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The Implication of Irreversible Pollution on the Relation between Growth and the Environment: The Degenerate Kuznets Curve

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The Implication of Irreversible Pollution on the Relation between Growth and the Environment: The Degenerate Kuznets Curve[1]

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ABSTRACT.- We develop an overlapping generations model where consumption is the source of polluting emissions. Pollution stock accumulates with emissions but is partially assimilated by nature at each period. The regeneration capacity of nature is limited and vanishes beyond a critical level of pollution. We first show that there exist multiple equilibria. More importantly, some of them exhibit irreversible pollution levels although an abatement activity is effective. Thus, the simple fact to engage in maintenance does not necessarily suffice to protect the economy against the convergence toward a steady state having the properties of an ecological and economic poverty trap. In contrast with earlier related studies, the emergence of the environmental Kuznets curve is no longer the rule. We rather detect a sort of degenerate Kuznets curve that corresponds to the equilibrium trajectory leading to the irreversible solution.

Key words: overlapping generations, irreversible pollution, poverty trap, environmental Kuznets curve

JEL codes: Q56, D62, D91.

1 Introduction

In the earlier 1990's, an empirical litterature has developed with the purpose to study the relation between economic growth and pollution (World Bank (1992), Holtz-Eakin and Selden (1992), Grossman and Krueger (1993), (1995), Selden and Song (1994)). These works detect an inverted U-shaped relation between the income and the concentration of some air (SO_2, CO, NO_x) and water (nitrates, heavy metals, fecal coliform) pollutants. The intuition behind the emergence of such a relation, commonly called the environmental Kuznets curve (EKC), is the following. In the first steps of industrialization, pollution grows rapidly since on the one hand, the priority is given to wealth accumulation and on the other hand, the agents are more concerned with their employment and their revenue than with the quality of air or water (Dasgupta *et al.* (2002)). In more advanced development stages, as soon as revenues increase, the agents attach more value to the environment and, it justifies the implementation, by the competent authority, of a pollution regulation intended for reducing its damages.

The EKC has given rise to a controversy among the economists, in both empirical and theoretical disciplines. Recent empirical studies show that this relation is generally valid for the aforementioned short-lived pollutants although the results are more mitigated for water polluants (Paudel *et al.* (2005)). Moreover, stock pollutants, like CO_2, rather seem to be monotonically and positively related to income (exceptions are the studies of Carson *et al.* (1997) or Schmalensee *et al.* (1998) that also detect an EKC for CO_2). Theoretical works focus on the factors capable of explaining the emergence of this relation. For instance, Andreoni and Levinson (2001) state, in a simple static model, that increasing returns in abatement are the origin of the EKC. In growth models, John and Pecchenino (1994), Selden and Song (1995) or Stokey (1998) show that the EKC comes from a regime switching in abatement activities or, is due to the adoption of a less polluting technology. In John and Pecchenino (1994), the EKC results precisely from the existence of two distinct development stages. In the first phase (for low levels of income and pollution), the agents favour consumption and capital accumulation. In other words, they choose not to engage in maintenance and, economic growth is associated with the degradation of environmental quality. Once the economy reaches a sufficient level of wealth and/or suffers from important environmental damages, the agents are willing to clean up the environment. During the second period, growth is then accompanied by a continuous improvement (respectively fall) in the environment (respectively in pollution). Finally, the synthesis of these two stages gives a relation between growth and the environment having the same qualitative properties as the EKC.

The EKC has been for a long time the subject of an instrumentalization, by some economists, meant to ban any form of pollution regulation and, to promote growth as the only way to maintain a decent environment (Beckerman (1992), Bartlett (1994)). Dasgupta and Mäler (2002) reject the idea according to which it is possible to pollute as much as one wants today since, once the economy is rich, it is always possible to reverse by compensating for the past damages caused to the environment. This criticism is based on the notion of irreversibility of environmental damages introduced in earlier works in biology and ecology (Holling (1973), Peterman (1980)). These studies show that some ecosystems can possess more than one stable equilibrium. Multiplicity implies that when submitted to strong perturbations, these ecosystems are unable to recover their original state. The most famous evidence of irreversibility is given by the shallow lakes example but, as mentioned by Dasgupta and Mäler (2002), this notion can also be extended to global problems like the repercussions of greenhouse gases emissions on climate.

This concept has given rise to criticism of the assumption of a constant rate of pollution assimilation that is used, almost systematically, to describe pollution accumulation in growth models (see Keeler *et al.* (1971), Van der Ploeg and Withagen (1991), Gradus and Smulders (1993), John and Pecchenino (1994) among others). Many authors (Forster (1975), Comolli (1977) and Dasgupta (1982)) have then proposed to give up this assumption for a new formulation of the decay function incorporating the idea that high pollution levels alter drastically the recovery process of nature. In fact, from their viewpoint, it is unreasonable to think that the more the level of pollution, the greater the nature ability to absorb pollution. This proposition has given birth to a litterature that rather considers an inverted U-shaped assimilation function. Such a function notably gives an account of the finite regeneration capacity of nature that vanishes beyond a critical threshold of pollution. The major consequence of this formulation, in optimal control and optimal growth models, lies in the existence of multiple equilibra among which some are associated with irreversible pollution (Forster (1975), Cesar and de Zeeuw (1994), Tahvonen and Withagen (1996) or Toman and Withagen (2000)).

In this paper, we keep in mind the Dasgupta and Mäler (2002)'s statement that the EKC must be rejected when one admits the potential irreversibility of environmental damages. Our purpose is to measure the repercussions of irreversibility on the relation between growth and the environment. More precisely, we wonder why irreversibility may challenge the emergence of the EKC as it is depicted in John and Pecchenino (1994). The intuition being that obtaining such a relation, in their model with a stock pollutant, is widely conditioned by the assumption of a constant rate of decay. We then generalize John and Pecchenino (1994)'s overlapping generations model by replacing the controversial assumption of a constant assimilation with a decay function similar to the one used in Forster (1975).

In this setting, the equilibrium analysis reveals three important features of the model. First, there exist multiple equilibria with diametrically opposite properties. Second, some of them are characterized by irreversible pollution although an abatement activity is effective. In contrast with John and Pecchenino (1994), the latter result implies that it will not always be possible to reverse in terms of the control of man-made environmental damages. In fact, during the development stages where the agents do not have enough incentive to abate pollution, economic growth goes hand in hand with the accumulation of an ecological debt. But, in all likelihood, the accumulated debt may be such that, once the agents choose to engage in maintenance, this activity does not suffice to prevent an irreparable worsening of the environment. This, in turn, is associated with a stage of economic recession that drives the economy toward an ecological and economic poverty trap. Therefore, we highlight a new possible explanation of the emergence of poverty traps[1] which is based on the existence of a threshold effect in the regeneration law of nature. Finally, the dynamic analysis shows that the EKC is no longer the rule. We proceed to numerical simulations and rather detect a degenerate EKC corresponding to the equilibrium trajectory that leads to the long run solution with irreversible pollution.

The paper is organized as follows. Section 2 sets out the model. Section 3 consists in a detailed analysis of the equilibrium. In section 4, we illustrate our results with numerical simulations. Finally, section 5 concludes.

2 The model

We developp an overlapping generations model *à la* Allais (1947), Samuelson (1958) and Diamond (1965). In a perfectly competitive world, the firms produce a single homogenous good used both for consumption and investment. Consumption generates polluting harmful emissions.

2.1 The environment

In the absence of human activity, pollution accumulation, for non negative levels of the stock P_t, is described by the following equation:

$$P_{t+1} = P_t - \Gamma(P_t) \tag{1}$$

where $\Gamma(P_t)$ corresponds to the natural decay function that gives the amount of pollution assimilated by nature each period. Nature's ability to absorb pollution depends on the level of pollutant concentration. More precisely, our aim is to express the idea that too high levels of pollution alter the environment's recovery process in an irreversible way. Therefore, following Forster (1975), Cesar and de Zeeuw (1994) and Tahvonen and Withagen (1996), we assume an inverted U-shape decay function (see fig.1). Its properties, summarized in the assumption below, give an account of the potential irreversibility of environmental damages caused by pollution:

Assumption 1. *The decay function* $\Gamma(P) : \mathbb{R}^+ \to \mathbb{R}^+$ *(with* $\Gamma(0) = 0$*) is concave (*$\Gamma''(P) \leq 0$*), first incresing until a level* $\tilde{P}$ *then, decreasing until the pollution reaches the irreversibility threshold* $\bar{P}$ *(*$\Gamma'(P) \geq 0 \ \forall P \leq \tilde{P}$, $\Gamma'(P) < 0 \ \forall P \in]\tilde{P}, \bar{P}[$ *with* $\tilde{P} < \bar{P}$*). Beyond this value, assimilation is nill:* $\Gamma(P) = 0 \ \forall P \geq \bar{P}$. *We also assume that the amount of pollution assimilated at each period is lower than the stock i.e.* $\Gamma(P) < P \ \forall P > 0$.

For low pollution levels ($P_t \leq \tilde{P}$), the volume of pollution absorbed by nature is first growing with the stock. Then, beyond the turning point $\tilde{P}$, the regeneration capacity starts to decline and

[1]see Azariadis and Stachurski (2005) for a survey on the poverty traps litterature.

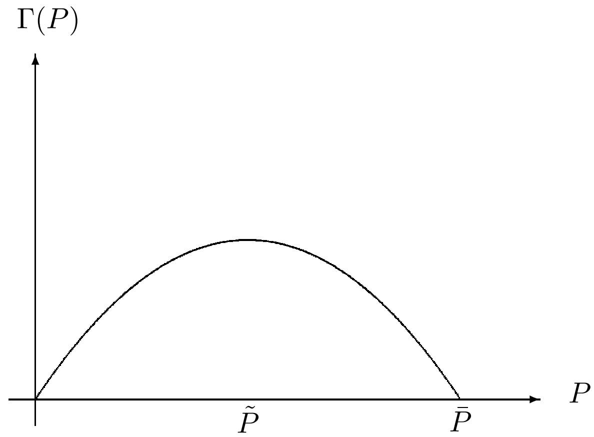

Figure 1: The assimilation function

assimilation decreases with the pollutant concentration. Finally, as soon as pollution reaches the critical level $\bar{P}$, the natural rate of decay is nill and pollution accumulation becomes irreversible. In other words, once the stock of pollution has achieved the critical threshold $\bar{P}$, the recovery process of nature is completely and permanently overwhelmed.

The pollution affects the quality of the environment. We define an index of environmental quality Q_t as the difference between a positive constant $\bar{Q}$ and the stock of pollution: $Q_t = \bar{Q} - P_t$. The level $\bar{Q}$ represents the highest stationary level of environment reached when pollution is nill. Assuming non negative pollution levels boils down to consider that $\bar{Q}$ is the upper bound of the domain of definition of Q_t. Environmental quality evolves according to the following dynamics:

$$Q_{t+1} = N(Q_t) \tag{2}$$

where $N(Q_t) :] - \infty, \bar{Q}] \rightarrow] - \infty, \bar{Q}]$ corresponds to a sort of law of motion of the environment. This function is defined piecewise:

$$N(Q_t) = \begin{cases} Q_t & \forall Q_t \leq \bar{Q} - \bar{P} \\ Q_t + \Gamma(\bar{Q} - Q_t) & \forall Q_t \in]Q - \bar{P}, \bar{Q}] \end{cases} \tag{3}$$

Its properties derive immediatly from those of the decay function. For low levels of environment $Q_t \leq \bar{Q} - \bar{P}$ (corresponding to irreversible pollution), it is simply linear. Beyond the threshold level $\bar{Q} - \bar{P}$, this function is increasing in Q_t and concave (for more details, see appendix A.1).

We now turn to the analysis of the private agents' choices and trade-off.

2.2 Production

Under perfect competition, the firms produce the final good Y_t with a constant return to scale technology using labor L_t and capital K_t:

$$Y_t = F(K_t, L_t) \tag{4}$$

Assumption 2. *The production function $F(K, L) : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is $\mathbb{C}^2$ and satisfies the following properties. It is increasing with respect to each argument, homogeneous of degree one and concave ($F_i > 0$, $F_{ii} \leq 0$ for $i = 1, 2$). The cross derivative is positive: $F_{12} > 0$. In addition, $F()$ satisfies Inada conditions for capital: $F(0, L) = 0$, $\lim_{K \to 0} F_1(K, L) = +\infty$ and* $\lim_{K \to +\infty} F_1(K, L) = 0$.

The homogeneity of degree one allows us to rewrite the production function in intensive terms: $F(K_t, L_t) = L_t F(k_t, 1) = L_t f(k_t)$ with $k_t = K_t/L_t$ the capital-labor ratio. The output per capita $f(k_t)$ is also monotonically increasing and concave in k_t. We further assume that it satisfies the Inada conditions: $f(0) = 0$, $\lim_{k \to 0} f'(k) = +\infty$ and $\lim_{k \to +\infty} f'(k) = 0$.

The capital depreciates at a constant rate $\delta < 1$. Firms maximise profits, taking the price of inputs as given:

$$\max_{K_t, L_t} F(K_t, L_t) - r_t K_t - w_t L_t - \delta K_t \tag{5}$$

where w_t represents the wage rate while r_t is the real rental rate of capital.

The first order conditions for profit maximization, expressed in terms of per capita variables, write :

$$w_t = f(k_t) - k_t f'(k_t) \tag{6}$$

$$r_t = f'(k_t) - \delta \tag{7}$$

2.3 The households

We consider an infinite horizon economy composed of finite-lived agents. A new generation is born at each period $t = 1, 2, ...,$ and lives for two periods: youth and old age. There is no population growth and the size of a generation is normalized to unity $N \equiv 1$. The young agent born at period t is endowed with one unit of labor which he (she) supplies to firms ineslatically for a real wage w_t. He (she) allocates this wage to savings s_t and maintenance m_t[2,3]. When retired, the agent supplies his (her) savings to firms and earns the return of savings $R_{t+1}s_t$ (with $R_{t+1} = 1 + r_{t+1}$ the interest factor). His (her) income is entirely devoted to consumption c_{t+1}. The two budget constraints faced by the agent respectively write:

$$w_t = s_t + m_t \tag{8}$$

[2] It is possible to reinterpret m_t as a tax levied by a one period lived government in order to finance the abatement activity, for the benefit of agents living during its period of office (John *et al.* (1995)).

[3] We do not consider any first period consumption. This simplifying assumption allows us to focus on the crucial trade-off between final good and environmental good consumptions (see next page the representative agent problem). Anyway, adding a first period consumption would not change our qualitative results.

$$c_{t+1} = R_{t+1} s_t \tag{9}$$

The preferences of the agent born at date t are defined on old age consumption and environmental quality. They are described by the utility function $U(c_{t+1}, Q_{t+1})$.

Assumption 3. *The utility function* $U(c, Q) : \mathbb{R}^+ \times] - \infty, \bar{Q}] \to \mathbb{R}$ *is* $\mathbb{C}^2$*. It is increasing and concave with respect to each argument:* $U_1 \geq 0$*,* $U_2 \geq 0$*,* $U_{11}, U_{22} \leq 0$*. The cross derivative is positive* $U_{12} \geq 0$[4]*. We further assume that* $\lim_{c \to 0} U_1(c, P) = +\infty$*.*

The quality of the environment is degraded by pollution. Following John and Pecchenino (1994) and John *et al.* (1995), we identify the source of polluting emissions, at period t, as the consumption of the old c_t. Emissions contribute to the accumulation of the pollutant stock. It is also possible to control the periodic flux of emissions and to improve environmental quality through the abatement expenditures of the young m_t. Real emissions are simply represented by the following linear function: $E(c_t, m_t) = \beta c_t - \gamma m_t$ with $0 \leq \beta, \gamma < 1$. In the presence of human activity, the law of motion of the environment then becomes:

$$Q_{t+1} = N(Q_t) - E(c_t, m_t) \tag{10}$$

We shall note that our model differs from John and Pecchenino (1994) who use the constant rate of decay assumption in order to modelize environmental dynamics[5].

In this framework, the households typically face an intergenerational externality. When the old consumes, he (she) does not take into account the negative repercussions of his (her) choice on future generations, going through the environmental quality bequeathed to them. In the same way, when the young chooses the amount of resources to devote to maintenance, he (she) only cares about the environment he (she) will enjoy in old age. But, the agent ignores future benefits of his (her) investment.

The role played by the intergenerational dimension is clearly more important than in John and Pecchenino (1994)'s model. In fact, the present generations decisions make future generations run the risk of facing an incurably degraded environment (because of the potential irreversibility of pollution).

The representative agent born at date t shares his (her) first period income among savings (which determines the consumption of the final good) and abatement (which influences the

[4]This assumption expresses a sort of complementarity between consumption and environmental quality in the agent's utility. An increase in the environment enhances the marginal utility of consumption and implies that the agent has a higher desire to consume. The alternative assumption $U_{12} < 0$ reflects, on the contrary, that the two goods are substitutes.

[5]In fact, they use the following specification: $Q_{t+1} = (1 - \Gamma) Q_t - \beta c_t + \gamma m_t$ where $0 \leq \Gamma < 1$ represents the constant rate of assimilation.

"consumption" of the environmental public good) in order to maximise his (her) lifetime utility. Taking as given prices and environmental quality at the beginning of period t, the problem writes:

$$\max_{s_t, m_t, c_{t+1}} U(c_{t+1}, Q_{t+1})$$

subject to,

$$\begin{cases} w_t = s_t + m_t \\ c_{t+1} = R_{t+1} s_t \\ Q_{t+1} = N(Q_t) - E(c_t, m_t) \\ m_t \geq 0 \end{cases}$$

The first order conditions read:

$$-R_{t+1}U_1(c_{t+1}, Q_{t+1}) + \gamma U_2(c_{t+1}, Q_{t+1}) + \mu = 0 \quad (11)$$

$$\mu m_t = 0 \quad (12)$$

with $\mu \geq 0$, the associated Lagrange multiplier.

Since there is a non negativity constraint on m_t, we have to distinguish the case where abatement is ineffective *i.e*, $m_t = 0$, from the one where the agents choose to engage in maintenance *i.e*, $m_t \geq 0$. Moreover, this study must also be divided in two sub-cases depending on whether or not, environmental quality has crossed the irreversibility threshold $\bar{Q} - \bar{P}$.

In the following section, we characterize the competitive equilibrium by analyzing separately the four possible cases, enumerated above, for which the model exhibits quite distinct economic and environmental dynamics.

3 The competitive equilibrium

3.1 Positive maintenance equilibrium

When the households' problem admits an interior solution, the FOC rewrites:

$$R_{t+1}U_1(c_{t+1}, Q_{t+1}) = \gamma U_2(c_{t+1}, Q_{t+1}) \quad (13)$$

This condition expresses the trade-off inherent in the abatement decision at period t. An increase in the invesment m_t is a mean to improve environmental quality and thus, to enhance welfare (right hand side). However, it also implies a fall in the non-environmental component of utility (left hand side) since both savings and old age consumption decrease. Consequently, the agent chooses m_t to equate the marginal benefit of maintenance to its marginal cost.

We now define the intertemporal equilibrium with perfect foresight:

Definition 1.

A competitive equilibrium is a sequence of per capita variables $\{c_t, m_t, s_t\}$*, a sequence of aggregate variables* $\{L_t, K_t, Q_t\}$ *and a sequence of prices* $\{R_t, w_t\}$ *such that:*

i/ households and firms are at their optimum: the FOC (13) and the two conditions (6)- (7), for profit maximization, are satisfied,

ii/ all markets clear: $L_t = N = 1$ *on the labor market and* $K_{t+1} = s_t (= k_{t+1})$ *on the capital market,*

iii/ budget constraints (8) and (9) are satisfied,

iv/ the dynamics of environmental quality are given by (10).

The equilibrium analysis consists in considering the system of equations (6)-(13) and the market clearing conditions. Combining these equations yields the expression of consumption and maintenance decisions as functions of the capital stock

$$c_t = (1 - \delta)k_t + k_t f'(k_t) \tag{14}$$

$$m_t = f(k_t) - k_t f'(k_t) - k_{t+1} \tag{15}$$

and, the emission function $E(c_t, m_t)$ rewrites:

$$E(k_t, k_{t+1}) = \beta \left((1 - \delta)k_t + k_t f'(k_t)\right) - \gamma \left(f(k_t) - k_t f'(k_t) - k_{t+1}\right) \tag{16}$$

We next define the capital share of output and the elasticity of substitution between capital and labor as follows:

$$s(k_t) = \frac{k_t f'(k_t)}{f(k_t)} \tag{17}$$

$$\sigma(k_t) = -\frac{(1 - s(k_t))f'(k_t)}{k_t f''(k_t)} \tag{18}$$

Then, we set two conditions concerning the properties of the production function.

Assumption 4. *The production function satisfies the following conditions:*

$$\lim_{k_t \to 0} \frac{f(k_t) - k_t f'(k_t)}{k_t} > 1 \tag{19}$$

$$\sigma(k_t) \geq 1 - s(k_t) \tag{20}$$

Condition (19) is analogous to the *no catching point* condition, presented in De La Croix and Michel (2002), according to which the first unit of capital must be sufficiently efficient, in terms of labor productivity (recall that the numerator in (19) corresponds to the wage $w(k_t)$), to avoid the trivial equilibrium associated with $k = 0$. This assumption guarantees that the stationary maintenance is positive in the neighborhood of zero. The second inequality (20) states that the elasticity of substitution between capital and labor is higher than the labor share of output[6]. It is a sufficient condition ensuring that the consumption function is increasing in k.

By substituting equations (14) and (15) into (13), we get the following equation:

$$R(k_{t+1})U_1(c(k_{t+1}),Q_{t+1}) - \gamma U_2(c(k_{t+1}),Q_{t+1}) = 0 \tag{21}$$

This equation corresponds to an equilibrium relation,

$$Q_t = Q^e(k_t) \tag{22}$$

which is valid for any t and increasing[7]: $Q^{e\prime}(k_t) \geq 0$. It governs the dynamics in the whole positive maintenance space, regardless of the level of environmental quality (reversible or not).

Finally, the substitution of equation (16) into the law of motion for the environment (10), together with (21), completely characterizes the equilibrium dynamics:

$$\begin{cases} R(k_{t+1})U_1(c(k_{t+1}),Q_{t+1}) - \gamma U_2(c(k_{t+1}),Q_{t+1}) = 0 \\ Q_{t+1} = N(Q_t) - E(k_t,k_{t+1}) \end{cases} \tag{23}$$

In the next step of our analysis, we focus on the properties of the equilibrium depending on whether or not, the economy has achieved the critical threshold $\bar{Q} - \bar{P}$.

Steady state analysis: A steady state (k,Q) is solution of the following system:

$$\begin{cases} R(k)U_1(c(k),Q) - \gamma U_2(c(k),Q) = 0 \\ Q - N(Q) = -E(k) \end{cases} \tag{24}$$

The existence conditions for an interior solution are summarized in the following proposition. We restrict our study to the interval $[0,\bar{k}]$ on which stationary maintenance is necessarily non negative: $m(k) \geq 0$ (see appendix $A.1$ for the definition of the upper bound $\bar{k}$).

Proposition 1.
i/ Under the condition

$$\lim_{k\to 0} \frac{r(k)}{m'(k)} < \frac{\beta+\gamma}{\beta} \tag{25}$$

[6]This condition seems quite reasonable since most of the estimations of the labor share and the elasticity of substitution give values respectively comprised in the range $[0.6, 0.7]$ and closed to 1. An example of production function satisfying this condition is the Cobb-Douglas technology. We refer the reader to appendix $A.1$ for a detailed analysis of the properties of relations (14), (15) and (16).

[7]By the implicit functions theorem, we have:

$$Q^{e\prime}(k_t) = -\frac{R'U_1 + Rc'U_{11} - \gamma c' U_{12}}{RU_{12 -} \gamma U_{22}}$$

and, under the set of our hypothesis, this derivative is positive.

there exists a steady state characterized by a level of environmental quality less than or equal to the threshold value $\bar{Q} - \bar{P}$,

ii/ Under the two additional conditions,

$$\sup_{P \in [0,\bar{P}]} \{\Gamma(P)\} \geq \sup_{k \in [0,\bar{k}]} \{E(k)\} \tag{26}$$

$$Q^e(\bar{k}) \geq \bar{Q} - \tilde{P} \tag{27}$$

there also exists a steady state with a level of environment above $\bar{Q} - \bar{P}$.

Proof. see appendix $A.2$. ■

Condition (25) is sufficient to proove the existence of an interior solution associated with irreversible pollution. This inequality implies that stationary emissions $E(k)$ are negative in the neighborhood of 0 (since $E(0) = 0$ and $E'(0) < 0$). Considering negative emissions boils down to assume that there exists a man-made environment. This case (picked out by Forster (1975) among others), is all the more likely in our model since the impact of an increase in k on maintenance exceeds its repercussions on consumption[8]. It also suppose the difference between the parameters of emissions diffusion β and abatement efficiency γ is relatively tight (when $\beta \geq \gamma$). The additional condition (26), for a solution with reversible pollution, corresponds to a rewriting, in our general equilibrium framework, of the condition used by Tahvonen and Withagen (1996). It conveys the idea that the maximum potential of natural assimilation is intrinsically higher than the maximum volume of emissions on the significant domains of variation for k and Q. Finally, (27) is a technical condition that precisely ensures some correspondance between the domains of definition of the studied variables. Without loss of generality, it allows us to deal with the analysis of existence.

Thus, the major implication of the inverted U-shape assumption, for the decay function, is the existence of multiple equilibria among which some are associated with the irreversibility of pollution. We find a quite similar result as the one stated in partial equilibrium (or simplified general equilibrium) studies that introduce this type of formulation of the recovery process in optimal control models (see Forster (1975), Tahvonen and Withagen (1996) or Toman and Withagen (2000)).

Local dynamics: We linearize the system (23) around a steady state (k^*, Q^*) and we get:

$$\begin{cases} dQ_{t+1} = Q^{e\prime}(k^*)dk_{t+1} \\ dQ_{t+1} = N'(Q^*)dQ_t - (E'(k^*) - \gamma)dk_t - \gamma dk_{t+1} \end{cases} \tag{28}$$

[8]If one refers to the Cobb-douglas example, then, it is straighforward that

$$\lim_{k \to 0} \frac{c(k)}{m(k)} < 1$$

as soon as the capital share of output is less than 1/2. Since $c(0) = m(0) = 0$, it implies that an increase in k has a stronger impact on $m()$ than on $c()$.

The manipulation of expressions in (28) gives the linearized system in the two state variables and allows us to study local stability. Once again, we distinguish the case $Q^* \leq \bar{Q} - \bar{P}$ from the opposite.

Proposition 2.
i/ An irreversible steady state is stable if and only if:

$$0 < E'(k^*) \leq 2(\gamma + Q^{e\prime}(k^*)) \tag{29}$$

ii/ when pollution is reversible, the necessary and sufficient condition becomes

$$(N'(Q^*) - 1)Q^{e\prime}(k^*) < E'(k^*) \leq \gamma + (1 + N'(Q^*))Q^{e\prime}(k^*) \tag{30}$$

Proof. see appendix $B.1$. ■

Condition (29) imposes that the derivative of the emission function relative to capital is positive. It also supposes that the impact of a rise in capital on emissions is less than a bound defined particularly by its impact on environmental quality at equilibrium (measured by $Q^{e\prime}(k^*)$). This condition ensures that the economy is able to assimilate a capital shock and to recover in a few periods its original state. When the level of pollution is below the critical point $\bar{P}$, the condition (30) generalizes (29). We shall note that the inequality $N'(Q^*) \leq 1$ is sufficient to guarantee that the repercussions of a shock on environmental quality are absorbed from periods to periods[9].

To summarize, the analysis of interior solutions reveals the existence of multiple equilibria whose properties are diametrically opposite since some of them exhibit an irremediably degraded environment while the others are associated with sustainable environment. The coexistence of these two types of solution shows that the agents' investment in abatement is not sufficient enough to protect the economy from the coming of an environmentally poor long term state. This result clearly differs from John and Pecchenino (1994) who state that maintenance is a "sufficient condition" for the improvement of environmental quality.

A synthesis of equilibrium properties also shows that a process of unregulated growth can drive the economy toward a poverty trap both economic and ecological. In fact, "irreversible" steady states are characterized not only by a level of environmental quality below the irreversibility threshold $\bar{Q} - \bar{P}$ but also, by a capital stock which is lower than the level achieved at any reversible solution. This result contributes to the growing litterature on poverty traps. Since the seminal paper of Azariadis and Drazen (1990), numerous studies have tried to determine

[9] The linear formulation of environmental dynamics (see note 4) does not allow to catch this new effect. The specificity (and limits) of this approach lies in the fact that nature is always able to assimilate a shock on the environment which returns to its stationary level in a finite time. On the contrary, in our framework, if $N'(Q) > 1$ then, this shock echoes in a more than proportional manner and the stability property is lost.

the factors explaining the emergence of such states (see notably Azariadis and Stachurski (2005) for a survey). Among the arguments frequently invoked, are an insufficient investment in human capital or the existence of threshold externalities affecting production or education technologies[10]. Here, we propose a new possible source of traps that comes from the potential irreversibility of pollution. The critical value $\bar{P}$ precisely implies the existence of a threshold effect playing through the regeneration law of nature. Moreover, we obtain a solution that exhibits a double characteristic of trap since it holds simultaneously a low level of wealth and a high concentration of pollution.

The fact to know if the economy will be dragged down into the poverty trap or if, on the contrary, it will enjoy a stage of economic growth combined with an increase in the environment depends obviously on its initial location. Any economy with initially low levels of wealth and environment would probably have the greatest difficulties in escaping from this impoverishment area. In section 4, we will illustrate our purpose with numerical simulations of the basins of attraction for each type of solution.

3.2 Zero maintenance equilibrium

We now analyze the equilibrium when the non negativity constraint on maintenance is binding *i.e*, $m_t = 0$. A justification for this study is that some economies, the less developed ones, may not be concerned with the protection of the environment and rather favor wealth accumulation. In other words, these economies, in the first stages of development, may be too poor to have the incentive to abate (Dasgupta *et al.* (2002)).

Definition 2.

A zero maintenance equilibrium with perfect foresight is given by the sequence of per capita variables $\{c_t, s_t\}$,*the sequence of aggregate variables* $\{L_t, K_t, Q_t\}$ *and the sequence of prices* $\{R_t, w_t\}$ *such that:*

i/ the households save the whole of their wage: $w_t = s_t$ *and, the two conditions (6)-(7), for profit maximization, are satisfyied,*

ii/ all markets clear: $L_t = N = 1$ *on the labor market and* $k_{t+1} = s_t$ *on the capital market,*

iii/ the dynamics of environmental quality are given by (10).

In this context, the representative agent does not face any trade-off since the constraint's weight is such that his (her) whole first period income is allocated to savings. The equilibrium analysis goes through the study of a system of equations similar to the one detailed in the previous section except that the FOC (13) is replaced by $m_t = 0$.

[10]These externalities imply that the production of education, for instance, first exhibits decreasing returns to scale for low levels of human capital. Then, above a critical value of the stock, there is a change in the technology and returns to scale become constant and even increasing.

We immediatly obtain the expression of capital accumulation in equilibrium,

$$k_{t+1} = f(k_t) - k_t f'(k_t) \tag{31}$$

and, one may note that this equation is independent of the evolution of the quality of the environment (which is still conditioned by the fact to know whether or not pollution is reversible). Emissions only depend on consumption: $E(k_t) = \beta c(k_t)$. The consumption decision, at equilibrium, remains unchanged and corresponds to (14).

The dynamics are then characterized by (31) and the equation describing the evolution of environmental quality:

$$Q_{t+1} = N(Q_t) - E(k_t) \tag{32}$$

Our analysis of the corner equilibrium follows the same steps that the ones developed in section 2.

Steady state analysis: A steady state now solves the following system of equations:

$$\begin{cases} k = f(k) - kf'(k) \\ Q - N(Q) = -E(k) \end{cases} \tag{33}$$

Proposition 3.

i/ there is no steady state associated with both zero maintenance and irreversible pollution,

ii/ under the conditions (19) and (26), there exists a corner steady state that exhibits sustainable environment.

Proof. see appendix $A.2$. ■

The lack of steady state characterized by irreversibility of pollution is explained by the environment inability to stabilize to a constant long run level. In fact, when the agents choose not to engage in maintenance, environmental quality deteriorates perpetually since the assimilation capacity of nature is nill and there is no force able to compensate for polluting emissions. The only way to stop its degradation is to cease consuming and, even more, to stop all productive activity. But, under our whole assumptions (on preferences and technology), this limit case can be excluded.

It is also possible to specify the number of corner solutions with reversible pollution since the conditions (19) and (26) are from now on necessary and sufficient.

Corollary. *As soon as the inequality (26) is strict, there exist exactly two solutions for* $k \in [0, \bar{k}]$.

Proof. see appendix $A.2$. ■

The analysis lies now in the study of local stability of zero maintenance reversible equilibria.

Local dynamics: The system (31)-(32) linearized around a steady state (k^*, Q^*) writes :

$$\begin{cases} dk_{t+1} = w'(k^*)dk_t \\ dQ_{t+1} = N'(Q^*)dQ_t - \beta c'(k^*)dk_t \end{cases} \tag{34}$$

and, we immediatly get the stability result.

Proposition 4.
The necessary and sufficient conditions for stability are the following:

$$w'(k^*) < 1 \tag{35}$$

$$N'(Q^*) < 1 \tag{36}$$

If the inequality (26) is strict, then the "high" steady state is locally stable while the "low"solution is unstable.

Proof. see appendix $B.2$ ■

Condition (35) implies that the economy is able to absorb a capital shock whereas inequality (36) expresses, once again, the environment capacity to recover its original state when submitted to an exogenous perturbation.

In fact, the stationary level k^*, for any constrained solution, equals to the upper bound of the interval $[0, \bar{k}]$. Now, from the definition of $\bar{k}$, we know that $w'(\bar{k}) < 1$ (see appendix $A.1$). Moreover, if (26) is strict, then the two steady states are associated with environmental quality levels that are necessarely located on both sides of the remarkable value $\bar{Q} - \tilde{P}$ (such that $N'(\bar{Q} - \tilde{P}) = 1$). Therefore, from the dynamic law of the environment, the low solution is unstable while the other is stable.

The analysis reveals two important features of the model. First, there exist multiple equilibria not only for the interior solution but also, for the zero maintenance case. Second, some interior steady states are characterized by irreversible pollution while all the corner equilibria exhibit reversibility.

We now turn to the study of the eligibility problem. This analysis boils down to examine the different equilibria location with respect to the two frontiers delimiting on the one hand, the interior area from the constrained one and, on the other hand, the irreversible pollution space from the reversible zone.

3.3 The frontier case

The frontier that distinguishes the zero maintenance case from the situation where the agents are willing to abate is defined by the following equation (obtained by subtituting $m_t = 0$ and $\mu = 0$ in (11)):

$$R(w(k_t))U_1(c(w(k_t)), N(Q_t) - \beta c(k_t)) - \gamma U_2(c(w(k_t)), N(Q_t) - \beta c(k_t)) = 0 \tag{37}$$

Equation (37) characterizes an increasing function relating the two stock variables, $Q_t = Q^f(k_t)$[11], which corresponds to the real separation of the $k - Q$ space between the two regions. This frontier represents the set of points where the agents are indifferent whether or not they invest in abatement expenditures. When the economy is located above this manifold, environmental quality is sufficiently good and/or capital stock is so low that the agents choose not to engage in maintenance. On the contrary, when economic activity takes place in the region below the frontier, the agents decide to maintain the environment. Even if the qualitative properties of $Q^f(k_t)$ are relatively invariant, we shall note that the level of environment, in conditioning the expression of $N(Q_t)$, influences its particular curvature. The second frontier, delimiting reversible levels of pollution from the irreversibility region is precisely given, in the $k - Q$ space, by the horizontal $Q = \bar{Q} - \bar{P}$.

The location of the steady states with respect to these frontiers is crucial when we are concerned with the question of eligibility. In fact, we have studied the four dynamic system corresponding to each possible region of the $k - Q$ space. We have next established the existence of solutions for three of these sub-spaces. But, it is possible that, during the convergence toward a stable solution of a determined zone, the equilibrium trajectory crosses one or the other frontier before reaching the steady state. Now, as soon as the trajectory goes through one of the two frontiers, the dynamics are governed by a new system totally different from the one valid in the previous region. In other words, the stable solution in consideration is not eligible since, once the frontier is crossed, the economy will converge to another stable solution associated with the new significant dynamics.

In order to have a better understanding of the dynamic behaviour of the economy, we proceed, in the next section, to numerical simulations intended for illustrating some noteworthy equilibrium trajectories. Finally, we discuss the possibility that an environmental Kuznets curve arises from the model.

4 Discussion

The purpose of this section is to isolate the impact of a inverted U-shaped decay function on environmental and economic dynamics. The key question is to know whether or not the potential irreversibility of pollution challenges the main result of John and Pecchenino (1994), that is, the existence of an environmental Kuznets curve (EKC). To answer this question, we have recourse to a numerical example characterized by the following functional forms:

[11] From equation (37), using the implicit functions theorem gives

$$Q^{f\prime}(k) = \frac{R'w'U_1 + Rc'w'U_{11} - \beta c'U_{12} - \gamma c'(w'U_{12} - \beta U_{22})}{N'(\gamma U_{22} - RU_{12})}$$

and, under all our assumptions, this ratio is positive.

The decay function is described, for $P_t \geq 0$, by a function defined piecewise:

$$\Gamma(P_t) = \begin{cases} \theta P_t(\bar{P} - P_t) & \forall P_t < \bar{P} \\ 0 & P_t \geq \bar{P} \end{cases}$$

the volume of pollution assimilated is first increasing in the stock until the level $\bar{P}/2$ next, it is decreasing. Beyong the critical threshold $\bar{P}$, the natural capacity to absorb pollution vanishes.

The environmental "law of motion" is deduced from the previous expression:

$$N(Q_t) = \begin{cases} Q_t & \forall Q_t \leq \bar{Q} - \bar{P} \\ Q_t + \theta(\bar{Q} - Q_t)(\bar{P} - (\bar{Q} - Q_t)) & \forall Q_t \in]\bar{Q} - \bar{P}, \bar{Q}] \end{cases}$$

The emissions are given by $E_t = \beta c_t - \gamma m_t$.

By analogy with John and Pecchenino (1994), we use a Cobb-Douglas technology:

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha}$$

with $A > 0$ a scale parameter.

Finally, household's preferences are characterized by a separable utility function[12]. This function is growing and concave in consumption and the environment (for $Q \leq \bar{Q}$):

$$U(c_{t+1}, Q_{t+1}) = \log c_{t+1} - \frac{1}{2}(\bar{Q} - Q_{t+1})^2$$

By using the following set of parameter values,

$$\left\{A = 2.52, \theta = 0.09, \gamma = 0.2, \beta = 0.3, \alpha = 0.3, \delta = 0.6, \bar{P} = 5, \bar{Q} = 7\right\}$$

it is possible to show that there exist only two eligible steady states[13]. These solutions are located in the positive maintenance space. But, the first is associated with reversible pollution while the other exhibits irreversibility.

We then proceed to numerical simulations in order to illustrate the basins of attraction and some equilibrium trajectories of the stable and eligible solutions.

We first present the basins of attraction of each solution (see fig.2). The partition in the $k-Q$ space is straighforward: starting from any point (K_0, Q_0) located in the upper and dark (resp. lower and bright) area, the economy reaches, in the long run, the reversible (resp. irreversible) steady state. Therefore, when Q_0 is lower than the critical level $\bar{Q} - \bar{P}$, the dynamics lead, with certainty, the economy into the environmentaly poor steady state. On the contrary, an economy, intially endowed with a sufficient amount of environment, will enjoy, in the long run, a safe environment.

More importantly, we shall note that the set of initial points characterized by reversible pollution and associated with a convergence toward the irreversible state is not empty (see fig.3).

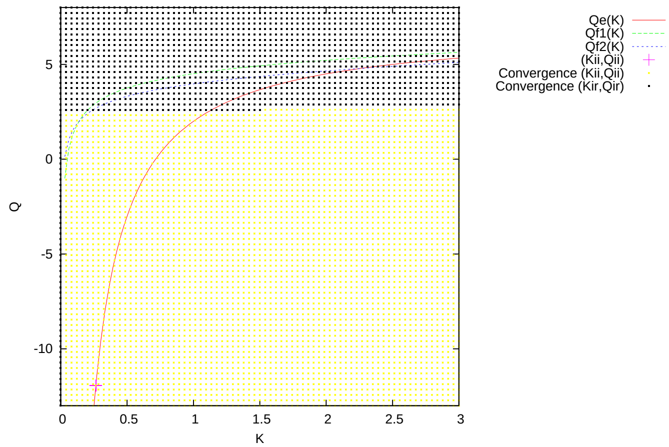

Figure 2: The basins of attraction. The figure also contains the equilibrium relation $Q^e(k_t)$ and the frontiers $Q_1^f(k_t)$ and $Q_2^f(k_t)$ delimiting the interior zone (below) from the constraint case (above). The frontier $Q_1^f(k_t)$ (resp. $Q_2^f(k_t)$) is valid in the irreversible (resp. réversible) pollution area. The frontier that corresponds to the critical thresold is simply defined by the horizontal $Q = \bar{Q} - \bar{P} = 2$. The two solutions are located on $Q^f(k_t)$. The irreversible state (K_{ii}, Q_{ii}) is represented by the "+" while the reversible (K_{ir}, Q_{ir}) does not explicitly appear (it is located just before the intersection between $Q_2^f(k_t)$ and $Q^e(k_t)$).

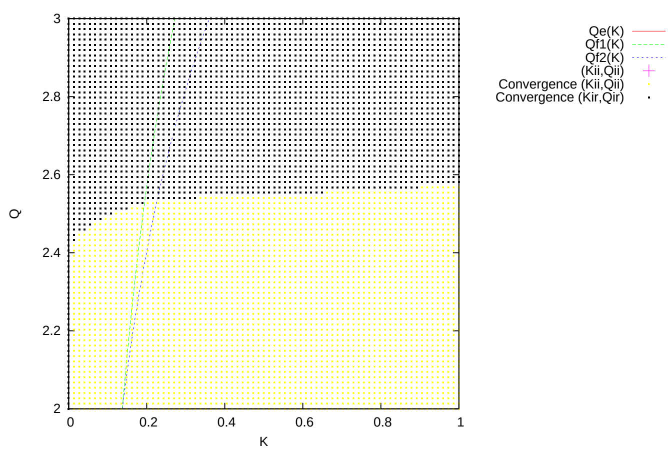

Figure 3: Set of points located in the reversible region but converging toward the irreversible state (left inferior region).

It means that an economy may be attracted to this kind of steady state despite its relatively high endowment in environmental quality.

We now focus on the qualitative properties of the equilibrium trajectories governing the dynamics. The analysis reveals very distinct features of our model in comparison with John and Pecchenino (1994)[14]. By analogy with their approach, we pay close attention to initial conditions belonging to the zero maintenance space. In addition, we only consider states (K_0, Q_0) associated with reversible pollution. The simulations of equilibrium trajectories show that the emergence of an EKC is no longer the rule. We rather point up two types of trajectories.

The first trajectory (see fig.4), with an initial level of environment closed to its maximum level $\bar{Q}$, illustrates a monotonically decreasing convergence toward the reversible steady state. The endowment Q_0 is such that the economy remains in the zero maintenance area during almost all the transition. The agents does not allocate any resource to maintenance and the economy enjoys a phase of sustained growth. In turn, the quality of the environment continuously declines. The degradation is first relatively slow but, as soon as capital approaches its stationary level, polluting emissions cause a severe fall in environmental quality (until its stabilization). We also pick out a turning point (when the frontier $Q_2^f(k_t)$ is crossed) from which the economy starts to abate. But, this effort does not allows to stop environmental deterioration.

The other striking trajectory, with diametrically opposite properties, is obtained for low, but still higher than the threshold, levels of environment (see fig.5). This trajectory corresponds to the case where the economy will be dragged down into the ecological and economic poverty trap. The intuition behind the emergence of this degenerate EKC is the following. Starting from an initial state with relatively low levels of capital and environment, we first observe an economic development phase going together with a (slow) worsening in environmental quality. During this period, the agents benefit from a sufficient environment and choose not to abate. They rather favour consumption and wealth accumulation. The economy gets rich but also accumulates an ecological debt that will be future generations' responsability. Hovewer, since the agents do not take into account intergenerational externalities, these liabilities will exceed the gains of a higher wealth. Thus, once the economy has crossed the frontier $Q^f(k_t)$, the simple fact to engage in maintenance will not suffice to absorb the ecological debt and, consequently, to avoid the

[12]Our qualitative results do not depend on the (simplifying) assumption of separability.

[13]Parameter restrictions and equilibrium properties, for the numerical example, are summarized in appendix D.

[14]In John and Pecchenino (1994), the authors detect a V-shaped relation between capital and environmental quality during the convergence toward the interior steady state. This EKC is the result of a breakdown in the abatement activity. Starting from a point located in the zero maintenance region, the agents have first no incentive to invest in maintenance and capital accumulation goes hand in hand with a degradation of the environment. When pollution emissions are sufficiently damaging (or equivalenlty, once the trajectory crosses the frontier $Q^f(k_t)$), the agents decide to engage in maintenance and, in this second phase, environmental quality is increasing in capital until the positive maintenance steady state is reached. Finally, by combining these two different phases of development, the authors get a sort of EKC.

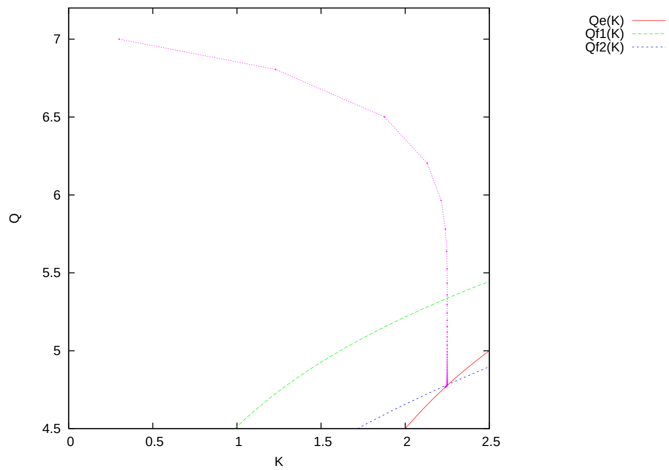

Figure 4: Equilibrium trajectory associated with the reversible steady state

convergence toward the environmentally poor steady state.

During the second phase, the agents are willing to devote a sizeable share of their resources to abatement in order to maintain a decent environment. However, maintenance is not enough to compensate for the harmful effect of polluting emissions which is exacerbated by the weakness of the natural regeneration capacity[15]. Moreover, abatement expenditures are done to the detriment of savings implying a rupture in the capital accumulation process. Therefore, at each date, capital and the environment reach a lower level.

This apoverishment mechanism finally re-occurs, from periods to periods, and the equilibrium trajectory will cross the irreversibility threshold. In this context, the economy is unable to stop the fall in environmental quality. In turn, a phase of economic recession takes place since the agents devote always more resources to maintenance in order to limit the unbroken rise in emissions. In the very long run, the economy reaches the steady state characterized by both a level of capital almost nill and a negative environmental quality.

The qualitative properties of this noteworthy trajectory echo the Dasgupta and Mäler (2002)'s warning against any hasty interpretation of the EKC. Under the assumption of a constant rate of decay, the economy can pollute with complete inpunity since it will be always able to compensate for the past damages caused to the environment, with the support of a natural assimilation growing in the pollution stock. On the contrary, once the potential irreversibility of polllution is taken into account, the exceeding of critical thresholds of environmental damage implies that the economy can not lean on the natural regeneration process anymore. Finally, it fails to absorb, on its own, the environmental debt accumulated with past polluting activities. The economy is then doomed to suffer an irreparable degradation of the environment.

[15]Here, the economy is located in the region corresponding to the decreasing part of the assimilation function.

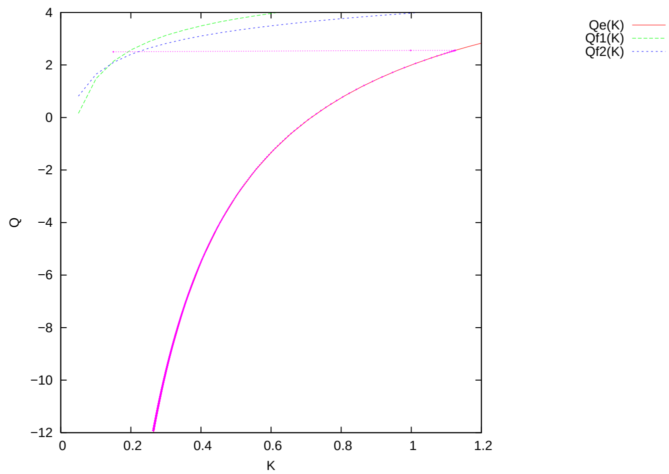

Figure 5: Equilibrium trajectory: the degenerate EKC

5 Conclusion

The purpose of this study is to confront the notion of irreversibility of pollution with the idea conveyed by the environmental Kuznets curve (EKC). Our analysis is intended for challenging the conclusions of John and Pecchenino (1994) who show that the equilibrium relation between growth and environmental quality presents the same characteristics as the EKC. The existence of such a relation, in their overlapping generations model, is based on a breakdown in abatement activity. This result supposes that it will be always possible to remedy the damages caused by pollution, in the first steps of economic development, provided that the economy devotes, once the need arises, a sufficient amount of resources to maintenance. In this paper, we wonder to know to what extend John and Pecchenino (1994)'s result is submitted to the assumption of a constant rate of pollution assimilation. Our approach is based on the Dasgupta and Mäler (2002)'s assertion according to which the concept of the EKC must be rejected from that time on one admits the potential irreversibility of environmental damages.

To answer this question, we generalize their framework by considering an inverted U-shaped assimilation function (similar to the one used by Forster (1975)). The equilibrium analysis reveals two important features of the model. First, there exist multiple equilibria with diametrically opposite properties. Second, some of them are associated with irreversible pollution although the abatement activity is effective. Therefore, the major implication of these results lies in the fact that an economy, having degraded the environment in too vast proportions by giving greater importance to economic growth, may be unable to reverse. In other words, the simple fact to engage in maintenance may not suffice to avoid the convergence toward a long run state having the characteristics of an ecological and economic poverty trap. This type of convergence, illustrated with numerical simulations, is similar to a degenerate EKC. We then confirm our intuition according to which the EKC, as it is depicted by John and Pecchenino (1994), is no longer the rule when the possibility of irreversibility is taken into account.

This result legitimates the intervention of public authorities in order to control pollution. In fact, environmental policy is the only recourse by means of which it will be possible to prevent the arrival of these environmentally (and economically) poor states. Therefore, a natural extension of our work consists in studying the optimal growth problem. The analysis should allow us to answer whether or not it might be optimal to let the environment perpetually degrades in a welfare maximizing perspective. In case of negative response, the key question is then to know what are the political instruments capable of directing the economy to the path of sustainable growth.

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Appendix

A. Proof of propositions 1 and 3

We first summarize the properties of functions that have a real influence on the study of existence. Next, we explicitly deal with the issue of existence by analyzing separately the four possibles cases.

A.1 Functions properties

We study in detail the properties of consumption, maintenance and emissions functions. We also present the behaviour of the law of motion of the environment.

The maintenance:

The analysis consists in studying the behaviour of the wage function $w(k) = f(k) - kf'(k)$ and, its location with respect to the first bisectrix since

$$m(k) = f(k) - kf'(k) - k$$

By definition, wage is non negative: $w(k) \geq 0 \ \forall k$ implies $f(k) \geq kf'(k) \geq 0$. Since $f(0) = 0$, we have $\lim_{k \to 0} kf'(k) = 0$ and $\lim_{k \to 0} w(k) = 0$. Its first derivative is positive: $w'(k) = -kf''(k) \geq 0 \ \forall k$ and, from the properties of the production function, we know that $\lim_{k \to +\infty} w(k)/k = \lim_{k \to +\infty} f(k)/k - f'(k) = 0$. The wage function is then located below the bisectrix when capital tends towards infinity. If we assume (assumption (19)): $\lim_{k \to 0} w(k)/k > 1$, then the order, in the neighborhood of 0, is reversed. Thus we can conclude that there exists at least one intersection between $w(k)$ and the bisectrix and, consequently, a positive finite value k such that $m(k) = 0$. We define:

$$\bar{k} = \inf\{k \in [0, +\infty[\ /w(k) = k\}$$

At this striking point, we have $w'(\bar{k}) < 1$ since the wage crosses the bisectrix from above.

The condition $m(k) \geq 0$, for an interior solution, necessitates to restrict our analysis of existence to the significant domain of variation of capital. More precisely, we decide to limit the study of existence to the interval defined by the lower intersection point $\bar{k}$ since we know that $\forall k \in [0, \bar{k}], \ m(k) \geq 0$[16].

On this interval, the maintenance function $m(k)$ behaves in the following manner. Since $m'(k) = -1 - kf''(k)$ and $\lim_{k \to 0} m(k) = \lim_{k \to \bar{k}} m(k) = 0$, it is possible to define $\breve{k} = \sup\{k \in [0, \bar{k}] \ /m'(k) = 0\}$ such that $\forall k \in [\breve{k}, \bar{k}]$, $m'(k) \leq 0$. We ignore *a priori* its behaviour on $[0, \breve{k}]$ and the sign of its second derivative (that depends on the third derivative of the production function).

The consumption function:

[16] In the particular case of a Cobb-Douglas technology, the intersection $\bar{k}$ is unique.

The consumption function writes:

$$c(k) = (1-\delta)k + kf'(k)$$

We know that $\forall k \in [0, \bar{k}]$, $c(k) \geq 0$ and, it is possible to compute its values at the bound of $[0, \bar{k}]$: $\lim_{k\to 0} c(k) = 0$ and $c(\bar{k}) = (1-\delta)\bar{k} + \bar{k}f'(\bar{k}) > 0$. The first derivative is given by $c'(k) = 1 - \delta + f'(k) + kf''(k)$ and we impose the following condition (assumption (20)):

$$\sigma(k) \geq 1 - s(k) \leftrightarrow f'(k) + kf''(k) \geq 0$$

with $\sigma(k)$, the elasticity of substitution, defined by (18) and $1 - s(k)$, the labor share of output (see (17)).

This assumption guarantees: $c'(k) > 0$ on $[0, \bar{k}]$.

The emissions function:

At the stationary equilibrium, polluting emissions are defined as a function of capital:

$$E(k) = \beta c(k) - \gamma m(k)$$

The levels of emission corresponding to each bound of the interval $[0, \bar{k}]$ equals to: $\lim_{k\to 0} E(k) = 0$ and $E(\bar{k}) = \beta c(\bar{k}) > 0$. It is easy to show that the emissions reach their maximum at the upper bound $\bar{k}$: $E(\bar{k}) = \max\{E(k) \;/\; k \in [0, \bar{k}]\}$.

From the properties of the functions $c(k)$ and $m(k)$, its first derivative, $E'(k) = \beta c'(k) - \gamma m'(k)$, is positive on the sub-interval $[\breve{k}, \bar{k}]$. Moreover, since $\lim_{k\to 0} m(k) = 0$ and $m(k) \geq 0$ $\forall k \in [0, \bar{k}]$, we necessarely have $m'(k) \geq 0$ in the neighborhood of 0. Now, $m'(k) \geq 0 \leftrightarrow 1 + kf''(k) \leq 0$ and $c'(k) > 0 \leftrightarrow 1 + kf''(k) > \delta - f'(k)$. The derivative can be rewritten as follows: $E'(k) = \beta(f'(k) - \delta) + (\beta + \gamma)(1 + kf''(k))$. We then formulate the following restriction (assumption (25)):

$$\lim_{k\to 0} \frac{\delta - f'(k)}{1 + kf''(k)} < \frac{\beta + \gamma}{\beta}$$

which is equivalent to:

$$\lim_{k\to 0} \frac{r(k)}{m'(k)} < \frac{\beta + \gamma}{\beta}$$

Under this additional condition, emissions are negative in the neighborhood of 0 because: $\lim_{k\to 0} E(k) = 0$ and $\lim_{k\to 0} E'(k) < 0$. This property interprets like a man-made production of environment. Formally, it means that there exists a positive value of capital for which emissions are nill:

$$\exists \hat{k} \in [0, \bar{k}] / E(k) = 0$$

The environmental law of motion:

The function $N(Q)$ is defined piecewise on the interval $]-\infty, \bar{Q}]$ (the upper bound comes from the fact that we consider only non negative pollution levels):

$$N(Q) = \begin{cases} Q & \forall Q \leq \bar{Q} - \bar{P} \\ Q + \Gamma(\bar{Q} - Q) & \forall \bar{Q} - \bar{P} < Q \leq \bar{Q} \end{cases}$$

For any value of Q lower than the critical threshold (that is $Q \leq \bar{Q} - \bar{P}$), $N(Q)$ is simply linear. Thus, we have $N(0) = 0$, $N(\bar{Q} - \bar{P}) = \bar{Q} - \bar{P}$ and $N'(Q) = 1$ $\forall Q \leq \bar{Q} - \bar{P}$.

Once the irreversibility frontier $\bar{Q} - \bar{P}$ is crossed, the natural regeneration process is effective and the law of motion becomes: $N(Q) = Q + \Gamma(\bar{Q} - Q)$. From the properties of the decay function $\Gamma(.)$, we know that $N(Q) > 0$ $\forall Q \in]\bar{Q} - \bar{P}, \bar{Q}]$, $\lim_{Q \to \bar{Q} - \bar{P}} N(Q) = \bar{Q} - \bar{P}$ and $N(\bar{Q}) = \bar{Q}$. The derivative $N'(Q) = 1 - \Gamma'(\bar{Q} - Q)$ is positive $\forall Q \leq \bar{Q}$. Il also appears that $N'(Q) \geq 1$ $\leftrightarrow Q \leq \bar{Q} - \tilde{P}$. Finally, this function is concave since $N''(Q) = \Gamma''(\bar{Q} - Q) \leq 0$.

We now turn to the study of existence.

A.2 Existence analysis

Positive maintenance equilibrium with irreversible pollution:

A steady state (k, Q) solves the following system:

$$\begin{cases} R(k)U_1(c(k), Q) - \gamma U_2(c(k), Q) = 0 \\ E(k) = 0 \end{cases}$$

Under assumption (25), the second equation admits an odd number (from the properties of $E(k)$) of positive solutions k^*_{ii}[17]. The set of these solutions exactly coincides with the set of values defined in appendix $A.1$: $\{k^*_{ii}\} = \{\hat{k} \in [0, \bar{k}] \;/\; E(k) = 0\}$. Moreover, the FOC defines, for any interior solution, a continuous and monotonically increasing equilibrium relation $Q_t = Q^e(k_t)$ valid for all t (see (22)). Substituting a solution k^*_{ii} in this equation gives the corresponding equilibrium value of environmental quality $Q^*_{ii} = Q^e(k^*_{ii})$.

Therefore, there exists at least one steady state associated with a positive maintenance and irreversible pollution. The eligibility of this solution (from the view point of its location with respect to the critical level $\bar{Q} - \bar{P}$), requires $Q^*_{ii} \leq \bar{Q} - \bar{P}$, which is equivalent to $k^*_{ii} \leq \underline{k}$ with $\underline{k} = (Q^e)^{-1}(\bar{Q} - \bar{P})$ since the relation $Q^e(k_t)$ is invertible.

Positive maintenance equilibrium with reversible pollution:

The system of equations (23) now writes:

$$\begin{cases} R(k)U_1(c(k), Q) - \gamma U_2(c(k), Q) = 0 \\ \Gamma(\bar{Q} - Q) = E(k) \end{cases}$$

[17] The index "i" (resp. "c") prevails for interior (resp. corner) solutions. The second subscript "i" (resp. "r") means that equilibrium pollution is irreversible (resp. reversible).

where the first equation still characterizes the equilibrium relation: $Q = Q^e(k)$.

The second equation can then be rewritten:

$$\Gamma(\bar{Q} - Q^e(k)) = E(k)$$

The analysis of existence boils down to compare the emissions function $E(k)$ behaviour with the one of the function $G(k) = \Gamma(\bar{Q} - Q^e(k))$. Moreover, considering the reversible pollution area impose to restrict the study to the interval $]\underline{k}, \bar{k}]$.

It is worth noting that there exists *a priori* a problem concerning the domains of definition of these two functions. In fact, $E(k)$ depends on conditions on technology, preferences and the law of pollution diffusion. But, $G(k)$ is also and especially characterized by the properties of the assimilation function. Thus, nothing guarantees the non-emptiness of the studied interval $]\underline{k}, \bar{k}]$ and, we will have to set conditions that ensures its significance.

$G(k)$ has the same qualitative behaviour as the assimilation function $\Gamma(\bar{Q} - Q)$ since $G'(k) = -Q^{e\prime}(k)\Gamma'(\bar{Q} - Q^e(k))$ with $Q^{e\prime}(k) \geq 0$. Thus, it is first increasing from $\underline{k}$ to $(Q^e)^{-1}(\bar{Q} - \tilde{P})$ and next, decreasing until $(Q^e)^{-1}(\bar{Q})$. We also have $G(k) \geq 0$ $\forall k \in]\underline{k}, (Q^e)^{-1}(\bar{Q})]$, $G(\underline{k}) = G((Q^e)^{-1}(\bar{Q})) = 0$ and $\max\{G(k) \ / \ k \in]\underline{k}, (Q^e)^{-1}(\bar{Q})]\} = G((Q^e)^{-1}(\bar{Q} - \tilde{P})) = \Gamma(\tilde{P})$.

We use a fourth assumption (condition (26) in proposition 1) that states that the maximum potential of natural assimilation is intrinsically higher than the maximum volume of emissions on the significant domains of variation for k and Q. Formally, this assumption reads

$$\sup_{P \in [0, \bar{P}]} \{\Gamma(P)\} \geq \sup_{k \in [0, \bar{k}]} \{E(k)\}$$

that is,

$$\Gamma(\tilde{P}) \geq E(\bar{k})$$

So as to determine the ranking between the functions $G(k)$ and $E(k)$ at the lower bound $\underline{k}$, it is possible to refer to the (partial) eligibility condition, for interior and irreversible solutions, that imposes $k^*_{ii} \leq \underline{k}$. If all these solutions are eligible, then $k^{*s}_{ii} = \sup\{\hat{k} \in [0, \bar{k}] \ / \ E(k) = 0\} \leq \underline{k}$. Therefore, we deduce from this eligibility result that $E(\underline{k}) > G(\underline{k})$ (since $E(\bar{k}) > 0$ implies $E(k) > 0$ $\forall k \geq \underline{k}$).

Otherwise, it is sufficient that the lower bound is contained between two successive odd then even solutions k^*_{ii} to obtain $E(\underline{k}) > G(\underline{k})$.

In these two cases, the ranking in $\underline{k}$ is known and, if we state that $\bar{k} \geq \tilde{k}$ with $\tilde{k} = (Q^e)^{-1}(\bar{Q} - \tilde{P})$ (which is equivalents to the condition (27) in proposition 1: $Q^e(\bar{k}) \geq \bar{Q} - \tilde{P}$), then we guarantee not only the non-emptiness of the interval studied (since $\underline{k} < \bar{k}$) but also, the existence of an intersection between the curves $E(k)$ and $G(k)$. In fact, under our fourth hypothesis, the order is inverted at the noteworthy point $\tilde{k} : E(\tilde{k}) \leq G(\tilde{k})$.

Consequenly, we have given here conditions that ensure the existence of an interior steady state with reversible pollution (k^*_{ir}, Q^*_{ir}). Finally, we shall note that this type of solution is associated with levels of capital and the environment that are necessarily higher than the respective levels at any interior but irreversible solution.

Zero maintenance equilibrium:

When the non negativity constraint on m is binding, the stationary system of equations rewrites:

$$\begin{cases} k = f(k) - kf'(k) \\ \Gamma(\bar{Q} - Q) = \beta c(k) \end{cases}$$

If pollution is irreversible, then the second equation becomes $c(k) = 0$. Therefore, we can conclude that the system does not admit a solution since the assumption $\lim_{c\to 0} U_1(c, Q) = +\infty$ allows us to exclude the limit case where $k = 0$. In other words, there is no zero maintenance steady state that exhibits irreversible pollution.

In the reversible case, for $Q \in]\bar{Q} - \bar{P}, \bar{Q}]$, under the assumption (19), there exists only one positive value of capital that solves the first equation on the interval $[0, \bar{k}]$. This solution exactly corresponds to the upper bound $\bar{k}$ (such that $m(k) = 0$): $k^*_{cr} = \bar{k}$. Now, we have to compute the corresponding level of environmental quality by solving the following equation: $\Gamma(\bar{Q}-Q) = \beta c(\bar{k})$. Under the properties of $\Gamma(\bar{Q} - Q)$, we deduce from the fourth assumption that there exists one (or two if the inequality (26) is strict) equilibrium value Q^*_{cr} associated with k^*_{cr}. The uniqueness of equilibrium implies that $Q^*_{cr} = \bar{Q} - \tilde{P} > \bar{Q} - \bar{P}$ otherwise, the ranking is such that $Q^{*+}_{cr} > \bar{Q} - \tilde{P} > Q^{*-}_{cr} > \bar{Q} - \bar{P}$.

To summarize, there exists at least one and, at most two steady states with zero maintenance and reversible pollution.

B. Proof of propositions 2 and 4

Here, we study local stability for the three types of steady states .

B.1 Interior solutions:

We first analyze the general case where the regeneration process of nature is effective. Then, we can easily determine the stability conditions for the solution characterized by irreversible pollution (which is simply the particular case where $N'(Q) = 1$).

Reversible pollution: linearizing the equilibrium dynamics (23) gives the system of equations (28). From this system, we immediatly get the jacobian matrix:

$$J = \begin{pmatrix} -\frac{E'(k)-\gamma}{Q^{e\prime}(k)+\gamma} & \frac{N'(Q)}{Q^{e\prime}(k)+\gamma} \\ -\frac{(E'(k)-\gamma)Q^{e\prime}(k)}{Q^{e\prime}(k)+\gamma} & \frac{N'(Q)Q^{e\prime}(k)}{Q^{e\prime}(k)+\gamma} \end{pmatrix}$$

Then, we compute the characteristic polynomial:

$$P(\lambda) = \lambda^2 - tra(J)\lambda + det(J)$$

We know that the trace corresponds to the sum of the (real parts) eigenvalues of J and, the determinant is the product, in modulus, of the eigenvalues. Stability requires that the two roots of $P(\lambda)$ are located into the unit circle, all other configuration being unstable.

Now, it is clear that $det(J) = 0$. In fact, the studied dynamics boil down to a one dimensional system because of the existence of the equilibrium relation (22). Therefore, the first eigenvalue is nill $\lambda_1 = 0$ while the second is equal to the trace $\lambda_2 = tra(J)$ with,

$$tra(J) = \frac{N'(Q)Q^{e\prime}(k) - (E'(k) - \gamma)}{Q^{e\prime}(k) + \gamma}$$

We summarize all possible cases:

1/ if $(N'(Q) - 1)Q^{e\prime}(k) < E'(k) \leq \gamma + N'(Q)Q^{e\prime}(k)$ then $1 > tra(J) \geq 0$

The conditions $E'(k) \geq 0$ and $N'(Q) \leq 1$ (with one of the two inequalities being strict) suffice to satisfy the necessary stability condition $E'(k) > (N'(Q) - 1)Q^{e\prime}(k)$.

2/ if $\gamma + N'(Q)Q^{e\prime}(k) < E'(k) < 2\gamma + (1 + N'(Q))Q^{e\prime}(k)$ then $tra(J) < 0$ and $|tra(J)| < 1$.

Thus, the double condition

$$(N'(Q) - 1)Q^{e\prime}(k) < E'(k) < 2\gamma + (1 + N'(Q))Q^{e\prime}(k)$$

defines an interval of variation, for the emissions function derivative, on which local stability of any interior reversible steady state is guaranteed. More precisely, the convergence is monotonic in the first case whereas it is oscillatory in the second one.

Irreversible pollution: this case is obtained by fixing $N'(Q) = 1$. Following the same processes, it appears that stability of interior and irreversible solutions necessitates the validation of the following condition:

$$0 < E'(k) < 2(\gamma + Q^{e\prime}(k))$$

B.2 Reversible corner solution:

The Jacobian now writes

$$J = \begin{pmatrix} w'(k) & 0 \\ -\beta c'(k) & N'(Q) \end{pmatrix}$$

Consequently, the conditions $w'(k) < 1$ and $N'(Q) < 1$ are necessary and sufficient for stability. The first condition is satisfied since $k^*_{cr} = \bar{k}$ and $w'(\bar{k}) < 1$. Moreover, if there exists a single solution, then $Q^*_{cr} = \bar{Q} - \tilde{P}$; now, we deduce from $N'(\bar{Q} - \tilde{P}) = 1$ that this steady state is unstable. As soon as there exists two solutions, considering their location with respect to $\bar{Q} - \tilde{P}$ ($Q^{*1}_{cr} > \bar{Q} - \tilde{P} > Q^{*2}_{cr}$), the "low" steady state is unstable (because $N'(Q^{*2}_{cr}) > 1$) while the other is locally stable ($N'(Q^{*1}_{cr}) < 1$).

D. Simulations

Parameter restrictions:

We impose a restriction concerning the parameters of the technology and the emissions function: $\gamma(1-\alpha)-\beta\alpha \geq 0$. We also fix the domain of variation of the scale parameter: $A \in [\underline{A}, \bar{A}]$ with

$$\underline{A} = \left(\frac{2}{\gamma \bar{P}}\right)^{1-\alpha} \frac{1}{1-\alpha}$$

and,

$$\bar{A} = \left(\frac{\theta \bar{P}^2 (1-\alpha)}{4\beta((1-\delta)(1-\alpha)+\alpha)}\right)^{1-\alpha} \frac{1}{1-\alpha}$$

These conditions cover the whole of our assumptions made in the general setting.

Equilibrium properties:

In this framework, the equilibrium analysis reveals the following features. There exist five steady states:

- one interior irreversible locally stable solution,
- two interior reversible solutions, the "low" being unstable whereas the "high" is stable,
- two corner reversibles solutions with the same configuration for stability.

It is possible to compute analytically the global dynamics characterizing the four possible regions.

Moreover, by using the following set of parameter values,

$$\left\{A = 2.52, \theta = 0.09, \gamma = 0.2, \beta = 0.3, \alpha = 0.3, \delta = 0.6, \bar{P} = 5, \bar{Q} = 7\right\}$$

we show that there exist only two steady states that satisfy the criterium of eligibility: the interior irreversible solution and the high interior reversible solution (constrained reversibles solutions are uneligible in the sense that they are located in the positive maintenance region).

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