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Stable International Agreements on Transfrontier Pollution with Ratification Constraints

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Stable International Agreements on Transfrontier Pollution with Ratification Constraints*

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Abstract

International agreements on transfrontier pollution require approval by domestic political institutions. In this paper we employ a voting game theoretic model to characterize the stability of such agreements when each country's participation is conditioned upon a domestic ratification vote. To describe the pre-treaty or no treaty international situation, we propose a concept of (noncooperative) political equilibrium and prove its existence. We then move to the diplomatic level, and employ a coalition formation game to show that there exist cooperative joint policies, yielding a treaty, that are ratified by all countries and that can be considered *stable* at the international level. In particular we exhibit a unique stable agreement for the grand coalition, inducing a (computable) allocation that has a natural equilibrium interpretation for the international economy.

Keywords: Voting Games, Coalition Formation, International Cooperation, Pollution, Political Equilibrium.

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1 Introduction

International agreements on environmental standards usually require the approval of domestic political institutions. Once an agreement is found at the international level, its prescriptions must be translated into domestic laws through a ratification process. The fact that negotiating countries are in all respects sovereign and independent decision makers, makes ratification a substantial element (possibly a constraint) in the decisional process. The difficulty of attaining the full commitment of many countries in actual cooperation problems (as, for instance, at the Rio and Kyoto conferences on Climate Change) may be partially explained as the effect of such domestic political constraints on the decisions of countries' political leaders.

The *stability* of an international agreement has been identified in the literature with the properties of various equilibrium concepts in game theoretic models of cooperation. Part of this literature has looked at the possibility of "full" cooperation, i.e., cooperation among all involved countries. Some of these works have studied the *core* of cooperative games representing the decisional process at the international level (see Chander and Tulkens 1992, 1995 and 1997, M*ä*ler 1989, Kaitala-M*ä*ler-Tulkens 1995). Core agreements are "stable" solutions to the negotiation problem in that no coalition of countries is able to induce a preferred outcome by its own means. Other contributions have studied the possibility of the formation of smaller coalitions: see, for example, Carraro and Siniscalco (1993), Barrett (1994) and Hoel and Schneider (1997)). Both approaches lack an institutional specification of the collective decision processes involved at the domestic levels. Countries' representatives are able to choose among all technologically feasible domestic policies in the attempt of maximizing aggregate domestic welfare. Domestic politics and decisional procedures do not play any role and, in particular, do not impose any constraint on the set of feasible policies.[1]

This paper studies the effect of the domestic institutions of ratification on the stability of international environmental agreements in an economy of the type studied by Chander and Tulkens (1997), in which domestic production activities have, as a by-product, the emission of some transboundary pollutant.

We assume that in the absence of international cooperation, each domestic parliament independently determines *by voting* the level of domestic environmental regulation. We formally describe this pre-treaty (or no-treaty) state of the economy by means of the concept of

[1]A notable exception is the paper by Haller and Holden (1997) on the effect of different ratification rules on countries' international bargaining power.

International Non-cooperative Political Equilibrium (INPE). We prove existence and uniqueness of the INPE for our economy.

We then study international cooperation taking as a status quo the INPE of the economy. The key element of our analysis is that an international agreement, defined as an emission abatement plan and some rule to share the associated costs among the involved countries, becomes effective in a country only if it is ratified by its parliament. Therefore, the only feasible agreements are those which are ratified by all signatories. We show that for each configuration of coalitions (of countries) in the international economy (i.e., for each "coalition structure"), there exists a unique collection of agreements - one for each coalition of cooperating countries - which are simultaneously ratified.

We then turn to the analysis of the incentives of national delegates to sign agreements at the international level. We assume that national delegates act on behalf of a supporting majority, maximizing the aggregate payoff of its members. The crucial assumption at this stage is that in the design of an international agreement, delegates anticipate the outcome of the ratification vote, and only consider agreements that would be eventually ratified by their national parliaments. Since each set of cooperating countries in a given coalition structure has only one agreement that would be ratified, delegates are able to order coalition structures by using the aggregate payoffs of their domestic supporting coalitions at the relevant ratified agreements.

These considerations motivate the use of a game of coalition formation, in which each delegate announces a coalition to which he wishes to belong. anticipating that this coalition will implement the unique ratified agreement (we use a specification of this game first introduced by Hart and Kurz (1983)). We look for coalitions structures that are *stable* to objections by subset of national delegates, identifying such stable structure with the set of strong Nash equilibria of the coalition formation game.

We find that the grand coalition is always a Strong Nash Equilibrium outcome, and the associated ratified international agreement shares abatement costs proportionally to the national incomes at the pre-treaty stage. Although other inefficient structures may emerge in equilibrium, we show that some degree of cooperation always occurs when domestic politicians maximize the aggregate welfare of their whole population. If politicians only maximize the welfare of their voters (that is, of their supporting majority), the complete absence of cooperation may occur as a Strong Nash Equilibrium, but never as a Strict Strong Nash Equilibrium.

The paper is organized as follows. In Section 2 we introduce the international economy. In Section 3 we formulate the voting game that describes the domestic decisional process, in the absence of international cooperation. In section 4 we prove existence and uniqueness of the international equilibrium resulting from these independent domestic policies. In Section 5 we study international cooperation: after an informal presentation of the decisional structure, which is of diplomatic nature at this stage, we present the game that bears upon the formation of coalitions among countries and prove our main result on politically stable international environmental agreements. Section 6 concludes the paper by summarizing its main points, comparing them with some of the alternative approaches mentioned above, and pointing towards generalization of our results for a larger class of preferences.

2 The International Economy

We consider an international economy E with a set K of countries, indexed by $k = 1, ..., \bar{k}$, a single private good and ambient pollution, which is the outcome of the discharges emitted by the countries as a by-product of their private good production.

2.1 Components of E

The elements of the economy are as follows.

- *Agents.* The set of individual economic agents (citizens) is denoted by $I = \{1, ..., i, ..., n\}$. The agents are partitioned into $\bar{k}$ countries. B_k denotes the set of agents living in country k, with $|B_k| = n_k$.
- *Commodities.* There are three types of commodities in the economy: a private good $x \geq 0$; pollutant discharges $p_k \geq 0$ occurring in country k, $k \in K$, with $p = (p_1, p_2, ..., p_{\bar{k}})$ denoting the vector of emissions occurring in all countries, and ambient pollution $z \leq 0$.
- *Ecological Transfer Function.* Countries' discharges determine linearly and additively the amount of ambient pollution, according to the relation

  $$z = -\sum_{k \in K} p_k.$$

  We will sometimes use the notation $z(p)$.

- *Production Technology.* Each country k produces a positive amount of the private good, denoted by the value of the production function[2] $g_k(p_k)$. We denote by $g_k'(p_k)$ and $g_k''(p_k)$ the first and second derivative of g_k.

  We assume the following:

  Assumption 1. $g_k(p_k) \geq 0$; $g_k'(p_k) \geq 0$; $g_k''(p_k) \leq 0$ *for all* $p_k \geq 0$;

  Assumption 2. $\exists p_k^0$ *such that* $g_k'(p_k^0) = 0$ *if and only if* $p_k \geq p_k^0$. Moreover, $g_k'(0) = +\infty$.

The level p_k^0 measures the maximal amount of emissions that are economically valuable: above this level, additional increases in emissions do not increase production. The level p_k^0 can be interpreted as a technological constraint due to unspecified inputs other than pollution.

- *Preferences.* Each agent $i \in I$ has a utility function $u_i(z, x_i)$ defined on $\mathbf{R}_- \times \mathbf{R}_+$, satisfying:

  Assumption 3. $u_i(z, x_i) = v(z) \cdot x_i$;

  Assumption 4. $v(z)$ *is twice differentiable, with* $v(z) > 0$, $\infty > v'(z) > 0$ *and* $v''(z) \leq 0$ for all $z \geq 0$, where v' and v'' are the first and second derivatives of v..

By Assumption 3 any difference in the way agents value the environmental quality is only due to differences in the consumption level of the private good. In other words, we assume that there exists a fundamental valuation of the ambient quality which is common to all agents in the economy E, and is represented by the functional form $v(z)$.

- *Individual incomes.* For each $k \in K$, agent $i \in B_k$ is allocated a share θ_i^k (with $0 \leq \theta_i^k \leq 1$ and $\sum_{i \in B_k} \theta_i^k = 1$) of the private good $g_k(p_k)$ produced in his country. The value $\theta_i^k \cdot g_k(p_k)$ is the private income of agent i living in country k. We will denote by θ^k the vector $\left(\theta_1^k, .., \theta_{n_k}^k\right)$. The vector θ^k represents the only source of heterogeneity in this model, where all agents have the same preference ordering, and in which agents with the same private consumption share the same utility level. We also remark that the amount $\theta_i^k \cdot g_k(p_k)$ does not identify the consumption x_i of agent i, but rather his endowment of private good. As we will see, private consumption will be co-determined by taxation and, possibly, by transfers.

  Assumption 5. $\theta_i^k > 0$ *for all* $i \in B_k$ *and all* $k \in K$.

[2] We abstract here from all inputs of production other than polluting discharges.

Definition 1 *A* **feasible state** *of the international economy* E *is a vector* $(z, p, x) = (z,\ p_1, \cdots, p_{\bar{k}},\ x_1, \cdots, x_n) \in \mathbf{R}_- \times \mathbf{R}_+^{\bar{k}} \times \mathbf{R}_+^n$ *such that*

$$\begin{aligned} \sum_{k \in K} g_k\left(p_k\right) &\geq \sum_{i \in I} x_i \\ z &= -\sum_{k \in K} p_k. \end{aligned}$$

For any feasible state (z, p, x), the pair (p, x) is called an allocation.

Definition 2 *A* **Pareto optimum** *of the economy* E *is a feasible state* (z, p, x) *such that there exists no other feasible state* (z', p', x') *such that* $u_i(z, x_i) \leq u_i(z', x_i')$ *for all* $i \in I$ *and* $u_h(z, x_h) < u_h(z', x_h')$ *for some* $h \in I$.

We now define an equilibrium concept for the economy E that will prove useful in the following. We will refer to the abatement cost function $C_k(p_k)$, defined for all k by the expression $\left[g_k\left(p_k^0\right) - g_k\left(p_k\right)\right]$.

Definition 3 *A* **ratio equilibrium** *of the economy* E *is a triple* (p, x, r) *in which* $r = (r_1, ..., r_n)$ *is a cost sharing ratio, with* $r_i = (r_i^1, ..., r_i^{\bar{k}})$*, such that for each* k $\sum_{i \in I} r_i^k = 1$ *and such that for all* $i \in B_k$ *and all* $k \in K$:

$$\begin{aligned} x_i &= \theta_i^k g_k\left(p_k^0\right) - \sum_{k=1}^{\bar{k}} r_i^k C(p_k); \\ u_i(-\sum_{k=1}^{\bar{k}} p_k, x_i) &\geq u_i(-\sum_{k=1}^{\bar{k}} p_k', x_i') \quad \forall\,(p', x_i') : x_i' \leq \theta_i^k g_k\left(p_k^0\right) - \sum_{k=1}^{\bar{k}} r_i^k C(p_k'). \end{aligned}$$

An equilibrium ratio r is a cost sharing vector with the property of inducing the same demand for emissions by all the agents in the economy. A property of ratio equilibria is that they always induce an Pareto Optimum of the economy. The converse is obviously not true, since some Pareto Optimal states distribute private good consumption in a way which is not compatible with the equilibrium constraint of the above definition. Additive of the ecological transfer function implies that at every ratio equilibrium (p, x, r) we have $r_i^k = r_i^j$ for all i and for all $j, k \in K$.

The next Lemma, recording a uniqueness property of the set of Pareto optima of the economy E, is basically a restatement of Proposition 1 in Chander and Tulkens (1997). Since the economy E considered in the present paper differs from the one considered there in that the functional form of utility functions is not linearly separable in the private good, their uniqueness result needs be re-established for this case (see Appendix for the proof).

Lemma 1 *Let* (z, p, x) *and* (z', p', x') *be two Pareto Optima of* E. *Then* $p = p'$.

Proof. Appendix.

2.2 Sub-economies $E_k(\bar{p}_{-k})$

In what follows, it will be useful to consider some variations on the economy E.

For all $k \in K$, we denote by $E_k(\bar{p}_{-k})$ the sub-economy obtained by restricting the economy E to the set of agents B_k and a given vector of emissions $\bar{p}_{-k} = (\bar{p}_1, ..., \bar{p}_{k-1}, \bar{p}_{k+1}, ..., \bar{p}_{\bar{k}})$ of countries other than k. A feasible state of the sub-economy $E_k(\bar{p}_{-k})$ is a vector $(z, p, x_k) \in \mathbf{R}_- \times \mathbf{R}_+^{\bar{k}} \times \mathbf{R}_+^{n_k}$ such that

$$\begin{aligned} g_k(p_k) &\geq \sum_{i \in B_k} x_i; \\ p_{-k} &= \bar{p}_{-k} \\ z &= -\sum_{j \neq k} \bar{p}_j - p_k. \end{aligned}$$

For any such feasible state, the pair (p_k, x_k) is called an allocation for $E_k(\bar{p}_{-k})$.

The definition of a ratio equilibrium directly applies to the sub-economy $E_k(\bar{p}_{-k})$.

Lemma 2 *The sub-economy* $E_k(\bar{p}_{-k})$ *admits a unique ratio equilibrium* (p_k^*, x_k^*, r_k^*), *inducing a Pareto optimum of the economy* $E_k(\bar{p}_{-k})$, *with* $r_i^{k*} = \theta_i^k$ *for all* i.

Proof. Appendix.

Some consistency relations between the sets of ratio equilibria of E and of the economies $E_k(\bar{p}_{-k})$, $k = 1, ..., \bar{k}$, will be established and used later on.

3 Domestic Decision Making

3.1 The Private Sector

If we assume that within each country k the private good is produced by the private sector, the level of emissions p_k^0 may be thought of as the outcome of the absence of any environmental regulation, be it domestic or international, in country k. In this case, the amount $g_k(p_k^0)$ of private good is produced and consumed.

3.2 The Public Sector

Countries are organized democratically. A legislative body decides by voting the level of domestic environmental regulation by fixing a maximal amount of emissions.

3.2.1 The Voting Game G_k

We formally represent the voting procedure within country k by means of the voting game $G_k\left(B_k, W_k^d; p_{-k}\right)$, in which B_k is the set of players (members of parliament), p_{-k} denotes the vector $(p_1, .., p_{k-1}, p_{k+1}, .., p_{\bar{k}})$ of emissions outside country k and $W_k^d \subseteq 2^{B_k}$ is the set of winning coalitions, *i.e.*, the coalitions that are decisive on domestic issues for the population of country k. In the case of simple majority rule, the set W_k^d contains all the coalitions that contain a majority of the population in k. The fact that the game is defined for a given vector p_{-k} of external emissions reflects the assumption that the players' payoffs are defined on the states of the whole international economy E. For short we henceforth write $G_k(p_{-k})$ for $G_k\left(B_k, W_k^d; p_{-k}\right)$.

We make the following assumptions on voting rules:

Assumption 6. (non-dictatorship) $\forall i \in B_k, \forall k \in K$:

$$B_k \backslash \{i\} \in W_k^d$$

Assumption 7. (monotonicity) $\forall i \in B_k, \forall k \in K$:

$$S \in W_k^d \text{ and } S \subset T \Rightarrow T \in W_k^d.$$

We remark that the above two properties are the minimal requirement for our results. They do not rule out the case in which winning coalitions count less than the majority of voters. The properness property[3], although not needed for the formal derivation of our results, would rule out such undesirable cases.

A **strategy** for a winning coalition $S_k \in W_k^d$ is a level of domestic emissions p_k. Given p_k, the distributive vector θ^k imputes a well defined level of consumption to each agent in country k. Coalitions not in W_k^d have an empty strategy set. We assume that agents belonging to a winning coalition can operate any transfers of private good among them[4], so that coalition

[3]The set W is proper if whenver S is winning its complement set is not.

[4]This framework is essentially the one used by Nakayama (1977), in which winning coalitions can choose the desired level of public goods and have to finance it proportionally to their relative incomes.

S can induce any feasible state (z, p, x_k) of the sub-economy $E_k(\bar{p}_{-k})$ such that

$$\begin{aligned} \sum_{i \in S} x_i &= \sum_{i \in S} \theta_i^k \cdot g_k(p_k) \\ x_h &= \theta_h^k \cdot g_k(p_k) \ \forall h \in B_k \backslash S. \end{aligned} \tag{1}$$

We say that coalition $S \in W_k^d$ *improves* upon the allocation (p_k, x_k) in $G_k(\bar{p}_{-k})$ if it can induce a state of the economy that all members prefer to (p_k, x_k), with strict preference for at least one member.

Definition 4 *The core of the voting game $G_k(\bar{p}_{-k})$ is the set of allocations (p_k, x_k) that no coalition can improve upon.*

3.2.2 Political Equilibrium in Country k

The core of any voting game $G_k(\bar{p}_{-k})$ has the property of being a stable collective decision in the parliamentary debate. We therefore define a political equilibrium in country k any state of the sub-economy $E_k(\bar{p}_{-k})$ induced by a core allocation in the game $G_k(\bar{p}_{-k})$.

Definition 5 *The feasible state (z, p, x_k) of the sub-economy $E_k(\bar{p}_{-k})$ is a* **political equilibrium** *for $E_k(\bar{p}_{-k})$ if and only if (p_k, x_k) belongs to the core of the associated voting game $G_k(\bar{p}_{-k})$.*

The next Proposition fully characterizes the political equilibria of country k for any given vector of external emissions p_{-k}.

Proposition 1 *The state (z, p, x_k) of the sub-economy $E_k(\bar{p}_{-k})$, in which p_k is the unique Pareto optimal emission level and $x_i = \theta_i^k g_k(p_k)$ for all $i \in B_k$, is the unique political equilibrium for the sub-economy $E_k\left(p_{-k}^*\right)$.*

Proof. Appendix.

Remark: In a political equilibrium no transfers of private good take place, and each agent consumes exactly the amount of private good determined by the efficient emission level of the restricted economy and by his distributive parameter.

4 International Non-Cooperative Political Equilibrium

Once we have determined the political equilibrium within each country as a function of the vector of external emissions, it is possible to characterize which states of the economy are expected to occur in the absence of international coordination of policies. Any such state must be such that all countries are simultaneously at a domestic political equilibrium.

Definition 6 *An* **International Noncooperative Political Equilibrium (INPE)** *is a state of the economy* $(\bar{z}, \bar{p}, \bar{x})$ *such that for all* k *in* K *the state* $(\bar{z}, \bar{p}, \bar{x}_k)$ *is a political equilibrium of the economy* $E_k(\bar{p}_{-k})$*.*

The INPE may be considered as representing a no-treaty or pre-treaty equilibrium, in the sense that it describes the outcome of national policies in absence of coordination. We here remark that because of the uniqueness of Pareto Optimal emission policies within each country, the INPE prescribes the same emissions vector that would obtain in a model of central planners, each maximizing the aggregate payoff of his domestic agents (this is the case, for instance, of the model studied in Chander and Tulkens (1997)). However, the domestic political constraints implicit in the definition of a INPE fully determine the domestic distribution of private consumption, which is not determined in Chander and Tulkens (1977).

Proposition 2 *There exists a unique INPE for the economy* E*.*

Proof. Appendix.

By comparing the first order conditions characterizing the INPE and the Pareto Optimal state of the economy E (these are necessary and sufficient conditions by assumptions 1-4, see also the proofs of lemma 1 and 2) we deduce that the INPE is generically not efficient. Indeed, these conditions write for any efficient state (z^*, p^*, x^*) as

$$\frac{v'(z^*)}{v(z^*)} \sum_{j \in K} g_j(p_j^*) = g_k'(p_k^*), \quad \forall k \in K$$

and for the unique INPE as

$$\frac{v'(\bar{z})}{v(\bar{z})} g_k(\bar{p}_k) = g_k'(\bar{p}_k), \quad \forall k \in K.$$

Since under the present assumptions production levels are always positive in any efficient state, inefficiently high aggregate emission levels are associated with the INPE. This type of properties are explored in details in the next section.

5 International Cooperation

5.1 An Informal Discussion

The INPE can be considered as the predictable outcome in the economy E if countries do not communicate and coordinate their domestic policies. However, the inefficiency of the INPE provides countries with incentives to promote some sort of international cooperation. Such coordinated actions are carried out by means of international agreements, *i.e.,* cooperative plans in which countries commit themselves to specific emission abatement plans as well as to cost-sharing schemes.

Definition 7 *An* **International Agreement** *(I.A.) among the countries of the set K is a pair $(\Delta p, \alpha)$ consisting (i) of a vector of emission changes $\Delta p = (\Delta p_1, ..., \Delta p_{\bar{k}})$ with respect to the INPE levels, with $\Delta p_k \in \left[-\bar{p}_k, p_k^0 - \bar{p}_k\right]$ for all $k \in K$, and (ii) of a total cost sharing rule $\alpha = (\alpha_1, ..., \alpha_{\bar{k}})$ such that $\alpha_k \in [0, 1]$ for all $k \in K$ and $\sum_{k \in K} \alpha_k = 1$.*

An I.A. thus prescribes changes in emissions with respect to those prevailing at the INPE, as well as a sharing rule among countries for the aggregate cost involved. In terms of forgone consumption of the private good, this cost is given by

$$C(\Delta p) \equiv \sum_{k \in K} \left[g_k(\bar{p}_k) - g_k(\bar{p}_k + \Delta p_k)\right],$$

while the induced ambient quality is:

$$z(\Delta p) = -\sum_{k \in K} (\bar{p}_k + \Delta p_k).$$

Institutionally, for an I.A. to come into existence, it must be the result of some collective decision process that comprises at least two levels: (i) the signature (or diplomatic) level, consisting of the adoption of the agreement's content by (delegates of) the countries involved; and (ii) the ratification (or political) level, consisting of the acceptance of that content within each of the countries involved.

In our analysis below, the ratification level is assumed to take place through voting on proposed agreements in each country. Domestic winning coalitions can object to a proposed IA by either rejecting it, in which case the economy remains at the no-treaty (INPE) state, or by proposing some alternative emissions vector. The mathematical models we use to describe the ratification stage is a cooperative *voting game* played by the committee of parliamentary members. The solution concept that identifies the ratified agreements is the *core*.

As far as the signature level is concerned, we assume that each country is represented by a delegate, and we consider that for a proposed agreement to be adopted by the delegates it must be both ratified in all countries. Moreover, in order to be adopted, an agreement must be coalitionally rational in the following sense: no set of delegates find it preferable to engage in a different agreement *that they could get ratified* in their respective countries. The two levels are intimately related through the fact that the ratification level sets limitations to the proposals that can be considered by the delegates, both as final outcome of cooperation and as conceivable deviations from it. We represent the diplomatic signature level as a *coalition formation game,* in which delegates propose collations, and payoffs are given by the unique core allocations of the ratification voting games.

We show that the grand coalition is a Strong Nash Equilibrium of the coalition formation game, implying emission abatement plans ratified by all countries, and inducing a Pareto optimum of the (world) economy E. Moreover, although (inefficient) outcomes with several coexisting partial agreements are not ruled out in equilibrium, some degree of cooperation always emerges when political delegates maximize the aggregate welfare of their citizens..

5.2 Politics: The Ratification Voting Game

For any I.A. involving all countries, we denote by $G_k\left(B_k, W_k^r, \alpha_k\right)$ the domestic ratification voting game in country k bearing on an international agreement that imputes to that country the cost share α_k. For a winning coalition $S_k \in W_k^r$ a strategy is any vector of abatements Δp and possibly transfers among its members, with total imputed cost

$$\sum_{i \in S_k} \theta_i^k \alpha_k C\left(\Delta p\right).$$

Note that we are including as a feasible strategy for a domestic winning coalition S_k the strategy $\Delta p' = 0$ inducing the INPE state of the economy. If this strategy is adopted, the cooperation process is rejected at the ratification stage.

Individual payoffs yield the following expression for coalition S_k's worth:

$$v\left(z\left(\Delta p\right)\right) \sum_{i \in S_k} \left[\theta_i^k g_k\left(\bar{p}_k\right) - \theta_i^k \alpha_k C\left(\Delta p\right)\right].$$

Definition 8 *We say that the I.A.* $(\Delta p^*, \alpha^*)$ *is* ratified by country k *if for some vector of transfers* $\tau_k^* = \left(\tau_i^{k*}\right)_{i \in B_k}$ *such that* $\sum_{i \in B_k} \tau_i^{k*} = 0$ *the allocation induced in the sub-economy* $E_k(\bar{p}_{-k} + \Delta p^*_{-k})$ *by the triple* $(\Delta p^*, \alpha_k^*, \tau_k^*)$ *is in the core of the game* $G_k\left(B_k, W_k^r, \alpha_k^*\right)$*. An I.A. is simply* ratified *if it is ratified by all countries.*

The unique ratified I.A. is characterized in the next proposition.

Proposition 3 *The I.A.* $(\Delta p^*, \alpha^*)$ *such that:*
1) $(\bar{p} + \Delta p^*)$ *is the efficient emissions vector of the economy* E*;*
2) $\alpha_k^* = \dfrac{g_k\left(\bar{p}_k\right)}{\sum_{j \in K} g_j\left(\bar{p}_j\right)}$ *for all* $k \in K$*,*
is the unique ratified international agreement. Moreover, within each country k *the associated transfers scheme* τ_k^* *is such that* $\tau_i^{k*} = 0$ *for all* $i \in B_k$*.*

Proof. Appendix.

Proposition 3 shows that the unique ratified international agreements prescribes the efficient emission levels and shares total costs proportionally to the relative income levels at the pre-treaty INPE.

The above definition and characterization can be applied to partial agreements within a subcoalition T of countries. Following the definition and letting α_T^* denote a cost sharing vector for countries in T, we say that the I.A. $(\Delta p_T^*, \alpha_T^*)$ is *ratified* by the coalition of countries $T \subset K$ given the emissions vector $\Delta p_{K \backslash T}$ if for all $k \in T$ there exists some vector of transfers $\tau_k^* = \left(\tau_i^{k*}\right)_{i \in B_k}$ such that the allocation induced by the vector $\left(\Delta p_T^*, \Delta p_{K \backslash T}, \alpha_k^*, \tau_k^*\right)$ is in the core if the game $G_k\left(B_k, W_k^r, \alpha_k^*, \Delta p_{K \backslash T}\right)$.

Proposition 3 easily extends as follows.

Proposition 4 *The partial agreement.* $(\Delta p_T^*, \alpha_T^*)$ *such that:*
1) $(\bar{p}_T + \Delta p_T^*)$ *is the efficient emissions vector of the economy* $E_T\left(\bar{p}_{K \backslash T} + \Delta p_{K \backslash T}\right)$*;*
2) $\alpha_k^* = \dfrac{g_k\left(\bar{p}_k\right)}{\sum_{j \in T} g_j\left(\bar{p}_j\right)}$ *for all* $k \in T$*,*
is the unique ratified partial agreement for the set of countries T *given* $\Delta p_{K \backslash T}$*. Moreover, in each country* $k \in T$ *the associated transfers scheme* τ_k^* *is such that* $\tau_i^{k*} = 0$ *for all* $i \in B_k$*.*

5.3 Diplomacy: The Coalition Formation Game

We now move to the international cooperation process itself. We wish to consider a model of cooperation in which national delegates only consider agreements which would eventually be ratified by their parliaments. As the previous section has shown, this restriction leaves national delegates with the sole choice of which coalition they wish to form, since once this choice is made, the ratified agreement is uniquely determined. This remark motivates us to model delegates' diplomatic behaviour by means of a coalition formation game, in which delegates consider different "partners" at the international stage, anticipating the effect of their choices on the payoff of the domestic winning coalition they represent.

The game we consider was first introduced by Hart and Kurz (1983) as the Γ coalition formation game. The set of players is K (all national delegates), with $S_k^* \in W_k^d$ denoting the winning coalition represented by the k-th delegate (the coalition in power in country k). Players act simultaneously. Each player $k \in K$ announces a coalition $T_k \subset K$ to which he wishes to belong. A strategy for player k is denoted by σ_k.

5.3.1 From Strategies to Coalition Structures

Once a profile of strategies $\sigma = (T_1, ..., T_{\bar{k}})$ is announced, players must be able to predict which coalitions will form in the system. Since the coalitions announced by the $\bar{k}$ players may not lead to a partition of the set K (or, in other words, players' wishes may not be compatible), a rule mapping strategy profiles into partitions of K is needed.

We will adopt the "gamma" rule, proposed by Hart and Kurz, predicting that coalition T effectively forms only if all of its members have announced precisely T.[5] Formally, the profile σ induces the cooperation structure

$$\pi(\sigma) = \left\{T_\sigma^k : k \in K\right\}$$

where

$$T_\sigma^k = \begin{cases} T_k \quad if \quad T_k = T_j \quad for\ all\ j \in T_k \\ \{k\} \quad otherwise \end{cases} .$$

[5] This game has been studied under the name "Simultaneous Coalition Unanamity Game", see Yi (1997). Hart and Kurz also consider the more permissive "delta" rule, allowing all players that have announced the same coalition to stay together.

Under this rule, defections from a coalition induce the remaining players to split up as singletons[6]. In particular, any joint deviation $\bar{\sigma}_T = (T, ..., T)$ by a coalition of players T from, e.g., the strategy profile $\sigma = (K, ..., K)$ induces a coalition structure

$$\pi\left(\bar{\sigma}_T, \sigma_{K\setminus T}\right) = \left(T, \{j\}_{j\in K\setminus T}\right),$$

in which the unique smaller coalition T forms.

5.3.2 Payoffs

We now define an imputation rule, specifying the players' payoffs for each possible coalition structure. This, together with the coalition formation rule, will yield a well defined game.

Since we are only interested in ratified agreements, we associate with each coalition structure $\pi = (T_1, ..., T_m)$ a series of partial agreements, one for each element of π, with the property of being all simultaneously ratified. This leads to:

Definition 9 *The vector of partial agreements* $((\Delta\tilde{p}_1, \tilde{\alpha}_1), ..., (\Delta\tilde{p}_m, \tilde{\alpha}_m))$ *is a* **Partial Agreements Equilibrium** *(PAE) for the coalition structure* $\pi = (T_1, ..., T_m)$ *if* $(\Delta\tilde{p}_h, \tilde{\alpha}_h)$ *is a ratified partial agreement for* T_h *given* $\Delta\tilde{p}_{K\setminus T_h}$*, for all* $h = 1, ..., m$.

A PAE consists of a set partial agreements that are simultaneously ratified by all cooperating countries in the cooperation structure π.

Lemma 3 *For each coalition structure* π *there exists a unique PAE w.r.t.* π*.*

Proof. Appendix.

The utility levels induced on the economy E by the PAE for the members of the cooperation structure π are used to define the payoffs in the game Γ. In particular, the payoff of delegate k when the profile of strategies σ is played is given by

$$u_k(\sigma) \equiv v\left(-\sum_{j\in K}(\bar{p}_j + \Delta\tilde{p}_j)\right) \cdot \sum_{i\in S_k^*} x_i(\Delta\tilde{p}, \tilde{\alpha}_k), \tag{2}$$

where $(\Delta\tilde{p}, \tilde{\alpha})$ is the PAE with respect to $\pi(\sigma)$. The fact that in (2) the sum of private consumptions is taken over players in S_k^* formally represents the assumption that each delegate behaves on behalf of the domestic winning coalition he represents.

[6] In particular, defections from the grand coalition lead to the formation of a unique, smaller coalition. A similar and closely related assumption underlies the concept of γ core studied in Chander and Tulkens (1997). We shall discuss the relation of the present paper with their work in our conclusion.

5.3.3 Strong Nash Equilibria of the Game

When seen as outcomes of a coalition formation game, equilibrium coalition structures identify stable agreements. In particular, Strong Nash Equilibria of the game Γ are strategy profiles with the property of being immune from both individual and coalitional deviations.

Definition 10 *A* **Strong Nash Equilibrium** *of the coalition formation game Γ is a profile of strategies σ^* such that there exists no coalition $T \subseteq K$ with a vector of strategies σ_T such that for all $k \in T$*

$$u_k \left(\sigma_T, \sigma^*_{N \backslash T} \right) \geq u_k \left(\sigma^* \right)$$

and for at least one $j \in T$

$$u_j \left(\sigma_T, \sigma^*_{N \backslash T} \right) > u_j \left(\sigma^* \right).$$

Equilibrium coalition structures identify politically stable agreements. We will now assert that the grand coalition always obtains as a Strong Nash Equilibrium outcome of the game Γ.

Theorem. *The strategy profile $\sigma^* = (K, ..., K)$, in which all players choose the grand coalition, is a Strong Nash Equilibrium of the game Γ.*

This directly implies that the unique I.A. ratified by all countries is also immune from deviations by means of national leaders. In this sense, this agreement can be legitimately expected to be proposed (and ratified) at national levels.

We prove the theorem in the appendix, under an additional assumption on total cost of cooperation, closely related to assumption 1", defined on preferences, used in Chander and Tulkens (1997).

Assumption 8: Let $T \subset K$ be such that $|T| \geq 2$, and let $\pi(T)$ be any partition of T. Let $\pi = \left\{ \pi(T), \{j\}_{j \in K \backslash T} \right\}$ denote the cooperation structure in which all countries outside $\pi(T)$ appear as singletons. Then the aggregate abatement cost of at least one element T_j of $\pi(T)$ at the Partial Agreement Equilibrium $(\Delta p, \alpha_T)$ w.r.t. π is weakly greater than at the INPE. Formally,

$$\sum_{k \in T_j} \left[g_k \left(\bar{p}_k \right) - g_k \left(\bar{p}_k + \Delta p_k \right) \right] \geq 0.$$

Assumption 8 imposes a constraint on the way in which welfare improvements are attained through cooperation. It requires that if some sets of countries cooperate, then at least one of them does not obtain a higher level of private consumption than at the noncooperative equilibrium. In other words, the benefits of international cooperation must be, at least for one set of cooperating countries, not in terms of higher consumption levels but rather in terms of a higher environmental quality. This assumption is always satisfied if countries have the same production technology and\or constant returns to scale.

One final issue to be addressed is whether other coalition structures than the grand coalition may occur as equilibria of the game Γ - equilibria that would necessary be inefficient in view of the uniqueness property of the strategy adopted by any coalition of delegates. Let us consider in particular the most extreme case of inefficiency, namely the complete absence of cooperation, here represented by the coalition structure $\bar{\pi}$ consisting of all countries as singletons: can it be an equilibrium outcome of the game Γ?

It is instructive to deal first with the case in which domestic delegates maximize the aggregate welfare of their citizens (in terms of the game Γ, the case in which $S_k^* = B_k$ for all $k = 1, 2, ..., \bar{k}$). Let $\sigma^* = (K, K, ..., K)$ and $\bar{\sigma}$ be any strategy profile inducing the coalition structure $\bar{\pi}$. The uniqueness of the Pareto optimum of the economy E (proved in lemma 1), together with the characterization result of proposition 3, imply that:

$$\sum_{k \in K} u_k(\sigma^*) = \sum_{i \in I} u_i(z^*, x_i^*) > \sum_{i \in I} u_i(\bar{z}, \bar{x}_i) = \sum_{k \in K} u_k(\bar{\sigma}). \tag{3}$$

Note also that since the international agreement $(\Delta p^*, \alpha^*)$, induced by the profile σ^*, satisfies the conditions for a ratio equilibrium of the economy $\bar{E}$ (obtained from E by considering the INPE as initial endowment), the induced allocation is individually rational for all agents in the economy, in the sense that it is weakly preferred to the INPE allocation. This leads to the following inequalities:

$$u_i(z^*, x_i^*) \geq u_i(\bar{z}, \bar{x}_i) \ \ \forall i \in I, \tag{4}$$

implying that

$$u_k(\sigma^*) \geq u_k(\bar{\sigma}) \ \ \forall k \in K. \tag{5}$$

Conditions (3) and (4) imply that for some agent $i^* \in I$

$$u_{i^*}(z^*, x_i^*) > u_{i^*}(\bar{z}, \bar{x}_i). \tag{6}$$

Since we are assuming that $i^* \in B_k$ for some k, we conclude that for some k:

$$u_k(\sigma^*) > u_k(\bar{\sigma}). \tag{7}$$

Conditions (5) and (7) directly imply the following proposition.

Proposition 5 *Let $S_k^* = B_k$ for all $k = 1, 2, ..., \bar{k}$. Then, the coalition structure $\bar{\pi}$ in which no cooperation occurs is never a Strong Nash Equilibrium of the game Γ.*

Thus, if political delegates maximize their countries' aggregate welfare, an international equilibrium must always contain some degree of cooperation. By contrast, if lack of cooperation prevails, it can only be imputed to the fact that political delegates do not represent the totality of their population but only a majority of it. To see how this may undermine the result of proposition 5, consider again condition (3). If $S_k^* \subset B_k$ for some k, we obtain

$$\sum_{k \in K} u_k(\sigma^*) \neq \sum_{i \in I} u_i(z^*, x_i^*),$$

so that condition (3) can only be stated in the following form:

$$\sum_{i \in I} u_i(z^*, x_i^*) > \sum_{i \in I} u_i(\bar{z}, \bar{x}_i). \tag{8}$$

Again, we can use (8) to conclude that some i^* exists for which condition (6) holds. However, it may now be the case that (6) only holds for one agent $i^* \in B_k$ for some $k \in K$ for which $i^* \notin S_k^*$. If this is the case, no incumbent winning coalition strictly prefers the efficient outcome (z^*, x^*) to the INPE allocation, and the proof of proposition 5 does not extend.

Notice that when complete non cooperation arises in equilibrium, it is because all members of incumbent winning coalitions are as well off as at the efficient outcome (z^*, x^*), while the "minority" agents are prevented from exploiting the surplus of cooperation. In this sense it can be argued that inefficiency is here strictly due to the political nature of delegates strategies.

These arguments show that no cooperation may be a stable outcome, in the particular sense of Strong Nash Equilibria. However, condition ((4)) also implies that the set K of delegates must either prefer full cooperation to the complete absence of cooperation, or be indifferent between these two outcomes. If we define the notion of Strict Strong Nash Equilibrium by relaxing the requirement of strict improvement of at least one player in definition 10 above[7], the following directly follows:

[7]We are here extending the notion of Strict Nash Equilibrium to coalitional deviations, considered in the Strong Nash Equilibirum concept. Intuitively, the strictness refinement requires that players can only do worse by changing their strategies from the equilibrium.

Proposition 6 *The coalition structure $\bar{\pi}$ in which no cooperation occurs is never a Strict Strong Nash Equilibrium of the game Γ.*

6 Conclusions

In this paper we have looked at international agreements that satisfy two stability requirements: they are a stable solution of the international negotiation process *and* they are domestically stable in the sense that they are ratified by all parliaments. We identify a unique I.A. among the whole set of countries, with the following properties:

1. It prescribes the efficient emissions levels for the international economy (lemma 2);
2. It shares abatement costs among countries proportionally to the relative incomes at the INPE (proposition 2);
3. Domestically, no transfers occur, and each agent consumes the amount of private good determined by his distributive parameter and by his country's cost share (proposition 2).

Our main theorem establishes that if this agreement is chosen, then the grand coalition is a stable outcome of a suitably defined coalition formation game. Moreover, although (inefficient) cooperation structure with several coexisting coalitions are not ruled out in equilibrium, some degree of cooperation always emerges when political delegates represent the totality of their population.

The specific cost sharing rule implied by the stable I.A. in the present paper should be related with the core-stable allocation identified by Chander and Tulkens (1997) for a similar economy with quasilinear preferences. In both papers, the way in which costs are imputed in equilibrium satisfy the property of the "ratio equilibrium", introduced for an economy with public goods by Kaneko (1977). More precisely, both papers propose the ratio equilibrium of the economy $\bar{E}$, obtained from the economy E by considering the INPE as initial endowment. The induced allocation has the nice feature of being *computable*, requiring, in the present paper, the only information of aggregate income levels at the no-treaty state of the economy. While in Chander and Tulkens (1997) this allocation is shown only *to belong to the core* of the international economy (among possibly other ones), in the present paper it is shown to characterize the *unique stable agreement* among the whole set of countries. This difference is due to the introduction of voting as domestic decision process, replacing the traditional aggregate utility maximization within each coalition. Since objections are "easier" for winning coalitions than for unanimous coalitions, all allocations other than the ratio equilibrium are objected to in the present paper, while some of them may still be stable in Chander and Tulkens (1997). In contrast, while no inefficient outcome was stable in the core-theoretic analysis (mainly due to the possibility of the benevolent delegates to operate any desired transfers scheme), here inefficient cooperation structures may emerge for the impossibility of operating such transfers of private good, needed to attain Pareto improvements.

A final word must be spent on the robustness of our result to larger classes of preferences. The special class adopted in this paper simplifies the analysis in three respects. Firstly, it is responsible for the uniqueness of the various solution concepts adopted in the paper. Second, equilibrium ratios of the sub-economies coincide with the distributional vectors θ, making the present environment equivalent to one of linear income taxation and allowing for a nonempty set of political equilibria. Third, the transferable utility property of preferences allowed us to determine the payoffs of national delegates as the aggregate utility of the supporting winning coalition. Our main results would still carry over to a more general class of preferences, requiring monotonicity of preferences in the private good, normality of the public good "ambient quality". Our characterization of the stable agreement, on the contrary, is strictly related to the specific form of preferences we have adopted. Although politically stable I.A. would still satisfy the ratio equilibrium property, cost shares would not be directly related to national incomes at the INPE.

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APPENDIX

Proof of Lemma 1

Assumptions 1-4 ensure that efficient states of the economy E are all associated with points in the interior of the sets $[0, p_k^0]$, for all $k \in K$. In fact, $p_k = 0$ is never an efficient emission level, since $v'(z(0, p_{-k}))$ is bounded and $g_k'(0) = +\infty$. Similarly, p_k^0 is never an efficient emission level, since $g'(p_k^0) = 0$ and $v'(z(p_k^0, p_{-k}) > 0$. Efficient emission vectors maximize the aggregate welfare of the economy E, given by the expression

$$v\left(z\right) \sum_{j \in K} g_j(p_j). \tag{9}$$

By assumption 1-4, (9) is a concave function of p_k, for all for all $k \in K$. Therefore, Samuelson's conditions are necessary and sufficient for an efficient emission vector. These conditions imply that for all $k \in K$:

$$\begin{aligned} \left(\frac{v'\left(z\right)}{v\left(z\right)}\right) \cdot \sum_{j \in K} g_j(p_j) &= g_k'(p_k) \\ \left(\frac{v'\left(z'\right)}{v\left(z'\right)}\right) \cdot \sum_{j \in K} g_j(p_j') &= g_k'(p_k'). \end{aligned} \tag{10}$$

Suppose now that $p \neq p'$ and, w.l.g., $p_k > p_k'$. By concavity of technology, $g_k'(p_k) \leq g_k'(p_k')$. Since, by condition (10), $g_k'(p_k) = g_j'(p_j)$ for all $j, k \in K$ and $g_k'(p_k') = g_j'(p_j')$ for all $j, k \in K$, this would imply that $p_j \geq p_j'$ for all $j \in K$, and thus $z' > z$. It follows from strict monotonicity of g_j for all $j \in K$ that

$$\sum_{j \in K} g_j(p_j) > \sum_{j \in K} g_j(p_j') \tag{11}$$

and by concavity of v w.r.t. z that

$$\left(\frac{v'\left(z\right)}{v\left(z\right)}\right) \geq \left(\frac{v'\left(z'\right)}{v\left(z'\right)}\right). \tag{12}$$

Conditions (10), (11) and (12) contradict the requirement that $g_j'(p_j) \leq g_j'(p_j')$. It follows that $p_k = p_k'$ for all $k \in K$.

Proof of Lemma 2

A ratio equilibrium is a triple (p_k, x_k, r_k) such that every agent $i \in B_k$ demands the same vector p_k facing the ditributive vector r_i. Agen $i \in B_k$ demanding the emision p_k and facing the ratio r_i consumes the amount $x_i = \theta_i^k g_k(p_k^0) - r_i \left[g_k\left(p_k^0\right) - g_k\left(p_k\right)\right]$. Therefore, agent i faces the following problem

$$\max_{p_k} v\left(z\left(p_k, \bar{p}_{-k}\right)\right) \cdot \left[\theta_i^k g_k(p_k^0) - r_i \left[g_k\left(p_k^0\right) - g_k\left(p_k\right)\right]\right]. \tag{13}$$

The maximand is a concave function of p_k by assumptions 1-4. Moreover, by the arguments used in the previous lemma to show that efficient emission vectors are interior, we know that $p_k = 0$ is never a solution of (13). First order conditions yield

$$-v'\left(z\left(p_k, \bar{p}_{-k}\right)\right)\left[\theta_i^k g_k(p_k^0) - r_i \left[g_k\left(p_k^0\right) - g_k\left(p_k\right)\right]\right] + v\left(z\left(p_k, \bar{p}_{-k}\right)\right) \cdot r_i \cdot g_k'\left(p_k\right) = 0, \tag{14}$$

from which

$$r_i = \frac{v'\left(z\left(p_k, \bar{p}_{-k}\right)\right)\theta_i^k g_k(p_k^0)}{v'\left(z\left(p_k, \bar{p}_{-k}\right)\right)\left[g_k\left(p_k^0\right) - g_k\left(p_k\right)\right] + v\left(z\left(p_k, \bar{p}_{-k}\right)\right) g_k'\left(p_k\right)}. \tag{15}$$

By imposing the condition $\sum_{i \in B_k} r_i = 1$, we get

$$\sum_{i \in B_k} \frac{v'\left(z\left(p_k, \bar{p}_{-k}\right)\right)\theta_i^k g_k(p_k^0)}{v'\left(z\left(p_k, \bar{p}_{-k}\right)\right)\left[g_k\left(p_k^0\right) - g_k\left(p_k\right)\right] + v\left(z\left(p_k, \bar{p}_{-k}\right)\right) g_k'\left(p_k\right)} = 1 \tag{16}$$

from which, using the fact that $\sum_{i \in B_k} \theta_i^k = 1$, we get

$$\frac{v'\left(z\left(p_k, \bar{p}_{-k}\right)\right) g_k(p_k^0)}{v'\left(z\left(p_k, \bar{p}_{-k}\right)\right)\left[g_k\left(p_k^0\right) - g_k\left(p_k\right)\right] + v\left(z\left(p_k, \bar{p}_{-k}\right)\right) g_k'\left(p_k\right)} = 1 \tag{17}$$

yielding, together with (15),

$$r_i^* = \theta_i^k.$$

The fact that p^* is the efficient vector of the economy $E_k(\bar{p}_{-k})$ comes from the fact that ratio equilibria trivially satisfy the Samuelson's conditions for that subeconomy.

Proof of Proposition 1.

We know by lemma 2 that the distributive parameter θ^k is the unique vector of equilibrium ratios of the sub-economy $E_k\left(\bar{p}_{-k}\right)$. We also know by theorems 1 and 2 in Hirokawa (1992) that the core of the voting game $G_k\left(\bar{p}_{-k}\right)$ coincides with the set of ratio equilibrium allocations of the sub-economy $E_k\left(\bar{p}_{-k}\right)$. It follows that the unique political equilibrium is the state of the economy associated with the ratio equilibrium allocation of $E_k\left(\bar{p}_{-k}\right)$.

Proof of Proposition 2

Existence: We denote by $f_j(p_{-j})$ the Pareto efficient level of emissions in country j given the levels p_{-j}. Let also $f(p)$ be the kth product of the functions $f_j(p_{-j})$ for $j = 1, ..., \bar{k}$. A fixed point p^* of the map $f(p)$ is a point p^* such that $p^* \in f(p^*)$. By definition 6 and proposition 1, if p^* is a fixed point of f then the pair $\left(p^*, \left(\theta_i^k g_k\left(p_k^*\right)\right)_{i \in I}\right)$ is an INPE. By Kakutani fixed point theorem, f admits a fixed point if it is upper hemi continuous, covex valued and defined on a nonempty, compact and convex set. As the product maintains these properties, it is enough to check these conditions on each projection map $f_j(p_{-j})$. Since the domain of f_j is the closed, convex and non-empty set $\prod_{k \neq j}([0, \bar{p}_k])$ and since f is a function by lemma 1, we just need to show upper hemi continuity of f, i.e., of the efficient value p_j of the economy $E_j(p_{-j})$ as a function of p_{-j}. This directly follows from continuity of v and g_k.

Uniqueness: Assume that there exist two INPE $(p, x) \neq (p', x')$. Let z and z' be the induced amounts of ambient pollution. By the characterization of INPE, for all $k \in K$:

$$\begin{aligned} \left(\frac{v'(z)}{v(z)}\right) g_k(p_k) &= g_k'(p_k) \\ \left(\frac{v'(z')}{v(z')}\right) g_k(p_k') &= g_k'(p_k'). \end{aligned} \tag{18}$$

By the assumptions that $g_k' \geq 0$, $g_k'' \leq 0$, $v' \geq 0$ and $v'' \leq 0$ the following implications hold:

$$p_k' \geq p_k \Rightarrow g_k'(p_k') \leq g_k'(p_k) \Rightarrow \left(\frac{v'(z')}{v(z')}\right) \leq \left(\frac{v'(z)}{v(z)}\right) \Rightarrow z' \geq z.$$

Then, for some $j \neq k$ it must be that $p_j' \leq p_j$, implying, by the same series of implications, that $z' \leq z$. The two inequalities together yield that $z' = z$. Then, in any INPE the aggregate ambient pollution is the same. Suppose now that $p_k' > p_k$ for some k. Then, by concavity, $g_k'(p_k') \leq g_k'(p_k)$ and, by strict monotonicity, $g_k(p_k') > g_k(p_k)$. These two facts, together with the fact that $z' = z$ and the two first order conditions in (18), imply a contradiction.

Proof of Proposition 3.

Let $\bar{E}$ be the economy derived by E considering the INPE as initial endowment. In terms of Ito and Kaneko (1981), $\bar{E}$ is defined by considering the level of emissions at the INPE as *allowance level*, and individual incomes at the levels defined by the INPE production and by the distributinve vector θ. Agen i demanding the vector p and facing the ratio r_i consumes the amount $x_i = \theta_i^k g_k(\bar{p}_k) - r_i \sum_{j \in K} [g_j(\bar{p}_j) - g_j(p_j)]$. A ratio equilibrium for this economy is a triple (p, x, r) such that every agent i demands vector p facing the ditributive vector r_i. We first show that the triple (p^*, x^*, r^*), where p^* is the efficient vector of the economy E,

$$r_i^* = \theta_i^k \frac{g_k(\bar{p}_k)}{\sum\limits_{j\in K} g_j\left(\bar{p}_j\right)}$$

and for $i \in B_k$

$$x_i^* = \theta_i^k g_k(\bar{p}_k) - r_i^* \sum_{j\in K} \left[g_j\left(\bar{p}_j\right) - g_j\left(p_j^*\right)\right]$$

is the unique ratio equilibrium of $\bar{E}$. Agent i faces the following problem

$$\max_a v\left(z\left(p\right)\right) \cdot \left[\theta_i^k g_k(\bar{p}_k) - \sum_{j\in K} r_i^j \left[g_j\left(\bar{p}_j\right) - g_j\left(\bar{p}_j + \Delta p_j\right)\right]\right].$$

By the first order conditions we get

$$v'\left(z\left(p\right)\right) \left[\theta_i^k g_k(\bar{p}_k) - \sum_{j\in K} r_i^j \left[g_j\left(\bar{p}_j\right) - g_j\left(p_j\right)\right]\right] - v\left(z\left(p\right)\right) \cdot r_i^k \cdot g_k'(p_k),\ \forall k$$

from which, for all $l, m \in K$

$$r_i^j = \frac{v'\left(z\left(p\right)\right)\theta_i^k g_k(\bar{p}_k)}{v'\left(z\left(p\right)\right) \sum\limits_{j\in K} \left[g_j\left(\bar{p}_j\right) - g_j\left(p_j\right)\right] + v\left(z\left(p\right)\right) g_k'(p_k)}$$

$$r_i^m = \frac{v'\left(z\left(p\right)\right)\theta_i^k g_k(\bar{p}_k)}{v'\left(z\left(p\right)\right) \sum\limits_{j\in K} \left[g_j\left(\bar{p}_j\right) - g_j\left(p_j\right)\right] + v\left(z\left(p\right)\right) g_m'(p_m).}$$

Since Pareto Optimality implies that

$$g_k'(p_k) = g_m'(p_m)$$

it follows that in equilibrium $r_i^k = r_i^m = r_i, \forall j, m \in K$, $\forall i \in I$. By imposing the condition $\sum\limits_{i\in I} r_i = 1$, we get

$$\sum_{i\in I} \frac{v'\left(z\left(p\right)\right)\theta_i^k g_k(\bar{p}_k)}{v'\left(z\left(p\right)\right) \sum\limits_{j\in K} \left[g_j\left(\bar{p}_j\right) - g_j\left(p_j\right)\right] + v\left(z\left(p\right)\right) g_k'(p_k)} = 1$$

$$\left[\frac{v'\left(z\left(p\right)\right)}{v\left(z\left(p\right)\right) g_k'(p_k) + v'\left(z\left(p\right)\right) \sum\limits_{j\in K} \left[g_j\left(\bar{p}_j\right) - g_j\left(p_j\right)\right]}\right] \sum_{j\in K} g_j(\bar{p}_j) = 1$$

from which

$$r_i^* = \theta_i^k \frac{g_k(\bar{p}_k)}{\sum\limits_{j \in K} g_j(\bar{p}_j)}.$$

It can be easily checked by means of the relevant first order conditions that the vector p^* is indeed the efficient emission vector of the economy E.

We can again apply the results of lemmas 5 and 6 to conclude that α^* is the only vector inducing the same vector of emissions changes as a ratio equilibrium of every sub economy $\bar{E}(\alpha_k^*)$. The result then follows from theorems 1 and 2 in Hirokawa (1992).

Proof of Lemma 3

Existence of the PAE can then be proved by direct application of the formal argument used in the proof of existence of a INPE. In this respect, note that in the case of the PAE, each group of countries belonging to the same element of π jointly choose their vector of emissions, while in the case of the INPE each country is choosing a single level of emission. Since by Lemma 4 every element of π is choosing the unique efficient level of emissions in any PAE, the existence proof for the INPE, relying on Kakutani's fixed point theorem, can be applied, provided upper hemi continuity is preserved. In this respect, the same argument used in the proof of proposition 2 extends. Similarly, the argument for uniqueness used in proposition 2 carries over to this case.

Theorem. *The strategy profile $\sigma^* = (K, ..., K)$, in which all players choose the grand coalition, is a Strong Nash Equilibrium of the game Γ.*

We will first prove three preparatpry lemmas.

The first extends to the present setting the characterization results of Proposition 1 in Chander and Tulkens (1997). Let $\pi = \left(\pi(T), \{j\}_{j \in K \setminus T}\right)$ be a coalition structure on the set K obtained by considering an arbitrary partition $\pi(T)$ of the arbitrary set $T \subset K$ and all the elements in $K \setminus T$ as singletons. Let $(\Delta\tilde{p}, \tilde{\alpha}) = \left((\Delta\tilde{p}_T, \tilde{\alpha}_T), (\Delta\tilde{p}_j)_{j \notin T}\right)$ be the PAE with respect to π.

Lemma 4 **a)** $\left(\bar{p}_{T_j} + \Delta\tilde{p}_{T_j}\right)$ *is the efficient emissions vector of the economy* $E_{T_j}\left(\bar{p}_{K \setminus T_j} + \Delta\tilde{p}_{K \setminus T_j}\right)$ *for all* $T_j \in \pi(T)$, *and* $\Delta\tilde{p}_j$ *is the efficient emissions vector of the economy* $E_j\left(\bar{p}_{K \setminus j} + \Delta\tilde{p}_{K \setminus j}\right)$ *for all* $j \in K \setminus T$;

b) *the total emissions induced by the vector* $\Delta\tilde{p}$ *are smaller than or equal to the INPE emissions;*

c) *the emissions level of each country in* $K\backslash T$ *at* $(\Delta\tilde{p}, \tilde{\alpha})$ *is greater than or equal to its INPE level. Moreover, the aggregate emissions of the countries in* T *is not greater than at the INPE.*

Proof of Lemma 4

a) Directly implied by Lemma 4.

b) Let $\tilde{p} = \bar{p} + \Delta\tilde{p}$. First order optimality conditions imply that for all $j \notin T$:

$$\begin{aligned}\left(\frac{v'(\bar{z})}{v(\bar{z})}\right) \cdot g_j(\bar{p}_j) &= g_j'(\bar{p}_j) \\ \left(\frac{v'(\tilde{z})}{v(\tilde{z})}\right) \cdot g_j(\tilde{p}_j) &= g_j'(\tilde{p}_j).\end{aligned} \tag{19}$$

Suppose now that $\bar{z} > \tilde{z}$; then by strict concavity of v in z we have

$$\frac{v'(\bar{z})}{v(\bar{z})} \leq \frac{v'(\tilde{z})}{v(\tilde{z})}.$$

Using (19) we get

$$\frac{g_j'(\bar{p}_j)}{g_j(\bar{p}_j)} \leq \frac{g_j'(\tilde{p}_j)}{g_j(\tilde{p}_j)}.$$

Since the term $\frac{g_j'(p_j)}{g_j(p_j)}$ is decreasing in p_j by concavity of g_j, we get that $\bar{z} > \tilde{z} \Rightarrow \tilde{p}_j \leq \bar{p}_j$ for every $j \in K\backslash T$.

Consider now the partition $\pi(T)$. By point a) it follows that for all $k \in T_m$ and all $T_m \in \pi(T)$:

$$\frac{v'(\tilde{z})}{v(\tilde{z})} \sum_{j \in T_m} g_j(\tilde{p}_j) = g_k'(\tilde{p}_k)$$

and

$$\frac{v'(\bar{z})}{v(\bar{z})} g_k(\bar{p}_k) = g_k'(\bar{p}_k).$$

If $\bar{z} > z$ then, by similar arguments to the one used above we get

$$\frac{g_k'(\bar{p}_k)}{g_k(\bar{p}_k)} \leq \frac{g_k'(\tilde{p}_k)}{\sum\limits_{j \in T_m} g_j(\tilde{p}_j)}.$$

Rewriting the term $\sum\limits_{j \in T_m} g_j(\tilde{p}_j)$ as $\left[\sum\limits_{j \in T_m \backslash k} g_j(\tilde{p}_j) + g_k(\tilde{p}_k)\right]$ and using the fact that

$$\sum_{j \in T_m \backslash k} g_j(\tilde{p}_j) \geq 0$$

we obtain the following inequality

$$\frac{g_k'(\bar{p}_j)}{g_k(\bar{p}_j)} \leq \frac{g_k'(\tilde{p}_j)}{g_k(\tilde{p}_j)}$$

which implies that $\bar{p}_k \geq \tilde{p}_k$ for all $k \in T_m$. The two results together imply that $\bar{z} \leq \tilde{z}$, which contradicts the assumption. Then it must be that $\tilde{z} \geq \bar{z}$.

c) Suppose that $\tilde{p}_j < \bar{p}_j$ for some country $j \in K \backslash T$. Concavity of g_j implies

$$g_j'(\tilde{p}_j) \geq g_j'(\bar{p}_j)$$

or, by points a) and b),

$$\frac{v'(\tilde{z})\, g_j(\tilde{p}_j)}{v(\tilde{z})} \leq \frac{v'(\bar{z})\, g_j(\bar{p}_j)}{v(\bar{z})}. \tag{20}$$

Again using the definition of z and the fact that, by point b), $\bar{z} \leq \tilde{z}$, we conclude that (20) implies

$$g_j(\bar{p}_j) \leq g_j(\tilde{p}_j)$$

which, by the fact that g_j is monotonically increasing, implies a contradiction. This fact, together with point b), imply that aggregate emissions of countries in T are smaller at the PAE than at the INPE.

The next two lemmas establish consistency properties of the set of ratio equilibria. For a given real number $\alpha_k \in (0,1]$, let $E_k(\alpha_k)$ denote the economy with set of agents B_k and all other fundamentals as in E, and in which the cost function is given by $\alpha_k C_j(p_j)$ for all $j = 1, 2, ..., \bar{k}$.

Lemma 5 *(van den Nouweland, Tijs and Wooders). If (p^*, x^*, r^*) is a ratio equilibrium of the economy E and $\alpha_k^* = \sum\limits_{i \in B_k} r_i^*$, then $(p^*, x_k^*, \left(\frac{r_i^*}{\alpha_k^*}\right)_{i \in B_K})$ is a ratio equilibrium of $E_k(\alpha_k^*)$.*

Lemma 6 *Let $\alpha_1^*, ..., \alpha_{\bar{k}}^*$ be such that $\sum_k \alpha_k^* = 1$. If there exists p^* and τ^* such that (p^*, x_k^*, τ_k^*) is a ratio equilibrium of $E_k(\alpha_k^*)$ for all k, then there is a ratio equilibrium (p^*, x^*, r^*) of the economy E such that $\sum\limits_{i \in B_k} r_i^* = \alpha_k^*$ and $\tau_k^* = \left(\frac{r_i^*}{\alpha_k^*}\right)_{i \in B_K}$.*

Proof of Lemma 6.

Since (p^*, x_k^*, τ_k^*) is a ratio equilibrium of $E_k(\alpha_k^*)$ for all k, we can write that for all $i \in B_k$, for all $k \in K$ and for all p :

$$u_i(p^*, x^*) \geq u_i(p, \theta_i g_k\left(p_k^0\right) - \tau_i^* \alpha_k^* \sum_{j=1}^{\bar{k}} C(p_j)).$$

Since $\sum_{i\in B_k} \tau_i^* = 1$ for all k and $\sum_{k\in K} \alpha_k^* = 1$ it follows that

$$\sum_{k\in K}\sum_{i\in B_k} \tau_i^* \alpha_k^* = \alpha_1^* \sum_{i\in B_1} \tau_i^* + ... + \alpha_{\bar{k}}^* \sum_{i\in B_{\bar{k}}} \tau_i^* = 1$$

so that $(p, x^*, (\tau^* \alpha^*), x^*)$ is a ratio equilibrium of the economy E. The facts that $\alpha_k^* = \sum_{i\in B_k} r_i^*$ and $\tau_k^* = \left(r_{i,k}^*\right)_{i\in B_K}$ follow directly from the definition of $(\tau^* \alpha^*)$. In particular, for the economy E we have that $\alpha_k^* = \sum_{i\in B_k} \theta_i^k$ for all k is the unique vector compatible with a ratio equilibrium in each sub economy $E_k(\alpha_k^*)$.

Proof of the Theorem.

We are now ready to prove the theorem. We proceed by contradiction. Suppose that some coalition of players T improves upon the strategy profile σ^* by means of the alternative profile σ_T. Denote by $(\Delta p^*, \alpha^*)$ the ratified I.A., and by $(\Delta\tilde{p}, \tilde{\alpha})$ the PAE w.r.t. coalition structure $\tilde{\pi} = \left(\pi(T), \{j\}_{j\in K\setminus T}\right)$ induced by the deviation of T and whose elements are the partition $\pi(T)$ of the set T and all the players outside T as singletons. We will use the notiation z^* and $\tilde{z}$ to indicate the induced environmental qualities. Using the definition of payoffs, the fact that T improves upon σ^* implies that $\forall i \in S_k^*$ and $\forall k \in T$:

$$v(\tilde{z})\, x_i(\Delta\tilde{p}, \tilde{\alpha}_T) > v(z^*)\, x_i(\Delta p^*, \alpha^*). \tag{21}$$

By Lemma 4 we know that $\Delta\tilde{p}_k \geq 0$ for all $k \in K\setminus T$. Denoting by $\mathbf{0}_{K\setminus T}$ the vector of zero changes in emissions of countries in $K\setminus T$, this, together with monotonicity of v, implies

$$v\left(z\left(\Delta\tilde{p}_T, \mathbf{0}_{K\setminus T}\right)\right) \geq v(z(\Delta\tilde{p})). \tag{22}$$

Inequalities (21) and (22) imply that

$$v\left(z\left(\Delta\tilde{p}_T, \mathbf{0}_{K\setminus T}\right)\right) \cdot x_i(\Delta\tilde{p}, \tilde{\alpha}_T) > v(z^*) \cdot x_i(\Delta p^*, \alpha^*) \tag{23}$$

We show that (23) implies a contradiction. The argument goes by showing that for all $i \in S_k^*$ and $\forall k \in T$

$$x_i(\Delta\tilde{p}, \tilde{\alpha}) > x_i\left(\Delta\tilde{p}_T, \mathbf{0}_{K\setminus T}, \alpha^*\right). \tag{24}$$

Suppose not, so that for some $i \in S_k^*$ and some $k \in T$:

$$x_i\left(\Delta\tilde{p}_T, \mathbf{0}_{K\setminus T}, \alpha^*\right) \geq x_i(\Delta\tilde{p}, \tilde{\alpha}). \tag{25}$$

By the equilibrium properties of the cost share vector α_k^*, we obtain (see lemmas 5 and 6 and proposition 3):

$$v\left(z^*\right)\cdot x_i\left(\Delta p^*,\alpha^*\right)\geq v\left(z\left(\Delta\tilde{p}_T,\mathbf{0}_{K\setminus T}\right)\right)\cdot x_i\left(\Delta\tilde{p}_T,\mathbf{0}_{K\setminus T},\alpha^*\right). \tag{26}$$

Using (25) and (26) we obtain a contradiction of (23).

We then use the definitions of $x_i\left(\Delta\tilde{p},\tilde{\alpha}\right)$ and of $x_i\left(\Delta\tilde{p}_T,\mathbf{0}_{K\setminus T},\alpha^*\right)$ and sum up (24) over $i\in S_k^*$ and $\forall k\in T_m$ for some $T_m\in\pi\left(T\right)$ to obtain:

$$\sum_{i\in S_k^*}\sum_{k\in T_m}\bar{x}_i-\theta_i^k\tilde{\alpha}_k\sum_{j\in T_m}\left(g_j\left(\bar{p}_j\right)-g_j\left(\bar{p}_j+\Delta\tilde{p}_j\right)\right)>\sum_{i\in S_k^*}\sum_{k\in T_m}\bar{x}_i-\theta_i^k\alpha_k^*\sum_{j\in T_m}\left(g_j\left(\bar{p}_j\right)-g_j\left(\bar{p}_j+\Delta\tilde{p}_j\right)\right),$$

or, more simply,

$$\sum_{i\in S_k^*}\sum_{k\in T_m}\theta_i^k\left(\alpha_k^*-\tilde{\alpha}_k\right)\sum_{j\in T_m}\left(g_j\left(\bar{p}_j\right)-g_j\left(\bar{p}_j+\Delta\tilde{p}_j\right)\right)>0.$$

Using now assumption 8 and the definitions of α_k^* and $\tilde{\alpha}_k$ we obtain

$$\sum_{i\in S_k^*}\sum_{k\in T_m}\theta_i^k\left[\frac{g_k\left(\bar{p}_k\right)}{\sum_{j\in K}g_j\left(\bar{p}_j\right)}-\frac{g_k\left(\bar{p}_k\right)}{\sum_{j\in T_m}g_j\left(\bar{p}_j\right)}\right]>0.$$

Note that in the above summation, all terms in brakets are weakly negative, since $\sum_{j\in K}g_j\left(\bar{p}_j\right)\geq\sum_{j\in T_m}g_j\left(\bar{p}_j\right)$. This implies a contradiction and concludes the proof.

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