29

Maintaining environmental quality for overlapping generations: Some Reflections on the US Sky Trust Initiative

Stéphane Lambrecht

May 2005

ENVIRONMENTAL ECONOMICS & MANAGEMENT MEMORANDUM

UCL
Université catholique de Louvain

Chair Lhoist Berghmans
in Environmental Economics
and Management

Center for Operations Research
and Econometrics (CORE)

Maintaining environmental quality for overlapping generations: Some Reflections on the US Sky Trust Initiative

Stéphane Lambrecht*

Abstract

Starting from the US Sky Trust claim that the "the sky belongs to us equally", this paper distinguishes two sources through which overlapping generations may consent to the use of the environment whom they are the owners: the common consent of all generations reached behind the Rawlsian veil of ignorance and the specific consents of generations born at different time periods. It proposes two institutions: a fund mandated to implement the common consent by auctioning permits to firms and a voting procedure to implement the specific consents by choosing each generation's preferred level of environmental maintenance. The analysis shows how the specific consent may be, each period, operative or inoperative and that there may be at most two switches between these two regimes on the transition path. Starting from the business as usual steady state, the introduction of these institutions always immediately increases the environmental quality, but the magnitude of this gain may be temporary and decrease if capital accumulation is strongly evicted by the policy. On the opposite, we stress a case in which the introduction of the policy has beneficial effects both on wealth and quality.

Keywords: OLG models, environmental quality

JEL Classification: D91, Q28

*Laboratoire MÉDEE, Faculté des Sciences Économiques et Sociales, Université des Sciences et Technologies de Lille, 59655 Villeneuve d'Ascq Cedex, France; e-mail: stephane.lambrecht@univ-lille1.fr and Center for Operations Reasearch and Econometrics (CORE), Université Catholique de Louvain, Belgium; e-mail: lambrecht@core.ucl.ac.be

Acknowledgments: The author is grateful to Denny Ellerman for his insights on the US Sky Trust Initiative, he also thanks Claude d'Aspremont, Thierry Bréchet, Philippe Michel, Pierre-André Jouvet, Gilles Rotillon, Lionel Ragot and Henry Tulkens for their comments. Finally he also benefited from discussions with Johan Eyckmans, Marc Germain and Vincent Vansteenberghe. The research presented in this paper was financed by the research program CLIMNEG 2, of the Belgian Office for Scientific, Technical and Cultural Affairs.

This paper presents research results of the Belgian Program CLIMNEG 2, initiated by the Belgian State, Prime Minister'sOffice, Science Policy Programming. The scientific responsability is assumed by the author.

Contents

1 Introduction **4**

2 The twofold consent and the institutional design **5**

3 The index of environmental quality **9**

4 The business-as-usual economy **12**

5 The economy with environmental policy **14**
- 5.1 The constitutional fund and the sale of permits 15
- 5.2 The voting procedure on the maintenance tax 15

6 Temporary equilibria **17**
- 6.1 The environmental maintenance in equilibrium 17
- 6.2 The zero-specific-consent temporary equilibria 19
- 6.3 The positive-specific-consent temporary equilibria 19

7 Characterization of the intertemporal equilibrium with perfect foresight **21**

8 The steady state equilibria **21**
- 8.1 The zero-specific-consent stationary equilibrium 21
- 8.2 The positive-specific-consent stationary equilibrium 22

9 Switches in the specific consent on the transition **23**

10 The transition from the BAU economy to the economy with policy **27**

11 Conclusion **30**

12 References **30**

A Appendices **31**
A.1 Proof of proposition 2.1 31
A.2 Proof of proposition 3.1 31
A.3 Proof of proposition 4.1 32
A.4 Proof of proposition 6.1 32
A.5 Proof of proposition 6.3 32
A.6 Proof of proposition 7.1 32
A.7 Proof of proposition 8.2 32
A.8 Proof of proposition 9.1 33
A.9 Proof of proposition 9.2 33
A.10 Proof of proposition 9.3 33
A.11 Proof of proposition 10.1 34

1 Introduction

Recently, in the debate over the global environmental issues implied by the exploding path of greenhouse gas emissions, a US private non-profit organization, the Sky Trust[1], defends the following idea: starting from the widely shared statement that the atmosphere's carbon absorption capacity is limited, they suggest to mandate a trust with the mission to administer a cap-and-trade program for carbon permits, organize auctions of these permits to polluting firms and redistribute the proceeds of the auctions equally to all citizens. The reason called upon for these transfers to the citizens is that "the sky belongs to all [the citizens] equally." So, according to them, the owners of the sky should be compensated for its use by polluters and, moreover, they should be compensated equally because they have in common to be equally members of the community of owners.

While the instrument of permits is well-known in environmental economics (Dales (1968), Montgomery (1972), Stokey (1998)) and is now being applied on a wide-scale for carbon emissions in Europe, the practice is essentially to give them for free to firms and to let them trade with each other to match their needs. The European Trading System allows for a maximum of 5% of the initial allocations to be auctioned during the firt period of the program. In the US, the Climate Stewardship Act of 2003 does not put any upper bound on the number of permits that will be distributed for free to firms.

This article has a twofold objective. First it grounds the Sky Trust design in an overlapping generations framework. This implies, on the one hand, to give a formal definition of the membership to the community of owners of the atmosphere and, on the other hand, to allow for the diversity of the "selves and egos" which characterizes this community. This is done by distinguishing two sources through which generations consent to the use of the environment: a consent common to all generations and another consent specific to any given generation. Second, this article proposes an institutional design whose aim is to reflect both the features of equal membership and diversity of the owners. This institutional design combines a fund, like the one proposed in the Sky Trust Initiative, with a voting procedure aimed at fixing the amount of resource invested in environmental maintenance. It is shown under which conditions the combination of these two institutions may solve the tradeoff between accumulation of wealth and preservation of the environment.

Section 2 presents the the twofold consent of the generations and the institutional design. Section 3 discusses the link between the dynamics of the environmental quality and the existence of the common consent of all generations.

[1] P. Barnes is the founder of the Sky Trust Initiative.

Section 4 studies the business-as-usual economy. Section 5 introduces the institutions to conduct environmental policy. Sections 6 to 8 present respectively the temporary equilibria, the characterization of the intertemporal equilibrium and the steady state equilibria. Section 9 deals with the dynamics and studies the regime switches which the economy experiences on the transitional path. Section 10 analyzes the effects on the equilibrium of the introduction of the environmental policy institutions starting from a steady state business-as-usual equilibrium. Finally section 11 concludes.

2 The twofold consent of the generations and the proposed institutional design

Underlying the Sky Trust assertion that "the sky belongs to all of us equally", there is the general idea that the environment can be considered as a global public good whose ownership falls on the whole mankind. Pollution then represents a negative externality which deteriorates the quantity of that public good. At the heart of the economic approach to environmental issues lies the question of how much pollution society would tolerate. This preferred level of pollution can be used as a target by policymakers in designing and implementing environmental policies.

The very concept of mankind ownership of the environment is however problematic because it abstracts from two relevant features in environmental issues: the flow of time and the finite length of human lives. Taking into account the first feature forces to envision many environmental policies in an intertemporal setting. The second feature implies that mankind is actually a community which undergoes a process of continuous population renewal. Being finitely-lived, many members of that community will never meet each other. When it exists, the connection between two community members does not last for their whole respective lifespans since these only partially overlap. Thus, contrary to a static view, the community of all mankind consists of a collection of "selves and egos" which seem to be connected to each other only if they overlap. These considerations then suggest that, as regards environmental issues, the concept of mankind ownership is an interesting one only if this community of owners is apprehended as a community of overlapping selves. Because such an overlapping structure may cause agents to neglect the long term impact of their decisions, Solow (1986) recommended the use of the overlapping generation model to study nature resource issues.

This article builds upon the class of overlapping generations growth model. We shall however defend the idea that, though imperfectly connected, all members of all generations share the feeling that they could be born at any point of time and we shall interpret this feeling of anonymous membership as a version of the Rawlsian veil of ignorance idea. In this article, we propose an institutional design to take into account both the overlapping and the anonymous characteristics of mankind community members.

The analysis starts as follows. We assume that all generations regard the environment as an intergenerational public good and polluting emissions E as a negative externality. Initially there are no policies to reduce the pollution externality. The economy is on its *laissez-faire* equilibrium path, which we will refer to as the business-as-usual (BAU) equilibrium. The BAU equilibrium is a path which is either stationary or non-stationary. In the case of a stationary path, the emissions are constant at the level E^{bau} and the environmental quality is stabilized, but at a non-optimal level.

In the case of a non-stationary BAU path, either the economy converge to a non-optimal steady state, or it collapses. This means that either polluting emissions follow an increasing path and the environment deteriorates but degradation dampens at the steady state, or emissions increase so much that they end up outreaching the maximum sustainable long run emissions $\bar{E}$. As a result, there is no steady state: the environment and the economy collapse.The

scope of this section is twofold. We discuss the motivations to improve upon the BAU equilibrium path and, simultaneously, we propose institutions which are a variety of the Sky Trust proposal design and represent means to improve upon the BAU.

The common consent and the constitutional fund

In general, there are several conceivable transitions from the BAU equilibrium to a regulated equilibrium. A severe reduction in the polluting emissions could be implemented at the beginning, followed by a growing emissions path only up to what the environment can absorb (Bréchet, Lambrecht and Prieur (2005)). Alternatively, a soft transition would consist in gradually tightening the constraint. We shall explore here a third possibility which boils down to a Rawlsian implementation of the Sky Trust initiative.At some period on the BAU

equilibrium path, all generations, both present and future ones, fictitiously meet to agree on how much to improve environmental quality. All generations are assumed to be ignorant of their date of birth and therefore share an anonymous membership to the community of owners of the atmosphere. This view is a version of Rawls's (1971) idea of the *veil of ignorance*. We assume that all members of all generations have the same utility function. Behind the veil of ignorance, everybody reasons as if he could be born at any point in time, including in bad times. We denote by *common consent* the *ex ante* agreement reached by these generations and give the following definition:

> **DEFINITION.** *The common consent is the path of maximum polluting emissions on which all generations could agree behind the veil of ignorance.*

We now characterize the properties of the common consent of all generations behind the veil of ignorance.

PROPOSITION 2.1. ***Property of the common consent***

Given the willingness to depart from the BAU equilibrium path, the common consent is characterized by a constant sequence of polluting emissions $(S)_{t=0}^{+\infty}$ *which verifies:*

$$S < \min\{E^{bau}, \bar{E}\} \tag{2.1}$$

Proof. *See appendix A.1.*

Admittedly, the fiction of the veil of ignorance enables to side-step the issue of the external effects of present decisions on future generations. But, more importantly, it does not imply that any path of emissions can be a candidate for the common agreement. Indeed the ignorance of the date of birth restricts the range of common-consent paths to be constant-emission paths avoiding the environmental disaster and improving upon the BAU stationary emissions. Obviously, this leaves room for a large variety of paths. We shall assume that the choice of a particular path, among all eligible paths, is exogenous.

To implement the common consent path of polluting emissions we assume the existence of a Sky Trust type of fund. Public authorities constitutionally mandate a *fund* to counter environmental degradation by regulating the source of emissions. We assume that the source of pollution is the firms' production. The instrument handled by the fund is a system of pollution permits. The firms need to possess permits to be allowed to pollute. The aggregate volume of permits issued each period must match the common consent emissions S. It is assumed that firms must pay for each unit of polluting emissions. This is equivalent to assume that no permits is given for free to the producing firms and that the fund sells all the permits issued in a given period (See for instance Jouvet, Michel and Rotillon (2005)). Given the time profile of the common consent path, the fund must issue a fixed volume S of pollution permits each period and this issue must be lower than the BAU emissions E^{bau} or the threshold collapse emissions $\bar{E}$.

The stationarity of the fund's policy instrument reflects the idea that all generations are treated equally and this equal treatment stems from the generations' common consent.

The specific consent and the voting procedure on maintenance

However the common consent policy target might appear *ex post* inadequate to a given generation born at some given period in the sequence of overlapping generations. Several reasons may be called upon to justify this inadequacy: uncertainty, changing tastes, changing wealth and environment. We shall focus on the last two reasons. Income per head changes across time because of capital accumulation. Depending on their date of birth, generations are more or less favored in terms of economic affluence. The changing environmental quality is another source of *ex post* heterogeneity among all generations. In general, this intergenerational heterogeneity is likely to affect the willingness to pay for the environment. Some generations may express a consent stronger than others. We denote by *specific consent* this *ex post* agreement and give the following definition.

> DEFINITION. *The specific consent is the amount of economic resource a given generation is willing to invest in the maintenance of the environment, at a given date.*

The specific consent is characterized by two features. First of all, the investment of the specific consent cannot be negative. In other words, no generation can "borrow" on environmental quality. This behaviour would break the common consent. As a consequence there is a non-negativity constraint on the specific consent. The second feature regards the amount invested. Depending on the degree of affluence and the level of environmental quality experienced at a given period, a generation will express a different specific consent. We summarize these features by the following proposition.

PROPOSITION 2.2. ***Property of the specific consent***

Given the common consent characterized by $(S)_{t=0}^{+\infty}$, the specific consent of the time t generation is a non-negative amount of economic resource invested in the maintenance of the environment:

$$m_t \geq 0, \ \forall t \tag{2.2}$$

On a non-stationary path of wealth and the environment, the sequence of the specific consents is a non-stationary path.

To deal with this source of *ex post* heterogeneity among generations, we shall assume the existence of a second institution in charge of preserving the environment. This institution consists in a *voting procedure*. It is organized each period to cooperatively determine the amount of investment in environmental maintenance each generation is willing to accept. The vote fixes the amount of resources to be levied on the households' income. These collected resources are then invested in the maintenance of the environment. What is here referred to as investment in maintenance are "end-of-pipe" measures like for instance carbon dioxide capture and sequestration. The voting procedure on maintenance operates like a flexibility mechanism. It allows those generations willing to go beyond the common consent to invest in the environment when young and enjoy, when old, a better environment than the one guaranteed by the common consent.

3 The index of environmental quality

It is instructive to compare our framework with those of other contributions to the literature on growth and the environment in OLG models. Like John and Pecchenino (1994), Ono (2002) assumes that the households' utility is positively related to a so-called index of "environmental quality". In Ono's (2002) contribution, the dynamic equation which governs the evolution of the environmental quality index is of the following type:

$$Q_{t+1} = (1 - \delta_Q)\, Q_t - \varepsilon E_t + \mu m_t \tag{3.1}$$

where $Q_t > 0$ is environmental quality at time t, $(1 - \delta_Q)$ is the rate at which the current environmental quality stock survives in the following period, ε is the rate at which time t emissions E_t decrease Q_{t+1}[2] and μ is the rate at which time t investment in maintenance m_t increases Q_{t+1}.

In this section, we first discuss why this index and its associated pattern of dynamics is incompatible with our approach to the twofold consent of the generations and we then derive an alternative index and pattern.

There cannot exist a common consent of all generations if environmental quality valued by individuals evolves according to the dynamics defined by (3.1) and if the investment in maintenance is equal to zero at each period. Under the above specification of the environmental dynamics and with $(m_t = 0, \forall t)$, environmental policies cannot rely only on the abatement of emissions E_t. Indeed, a quick inspection of the behavior of the environmental stock Q_t reveals that any positive emissions' ceiling S fails to guarantee a positive stationary environmental

[2]Emissions may come from consumption like in John and Pecchenino (1994) or from production like in Ono (2002).

quality Q. Hence a policy consisting in reducing emissions is unable to avoid the environmental collapse. As a consequence, there does not exist a common consent characterized by a stationary volume S, over which all generations could agree behind the veil of ignorance. Indeed, in that original position, no generation can exclude to be born in the catastrophic long run : any S is thus rejected by any generation. The common consent does not exist.

As a corollary of the inexistence of the common consent, the specific consent consisting in investing in the maintenance of the environment is forced to be positive in the long run.

To show this let us now add maintenance to the picture. On the transition path, maintenance may act as a complement of the emissions abatement. It can be positive or equal to zero and it can vary across time. When maintenance is equal to zero for some periods on the transition, only the emissions abatement is operative. Hence, as we showed, if zero maintenance persists, the economy necessarily converges toward collapse. Since any generation would be willing to avoid it, the desired maintenance will necessarily switch to a positive level after some time and stick to it in the long run. The specification defined by (3.1) is not compatible with our interpretation of maintenance in terms of the generation specific consent. Generations born in the long run are not free to invest or not in maintenance. In such a setting, the institutional voting procedure on maintenance is not a thoroughly flexible mechanism.

We need to assume that households value an index of environmental quality whose dynamics are compatible with the existence of a common consent. We propose to derive the index of environmental quality, as valued by households, from the process through which pollutants accumulate through time. As we shall see, an alternative pattern for the environmental dynamics is then derived which is compatible with our approach of the consent of generations.

To do this we first need to describe this process of pollutants accumulation. The concentrations of the polluting emissions in the atmosphere is considered as an intergenerational public bad. By accumulating in the atmosphere, emissions are the source of damages for generations far beyond the generation contemporaneous of the corresponding emissions. We model concentrations in the atmosphere as a single stock $M_t > 0$. The equation which governs the evolution of M_t is the following:

$$M_{t+1} = (1 - \delta_M) M_t + \varepsilon E_t \tag{3.2}$$

According to this equation, the current concentrations stock M_t survives in the next period but only up to a fraction $1-\delta_M$. Thus the parameter δ_M is a factor of natural decay. The parameter ε is, as above, the rate at which emissions E_t

increase the concentrations of the next period M_{t+1}. If emissions are equal to zero, the pollution stock M_t also tends to zero. This particular steady state of the stock M_t corresponds to the case in which the atmosphere is absolutely free from anthropogenic emissions[3]. At the opposite, the stock of concentrations may be arbitrarily large, even if temporarily.

PROPOSITION 3.1. ***The dynamics of the index of environmental quality***

Let equation (3.2) describe the evolution of the pollutant stock. Let $\overline{Q}$ define the threshold level of the pollutant stock above which the economy disappears. There exists a common consent of all generations if individuals value the following index of environmental quality:

$$Q_t = \overline{Q} - M_t \tag{3.3}$$

Given the dynamics of the pollutant stock M_t, the dynamics of the index of environmental quality Q_t is given by:

$$Q_{t+1} = (1 - \delta_M)\, Q_t + \delta_M \bar{Q} - \varepsilon E_t \tag{3.4}$$

The common consent exists and belongs to the set of constant-emission paths $(S)_{t=0}^{+\infty}$ for which:

$$S < \min\{E^{bau}, \frac{\delta_M}{\varepsilon}\overline{Q}\} \tag{3.5}$$

Proof. See appendix A.2

In the literature, at odd with our approach, the index of quality is generally not derived from an underlying pollutant stock equation. While the degree of concentrations of polluting emissions in the atmosphere is a matter of fact provided by climatologists, the index of environmental quality which enters the individuals' utility function is an economic concept. The way we construct it is simple. The definition (3.3) restricts the set of values which M_t can take to those compatible with economic activity, i.e. lower than the upper bound $\bar{Q}$. If the pollutant stock tends to $\overline{Q}$, the environmental index tends to zero. In the absence of anthropogenic emissions, the pollutant stock M_t tends to zero and the index of quality tends to $\overline{Q}$. Therefore the threshold $\bar{Q}$ can also be interpreted as the highest attainable environmental quality when emissions out of economic activity are zero.

[3]There normally remains a positive level of, e.g., greenhouse gas emissions in the atmosphere. Otherwise the temperature on earth would be freezing. It would be easy to generalize the setting to one with both anthropogenic and non-anthropogenic emissions. This extension would only bound the concentration M_t from below at a positive level, instead of zero, and would not change the results of the analysis.

It is easy to show that the common consent exist, i.e. that it belongs to a non-empty set. The index of environmental quality Q_t is defined in the interval $\left[0, \overline{Q}\right[$. If it exists, a steady state Q must belong to this interval. The maximum sustainable long run emissions $\bar{E}$ are obtained by equating the stationary quality Q to zero and are equal to $(\delta_M/\varepsilon)\,\overline{Q}$. If the BAU path goes toward collapse $\left(0 < \bar{E} < E^{bau}\right)$, the common consent of all generations is characterized by the set of all values of S smaller than $\bar{E}$. If a steady steady of the BAU path exists $\left(0 < E^{bau} \leq \bar{E}\right)$, the common consent belongs to the set defined by $S \leq E^{bau}$.

In order to take the specific consent into account, let us add maintenance with the term μm_t in the RHS of the dynamic equation (3.4) and assume a stationary maintenance $m \geq 0$. It then follows from the above discussion that, under the dynamics defined by (3.4), the specific consent to invest in environmental maintenance is not constrained to be positive in the long run. When it is equal to zero, we simply get the steady state $Q \in \left[0, \overline{Q}\right]$.

4 The business-as-usual economy

We now turn to the description of the economy. We consider a competitive economy with overlapping generations in which agents have perfect foresight. The population size is constant and normalized to unity: $N = 1$. In this section, we analyze the BAU economy. In the next section we introduce the two institutions described in section 2: the constitutional fund and the voting procedure.

Output, emissions and profits in the BAU economy

In the BAU economy, a representative firm produces a consumption/investment good Y_t according to a constant returns to scale technology. This production generates polluting emissions which, in turn, generate environmentally harmful atmospheric concentrations M_t. Beyond the level of concentrations $\overline{Q}$, production is annihilated. As long as the pollutant stock falls short this limit threshold, we define the production technology by a Cobb-Douglas type production function:

$$Y_t = \widetilde{A} K_t^{\alpha} L_t^{1-\alpha} \zeta_t \tag{4.1}$$

where $\widetilde{A} > 0$ is an index of productivity, $K_t \geq 0$ and $L_t \geq 0$ are capital and labour at time t and $\zeta_t \in (0,1)$ is the intensity of pollution. We assume $0 < \alpha < 1$. Polluting emissions are defined as follows:

$$E_t = Y_t \zeta_t^{\theta} \tag{4.2}$$

with $\theta > 0$. The intensity of pollution ζ_t may be written as a function of the ratio between emissions and output:

$$\zeta_t = \left(\frac{E_t}{\widetilde{A} K_t^{\alpha} L_t^{1-\alpha}} \right)^{1/(1+\theta)} \tag{4.3}$$

In the BAU economy, pollution is maximum because there is no environmental policy. This corresponds to the case for which $\zeta_t = 1$ and implies : $E_t = \tilde{A} K_t^{\alpha} L_t^{1-\alpha}$.

We assume that the capital is entirely depreciated in the production process. The representative firm is a price taker and maximizes its real profit, which is defined by

$$\pi_t = \tilde{A} K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - R_t K_t \tag{4.4}$$

The variables L_t and K_t are respectively labour and capital demands and w_t and R_t the real wage rate and interest factor, i.e. one plus the interest rate r_t. The first-order conditions of the firm's problem are the following:

$$w_t = w^{bau}(k_t) \equiv (1-\alpha) \tilde{A} k_t^{\alpha} \tag{4.5a}$$

$$R_t = R^{bau}(k_t) \equiv \alpha \tilde{A} k_t^{\alpha-1} \tag{4.5b}$$

where $k_t = K_t/L_t$ is the capital-labor ratio.

Preferences and the households' utility in the BAU economy

Since we want to compare the BAU economy with the regulated economy, we make simplifying assumptions on the BAU economy, in particular, on the opportunities open to households. In the BAU economy, households supply inelastically one unit of labour to the firm for a real wage rate w_t and save everything in productive capital: $w_t = s_t$. When old they consume all their capital income, principal and interest: $R_{t+1} s_t = d_{t+1}$, where R_{t+1} is the interest factor and d_{t+1} old age consumption. Households derive utility from old age consumption d_{t+1} and from environmental quality Q_{t+1} and their utility is assumed to be loglinear:

$$u_t = \log d_{t+1} + \gamma \log Q_{t+1} \tag{4.6}$$

where $\gamma > 0$ is the weight put on the log of environmental quality. Households maximize their utility under their budget constraint, taking prices as given. Optimal consumption is simply : $d_{t+1} = R_{t+1} w_t$.

The BAU equilibrium

In equilibrium, the labour market clears: $L_t = N_t = 1$. Savings of the young at time $t-1$, invested in capital, constitutes the current period capital stock: $s_{t-1} = K_t$. Since the aggregate labor supply is inelastic, the capital-labour ratio is equal to capital per head. Hence, together with the population size equals unity, we have $k_t = K_t/1$. So we get $s_{t-1} = k_t$. Equilibrium wage and interest factor are: $w_t = w^{bau}(k_t) \equiv (1-\alpha)\tilde{A}k_t^{\alpha}$ and $R_t = R^{bau}(k_t) \equiv \alpha\tilde{A}k_t^{\alpha-1}$, where k_t is the capital per head. Emissions are equal to output.

Hence the dynamics of k_t and those of Q_t are given by the following two equations:

$$k_{t+1} = (1-\alpha)\tilde{A}k_t^{\alpha} \tag{4.7}$$

$$Q_{t+1} = (1-\delta_M)Q_t + \delta_M\bar{Q} - \varepsilon\tilde{A}k_t^{\alpha} \tag{4.8}$$

A steady state of the BAU economy is a pair $\left(k^{bau}, Q^{bau}\right)$, with $k^{bau} > 0$ and $Q^{bau} > 0$, which solves (4.7) and (4.8). It is given by:

$$k^{bau} = \left[(1-\alpha)\tilde{A}\right]^{1/(1-\alpha)} \tag{4.9}$$

$$Q^{bau} = \bar{Q} - \frac{\varepsilon}{\delta_M}\tilde{A}\left(k^{bau}\right)^{\alpha} \tag{4.10}$$

Hence the following proposition holds.

PROPOSITION 4.1. ***Existence of the BAU steady state equilibrium***

There exists a steady state equilibrium in the BAU economy if and only if

$$\tilde{A} < \left(\frac{\delta_M}{\varepsilon(1-\alpha)^{\alpha/(1-\alpha)}}\bar{Q}\right)^{1-\alpha} \tag{4.11}$$

Proof. See appendix A.3

When firms are free to pollute, environmental quality in the long run may be dramatically low, or even tend to 0. The higher the output per head, the lower the environmental quality. It is easy to show that the steady state $\left(k^{bau}, Q^{bau}\right)$ is a sink. As an illustration, consider the effect of an increase in the index of overall productivity $\tilde{A}$. The higher the overall productivity index $\tilde{A}$, the higher the output and thus the higher polluting emissions. A higher overall productivity $\widetilde{A}$ is also associated with a steeper curve $Q^{bau} = \bar{Q} - (\varepsilon/\delta_M)\tilde{A}(k_t)^{\alpha}$. Stationary wealth and environmental quality evolve in opposite directions: each individual enjoys higher wealth but suffers from a more deteriorated environment.

5 The economy with environmental policy

We now introduce the institutions of the constitutional fund and the voting procedure, in charge of implementing, respectively, the common and the specific consent of the generations. We describe how the instruments handled by these institutions modify the agents' decisions. We assume from now on that there exists a BAU steady state equilibrium ($E^{bau} < \bar{E}$) and that the economy is at that steady state when these institutions are introduced.

5.1 The constitutional fund and the sale of permits

As Jouvet, Michel and Rotillon (2005) and in the spirit of the Sky Trust initiative, we assume that firms are obliged to pay for each unit of their emissions. Each period the constitutional fund is mandated to implement the common consent of all generations by inelastically supplying to the firms a global amount S of emissions permits, which satisfies the property of the common consent $S < E^{bau} = \tilde{A}\left(k^{bau}\right)^{\alpha}$. Denote by q_t the equilibrium price.

Thus the fund collects each period the amount $q_t S$. We asssume that $q_t S$ is entirely distributed during the same period to the young individuals (τ_t per head). With $N = 1$, this yields:

(5.1) $\quad \tau_t = q_t S$

Since there is no heterogeneity among firms, all firms will demand the same amount of permits at the price q_t. Let E_t be the demand for permits expressed by the representative firm. We need to re-write the production function in order to let appear the three production factors, capital, labour and emissions. Eliminating ζ_t from the production function, one gets

(5.2) $\quad Y_t = AK_t^{\alpha_K} L_t^{\alpha_L} E_t^{\alpha_E}$

with $A = \tilde{A}^{\theta/(1+\theta)}$, $\alpha_K = \alpha\theta/(1+\theta)$, $\alpha_L = (1-\alpha)\theta/(1+\theta)$ and $\alpha_E = 1/(1+\theta)$. Note that $\alpha_K + \alpha_L + \alpha_E = 1$. The share of capital and labour in production are reduced with respect to the BAU case: $\alpha_K < \alpha$ and $\alpha_L < 1-\alpha$. The firm's profit is then :

(5.3) $\quad \pi_t = AK_t^{\alpha_K} L_t^{\alpha_L} E_t^{\alpha_E} - R_t K_t - w_t L_t - q_t E_t$

The maximization of profit implies :

(5.4a) $\quad R_t = \alpha_K A k_t^{\alpha_K - 1} e_t^{\alpha_E} \equiv R(k_t, e_t)$

(5.4b) $\quad w_t = \alpha_L A k_t^{\alpha_K} e_t^{\alpha_E} \equiv w(k_t, e_t)$

(5.4c) $\quad q_t = \alpha_E A k_t^{\alpha_K} e_t^{\alpha_E - 1} \equiv q(k_t, e_t)$

with $k_t = K_t/L_t$ and $e_t = E_t/L_t$.

5.2 The voting procedure on the maintenance tax

The second institution is the voting procedure. This procedure cooperatively fixes the amount of the numeraire which, each period, is invested in the maintenance of the environment. To satisfy the properties of the specific consent it must be non-negative and maximize the welfare of the current generation. The tax which finances the investment modifies the individuals' decision making process. When young, individuals divide their income, wage income plus the fund transfer, $w_t + \tau_t$, between savings in productive capital s_t and maintenance of environmental quality m_t :

$$w_t + \tau_t = s_t + m_t \tag{5.5}$$

When old they consume their savings d_{t+1} :

$$R_{t+1} s_t = d_{t+1} \tag{5.6}$$

The technology of maintenance of environmental quality is linear: each unit of good invested increases quality by μ. With environmental maintenance the quality equation (3.4) becomes:

$$Q_{t+1} = (1 - \delta_M) Q_t + \delta_M \bar{Q} - \varepsilon E_t + \mu m_t \tag{5.7}$$

Note $N_t m_t = m_t$ since $N_t = 1$ for each t.

Given the simplifying assumption on the absence of consumption during individuals' youth and given prices, the specific consent characterized by the voted investment in maintenance fully characterizes the individuals' decision making. After substitution of the budget constraints and the definition of environmental quality Q_{t+1} in the utility function, the household's problem is equivalent to choose the non-negative maintenance tax which maximize utility, i.e.:

$$\max_{m_t \geq 0} \log[R_{t+1}(w_t - \tau_t - m_t)] + \gamma \log[(1 - \delta_M) Q_t + \delta_M \bar{Q} - \varepsilon E_t + \mu m_t] \tag{5.8}$$

subject to $m_t \geq 0$. Maintenance m_t belongs to the interval $[0, w_t + \tau_t]$. The specific consent is characterized by zero maintenance if marginal utility of consumption d_{t+1} is larger than marginal utility of quality at zero maintenance $(m_t = 0)$, i.e.

$$1/[w_t + \tau_t] \geq \gamma\mu / \left[(1 - \delta_M) Q_t + \delta_M \bar{Q} - \varepsilon E_t\right]$$

otherwise the specific consent is characterized by a strictly positive maintenance $(m_t > 0)$. The FOC's are given by

$$\frac{1}{w_t + \tau_t - m_t} \geq \frac{\mu\gamma}{(1 - \delta_M) Q_t + \delta_M \bar{Q} - \varepsilon E_t + \mu m_t} \tag{5.9}$$

with equality if $m_t > 0$. Alternatively, let $\tilde{m}_t$ be the solution of this equation holding with equality, i.e.

$$\tilde{m}_t = \frac{\gamma}{1 + \gamma} (w_t + \tau_t) - \frac{1}{\mu (1 + \gamma)} \left[(1 - \delta_M) Q_t + \delta_M \bar{Q} - \varepsilon E_t\right] \tag{5.10}$$

Let us refer to $\tilde{m}_t$ as the *desired* maintenance which may be positive or negative. Then the specific consent is characterized by

$$m_t = \max\{\tilde{m}_t, 0\} \tag{5.11}$$

It coincide with the desired maintenance if the latter is positive, otherwise is is equal to zero.

6 Temporary equilibria

We now study the equilibrium of a single period. We shall first derive the expression for the optimal maintenance in equilibrium. Then we shall study successively the temporary equilibrium with a zero specific consent, i.e. a constrained-maintenance time t equilibrium and then the temporary equilibrium with a positive specific consent, i.e. an unconstrained-maintenance time t equilibrium.

As in the BAU case, the supply of labour is inelastic and labour market equilibrium implies $L_t = 1$. The supply of capital at time t is predetermined: $s_{t-1} = k_t$. The new feature with respect to the BAU case is the market for permits. In equilibrium the demand for permits and the inelastic supply of permits must be equal: $E_t = S$, or in intensive terms: $e_t = S$. Equilibrium prices are given by

(6.1a) $$w_t = w(k_t, S) \equiv \alpha_L A k_t^{\alpha_K} S^{\alpha_E}$$

(6.1b) $$R_t = R(k_t, S) \equiv \alpha_K A k_t^{\alpha_K - 1} S^{\alpha_E}$$

(6.1c) $$q_t = q(k_t, S) \equiv \alpha_E A k_t^{\alpha_K} S^{\alpha_E - 1}$$

Also, the fund budget constraint must be balanced

(6.2) $$q(k_t, S) S = \tau_t$$

6.1 The environmental maintenance in equilibrium

Note that the first period income writes in equilibrium

(6.3) $$\begin{aligned} w(k_t, S) + q(k_t, S) S &= \alpha_L A k_t^{\alpha_K} S^{\alpha_E} + \alpha_E A k_t^{\alpha_K} S^{\alpha_E} \\ &= (1 - \alpha_K) A k_t^{\alpha_K} S^{\alpha_E} \end{aligned}$$

Thus a young individual receives a share $1 - \alpha_K$ of production as first-period income. Given $\{k_0, Q_0\}$, the time $t = 1$ equilibrium value of all variables can be computed with maintenance m_0. Indeed, with m_0 and $\{k_0, Q_0\}$, we compute next period capital intensity and environmental quality

(6.4) $$k_1 = w(k_0, S) + q(k_0, S) S - m_0$$

(6.5) $$Q_1 = (1 - \delta_M) Q_0 + \delta_M \bar{Q} - \varepsilon S + \mu m_0$$

and old-age consumption

(6.6) $$d_0 = R(k_0, S) k_0$$

Hence we obtain (k_1, Q_1), which are the data of the time $t = 1$ temporary equilibrium. This can be repeated at any time t. We characterize maintenance m_t in equilibrium.

PROPOSITION 6.1. ***Environmental maintenance in equilibrium and the non-negativity constraint***

The time t equilibrium desired maintenance $\tilde{m}_t$, is a function of the degree of concern for the environment γ, the volume of permits S and the capital stock and environmental quality inherited from the previous period (k_t, Q_t) :

(6.7)

$$\tilde{m}(\underset{+}{k_t}, \underset{-}{Q_t}, \underset{+}{S}, \underset{+}{\gamma}) = \frac{\gamma\,(1-\alpha_K)\,Ak_t^{\alpha_K} S^{\alpha_E}}{1+\gamma} - \frac{(1-\delta_M)\,Q_t + \delta_M \bar{Q} - \varepsilon S}{\mu\,(1+\gamma)}$$

The time t equilibrium maintenance m_t is given by:

$$m_t = \max\{0, \tilde{m}\,(k_t, Q_t, S, \gamma)\}\,. \tag{6.8}$$

The non-negativity condition on maintenance $\tilde{m}_t \geq 0$ is equivalent to

$$Q_t \leq \frac{\mu\gamma\,(1-\alpha_K)\,Ak_t^{\alpha_K} S^{\alpha_E}}{1-\delta_M} - \frac{\delta_M \bar{Q} - \varepsilon S}{1-\delta_M} \tag{6.9}$$

Proof. *See appendix A.4*

The time t equilibrium maintenance depends only on current prices (wage, permit price) and on the inherited environmental quality. The higher the sum of wage income and permits income ($(1-\alpha_K)Ak_t^{\alpha_K} S^{\alpha_E}$), the higher the maintenance of the environment. On the contrary, the higher the environmental quality, the lower the maintenance.

Thus, depending on the inherited capital stock and environmental quality, the generation specific consent is positive or equal to zero. The effect of capital is an income effect. The effect of the aggregate amount of permits S is twofold. First, like for capital, there is an income effect since S is a component of the first-period income. Second, the aggregate amount of permits has a substitution effect which is the following. The higher S, i.e. the higher the level of the common consent, the lower the index of quality Q_t and thus the higher the willingness to improve environmental quality, i.e. the higher the specific consent.

Let us summarize the above discussion. At any time t the economy experiences a unique equilibrium which is one of two types: either one with positive specific consent , i.e. with unconstrained maintenance or one with zero specific consent, i.e. with constrained maintenance. The two equations which govern the evolution of capital and environmental quality in equilibrium are

$$k_{t+1} = (1-\alpha_K)\,Ak_t^{\alpha_K} S^{\alpha_E} - \max\{\tilde{m}\,(k_t, Q_t, S)\,, 0\} \tag{6.10}$$

$$Q_{t+1} = (1-\delta_M)\,Q_t + \delta_M \bar{Q} - \varepsilon S + \mu \max\{\tilde{m}\,(k_t, Q_t, S)\,, 0\} \tag{6.11}$$

Let us now study separately these two types of temporary equilibria. We shall see that they have very different properties.

6.2 The zero-specific-consent temporary equilibria

We shall denote the variables in the zero specific consent equilibria with the upperscript "z", for "zero maintenance". From the above two equations it follows that, when the specific consent is zero, the two equilibrium dynamics of capital and environmental quality are given by $k_{t+1}^{z} = (1-\alpha_K)\,Ak_t^{\alpha_K}S^{\alpha_E}$ and $Q_{t+1}^{z} = (1-\delta_M)\,Q_t + \delta_M\bar{Q} - \varepsilon S$. The following proposition follows.

PROPOSITION 6.2. ***Characterization of zero specific consent temporary equilibria***

On the transition path, at a zero specific consent time t temporary equilibrium, environmental quality is independent of capital accumulation. Both capital and quality depend on the common consent of all generations, i.e. the volume of permits S. Capital accumulation is increasing in the common consent S whereas environmental quality is decreasing in S :

$$k_{t+1}^{z} = \xi^{z}(\underset{+}{k_t}, \underset{+}{S}) \equiv (1-\alpha_K)\,Ak_t^{\alpha_K}S^{\alpha_E} \tag{6.12}$$

$$Q_{t+1}^{z} = \phi^{z}(\underset{+}{Q_t}, \underset{-}{S}) \equiv (1-\delta_M)\,Q_t + \delta_M\bar{Q} - \varepsilon S \tag{6.13}$$

When the specific consent is zero, a lower common consent, i.e. tightening the constraint of abatement of emissions, increases environmental quality. But it does so at the expense of the income of the young and, hence, at the expense of capital accumulation. Equilibria with a positive specific consent have very different properties.

6.3 The positive-specific-consent temporary equilibria

We denote with the upperscript "p", for "positive maintenance", the variables in positive specific consent equilibria. The equations which govern the equilibrium dynamics of the capital stock and the environmental quality in positive specific consent equilibria are given by $k_{t+1}^{p} = (1-\alpha_K)\,Ak_t^{\alpha_K}S^{\alpha_E} - \tilde{m}_t$ and $Q_{t+1}^{p} = \phi^{p}(Q_t, S) \equiv \left[(1-\delta_M)\,Q_t + \delta_M\bar{Q} - \varepsilon S\right] + \mu\tilde{m}_t$. From the FOC's, at an equilibrium with positive maintenance the following holds $Q_{t+1}^{p} = \mu\gamma k_{t+1}^{p}$. Thus we plug this expression in the equation in Q_{t+1} and Q_t and then from the households' first period budget constraint, we substitute $(1-\alpha_K)\,Ak_t^{\alpha_K}S^{\alpha_E} - k_{t+1}$ for $\tilde{m}_t$. This yields $\mu\gamma k_{t+1} = (1-\delta_M)\,\gamma\mu k_t + \delta_M\bar{Q} - \varepsilon S + \mu\,(1-\alpha_K)\,Ak_t^{\alpha_K}S^{\alpha_E} - \mu k_{t+1}$. This is an equation in $k_{t+1} - k_t$.

PROPOSITION 6.3. ***Characterization of positive specific consent temporary equilibria***

On the transition path, at a positive specific consent time t temporary equilibrium, environmental quality is positively related to capital accumulation:

$$Q_{t+1}^{p} = \mu\gamma k_{t+1}^{p} \tag{6.14}$$

and capital accumulation verifies

$$\begin{aligned} k_{t+1}^{p} &= \xi^{p}(\underset{+}{k_t}, \underset{+/-}{S}) \\ &\equiv \frac{(1-\delta_M)\,\gamma}{1+\gamma}k_t + \frac{\delta_M\bar{Q} - \varepsilon S}{(1+\gamma)\,\mu} + \frac{1}{1+\gamma}\,(1-\alpha_K)\,Ak_t^{\alpha_K}S^{\alpha_E} \end{aligned} \tag{6.15}$$

The higher the households' taste for the environment, γ, and the higher the impact of maintenance, μ, the stronger the link between capital accumulation and environmental quality. Environmental quality and capital accumulation are both increasing in the common consent S if and only if S is sufficiently low:

$$S < \left(\varepsilon^{-1}\mu\alpha_E\,(1-\alpha_K)\,Ak_t^{\alpha_K}\right)^{1/(1-\alpha_E)} \tag{6.16}$$

i.e. if and only if the productivity of emissions is high enough:

$$\alpha_E A k_t^{\alpha_K} S^{\alpha_E - 1} > \left[\mu\,(1-\alpha_K)\right]^{-1}\varepsilon. \tag{6.17}$$

Proof. *See appendix A.5*

Hence, at a temporary equilibrium with positive specific consent, if the volume of permits corresponding to the common consent is increased marginally, there is a positive effect on both next period capital accumulation and environmental quality if emissions are scarce and hence their marginal productivity high. Indeed, the increase in S has two effects on capital: a substitution effect and an income effect. The substitution effect is the following: increasing S deteriorates environmental quality and this stimulates the specific consent, i.e. investment in maintenance becomes relatively more interesting in terms of utility. Thus, with an unchanged production, capital accumulation decreases and investment in the environment increases. The income effect comes from an increased production through higher S. It plays through the marginal productivity of emissions. If the income effect dominates the substitution effect, capital is positively affected. This may be called a scarcity rent effect.

7 Characterization of the intertemporal equilibrium with perfect foresight

In this section we characterize the intertemporal equilibrium with perfect foresight. In the dynamic equation of capital (6.10) and in the one of environmental quality (6.11), we replace desired maintenance $\tilde{m}(k_t, Q_t, S)$ by its definition. We obtain at each period one of these two dynamic systems: either

$$k_{t+1} = \xi^z(k_t, S) \equiv (1-\alpha_K) A k_t^{\alpha_K} S^{\alpha_E} \tag{7.1}$$

$$Q_{t+1} = \phi^z(Q_t, S) \equiv (1-\delta_M) Q_t + \delta_M \bar{Q} - \varepsilon S \tag{7.2}$$

for zero specific consent, or

$$k_{t+1} = \xi^p(k_t, S) \equiv \frac{(1-\delta_M)\gamma}{1+\gamma} k_t + \frac{1}{(1+\gamma)\mu}\left(\delta_M \bar{Q} - \varepsilon S\right) + \frac{1}{1+\gamma}(1-\alpha_K) A k_t^{\alpha_K} S^{\alpha_E} \tag{7.3}$$

$$Q_{t+1} = \mu\gamma\xi^p(k_t, S) \tag{7.4}$$

for positive specific consent. Both functions ξ^z and ξ^p are increasing in k_t. In the following proposition, we characterize the equilibrium dynamics of capital.

PROPOSITION 7.1. ***Characterization of the equilibrium dynamics of capital***

Given initial conditions (k_0, Q_0), the equilibrium dynamics of capital intensity are characterized by

$$k_{t+1} = \min\{\xi^z(k_t, S), \xi^p(k_t, S)\} \tag{7.5}$$

Proof. See appendix A.6

8 The steady state equilibria

8.1 The zero-specific-consent stationary equilibrium

A steady state with zero specific consent is a pair (k^z, Q^z) which violates the non-negativity condition (6.9). It is defined by $k^z = \left[(1-\alpha_K) A S^{\alpha_E}\right]^{1/(1-\alpha_K)}$ and $Q^z = \bar{Q} - (\varepsilon/\delta_M) S$. The steady Q^z is positive only if $S < (\delta_M/\varepsilon)\bar{Q}$, which is always true if there exists a steady state equilibrium of the BAU economy, which we assumed. Both the dynamics of capital and environmental quality, ξ^z and ϕ^z, are increasing and concave. The slope of these two dynamics around their steady state is less than unity. Indeed, k_{t+1}^z has infinite slope as k_t tends to 0 and zero slope as k_t tends to $+\infty$. As far as Q_t is concerned, as Q_t tends to zero, the RHS of the quality equation with zero maintenance tends to $\delta_M \bar{Q} - \varepsilon S$ which is strictly positive if $S < (\delta_M/\varepsilon)\bar{Q}$ and the derivative $dQ^z_{t+1}/dQ_t = 1 - \delta_M < 1$, $\forall Q_t$. Let us summarize by the following proposition.

PROPOSITION 8.1. ***Characterization of the zero specific consent steady state equilibrium***

Any zero specific consent steady state is a pair (k^z, Q^z), with $k^z > 0$ and $Q^z > 0$, which verifies

(8.1) $$Q^z > \frac{1-\alpha_K}{1-\delta_M}\mu\gamma A S^{\alpha_E}(k^z)^{\alpha_K} - (1-\delta_M)\left(\delta_M \bar{Q} - \varepsilon S\right)$$

(8.2) $$k^z = k^z(\underset{+}{S}) \equiv \left[(1-\alpha_K) A S^{\alpha_E}\right]^{1/(1-\alpha_K)}$$

(8.3) $$Q^z = Q^z(\underset{-}{S}) = \bar{Q} - \frac{\varepsilon}{\delta_M} S \geq 0$$

Such a steady state equilibrium is unique and locally stable. At that steady state equilibrium, the level of the capital stock increases with the common consent S and environmental quality decreases with S.

8.2 The positive-specific-consent stationary equilibrium

A steady state capital intensity with positive specific consent k^p, solves $k^p = \xi^p(k^p)$ or equivalently

(8.4) $$\delta_M \bar{Q} - \varepsilon S = \mu k^p (1 + \gamma\delta_M) - \mu(1-\alpha_K) A (k^p)^{\alpha_K} S^{\alpha_E}$$

The LHS of (8.4) is positive if $S \leq (\delta_M/\varepsilon)\bar{Q}$, which is always verified if there exists a BAU steady state equilibrium, as we assume. The RHS of (8.4) tends to 0 as k^p tends to 0. The derivative of the RHS with respect to k^p tends to $-\infty$ as k^p tends to 0, is equal to 0 for some value of k^p which solves $(1+\gamma\delta_M) = \alpha_K(1-\alpha_K) A (k^p)^{\alpha_K - 1} S^{\alpha_E}$ and tends to a positive constant $\mu(1+\gamma\delta_M)$ as k^p tends to $+\infty$. Hence the RHS of (8.4) is first negative, reaches a minimum and then increase and tends to $+\infty$ as k^p tends to $+\infty$. It is continuous. Thus, in the case $S \leq (\delta_M/\varepsilon)\bar{Q}$, there exists a unique k^p which solves this equation:

(8.5) $$k^p = k^p(S)$$

In the following proposition we state and prove the properties of the equilibrium stationary capital in the case of a long run positive specific consent.

PROPOSITION 8.2. ***Characterization of the positive specific consent steady state equilibrium***

Any unconstrained-maintenance steady state equilibrium is a pair (k^p, Q^p), with $k^p > 0$ and $Q^p = \mu\gamma k^p > 0$, which verifies

$$Q^p \leq \frac{1-\alpha_K}{1-\delta_M}\mu\gamma A S^{\alpha_E}\left(k^p\right)^{\alpha_K} - \left(1-\delta_M\right)\left(\delta_M \bar{Q} - \varepsilon S\right) \tag{8.6}$$

$$\delta_M \bar{Q} - \varepsilon S = \mu k^p \left(1+\gamma\delta_M\right) - \mu\left(1-\alpha_K\right) A \left(k^p\right)^{\alpha_K} S^{\alpha_E} \tag{8.7}$$

It is unique and locally stable. At this steady state equilibrium, the impact of a change in the common consent on the level of the capital stock is given by

$$\frac{dk^p}{dS} = \frac{-\varepsilon + \mu\alpha_E\left(1-\alpha_K\right) A \left(k^p\right)^{\alpha_K} S^{\alpha_E - 1}}{\mu\left(1+\gamma\delta_M\right) - \mu\alpha_K\left(1-\alpha_K\right) A\left(k^p\right)^{\alpha_K - 1} S^{\alpha_E}} \tag{8.8}$$

The impact of an increase in the common consent S is positive if and only if S is sufficiently low

$$S < \left(\varepsilon^{-1}\mu\alpha_E\left(1-\alpha_K\right) A k_t^{\alpha_K}\right)^{1/(1-\alpha_E)} \tag{8.9}$$

Proof. *See appendix A.7*

9 Switches in the specific consent on the transition

We now examine the possibility of a switch between one temporary equilibrium with zero specific consent to a temporary equilibrium with positive consent the period after. To do this, let us study the positions of the two loci of points ξ^p and ξ^z : $\lim_{k_t\to 0}\xi^p\left(k_t\right) = \left(1+\gamma\right)^{-1}\mu\left(\delta_M\bar{Q} - \varepsilon S\right) \geq \lim_{k_t\to 0}\xi^z\left(k_t\right) = 0$. Thus in the neighbourhood of zero the regime is the one with zero speific consent. The slope at $k_t = 0$ is given by

$$\begin{aligned}\lim_{k_t\to 0}\frac{\partial\xi^p(k_t)}{\partial k_t} &= \lim_{k_t\to 0}\left[\frac{\left(1-\delta_M\right)\gamma}{1+\gamma} + \frac{1}{1+\gamma}\alpha_K\left(1-\alpha_K\right) A k_t^{\alpha_K - 1} S^{\alpha_E}\right] \\ &= +\infty\end{aligned} \tag{9.1}$$

$$\lim_{k_t\to 0}\frac{\partial\xi^z(k_t)}{\partial k_t} = \lim_{k_t\to 0}\left[\alpha_K\left(1-\alpha_K\right) A k_t^{\alpha_K-1} S^{\alpha_E}\right] = +\infty \tag{9.2}$$

We know that

$$\begin{aligned}\tilde{m}_t \geq 0 \Leftrightarrow Q_t &\leq \left(1-\delta_M\right)^{-1}\left(\varepsilon S - \delta_M\bar{Q}\right) \\ &\quad + \left(1-\delta_M\right)^{-1}\left(1-\alpha_K\right)\mu\gamma A S^{\alpha_E} k_t^{\alpha_K}\end{aligned}$$

is also equivalent to $\xi^p\left(k_t\right) \leq \xi^z\left(k_t\right)$. The two loci may cross each other. How many times? Any crossing point solves $\xi^p\left(k_t\right) - \xi^z\left(k_t\right) = 0$, which is equivalent to

$$\delta_M\bar{Q} - \varepsilon S = \gamma\mu\left[\left(1-\alpha_K\right) A k_t^{\alpha_K} S^{\alpha_E} - \left(1-\delta_M\right) k_t\right] \tag{9.3}$$

The LHS of (9.3) is positive or zero since $S \leq \left(\delta_M/\varepsilon\right)\bar{Q}$. The RHS of (9.3) tends to zero as k_t tends to zero and tends to $-\infty$ as k_t tends to $+\infty$. It is first increasing and then decreasing. Its derivative is $\gamma\mu\alpha_K(1-\alpha_K)Ak_t^{\alpha_K-1}S^{\alpha_E} - \gamma\mu(1-\delta_M) > 0$. Hence it tends to $+\infty$ as k_t tends to zero and it tends to $-\gamma\mu(1-\delta_M)$ as k_t tends to $+\infty$. The maximum of the RHS is reached when capital intensity equals $\left[(1-\delta_M)^{-1}\alpha_K(1-\alpha_K)AS^{\alpha_E}\right]^{1/(1-\alpha_K)}$.

PROPOSITION 9.1. ***Regime switches in the specific consent on the transition***

There exist at most two regime switches in the specific consent on the equilibrium dynamics. Proof. See appendix A.8

The characteristics of the economy may be incompatible with the existence of any regime switch. In that case, the economy always experiences zero specific consent for any generation, including in the long run if the scale factor $\bar{Q}$ of the environmental equation is sufficiently higher than the scale factor of the production function A. Figure 1 depicts this case. The equilibrium dynamic is given by the $\xi^z(k_t)$ curve.

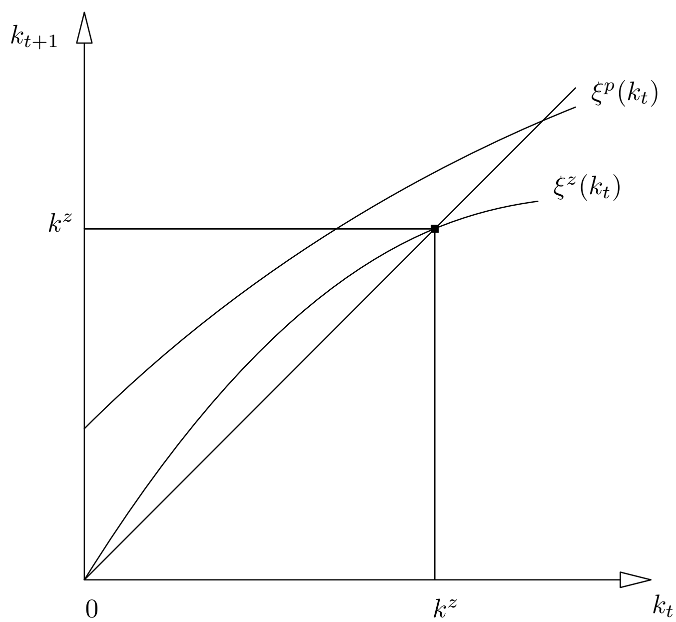

Figure 1: Equilibrium dynamics with no switch. Starting from the origin, the equilibrium dynamics is always located on the $\xi^z(k_t)$ curve.

A limit case is when the zero and the positive consent curve are tangent (see Figure 2).

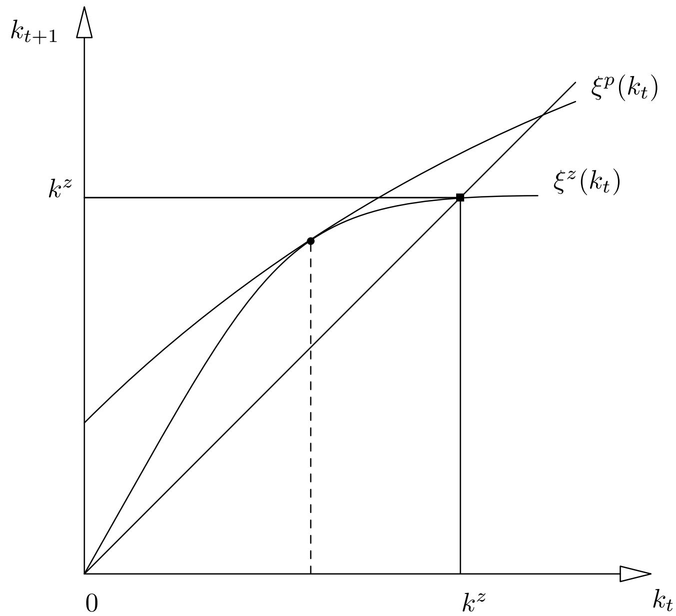

Figure 2: Equilibrium dynamics, limit case. Starting from the origin, the equilibrium dynamics is located on the $\xi^z(k_t)$ curve and passes through the tangency point with the curve $\xi^p(k_t)$.

Most of the time there will be two regime switches on the dynamics (see Figure 3 and 4). Assume there exist two regime switches on the dynamics. The economy may follow a path on which it switches only once. Single switch trajectories may occur when the dynamics cross twice, once above the 45° line (κ_l) and once below the 45° line (κ_h) (see Figure 3).

PROPOSITION 9.2.

When the two regime switches are such that $\xi^p(\kappa_l) > \kappa_l$ and $\xi^p(\kappa_h) < \kappa_h$ (figure 3), the economy converges to the positive specific consent steady state equilibrium k^p and it experiences a single regime switch if the initial capital lies in the zero specific consent area: $k_0 \in (0, \kappa_l) \cup (\kappa_h, +\infty)$ Proof. See appendix A.9

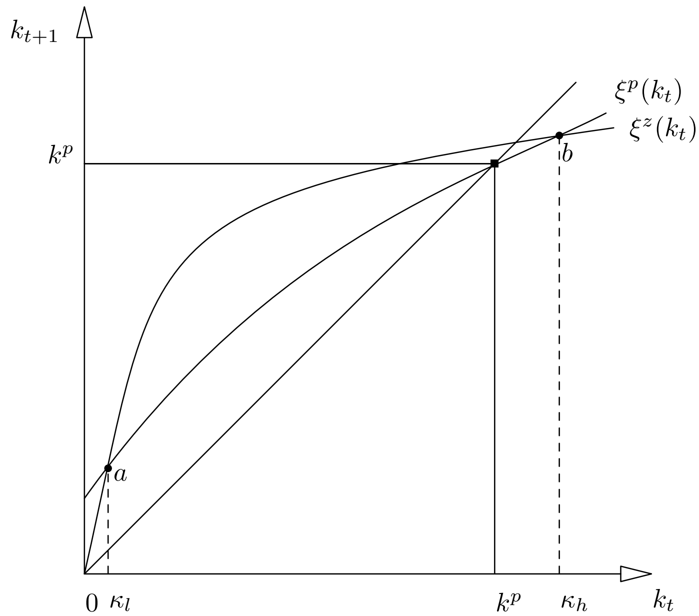

Figure 3 - Equilibrium dynamics with two switches, first case. Starting from the origin, the equilibrium dynamics is located on the $\xi^z(k_t)$ curve up to point a, then switches to the $\xi^p(k_t)$ curve up to point b and switches back to $\xi^z(k_t)$.

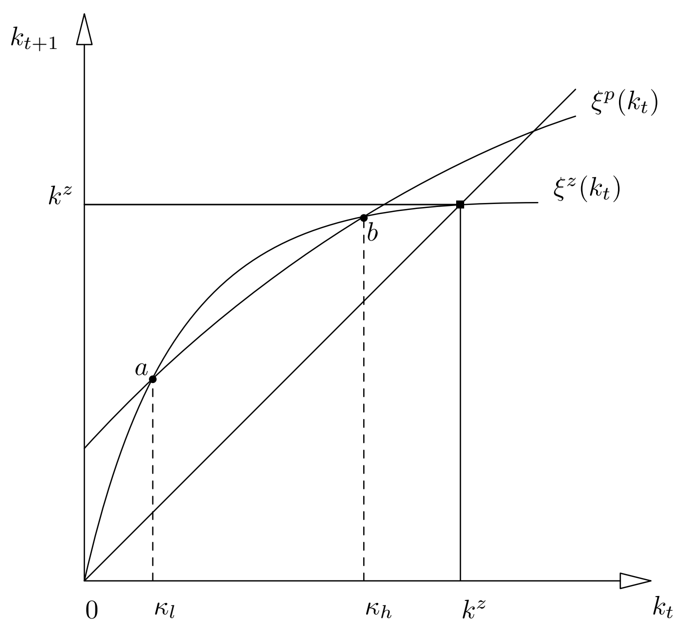

Figure 4 : Equilibrium dynamics with two switches, second case. Starting from the origin, the equilibrium dynamics is located on the $\xi^z(k_t)$ curve up to point a, then switches to the $\xi^p(k_t)$ curve up to point b and switches back to $\xi^z(k_t)$.

Double switch trajectories may only occur when the dynamics cross each other twice above the 45° line (see Figure 4).

PROPOSITION 9.3.

When the two regime switches are such that $\xi^p(\kappa_l) > \kappa_l$ and $\xi^p(\kappa_h) > \kappa_h$ (figure 4) and when initial capital intensity verifies $k_0 < \kappa_l$, the economy converges to the zero specific consent steady state equilibrium k^z and experiences a double regime switch : from zero to positive specific consent and then from positive to zero specific consent. *Proof.* *See appendix A.10*

All the cases where the steady state equilibrium is with zero specific consent (cases 1, 2 and 4) are at odd with Ono (2002) in which the specific consent cannot be zero in the long run.

10 The transition from the BAU economy to the economy with policy

Until now we have studied the effects of the consents of generations, on capital accumulation and the environment, when these consents and their respective institutions are already operating. In this section, we complete this analysis by studying the impact of the introduction of the two institutions starting from the BAU economy. Remind that the BAU economy is assumed to be initially at the BAU steady state equilibrium $\left(k^{bau}, Q^{bau}\right)$.

PROPOSITION 10.1. ***The transition from the BAU equilibrium to the equilibrium with policy***

Let the initial equilibrium be the business-as-usual steady state $\left(k^{bau}, Q^{bau}\right)$. At some given date t_0, assume that (i) a common consent is chosen, through the setting of a limit to emissions ($S < E^{bau}$), and implemented by a constitutional fund which is mandated to sell each period the volume S of permits to the firms and to redistribute the proceeds of the permits sales to the young households and that (ii) each period the specific consent is expressed through a voting procedure which determines the level of maintenance.

The environmental quality at time t_0 increases and converges to a higher level in the long run. The capital per head increases and converges to a higher level if and only if at time t_0

$$\min\left\{\xi^z\left(k^{bau}\right), \xi^p\left(k^{bau}\right)\right\} > k^{bau} \tag{10.1}$$

If this last condition does not hold, capital per head decreases and if in addition the specific consent is positive at time t_0, i.e. if $\xi^p\left(k^{bau}\right) < \xi^z\left(k^{bau}\right)$, environmental quality follows a non-monotonic path to its higher steady state equilibrium: it first overshoots it and then decreases. Proof.

See appendix A.11

It is thus easy to increase environmental quality in the short run (Q_{t_0+1}) but less easy to maintain this higher level in the long run. The critical factor is the impact on the wealth of the economy. If the policy harms seriously the capital accumulation process, there is a probability that the advantage gained in environmental quality partially melts down on the transition path. On the contrary, if the policy enhances the capital accumulation process, the short run gain on quality can be increased on the transition path. Here is an example of such a scenario.

EXAMPLE. *At the BAU equilibrium $k^{bau} = (1-\alpha)\tilde{A}\left(k^{bau}\right)^{\alpha}$. Suppose the specific consent remains equal to zero both in the short and the long run. This might be because for any level of capital intensity $\xi^z < \xi^p$. When the constitutional fund implements the common consent S it actually implements a steady value of the degree of pollution, $\zeta \in (0,1)$, (see section 4): $\zeta = \left(S/\tilde{A}\left(k^{bau}\right)^{\alpha}\right)^{1/(1+\theta)}$. Hence the aggregate volume of permits sold to the firms writes*

$$S = \zeta^{1+\theta}\tilde{A}\left(k^{bau}\right)^{\alpha}$$

with $0 < \zeta < 1$. As a result the contribution of permits to production, S^{α_E}, can be written as a function of the BAU emissions

$$S^{\alpha_E} = S^{1/(1+\theta)} = \zeta\left(\tilde{A}\left(k^{bau}\right)^{\alpha}\right)^{1/(1+\theta)}$$

Taking into account this expression for S^{α_E}, the time t_0+1 capital intensity can be re-written as

$$k_{t_0+1} = \xi^z\left(k^{bau}\right) \equiv \zeta\left(1-\alpha_K\right)\tilde{A}\left(k^{bau}\right)^{\alpha} \tag{10.2}$$

The comparison of k_{t_0+1} and k^{bau} shows that the common consent implemented by the fund not only increases environmental quality at time t_0+1 but also foster capital accumulation if the degree of pollution ζ is set in the following interval:

$$\frac{1-\alpha}{1-\alpha_K} < \zeta < 1 \tag{10.3}$$

On the transition, capital per head and environmental quality both jump up at time t_0+1 and then monotonically increase and converge to their new steady state (k^z, Q^z). The price of permits also follows a monotonically increasing path toward its steady value $q^z = q(k^z, S)$.

The interpretation of this example runs as follows. The parameter θ, which appears in $\alpha_K = \alpha\theta/(1+\theta)$, and comes from the emissions equation (4.2), influences the volume of emissions for a given output Y_t. It also influences the degree of the redistribution of output between factor owners, at the benefit of the young households.

The higher θ is, the lower the share of emissions in output α_E, the higher the one of capital α_K, hence the closer α_K is to α. In other words, the closer is the share of the young in production in the regulated economy $(1-\alpha_K)$, with respect to the share of the young in the BAU economy $(1-\alpha)$. This means that the redistribution of production to the young through permits sales recycling is low.

The range of ζ values verifying the above condition may be narrow. The smaller the redistribution effect to the young households, the smaller the range of ζ values over which both capital accumulation and environmental quality can be enhanced.

What is the immediate gain in environmental quality $(Q_{t_0+1} - Q^{bau})$and what is the long run gain $(Q^z - Q^{bau})$? It is easy to show that these gains are respectively:

$$Q_{t_0+1} - Q^{bau} = (1-\zeta^{1+\theta})\varepsilon E^{bau} \tag{10.4}$$

and

$$Q^z - Q^{bau} = \frac{1}{\delta_M}(1-\zeta^{1+\theta})\varepsilon E^{bau} \tag{10.5}$$

The short run gain is multiplied by a factor $1/\delta_M$ in the long run. So this is potentially important even if the short gain is low.

To get an idea of the magnitude of the interval $((1-\alpha)(1-\alpha_K), 1)$, and of the environmental gain, let us set $\alpha = 0.3$. Consider two values of θ: $\theta = 2$ and $\theta = 5$. The latter corresponds to the smallest redistribution of output. We respectively get:

$$\alpha_K = 0.2 \quad \alpha_E = 0.333 \quad (\frac{1-\alpha}{1-\alpha_K}, 1) = (0.875, 1) \tag{10.6}$$

and

$$\alpha_K = 0.25 \quad \alpha_E = 0.166 \quad (\frac{1-\alpha}{1-\alpha_K}, 1) = (0.933, 1) \tag{10.7}$$

Consider the last case with the smallest interval $(\theta = 5)$and assume that the common consent policy implemented is the one which leaves the capital intensity unchanged $(k_{t_0+1} = k_{t_0} = k^{bau})$. The choice of the degree of pollution is the lower bound of the interval: $\zeta = 0.933$. The short run gain in terms of the contribution of BAU emissions is then:

$$\frac{Q_{t_0+1} - Q^{bau}}{\varepsilon E^{bau}} = (1 - 0.9333^6) = 0.34 \tag{10.8}$$

This short study of the environmental effects of the common consent policy sheds light on the possibilities to leave capital accumulation unchanged while still improving environmental quality in a non-marginal fashion.

11 Conclusion

Starting from the US Sky Trust claim that the "the sky belong to us equally", this paper distinguishes two sources through which overlapping generations may consent to pay for the environment whom they are the owners: the common consent of all generations reached behind the Rawlsian veil of ignorance and the specific consent of generation born at a given period. It proposes two institutions: a fund, which implements the common consent, is mandated to sell permits to firms and to redistribute the proceeds to the young households and a voting procedure, designed to implement the specific consent, fixes a tax on the young households to finance an investment in environmental maintenance.

The analysis shows how the specific consent may be operative or inoperative on the transition path and that there may be at most two regime switches. It highlights the interaction between the common consent and the specific consent.

Starting from the BAU steady state, the introduction of the institutions of the fund and the voting procedure always immediately increases the environmental quality but this gain may be temporary and partially vanishes if capital accumulation is strongly evicted by the policy. On the opposite, we stress a case in which the introduction of the policy has beneficial effects both on wealth and quality, simply because the permits, on the one hand, help reducing the source of pollution and, on the other hand, generate a revenue which is redistributed to savers and thus enhance capital accumulation.

12 References

1. Bréchet, Th., S. Lambrecht and F. Prieur (2005), "Intergenerational transfers of pollution rights and growth", CORE Discussion Paper n°42.

2. Dales, J.H. (1968), Pollution, Property and Prices, Toronto: University of Toronto Press.

3. John, A. and Pecchenino R. (1994). "An overlapping generations model of growth and the environment", *The Economic Journal*, vol. 104, pp. 1393-1410.

4. Jouvet, P.-A., Ph. Michel and G. Rotillon (2005) , "Optimal growth with pollution: How to use pollution permits?", *Journal of Economic Dynamics and Control*,vol. 29 (9) pp. 1597-1609.

5. Montgomery, D. (1972), "Markets in Licenses and Efficient Pollution Control Program", *Journal of Economic Theory*, 5: 395-418.

6. Ono, T. (2002), "The effects of emission permits on growth and the environment", *Environmental and Resource Economics* 21, 75-87.

7. Rawls, J. (1971), "A Theory of Justice", Cambridge, MA: Bellknap Press.

8. Solow, R., (1986), "On the intergenerational allocation of nature resources", *Scandinavian Journal of Economics*, vol. 88, pp. 141-144.

9. Stokey, N. L. (1998), "Are there limits to growth?", *International Economic Review*, 39: 1-31.

A Appendices

A.1 Proof of proposition 2.1

By assumption, the common consent level of emissions S must take any value below the BAU long run level E^{bau}. Second, in case the BAU economy is heading toward the collapse, the common consent level of emissions must also be smaller than the disaster threshold $\bar{E}$. Indeed, if the policy does not make it possible to avoid the long run collapse, then behind the veil of ignorance no generation would accept to be among the long run sacrificed generations. Finally, the common consent level S must also be the same for all generations because no generation would accept to suffer a more deteriorated environment than the others.

A.2 Proof of proposition 3.1

By multiplying both sides of $M_{t+1} = (1-\delta_M) M_t + \varepsilon E_t - \mu m_t$ by -1, and then adding $(1-\delta_M)\bar{Q}$ to both sides, one obtains:

$$(A.1) \qquad Q_{t+1} = (1-\delta_M) Q_t + \delta_M \bar{Q} - \varepsilon E_t + \mu m_t$$

To prove the existence of the common consent, we set $m_t = 0$ and $E_t = S$ and stationarize the above equation of environmental quality. This yields:

$$(A.2) \qquad Q = \overline{Q} - \frac{\varepsilon}{\delta_M} S$$

Steady quality Q is positive for values of S verifying:

$$S < \frac{\delta_M}{\varepsilon}\overline{Q} \equiv \bar{E} \tag{A.3}$$

The RHS of this inequality defines the maximum steady emission compatible with the economic survival, $\bar{E}$.

A.3 Proof of proposition 4.1

The existence follows from the positivity condition on Q^{bau}

$$Q^{bau} = \bar{Q} - (\varepsilon/\delta_M)(1-\alpha)^{\alpha/(1-\alpha)} \tilde{A}^{1/(1-\alpha)} > 0 \tag{A.4}$$

which is equivalent to the condition:

$$\tilde{A} < \left[\frac{\delta_M}{\varepsilon(1-\alpha)^{\alpha/(1-\alpha)}}\bar{Q}\right]^{1-\alpha} \tag{A.5}$$

A.4 Proof of proposition 6.1

This follows from the first order condition valued in equilibrium.

A.5 Proof of proposition 6.3

The equation of $k_t - k_{t+1}^p$ is straightforward. The effect of an increase in S on the equilibrium k_{t+1}^p is given by the derivative $\partial k_{t+1}^p/\partial S$. It is positive if and only if $\alpha_E A k_t^{\alpha_K} S^{\alpha_E - 1} \geq \varepsilon / [\mu(1-\alpha_K)]$.

A.6 Proof of proposition 7.1

If $\tilde{m}(k_t, Q_t, S) \geq 0$ (which is equivalent to $Q_{t+1} = \mu\gamma k_{t+1}$), then k_{t+1} as a function of k_t is inferior to its value when $\tilde{m}(k_t, Q_t, S) < 0$. Hence, the equilibrium k_{t+1} should be the minimum of ξ^p and ξ^z.

A.7 Proof of proposition 8.2

Uniqueness follows from the discussion preceding the proposition. To show stability, let us examine the function ξ^p:

$$\begin{aligned} k_{t+1}^p = (1+\gamma)^{-1}(1-\delta_M)\gamma k_t + (1+\gamma)^{-1}\mu^{-1}\left(\delta_M\bar{Q} - \varepsilon S\right) \\ + (1+\gamma)^{-1}(1-\alpha_K) A k_t^{\alpha_K} S^{\alpha_E} \end{aligned}$$

It is increasing and concave and has a positive intercept. As k_t tends to 0, the slope tends to $+\infty$ and as k_t tends to $+\infty$ the slope tends to $(1+\gamma)^{-1}(1-\delta_M)\gamma$, which is less than unity. Thus at the unique steady state the slope is less than unity ($dk_{t+1}/dk_t < 1$). Re-write the dynamics in terms of difference : $\Xi(k_{t+1}, k_t, S) \equiv k_{t+1} - \xi^p(k_t) = 0$. The derivative dk^p/dS reads:

$$dk^p/dS = -\Xi'_S / \left(\Xi'_{k_{t+1}} + \Xi'_{k_t}\right)$$

valued at $k_{t+1} = k_t = k^p$. At the steady state, we have $dk_{t+1}/dk_t = -\Xi'_{k_t}/\Xi'_{k_{t+1}} < 1$, with $\Xi'_{k_{t+1}} > 0$ and $\Xi'_{k_t} < 0$. Equivalently $-\Xi'_{k_t} < \Xi'_{k_{t+1}} \Leftrightarrow \Xi'_{k_{t+1}} + \Xi'_{k_t} > 0$. Thus the sign of dk^p/dS is the same as the sign of the numerator which is positive if and only if $S < \left(\varepsilon^{-1}\mu\alpha_E(1-\alpha_K)Ak_t^{\alpha_K}\right)^{1/(1-\alpha_E)}$.

A.8 Proof of proposition 9.1

The dynamics ξ^p lie above the dynamics ξ^z for low values of k_t, if $S \leq (\delta_M/\varepsilon)\bar{Q}$. Both ξ^p and ξ^z are strictly increasing and strictly concave.

(i) either $\xi^p > \xi^z$, $\forall k_t$, and there is no switch, maintenance is always constrained $\delta_M\bar{Q} - \varepsilon S > \gamma\mu\left[(1-\alpha_K)Ak_t^{\alpha_K}S^{\alpha_E} - (1-\delta_M)k_t\right]$, $\forall k_t$; this happens for instance when A is taken as small as possible; the slope of ξ^p remains strictly positive as $k_t \to +\infty$ while the slope of ξ^z tends to 0;

(ii) either $\xi^p = \xi^z$ for a unique value at which maintenance is equal to 0 but unconstrained (limit case);

(iii) either $\xi^p > \xi^z$, $\forall k_t \in (0, \kappa_l)$, $\xi^p \leq \xi^z$, $\forall k_t \in [\kappa_l, \kappa_h]$ and $\xi^p > \xi^z$, $\forall k_t \in (\kappa_h, +\infty)$, i.e. maintenance is constrained for $k_t \in (0, \kappa_l)$, unconstrained for $k_t \in [\kappa_l, \kappa_h]$ and constrained for $k_t \in (\kappa_h, +\infty)$, i.e. $\delta_M\bar{Q} - \varepsilon S \gtreqless \gamma\mu\left[(1-\alpha_K)Ak_t^{\alpha_K}S^{\alpha_E} - (1-\delta_M)k_t\right]$, respectively for $\forall k_t \in (0, \kappa_l)$, $\in [\kappa_l, \kappa_h]$and $\in (\kappa_h, +\infty)$; beyond the first crossing there necessarily is a second crossing since the slope of ξ^p remains strictly positive as $k_t \to +\infty$;

(iv) three or more regime switches are impossible because of the strict concavity of both ξ^p and ξ^z.

A.9 Proof of proposition 9.2

This follows from continuity of the function ξ^p and from the fact it is strictly increasing and strictly concave.

A.10 Proof of proposition 9.3

This follows from continuity, increasingness, strict concavity and slope tending to 0 as $k_t \to +\infty$.

A.11 Proof of proposition 10.1

Since time t_0 capital intensity is equal to k^{bau}, time t_0+1 capital intensity is given by $k_{t_0+1} = \min\left\{\xi^z\left(k^{bau}\right), \xi^p\left(k^{bau}\right)\right\}$. Hence $\min\left\{\xi^z\left(k^{bau}\right), \xi^p\left(k^{bau}\right)\right\} > k^{bau}$ is equivalent to $k_{t_0+1} > k^{bau}$. Given monotonicity of $\min\{\xi^z, \xi^p\}$ the condition $k_{t_0+1} > k^{bau}$ is equivalent to $k_{t_0+i+1} > k_{t_0+i}$ and the slope of these dynamics implies convergence to a higher steady state. According to the same reasoning, $\min\left\{\xi^z\left(k^{bau}\right), \xi^p\left(k^{bau}\right)\right\} < k^{bau} \Leftrightarrow k_{t_0+1} < k^{bau}$ implies a downward shift of capital intensity followed by a monotonic convergence to a lower steady state. As for environmental quality, whatever the level of maintenance and whatever the level of capital intensity, since $S < E^{bau}$ and $\max\left\{\tilde{m}\left(k^{bau}, Q^{bau}\right), 0\right\} \geq 0$, environmental quality at time t_0+1 verifies $Q_{t_0+1} = (1-\delta_M)\, Q_{bau} + \delta_M \bar{Q} - \varepsilon S + \mu \max\left\{\tilde{m}\left(k^{bau}, Q^{bau}\right), 0\right\} > (1-\delta_M)\, Q_{bau} + \delta_M \bar{Q} - \varepsilon E^{bau} = Q^{bau}$. As for subsequent periods

(i) In the case of enhanced capital accumulation, environmental quality always increases until its new steady state whatever the regime of maintenance. Indeed, if maintenance is constrained the dynamics are independent of capital intensity, monotonic and with slope constant and less than unity: thus $Q_{t_0+1} > Q^{bau}$ implies $Q_{t_0+i+1} > Q_{t_0+i}$. If maintenance is unconstrained, the dynamics of environmental quality follow the dynamics of capital $(Q_{t+1} = \mu\gamma k_{t+1})$ which implies monotonic convergence to a higher level;

(ii) In the case of discouraged accumulation, Q_t still converges to a higher steady state since $Q^{bau} = \bar{Q} - (\varepsilon/\delta_M)\, E^{bau} < \bar{Q} - (\varepsilon/\delta_M)\, S = Q^z$ and if maintenance is unconstrained in the long run it implies $Q^z < Q^p$ (equilibrium $Q = \max\{Q^p, Q^z\}$), which implies $Q^{bau} < Q^p$.

As for non-monotonic convergence, if after the introduction of the policy not only $\min\left\{\xi^z\left(k^{bau}\right), \xi^p\left(k^{bau}\right)\right\} < k^{bau}$ but also $\xi^p\left(k^{bau}\right) < \xi^z\left(k^{bau}\right)$, the environmental quality, after the initial jump $Q_{t_0+1} > Q^{bau}$, follows the decreasing trajectory of capital which implies non-monotonic convergence.

Environmental Economics & Management Memoranda

38. Paul-Marie BOULANGER and Thierry BRECHET. Models for policy-making in sustainable development: The state of the art and perspectives for research. November 2005.

37. Johan EYCKMANS an Henry TULKENS. Optimal and Stable International Climate Agreements. October 2005. Reprint from "*Economic Aspects of Climate Change Policy : A European and Belgian Perspective*", a joint product of CES-K.U.Leuven and CORE-UCL, edited by Bert Willems, Johan Eyckmans and Stef Proost, published by ACCO, 3000 Leuven (Belgium)

36. Thierry BRECHET and Benoît LUSSIS. The Clean Development Mechanism in Belgian Climate Policy. October 2005. Reprint from "*Economic Aspects of Climate Change Policy : A European and Belgian Perspective*", a joint product of CES-K.U.Leuven and CORE-UCL, edited by Bert Willems, Johan Eyckmans and Stef Proost, published by ACCO, 3000 Leuven (Belgium)

35. Vincent VAN STEENBERGHE. The impact of banki!ng on permits prices and compliance costs. October 2005. Reprint from "*Economic Aspects of Climate Change Policy : A European and Belgian Perspective*", a joint product of CES-K.U.Leuven and CORE-UCL, edited by Bert Willems, Johan Eyckmans and Stef Proost, published by ACCO, 3000 Leuven (Belgium)

34. Johan EYCKMANS, Denise VAN REGEMORTER and Vincent VAN STENBERGHE. Kyoto-permit prices and compliajce gosts: an analysis with MacGEM. October 2005. Reprint from "*Economic Aspects of Climate Change Policy : A European and Belgian Perspective*", a joint product of CES-K.U.Leuven and CORE-UCL, edited by Bert Willems, Johan Eyckmans and Stef Proost, published by ACCO, 3000 Leuven (Belgium)

33. Johan EYCKMANS, Bert WILLEMS and Jean-Pascal VAN YPERSELE. Climate Change: Challenges for the World. October 2005. Reprint from "*Economic Aspects of Climate Change Policy : A European and Belgian Perspective*", a joint product of CES-K.U.Leuven and CORE-UCL, edited by Bert Willems, Johan Eyckmans and Stef Proost, published by ACCO, 3000 Leuven (Belgium)

32. Marc GERMAIN, Stef PROOST and Bert SAVEYN. The Belgian Burden Sharing. October 2005. Reprint from "*Economic Aspects of Climate Change Policy : A European and Belgian Perspective*", a joint product of CES-K.U.Leuven and CORE-UCL, edited by Bert Willems, Johan Eyckmans and Stef Proost, published by ACCO, 3000 Leuven (Belgium)

31. Ingmar SCHUMACHER. Reviewing Social Discounting within Intergenerational Moral Intuition. June 2005.

30. Stephane LAMBRECHT. The effects of a demographic shock in an OLG economy with pay-as-you-go pensions and property rights on the environment: the case of selfish households. January 2005.

29. Stephane LAMBRECHT. Maintaining environmental quality for overlapping generations: Some Reflections on the US Sky Trust Initiative. May 2005.

28. Thierry BRECHET, Benoît LUSSIS. The contribution of the Clean Development Mechanism to national climate policies. April 2005.

27. Thierry BRECHET, Stéphane LAMBRECHT, Fabien PRIEUR. Intergenerational transfers of pollution rights and growth. May 2005.

26. Maryse LABRIET, Richard LOULOU. From non-cooperative CO_2 abatement strategies to the optimal world cooperation: Results from the integrated MARKAL model. April 2005.

25. Marc GERMAIN, Vincent VAN STEENBERGHE, Alphonse MAGNUS. Optimal Policy with Tradable and Bankable Pollution Permits : Taking the Market Microstructure into Account.*Journal of Public Economy Theory*, 6(5), 2004, 737-757.

24. Marc GERMAIN, Stefano LOVO, Vincent VAN STEENBEGHE. De l'impact de la microstructure d'un marché de permis de polluer sur la politique environnementale. *Annales d'Economie et de Statistique*, n° 74 – 2004, 177-208..

23. Marc GERMAIN, Alphonse MAGNUS, Vincent VAN STEENBERGHE. Should developing countries participate in the Clean Development Mechanism under the Kyoto Protocol ? The low-hanging fruits and baseline issues. December 2004.

22. Thierry BRECHET et Paul-Marie BOULANGER. Le Mécanisme pour un Développement Propre, ou comment faire d'une pierre deux coups. *Regards Economiques*, Ires n° 27, janvier 2005.

21. Sergio CURRARINI & Henry TULKENS. Stable international agreements on transfrontier pollution with ratification constraints. In C. Carrarro and V. Fragnelli (eds.), *Game Practice and the Environment*. Cheltenham, Edward Elgar Publishing, 2004, 9-36. (also available as CORE Reprint 1715).

20. Agustin PEREZ-BARAHONA & Benteng ZOU. A comparative study of energy saving technical progress in a vintage capital model. December 2004.

19. Agustin PEREZ-BARAHONA & Benteng ZOU. Energy saving technological progress in a vintage capital model. December 2004.

18. Matthieu GLACHANT. Voluntary agreements under endogenous legislative threats and imperfect enforcement. November 2004.

17. Thierry BRECHET, Stéphane LAMBRECHT. Puzzling over sustainability: an equilibrium analysis. November 2004.

16. Vincent VAN STEENBERGHE. Core-stable and equitable allocations of greenhouse gas emission permits.. October 2004. (also available as CORE DP 2004/75)

15. Pierre-André JOUVET Philippe MICHEL, Pierre PESTIEAU. Public and private environmental spending. A political economy approach. September 2004. (also available as CORE DP 2004/68.)

14. Thierry BRECHET, Marc GERMAIN, Vincent VAN STEENBERGHE. The clean development mechanism under the Kyoto protocol and the 'low-hanging fruits' issue. July 2004. (also available as CORE DP 2004/81).

13. Thierry BRECHET, Philippe MICHEL. Environmental performance and equilibrium. July 2004. (also available as CORE DP 2004/72).

12. Luisito BERTINELLI, Eric STROBL. The Environmental Kuznets Curve semi-parametrically revisited. July 2004. (also available as CORE DP 2004/51).

11. Axel GOSSERIES, Vincent VAN STEENBERGHE. Pourquoi des marchés de permis de polluer ? Les enjeux économiques et éthiques de Kyoto. April 2004. (also available as IRES discussion paper n° 2004-21).

10. Vincent VAN STEENBERGHE. CO_2 Abatement costs and permits price : Exploring the impact of banking and the role of future commitments. December 2003. (also available as CORE DP 2003/98).

9. Katheline SCHUBERT. Eléments sur l'actualisation et l'environnement. March 2004.

8. Marc GERMAIN. Modélisations de marchés de permis de pollution. July 2003.

7. Marc GERMAIN. Le Mécanisme de Développement Propre : Impacts du principe d'additionalité et du choix de la baseline. January 2003.

6. Thierry BRECHET et Marc GERMAIN. Les affres de la modélisation. May 2002.

5. Marc GERMAIN and Vincent VAN STEENBERGHE. Constraining equitable allocations of tradable CO_2 emission quotas by acceptability, *Environmental and Resource Economics*, (26) 3, 2003.

4. Marc GERMAIN, Philippe TOINT, Henry TULKENS and Aart DE ZEEUW. Transfers to sustain dynamic core-theoretic cooperation in international stock pollutant control, *Journal of Economic Dynamics & Control*, (28) 1, 2003.

3. Thierry BRECHET, Marc GERMAIN et Philippe MONTFORT. Spécialisation internationale et partage de la charge en matière de réduction de la pollution. (also available as IRES discussion paper n°2003-19).

2. Olivier GODARD. Le risque climatique planétaire et la question de l'équité internationale dans l'attribution de quotas d'émission échangeable. May 2003.

1. Thierry BRECHET. Entreprise et environnement : des défis complémentaires ? March 2002.

Environmental Economics & Management Memorandum

Chair Lhoist Berghmans in Environmental Economics and Management
Center for Operations Research & Econometrics (CORE)
Université catholique de Louvain (UCL)
Voie du Roman Pays 34
B-1348 Louvain-la-Neuve, Belgium

Hard copies are available upon request : env@core.ucl.ac.be
Papers are available in pdf format on line : www.core.ucl.ac.be/chlhoist