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Voluntary agreements under endogenous legislative threats and imperfect enforcement

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Voluntary agreements under endogenous legislative threats and imperfect enforcement

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Abstract

The paper analyzes whether voluntary agreements with polluters (VAs) are able to achieve an efficient level of environmental protection when they are obtained under the legislative threat of a stricter pollution quota. The Congress responsible for setting the quota is subject to lobbying by the polluter. Furthermore, the VA contract is generally not legally binding in reality. We model accordingly the enforcement of the VA target. It is shown that a VA never achieves the socially optimal outcome. However, it leads to a more efficient level of pollution abatement than the legislative pollution quota in some cases. In particular, the lobbying responsiveness of the Congress plays an ambiguous role. When responsiveness is low, VAs dominates legislative intervention if the polluter experiences a low discount rate. The contrary is observed when responsiveness is high. These findings suggest a very selective use of VAs taking into account sector characteristics and the degree of lobbying influence in the Congress.

Keywords: environmental policy, voluntary agreements, bargaining, legislatures, enforcement

JEL classification: D72, Q28

1 Introduction

In the field of environmental policy, voluntary agreements (VAs) are probably the major policy innovation of the last decade. While they were marginal practices in a limited number of countries beforehand (e.g., in Germany, Japan), they are now used almost everywhere. They are particularly widespread in the field of climate change and waste policies. These approaches differ in certain respects but one common feature to all is that polluters voluntarily commit to pollution abatement activities. The use of the term "voluntary" has long been disputed since many agreements are in fact obtained under the threat of an alternative legislative intervention. The present paper focuses on such agreements preempting legislation.

The efficiency of the level of environmental protection achieved by VAs is a major practical concern. Many observers suspect that VAs bring very little genuine environmental improvements (e.g., see the recent review of the OECD, 2003). The suspicion is rooted in several features of actual VAs. First, they are voluntary suggesting that the polluters see the schemes as involving little abatement efforts. However, the argument does not apply as such to threat-driven VAs for which the relevant benchmark is not a 'do-nothing' scenario but a legislative threat. Firms do not enter in a VA because the scheme is cheap but because it is cheaper than the alternative legislative option. The fact that almost all VAs are not legally binding is a second source of suspicion. Therefore, the regulator has no legal tools to enforce the firms' commitments.[1]

In order to explore the seriousness of these concerns, we develop a model of VA between a polluter and a regulator in order to examine whether the VA can lead to an efficient level of pollution abatement and how this level compares to both the first best level and the level that might have been imposed legislatively. The threat is a pollution quota. The model explicitly describes the legislative process whose the quota is the outcome. We make the hypothesis that the polluter lobbies the Congress, thereby reducing the stringency of mandated abatement. Note that the existence of lobbying under the legislative route is a necessary condition for a VA to exist. Otherwise, the Congress would implement the first best legislation and the regulator would have absolutely no reason to rely on VAs. In addition, in contrast with the existing literature, we assume the VA contract to be non-binding. As a result, the regulator can only punish the polluter by implementing the threat in a future period in case of non compliance. In this context,

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[1] Another concern is that the benefit of threat preemption may have public good features. It can lead to free riding which possibly undermines the cooperation between polluters. For instance, it occurs in certain VAs which involve a group of firms, typically represented by a sector association. Our analysis is not tailored to address this issue.

the polluter's propensity to comply is basically driven by the discounting rate applied to the future punishment cost. In this politically constrained world, the regulator must choose between two evils: either using an imperfectly enforced VA with the necessity to reduce the environmental strictness of the VA relative to the first best to obtain the consent of the polluter and/or to increase compliance probability, or implementing a politically distorted pollution quota.

We find out that the efficiency of the VA relative to legislative intervention is ambiguous. In this respect, two variables are critical: the responsiveness of the Congress to lobbying and the discount rate of the polluter. VAs dominate legislative intervention in two contrasted contexts. First, they are efficient if (i) the Congress is weakly responsive to lobbying and (ii) the polluter discounts the future cost of the punishment at a low rate. Symmetrically, they are efficient in the opposite context in which responsiveness is high and the polluter's discount rate is high.

The underlying intuition can be summarized as follows. Key is that responsiveness to lobbying yields two opposite effects. The first direct effect is obvious: the lower the responsiveness, the more efficient the legislative quota in comparison with the VA target. The second is the indirect effect on VA enforcement. As the punishment consists in the implementation of the legislative quota in a future period, a strict quota also facilitates compliance with the VA by increasing sanction costs, and thereby enhances VA efficiency. In addition, note that the second effect decreases in the discount rate. As a result, the direct effect tends to dominate the indirect effect - meaning that legislation dominates VAs - when the discount rate is high while the contrary occur at lower discount rates.

The rest bof the paper is structured as follows. Section 2 discusses how the paper relates to the literature. In section 3, we consider the benchmark case in which the Regulator is perfectly informed about the compliance behavior of the polluter. We establish that results are ambiguous, except in the particular case where enforcement is perfect. In this case, the VA is systematically more efficient than the legislative quota. What drives this clear-cut is that the lobbying cost borne by the polluter under the legislative route is high enough to make him willing to a stricter VA than the legislative quota in order to avoid this cost. Section 4 develops the full model, assuming that information on the discount rate of the polluter is asymmetric. Section 5 summarizes the results and discuss policy implications.

1.1 Related research

As VAs have only been developed recently and by practitioners, the theoretical literature is still limited but it is growing rapidly. The majority of the papers deals with the case, similar to ours, where the motivation of the polluters to accept voluntary agreements is the preemption of future regulations. A few papers have analyzed VAs obtained in exchange of a subsidy (see for instance Carraro and Siniscalco, 1996, Lyon and Maxwell, 2003)[2]. The underlying economic mechanisms driving this types of VA are very different. In what follows, we restrict our discussion to preemptive VAs.

The standard approach to the problem is to assume that the regulator maximizes welfare but the existence of political constraints hinders the implemention of her first best policy through a traditional mandatory legislation.[3] In the paper by Segerson and Micelli (1999), the political constraint is reflected in a given exogenous probability p ($p < 1$) with which the threat is implemented. In a more recent model, Lyon & Maxwell (2003) complexify the Segerson and Miceli's approach by assuming that the probability decreases in the cost borne by the polluters under the threat scenario, suggesting that they lobby against the legislation. Another paper by Maxwell, Lyon and Hackett (2000) partly endogeneize the political constraint by modelling explicitly lobbying activities using the influence function pioneered by Becker (1983).

Our paper adopts this standard approach. We go further than Maxwell et al. (2000) in the endogenization of the political constraints by using the well established Grossman & Helpman approach of lobbying. Contrary to Becker where lobbying is represented as a function transforming lobbying expenditures to the policy outcome (1983), this approach in the political "black box" by modelling lobby groups and the regulator's decisions. This modelling choice has direct impacts on the results obtained (in particular, in Proposition 1).

Also, we make the additional assumption that the VA contract is non binding so that the only sanction available to the regulator is the ex post implementation of the legislation. As noticed above, almost all actual voluntary commitments present this property. This assumption leads to a more complex relationship between the strictness

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[2] In fact, the model by Lyon and Maxwell (2003) includes two voluntary agreements in a three-stage game. In the first stage, the firms unilaterally commit to abate pollution under the threat of a tax set in a second stage. In a third stage, the regulator offers a subsidy to companies willing to participate to a voluntary program. We discuss below the results of the paper pertaining to the premptive VA of the first stage.

[3] The only exception is Hansen's model (1999) where polluter negotiates a VA with a politically biased regulator under a legislative threat which could be implemented by a welfare maximizing Congress. The divergence between the regulator's and the Congress' objective situation creates the possibility for a mutually beneficial agreement when polluter's and regulator's interests are aligned.

of political distortions and VA efficiency. Political distortions negatively affect legislative outcomes, but they also damage the performance of the VA route by increasing the risk of non compliance.

We complete this brief review with the paper of Manzini and Mariotti (2003). They propose a model of preempting negotiated agreement involving several firms but with a very different focus from the papers discussed so far. Central is the assumption that the regulator experiences a disutility in case of legislative intervention. The model then takes for granted that a VA will emerge. The question is then how the surplus will be shared between the firms and the regulator. They answer the question by modelling very cautiously the bargaining procedure, showing that the outcome is essentially determined by the firm with the most aggressive attitude towards environmental control.The analysis is thus very complementary to the other papers in that they start the analysis at the point where the others stop.

2 Overview of the model

We depict a policy game with three players: a benevolent environmental regulator, a polluter and a Congress responsible for enacting legislation. The regulator R and the polluter P can agree to make a voluntary agreement specifying a pollution abatement level, denoted B, to be met by the polluter. In case of persisting disagreement, the regulator can ask the Congres to enact a legislation. What makes the problem non trivial is that the Congress is subject to lobbying.

The design of VAs encountered in reality is very diverse. The OECD distinguishes three broad categories (1999). Each type ultimately differs with respect to the degree of involvement of the regulator. Under public voluntary programs, the firms agree to make abatement efforts to meet goals which are established by the regulator. In a negotiated agreement, the polluter and the regulator jointly devise the commitments through bargaining. Under self-regulation or unilateral commitments, the polluter takes the initiative. He freely sets up a program of environmental actions without any formal influence from the regulator.

In our model, we essentially impose the participation constraints of the polluter and the regulator to hold so that the VA can either be a negotiated agreement or a public voluntary program. Self-regulation is also a possible application of the model since the preemption of legislation by self regulatory actions necessarily requires the implicit consent of the regulator. A further preliminary remark is that, in practice, certain VAs are signed with a coalition of polluters represented by an industrial branch association. The model is sufficiently general for the polluter to be either a single firm or an industry. In that case, we simply assume that the members of the coalition have solved their collective action problems.

Abating pollution entails a cost borne by the polluter which is described by an increasing and convex function $C(B)$. It also generates a benefit in terms of avoided environmental damage. For the sake of simplicity, we assume that the benefit is equal to the abatement level B. The linearity of the benefit function simplifies the analysis without altering any results. We further assume that $C\prime(0) < 1$ and $C(0) = 0$. These hypotheses imply that the first best policy of the regulator involved a level of abatement B^* defined by the interior solution:

$$C'(B^*) \equiv 1 \tag{1}$$

The agreement is obtained under the threat of an alternative policy. We endogeneize the threat by modelling explicitly the legislative process whose the threat policy is the outcome. More specifically, the regulator can require the Congress to enact a new legislation.[4] The legislation consists in a pollution quota that mandates a level of abatement L. We do not grant any cost advantage to the VA: abatement costs the same under the VA and under the legislative quota. Doing otherwise would make it too easy to reach conclusions about the superiority of the VA.

We suppose that lobbying affects the legislative process. It is a key assumption that creates the possibility for the VA to be an alternative option to legislation. In the absence of political distortions, the welfare maximizing Congress would simply enact the efficient quota B^* defined by Eq. (1). In this context, the Regulator would have no reason to rely on a VA preempting his ideal policy.

We adopt the lobbying approach popularized by Grossman and Helpman. More specifically, we assume that the polluter is the only lobby group exerting an influence in the Congress by making campaign contributions to a median legislator.[5] Contributions can be in kind - by working for the legislators, by communicating, or by convincing citizens - or in cash. We assume the median legislator is the agenda setter in the Congress. As in Grossman and Helpman (2001), he maximizes his probability of reelection facing an implicit challenger by maximizing a weighed sum of the campaign

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[4] Strictly speaking, the Regulator is not the agenda setter in that she does not propose the bill. This is the responsibility of the legislature.

[5] Thus, we consider the case analyzed Grossmann & Helpman in Chapter 7 of their recent book (2001). Instead, we could consider that the polluter competes with a green lobby group. This common agency framework would complexify the analysis without altering the results. The author has implemented this alternative assumption in a rent seeking model (Glachant, forthcoming).

contribution and the social welfare. What we have in mind is a democratically elected legislator that during a term in Congress collects campaign contributions he will use in a later, un-modelled, election. In this situation, he is facing a trade-off between (i) higher campaign contributions that help him to convince undecided or uninformed voters but at the cost of distorting policy choices in favor of the contributing group and (ii) a higher social welfare which increases the probability of re-election, given that voters take their welfare into consideration in their choice of candidate. Formally, the legislator's utility function is

$$V(L, x) = \lambda W(L) + (1 - \lambda)x, \tag{2}$$

where x is the campaign contribution offered to the Legislator and $\lambda \in [0, 1]$, the exogenously given weight that the Legislator places on social welfare relative to the campaign contribution.[6]

The timing of the legislative subgame is as follows.

- The Regulator initiates the legislative process by requiring the Congress to adopt a legislative quota.
- The Polluter offers the median Legislator a campaign contribution schedule $x(L)$ which is contingent on the legislative quota L which will be adopted.
- Then, the Legislator proposes and ratifies the quota L and receives from the polluter the contribution associated with the policy selected. Although this is not a one-stage game, the polluter cannot renege on his promise in the last stage.

We turn finally to the enforcement issue. In reality, VAs are not legally binding except a few cases in the Netherlands and in Belgium. As a result, the regulator can only punish non-complying polluters by implementing the legislative threat. Accordingly, we suppose that, if the polluter fails to meet B, the regulator initiates the legislative process leading to the quota L. As enforcement takes place in a future period, we discount the cost of the punishment. Assuming discount factor δ $(0 < \delta < 1)$, the polluter complies only if

$$C(B) \leq \delta \left[C(L) + x(L) \right] \tag{3}$$

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[6] We can show that, if we assign the agenda setting role to the (benevolent) Regulator, this would suppress the political distortions in the legislative route. Given the Legislator utility function of Eq.(2), the Regulator would propose the first best quota B^* which the legislator will adopt since, for any B and x, $V(L, x) \geq 0$.

Note that discounting potentially lead the polluter to adopt a "delaying" strategy whereby he signs a VA with intent not to comply, just to postpone legislative action. We develop two versions of the model. In the first version, δ is common knowledge whereas, in the second one, δ is a random variable whose realization is known by the polluter while the Regulator only knows its distribution.

The decision tree of Figure 1 summarizes the model.

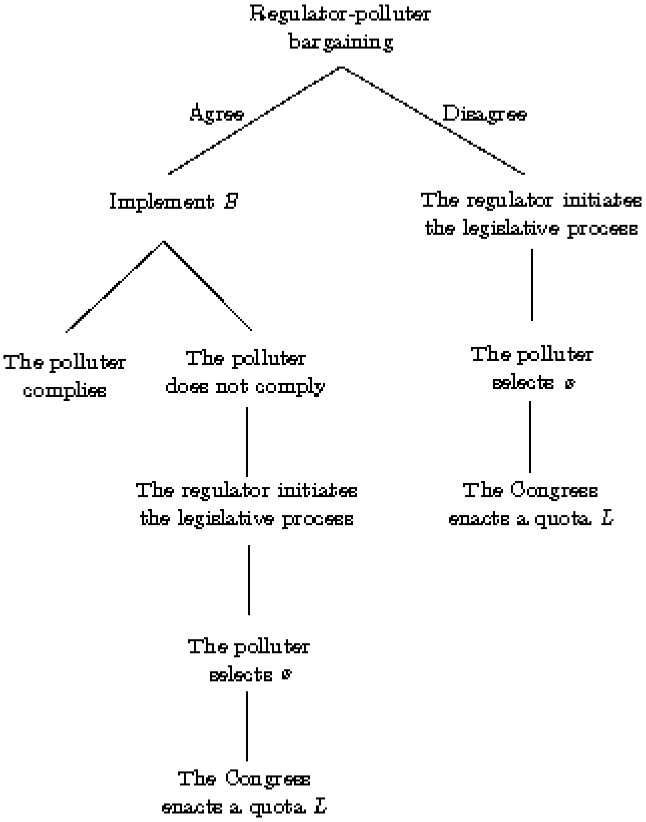

Figure 1: Decision tree of the game

3 The perfect enforcement case

As a benchmark, we first investigate the perfect enforcement case. The case is also interesting as such since perfect enforcement is the standard assumption in the existing literature.

3.1 The legislative sub game

We reason backward with the analysis of the legislative sub-game. Recall that the median Legislator's utility is $V(L,x) = \lambda W(B) + (1-\lambda)x$. Any feasible contribution should leave him with at least the same utility under the policy B as he would achieve with no contribution. Otherwise, he would reject the offer and implement his ideal policy when $x = 0$, that is B^*. Consider now the polluter. As his utility $C(L) + x$ decreases in x, he simply offers the legislator the lowest possible contribution x, which is implicitly defined by $V(L,x) = V(B^*,0)$. Developing and rearranging the equality, we obtain the equilibrium campaign contribution contingent on the quota which will be adopted:

$$x(L) = \frac{\lambda}{1-\lambda}\left[W(B^*) - W(L)\right] \tag{4}$$

Then substituting Eq.(4), the polluter minimizes

$$C(L) + \frac{\lambda}{1-\lambda}\left[W(B^*) - W(L)\right]$$

The solution of this program defines the quota L to be enacted by the Legislator contingent on the contribution $x(L)$. As the function is convex, we obtain a unique interior solution given by $C'(L) = \lambda$. Observe that, unsurprisingly, lobbying leads L to be less than the first best quota B^* (since $\lambda \leq 1$). We collect these findings in

Lemma 1 *The equilibrium legislation* L *is such that* $C'(L) = \lambda$ *while the equilibrium campaign contribution is* $x(L) = \frac{\lambda}{1-\lambda}\left[W(B^*) - W(L)\right]$.

3.2 The VA stage

We now analyze the VA sub game assuming perfect compliance. When bargaining, each player maximizes the difference between his(her) utility under the VA and the utility he(she) achieves in case of persisting disagreement. The payoff of the Regulator and of the Polluter is respectively

$$\begin{aligned} U_R(B) &= W(B) - W(L) && (5) \\ U_P(B) &= C(L) + x(L) - C(B) && (6) \end{aligned}$$

Note that, as usual in the political economy literature, we assume that the regulator does not care about the campaign contribution as it is a transfer between the polluter and the Congress.[7] Further, as the regulator maximizes welfare, any VA which satisfies his participation constraint VA improves de facto the welfare relative to the legislation. Proving the existence of a VA is thus sufficient for establishing the superiority of the VA over the legislation.

In this perspective, we consider now the set of the feasible agreements $\Gamma = \{B : U_P(B) \geq 0, U_R(B) \geq 0\}$ and establish that Γ is never empty, which implies that

Proposition 2 *When enforcement is perfect, there always exists a feasible VA between the Polluter and the Regulator. Moreover any feasible VAs yield a higher welfare than the legislation. However, the induced abatement is less than the first best level B^*.*

Proof. *First we show that $\Gamma \neq \varnothing$. As U_R and U_P are respectively increasing and decreasing in B, we just need to establish that the highest abatement level acceptable by the polluter is higher than the lowest level acceptable by the regulator. Denote B_R and B_P the reservation levels of the regulator and the polluter, respectively. By definition, $W(B_R) \equiv W(L)$ which implies $B_R = L$ since W is monotonic below B^*. Furthermore, substituting $B_R = L$ in $C(B_P) \equiv C(L) + x(L)$, we obtain $C(B_P) = C(B_R) + x(L)$. Then $B_P > B_R$ follows from the fact that $x(L) > 0$ and $C' > 0$.*

Efficiency of the VA follows directly from the regulator's participation constraint whereas $B < B^*$ follows from $B < L$ and $L \leq B^*$. ■

Even tough perfect enforcement is not a realistic assumption, the result is interesting in that it holds true when the lobbying influence on the Congress is almost zero, that is when $\lambda \rightarrow 1$. There always exist positive gains from trade for both parties which rooted in the existence of political distortions under the legislative route. A VA saves the lobbying cost borne by the polluter while the regulator avoids a politically distorted legislative quota. Of course, political distortions also occur under the VA since the participation constraint of the polluter needs to be satisfied. But the saving of lobbying cost is sufficiently high for the polluter to accept a target B under the VA higher than the legislation L. Also note that our results are robust to changes in the way lobbying is formalized.[8] The reason is that the proof of Proposition 1 is ultimately based on the very general property that the lobbying cost is strictly positive ($x(L) > 0$).

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[7] The alternative assumption in which the contribution is viewed as a cost included in the welfare function would not reverse the results. It would simply make more likely the emergence of VA since the Regulator would have an additional motive for accepting a BVA: to avoid the lobbying cost.

[8] By the way, we have established the same result in an alternative rent seeking setting (Glachant, forthcoming).

Proposition 1 is in sharp contrast with previous papers in which the efficiency ranking between the VA and the legislation is ambiguous depending on various parameters. A first explanation of this difference is that most previous contributions simply do not endogeize the political distortions so that the polluter bears zero lobbying cost.

However, this reason does not apply to Maxwell et al. (2000) which model consumers and polluters competing to influence the Congress, implying that the polluter bears a positive lobbying costs. Despite this property, the emergence of voluntary commitments is not systematic in their model. The reason of this difference lies in the lobbying approach they are using. They rely on the political influence function pioneered by Becker (1983). As explained previously, lobbying is a function that gives the mandatory level of abatement as a function of the lobbying costs borne by the polluter and the consumers. Preempting legislation is thus equivalent to preempting the entry of the consumers in the political arena. In this setting, voluntary actions are for example not profitable for the polluter when the consumers do not enter due to a too high entry cost. But introducing an active regulator in the game would reverse the result: in the absence of voluntary action, the regulator would initiate a legislative process, thereby restoring the necessity of positive voluntary actions by the polluter.

3.3 Efficiency properties

We complete the analysis of the perfect enforcement case by looking at the influence of the parameter λ on VA efficiency. We have already derived two efficiency results: first the VA always dominates the legislative route; second a VA never achieves the first best abatement level $B*$. In this subsection, we investigate how the responsiveness of the Congress to lobbying affects the efficiency of the VA outcome. Put differently, in what political conditions, a VA is a particularly efficient option in comparison with the legislation ? At this stage, perfect enforcement is assumed. The answer to the question is thus provisional but it enlighten the final results derived for the imperfect enforcement case in the next section.

We assume that VA outcome, denoted B^{VA}, is given by the Nash bargaining solution so that

$$B^{VA} = \underset{B \in \Gamma}{\arg\max} U_R(B) \cdot U_P(B)$$

In order to facilitate the analysis, we now specify the cost function using a quadratic form.

Assumption *The cost function has a quadratic form* $C(B) = \frac{1}{2}\theta B^2$*, where* θ *is a strictly positive parameter.*

B^{VA} is defined by the first order condition of the Nash product maximization program that follows

$$C'(B^{VA}) \cdot \left[W(B^{VA}) - W(L)\right] = W'(B^{VA}) \cdot \left[C(L) + x(L) - C(B)\right] \tag{7}$$

Then, we establish

Proposition 3 *The more responsive the Congress to lobbying, the more efficient a VA relative to the legislation* L*.*

Proof. *Using Eq(7), we write the difference of welfare of both options as follows* $\Delta \equiv W(B^{VA}) - W(L) = W'(B^{VA}) \cdot \left[C(L) + x(L) - C(B)\right]/C'(B^{VA})$*. Using Assumption XX and the envelop theorem, we differentiate the equation and obtain* $d\Delta/d\lambda = -\left(dB^{VA}/d\lambda\right)\left[\frac{U_P(B^{VA})}{\theta(B^{VA})^2} + \theta B^{VA}\right] - \frac{1}{2\theta B^{VA}}$*. The derivative* $dB^{VA}/d\lambda \geq 0$ *is positive since* $dB^{VA}/d\lambda = \left[C'(B^{VA})\left(\frac{1}{2\theta}(1-\lambda) + W(L)\right)\right] / \left[\theta U_P + \theta B^{VA} \cdot U_R(B^{VA}) + \theta B^{VA}\right]$*. Therefore,* $d\Delta/d\lambda$ *is negative.* ■

4 Imperfect enforcement of the VA

4.1 Existence of the VA

We now analyze the more realistic case in which the polluter does not always comply with the VA. We reason backward by considering the compliance subgame, assuming that a VA with the abatement level B has been agreed previously. Recall that the implementation of the threat is the only compliance incentive for the polluter. Accordingly, he complies only if the compliance cost is lower than the discounted cost associated with the legislative quota, that is

$$C(B) \leq \delta\left[C(L) + x(L)\right],$$

Turning next to the bargaining stage, the polluter's payoff obviously depends on his compliance behavior. In particular, if he fails to comply, the legislative threat is implemented leading to a cost $\delta\left[C(L) + x(L)\right]$. Accordingly, his payoff is given by:

$$U_P(B) = \begin{cases} C(L) + x(L) - C(B), & \text{if } C(B) \leq \delta\left[C(L) + x(L)\right] \\ (1-\delta)\left[C(L) + x(L)\right], & \text{otherwise} \end{cases}$$

Then, we have

Lemma 4 $U_P(B) > 0$ *is strictly positive for any* B.
Proof. *Obvious because* $\delta < 1$ ■

This property has crucial consequences in that the polluter is willing to participate to any VA. It is so because discounting makes the sanction cost $(1-\delta)\left[C(L)+x(L)\right]$ to be strictly less than the disagreement utility $C(L)+x(L)$ of the polluter. In this context, the polluter participates to a VA B either because it is less costly than legislation (when B is low), or because he anticipates he will not comply with (when B is higher).

Turning next to the bargaining payoff of the Regulator, key is the fact that she is not perfectly informed on the propensity of the polluter to comply with the VA. More specifically, we make the hypothesis that δ is a random variable whose realization is known only to the polluter when the game begins, but whose distribution is common knowledge. Hence, the Regulator only knows the probability of compliance. For the sake of simplicity, we assume that δ is uniformly distributed over $I = \left[\bar{\delta}-\sigma, \bar{\delta}+\sigma\right] \subset [0,1]$.[9] It will prove useful in what follows to denote $B^{\min}$ and $B^{\max}$ the abatement levels such that $\bar{\delta}-\sigma \equiv C(B^{\min})/\left[C(L)+x(L)\right]$ and $\bar{\delta}+\sigma \equiv C(B^{\max})/\left[C(L)+x(L)\right]$. The probability of compliance is then given by

$$\begin{aligned} p(B) &= \Pr\left(C(B) \leq \delta\left[C(L)+x(L)\right]\right) \\ &= \begin{cases} 0, & \text{if} \quad B \geq B^{\max} \\ \frac{1}{2\sigma}\left(\bar{\delta}+\sigma-\frac{C(B)}{C(L)+x(L)}\right), & \text{if} \quad B^{\min} < B < B^{\max} \\ 1, & \text{if} \quad B \leq B^{\min} \end{cases} \end{aligned} \tag{8}$$

Note the two corner cases. When $B > B^{\max}$, the compliance condition is not met for the polluter with the highest type $(\bar{\delta}+\sigma)$, implying that $p(B) = 0$. On the contrary, the regulator is certain that compliance occurs if $B < B^{\min}$ since the lowest type complies.

In the end, the payoff of the Regulator is given by

$$U_R(B) = p(B) \cdot W(B) - W(L), \tag{9}$$

Keeping the previous notations, the set of the feasible agreements is then $\Gamma = \{B : U_R(B) \geq 0\}$. If information was complete, the non emptiness of Γ would be a

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[9] The uniformity of the distribution simplifies the presentation of the results. The results will be valid with other distributions, assuming the cumulative and density are positive and increasing on the whole interval.

sufficient condition for a VA to emerge. By contrast, under asymmetric information, non-emptiness is not sufficient for ensuring the existence of *ex post* efficient bargaining outcomes when payoffs are correlated (see Muthoo, p 256-60, 1999 , for a general discussion).[10] Intuitively, it is so because the informed player (the polluter in our setting) has an incentive to manipulate the information he transmits to the uniformed player (the regulator). More precisely, he has an incentive to pretend he will comply with the VA. As the regulator is aware of this 'incentive to lie', the minimal level of abatement she might be willing to accept may be strictly less than the reservation level of the 'high type' polluter who complies with the VA.

However this general argument does not apply to our case. Since $U_R > 0$ for any $B \geq 0$ and any $\delta \in I$, the reservation level of the polluter never falls below the one of the Regulator. As a result, the set of the feasible agreements Γ is common knowledge and non-emptiness is a sufficient condition for a VA to exist. We establish the argument more rigorously in

Lemma 5 *If* Γ *is non-empty, then there exists a bargaining procedure such that bargaining yields an ex-post efficient Bayes Nash equilibrium.*

Proof. *Consider the following bargaining procedure. The polluter makes an offer to the regulator. If she accepts the offer, the agreement is struck and the game ends. But if she rejects the offer, then the game ends with no agreement. Letting* $\tilde{B}(\delta)$ *denoting the polluter's offer when his type is* δ*, the following set of strategies is a Bayes Nash equilibrium:* $\forall \delta \in I$, $\tilde{B}(\delta) = B^{\circ}$ *such that* $U_R(B^{\circ}) = 0$; *and the regulator accepts the offer. The outcome is obviously pareto efficient, because any deviation from* B° *makes one player worse off.* ■

Note that the bargaining procedure considered in the proof allocates all the bargaining power to the polluter. Symmetrically, it is easily shown that the bargaining outcome would also be ex post efficient under the hypothesis that the regulator has the bargaining power. She would make an offer maximizing her payoff which will be accepted since the polluter agrees over any VAs. Lemma 4 simplifies the analysis. We only need to identify the conditions under which Γ is non-empty.

It will prove useful in what follows to identify some general properties of the function $F(B) = \frac{1}{2\sigma}\left(\bar{\delta} + \sigma - \frac{C(B)}{C(L)+x(L)}\right) \cdot W(B)$. This function is similar to the first term of U_R,

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[10] A bargaining outcome is said ex post efficient if and only if after all the information is revealed the players' payoffs associated with the bargaining outcome are Pareto-efficient. Payoffs are said to be correlated when the piece of private information (here δ) affects both players' payoffs.

$p(B)W(B)$ except that $p(B)W(B) = W(B)$ below $B^{\min}$ and $p(B)W(B) = 0$ above $B^{\max}$. Then, we have

Lemma 6 *F admits a unique interior solution, denoted $\hat{B} = \arg\max F(B)$ in the interval $[0, B^{\max}]$.*

Proof. *The existence of a unique interior solution over $[0, B^{\max}]$ requires that U_R is strictly concave in $[0, B^{\max}]$, strictly increasing in the particular case where $B = 0$ and strictly decreasing when $B = B^{\max}$. Consider first the concavity of U_R.We can easily show that $U_R^{"}(B) = p"(B) \cdot W(B) + 2p'(B) \cdot W'(B) - p(B) \cdot C"(B)$ is positive. It is obvious that the first term $p"(B)W(B) = -\frac{1}{2\sigma}\frac{C"(B)}{C(L)+x(L)} \cdot W(B)$ is negative. The same is true for the last term $-p(B)C"(B)$. The second term is negative only if $W'(B) \geq 0$ since $p'(B) = -\frac{1}{2\sigma}\frac{C'(B)}{C(L)+x(L)} \leq 0$.In order to establish this, note that $C(L) + x(L) < C(B^*)$, meaning that the cost to the polluter without lobbying is necessarily lower than the cost with lobbying (otherwise, he would not lobby). Moreover from $p(B) \geq 0$ follows $\bar{\delta} + \sigma \geq C(B)/[C(L) + x(L)]$ and then $C(L) + x(L) \geq C(B)$ since $\bar{\delta} + \sigma \leq 1$. As a result $C(B) \leq C(B^*)$ implying that $W'(B) \geq 0$. Turning next to the sign of the marginal payoff in 0 and $B^{\max}$, we have $U_R'(B) = p'(B) \cdot W(B) + p(B) \cdot W'(B)$. Substituting $B = 0$ leads to $U_R'(0) = p(0) \cdot W'(0) = W'(0)$ which is strictly positive while $U_R'[B^{\max}] = p'(B^{\max}) \cdot W(B^{\max})$ is strictly negative.* ■

Then we draw two curves in Figure 2 in order to see how U_R looks like. The first one is W for which $W\prime = 0$ in B^*. The second one represents $F(B) = p(B)W(B)$ of which maximum is $\hat{B}$. Note that, in $B = 0$, $F'(0) = p(0) \cdot W'(0) = W'(0)$, meaning that the slope of the two curves are the same. However, then F rises faster than W since $F^{"}(B) = 2p'(0) \cdot W'(0) + W"(B) < W"(B)$ since $p' < 0$. As a result $F(B)$ is larger than $W(B)$ for small values of B. The two curves intersect in $B^{\min}$, where $F(B) = p(B)W(B) = W(B)$. In Figure 2, we depict the particular case where the maximum $\hat{B}$ is larger than $B^{\min}$. But the alternative case where $\hat{B}$ is less than $B^{\min}$ is also possible. We also have $F(B^{\max}) = 0$.Given these two curves, we can now represent in bold the function $p(B)W(B)$ and use Figure 2 to discuss when U_R is positive, or put differently, when a VA is feasible.

Consider first the two polar cases where $L < B^{\min}$ and $L > B^{\max}$:

- If $L < B^{\min}$, a VA B such that $B \in \left]L, B^{\min}\right[$ is feasible since $p(B) = 1$, implying that $U_R(B) = W(B) - W(L) > 0$. But VAs continue to be feasible with a B higher than $B^{\min}$ even though $p(B)$ falls below 1. Denoting $L^{\max}$ the abatement level such that $W(L) \equiv p(L^{\max})W(L^{\max})$ above $B^{\min}$, a VA B is feasible as long as $p(B)W(B) > p(L^{\max})W(L^{\max})$ as shown on Figure 2. Hence $\Gamma =]L, L^{\max}[$. In this situation, the regulator freely select her first best VA since the polluter's participation constraint is satisfied for all values of B. Hence, she chooses $B^{VA} = B^{\min}$.

- If $L > B^{\max}$, the analysis is straightforward since $p(B) = 0$ for any level of abatement $B > L$. It follows that $U_R(B) = -W(L)$ which is always negative. When the regulator is sure the VA will lead to zero abatement, legislation obviously dominates the voluntary approach.

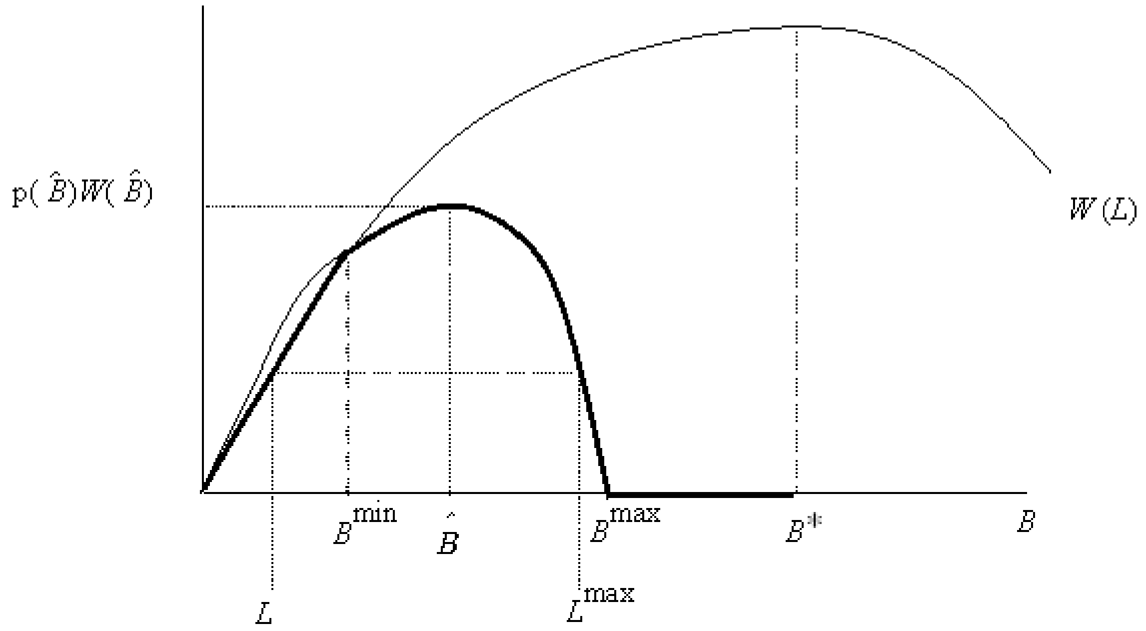

Figure 2: Feasible VAs when $L < B^{\min}$

Turning next to the intermediate case where $B^{\min} < L < B^{\max}$, first note that a necessary condition for Γ to be non-empty is that the maximum of F, $\hat{B}$, belongs to the interval $\left[B^{\min}, B^{\max}\right]$,which is equivalent to $U_R'(B^{\min}) > 0$. Otherwise, if $\hat{B}$ is strictly less than $B^{\min}$, the local maximum of F in the interval $\left[B^{\min}, B^{\max}\right]$ would correspond to the corner solution $B^{\min}$ and $U_R(B^{\min}) = W(B^{\min}) - W(L)$ would be negative since

$B^{\min} \leq L \leq B^{\max}$. As a consequence, the regulator's participation constraint would not be satisfied even at the local maximum of U_R. Figure 3 illustrates the point. It is similar to Figure 2 assuming, in particular, $U_R'(B^{\min}) > 0$. The only difference is that we represent a legislative abatement level denoted $\hat{L} \in \left[B^{\min}, B^{\max}\right]$ which is such that $W(\hat{L}) \equiv p(\hat{B})W(\hat{B})$. It is immediate that, for any $L \in \left]B^{\min}, \hat{L}\right]$, any VA B lying in between L and $L^{\max}$[defined by $W(L) \equiv p(L^{\max})W(L^{\max})$] yields a welfare $p(B)W(B)$ larger than the legislative welfare $W(L)$. In this situation, the equilibrium VA is thus given by $B^{VA} = \hat{B}$.

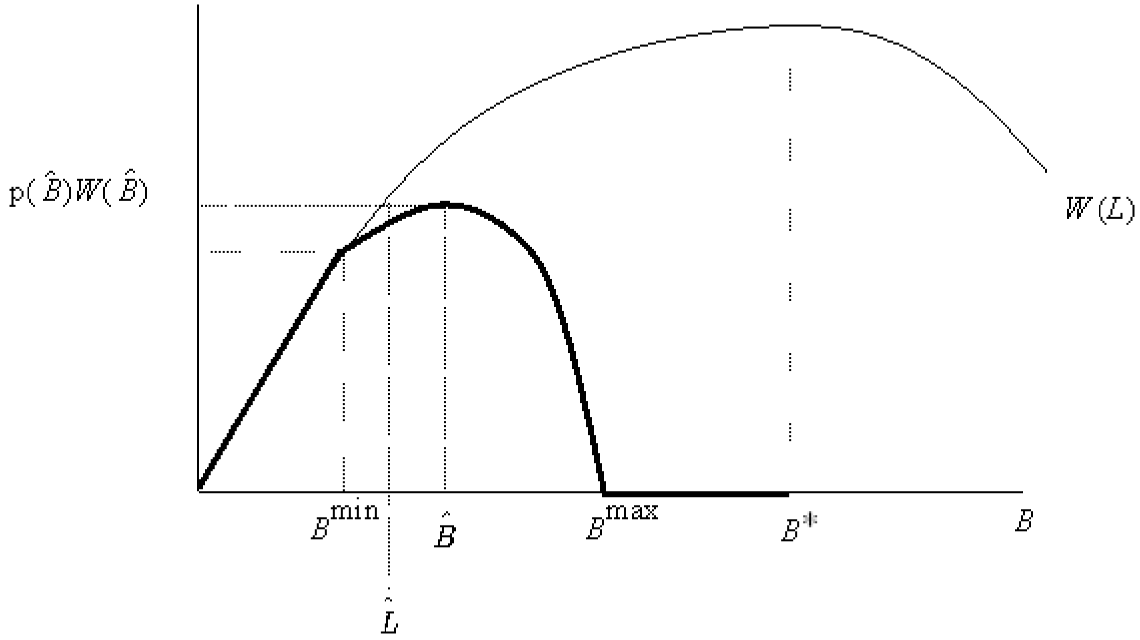

Figure 3: Feasible VAs when $B^{\min} < L < B^{\max}$

We collect these findings in

Proposition 7 *The existence of a (welfare improving) VA depends on the following properties:*

1) If $U_R'(B^{\min}) \leq 0$, *then there exists a VA for any legislation such that* $L \leq B^{\min}$.*The corresponding equilibrium VA is given by* $B^{VA} = B^{\min}$.

2) If $U_R'(B^{\min}) > 0$, *then a VA emerges for any legislation* L *such that* $L < \hat{L}$. *The equilibrium VA is then* $B^{VA} = \hat{B} = \arg\max p(B) \cdot W(B)$, *with the limit legislation* $\hat{L}$ *defined by* $W(\hat{L}) = p(\hat{B}) \cdot W(\hat{B})$ *and* $\hat{L} > \hat{B}$.

Contrary to the perfect enforcement case, voluntary commitments are not systematic anymore. In order to facilitate the interpretation of Proposition 7, we now use the quadratic cost function $C(B) = (\theta/2)B^2$.[11] In the proposition, the equilibrium VA depends on the sign of $U_R'(B^{\min})$.With the quadratic cost function, $B^{\min} = \frac{1}{\theta}\sqrt{\lambda\left(\bar{\delta}-\sigma\right)}$ and

$$U_R(B^{\min}) = \frac{1}{\theta}\left(\sqrt{\lambda\left(\bar{\delta}-\sigma\right)} - \frac{1}{2}\lambda\left(\bar{\delta}-\sigma\right) - \lambda\left(1-\frac{1}{2}\lambda\right)\right)$$

which implies that the sign of $U_R'(B^{\min})$ is the same as the one of the expression

$$\Omega \equiv 2\sigma - \bar{\delta} + \sqrt{\lambda(\bar{\delta}-\sigma)}\left(\frac{\bar{\delta}-3\sigma}{2}\right)$$

The sign of Ω is clearly ambiguous. For instance, if $\sigma = 0$, then $\Omega = \bar{\delta}\left(\sqrt{\lambda\bar{\delta}}/2 - 1\right)$ < 0. Alternatively, $\Omega = \bar{\delta} > 0$ if $\sigma = \bar{\delta}$. The limit case $\Omega = 0$ implicitly defines a value of λ, denoted λ_1,given by

$$\lambda_1 = \frac{4}{\bar{\delta}-\sigma}\left(\frac{2\sigma-\bar{\delta}}{3\sigma-\bar{\delta}}\right)^2 .$$

We also denote $\lambda_2 = \bar{\delta}-\sigma$ and $\hat{\lambda} = \theta\hat{L}$, and then draw λ_1, λ_2, and $\hat{\lambda}$ in a 2-axis graph in Figure 4 where the vertical axis is λ while the horizontal axis represents $\bar{\delta}$ as a function of σ.[12] As $\bar{\delta}$ is necessary larger than σ and $\bar{\delta}+\sigma$ is less than 1, $\bar{\delta}$ belongs to the interval $\left[\sigma, 1-\sigma\right]$. Then λ_1, λ_2, and $\hat{\lambda}$ define three intervals for δ.For the highest values of δ such that $\bar{\delta} > 2\sigma$, the compliance problem is not too severe (either because the discount factor δ is high or because the uncertainty surrounding compliance, as reflected by σ, is low). In this case, a VA emerges when the Congress is responsive to lobbying (a small λ). In equilibrium, the VA is $B^{\min}$, meaning that compliance is certain ($p(B^{\min}) = 1$). When the compliance problem is intermediate ($\bar{\delta} \in \left[1+2\sigma-\sqrt{1+\sigma^2}, 2\sigma\right]$), the VA still emerges for low values of λ, but depending on the severity of the compliance problem, the VA can either involve perfect compliance (when $B^{VA} = B^{\min}$) or imperfect compliance (when $B^{VA} = \hat{B}$, implying $p(\hat{B}) < 1$). Below $1+2\sigma-\sqrt{1+\sigma^2}$, compliance is very

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[11] Accordingly, we now have $L = \frac{\lambda}{\theta}$ and $C(L)+x(L) = \lambda/2\theta$. Furthermore, the compliance probability of Eq.(**??**) is now $\frac{1}{2\sigma}\left(\bar{\delta}+\sigma-\frac{(\theta B)^2}{\lambda}\right)$ for any $B \in \left[B^{\min}, B^{\max}\right]$

[12] Representing $\hat{\lambda}$ is less straightforward than λ_1 and λ_2 in that it is only defined implicitly by the equation $W(\hat{\lambda}/\theta) = p(\hat{B}) \cdot W(\hat{B})$. But the differentiation of this equation with respect to δ leads to $\partial\hat{\lambda}/\partial\delta = \theta W(\hat{B})/2\sigma W'(\hat{L})$ implying that $\hat{\lambda}$ increases in δ.Furthermore, when $\delta = \sigma$, we have $\hat{L} > 0$ and thus $\hat{\lambda} > 0$.Finally, in $\delta = 1+2\sigma-\sqrt{1+\sigma^2}$, $\hat{B} = B^{\min}$ which implies $B^{\min} = \hat{L}$. Thus $\hat{\lambda}$ intersects with both λ_1 and λ_2.

problematic and VA only emerges when the Congress is very responsive to lobbying. Furthermore, all the equilibrium VAs involve a risk of non compliance in this interval ($p(\hat{B}) < 1$). These lessons are summarized in

Proposition 8 *VAs are more likely when the Congress is strongly responsive to lobbying (a low* λ*) and when the compliance problem is not too severe (a high* $\bar{\delta}$ *and/or a low* σ*).*

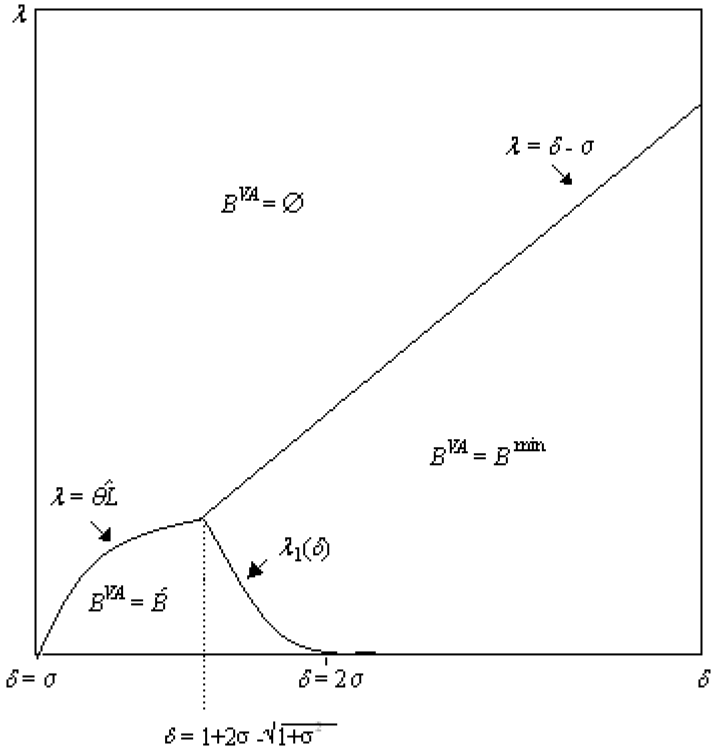

Figure 4

4.2 Efficiency properties

We complete the analysis by investigating the effect of the two key parameters λ and δ on the efficiency of VAs relative to legislation. The analysis is preliminary in that we will only analyze the simplest case where $B^{VA} = B^{\min}$.[13]

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[13] Simulations that we have already performed show that the results obtained when $B^{VA} = B^{\min}$ also apply to $B^{VA} = \hat{B}$.

Proposition 9 *The efficiency of VAs is reduced by high risks of non compliance (a low* δ*). The impact of the responsiveness of the Congress to lobbying is ambiguous. More precisely, the relationship is inverse U-shaped with a maximum welfare for intermediate values of* λ*.*
Proof. *See the Appendix.* ■

The first part of Proposition is not surprising: compliance concerns obviously reduce the efficiency of VAs. The ambiguity of the impact of λ is less intuitive. The result is due to the fact that increasing λ has two opposite effects. It yields a first negative effect on VA by increasing the level of mandated abatement L. But a strict quota also increases the cost of the punishment $\delta\left[C(L)+x(L)\right]$, thereby mitigating the compliance problem. Given this tradeoff, efficiency is maximized for intermediate levels of responsiveness. In the following, we denote $\lambda^{\max}$, the value of λ at which the equilibrium net welfare of the VA is maximized.

This explanation also suggests an interesting relationship between λ and δ. The second effect is increasing with δ since the cost of the punishment is $\delta\left[C(L)+x(L)\right]$.It follows that the higher the discount factor, the higher $\lambda^{\max}$. Put differently,

Corollary 10 *When the discount factor is increasing, that is when the VA compliance concern is decreasing, the level of lobbying responsiveness at which a VA is the most efficient is increasing.*

5 Conclusion

We have developed a model of voluntary agreement under the threat of a legislative quota. Key are two assumptions. First, the polluter is an active lobby group in the Congress influencing the making of the legislation. This political distortion creates the possibility for a mutually beneficial VA: the polluter saves lobbying cost while the regulator avoids the enactment of a politically distorted abatement quota. Second, the VA is non binding and thus, the regulator can only enforce the VA contract by implementing the legislation if the polluter fails to comply.

The results of the analysis are the following. First, we establish in the particular case of perfect enforcement - standard in the existing literature - that there always exists a VA yielding a more efficient level of abatement than the level which would have been imposed legislatively. However this level is strictly lower than the socially optimal level.

This result, albeit intermediate, suggests that the (potential) inefficiency of VAs do not lie in the mechanism triggering the emergence of VA: a legislative threat is an efficient way to give birth to voluntary abatement. This result also stresses the need to relax the perfect enforcement assumption when analyzing VAs.

In this perspective, when assuming imperfect enforcement and incomplete information, the ranking between VA and the legislative quota becomes ambiguous. In particular, it depends on the discount rate of the polluter (reflected by $\bar{\delta}$) and the degree of concern of the Congress to social welfare (λ). The likeliness of VAs is increasing with the discount factor and increasing with the degree of concern to social welfare.

As regards the impact of these 2 parameters on the efficiency of feasible VAs, we obtain the interesting result that the level of social welfare concern yielding the most efficient VA is increasing with the discount factor. The intuition underlying this result is related to the fact that political distortions in the Congress yield two opposite effects on VA efficiency. First, it increases the interest of VA by reducing the efficiency of legislation. Second, it raises VA efficiency by mitigating the compliance problem since the lehislative quota consitutes the sanction in case of non-compliance.

These findings bring interesting policy implications. First, they suggest a selective use of VAs in environmental policy taking into account sector characteristics and the responsiveness of the Congress to lobbying. In practice, there probably exists a negative correlation between the discount rate and lobbying. Firms with a high discount rate typically belongs to sectors where investments are very large and irreversible like heavy industries or energy producers. These sectors are typically very efficient in lobbying for various reasons, including the fact that they are large companies and that pollution abatement costs frequently represents a significant share of their total production costs. Our model predicts that VAs should be very frequent in these contexts. This is in accordance with reality where many VAs are in place in these sectors, particularly with the aim of reducing CO2 emissions.

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Appendix Proof of Proposition

We first investigate the sign of $\partial U^R(B^{\min})/\partial\delta$.We have:

$$U_R\left(B^{VA}=B^{\min}\right)=\frac{\sqrt{\lambda\left(\bar{\delta}-\sigma\right)}}{\theta}\left(1-\frac{\sqrt{\lambda\left(\bar{\delta}-\sigma\right)}}{2}\right)-\frac{\lambda}{\theta}\left(1-\frac{\lambda}{2}\right)$$

and thus

$$\frac{\partial U_R\left(B^{VA}=B^{\min}\right)}{\partial\bar{\delta}}=\frac{\lambda}{2\theta}\left(\frac{1}{\sqrt{\lambda\left(\delta-\sigma\right)}}-1\right)>0$$

Turning next to $\partial U^R(B^{VA})/\partial\lambda$, we have

$$\frac{\partial U_R\left(B^{\min}\right)}{\partial\lambda}=\frac{1}{2\theta}\left(\sqrt{\frac{\delta-\sigma}{\lambda}}-2\left(1-\lambda\right)-\left(\delta-\sigma\right)\right), \tag{10}$$

of which sign is ambiguous. For instance, for the least possible λ, that is $\lambda=0$, we have $\partial U_R\left(B^{\min}\right)/\ \partial\lambda\to+\infty$. Alternatively, for the highest feasible $\lambda=\delta-\sigma$, $\partial U_R\left(B^{\min}\right)/\ \partial\lambda=-\frac{1}{2\theta}\left(1-\lambda\right)<0$. The sign of $\partial U_R\left(B^{\min}\right)/\ \partial\lambda$ is the same as the one of

$$\Phi = \sqrt{\delta - \sigma} - 2\left(1 - \lambda\right)\sqrt{\lambda} - \left(\delta - \sigma\right)\sqrt{\lambda}$$

Denoting $A = \sqrt{\lambda}$, $\Phi = \sqrt{\delta - \sigma} - \left(2 + \left(\delta - \sigma\right)\right)A + 2A^3$ is a polynomial of degree 3. Identifying the roots is an uneasy task. However, the differentiation of Φ yields $-\left(2 + \left(\delta - \sigma\right)\right) + 6A^2$ of which sign is negative if $A^2 = \lambda < \frac{1}{3} + \frac{\delta - \sigma}{6}$ and then positive until $\lambda = \delta - \sigma$,meaning that $d\Phi/dA$ is U-shaped on the interval $[0, \delta - \sigma]$. Recall that $\partial U_R\left(B^{\min}\right) / \partial\lambda > 0$ in $\lambda = 0$ while $\partial U_R\left(B^{\min}\right) / \partial\lambda < 0$ in $\lambda = \delta - \sigma$. From these two properties and the fact that $d\Phi/dA$ is U-shaped follows that there exists a unique $\lambda\,\mathrm{max}$ such that $\partial U_R\left(B^{\min}\right) / \partial\lambda < 0$ below $\lambda\,\mathrm{max}$ and $\partial U_R\left(B^{\min}\right) / \partial\lambda > 0$ above.

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