our calculations suggest that the rationale used by Sauvy to define the optimal fertility could not take, as a unique social objective, the maximization of average social welfare in Wallonia at that time. Obviously, some other considerations, either for numbers, or for culture and knowledge, or for what he called power, were playing a role in his recommendations. Hence, before saying that he was right or wrong, we need first to consider how those other concerns could be properly included in an exhaustive economic analysis of optimal fertility. This remains to be done.

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