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Jakub GROWIEC & Ingmar SCHUMACHER

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ON TECHNICAL CHANGE IN THE ELASTICITIES OF RESOURCE INPUTS *

Jakub GROWIEC† Ingmar SCHUMACHER‡

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Abstract

This article considers a Ramsey–Hotelling economy whose production function takes a CES bundle of non-renewable and renewable resources as its input. We allow for exogenous technical change in the elasticity of substitution between these two types of resources as well as for technical change in the distribution parameter. We derive the sufficient conditions under which each type of technical change is able to deliver long-run growth even if initial conditions are unfavourable. Renewable resources are a potential backstop technology: they alone can produce output if the elasticity of substitution exceeds unity from some moment in time on. Short-run dynamics of the model are studied numerically. Our results provide new considerations for the debate on natural resources in production.

Keywords: Elasticity of substitution, Technical change, Biased technical change, Non-renewable resources, Renewable resources.
JEL Classification: Q20, Q30, O30.

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†CORE, Université Catholique de Louvain, 34 Voie du Roman Pays, 1348 Louvain-la-Neuve, Belgium; and Warsaw School of Economics, Warsaw, Poland. E-mail address: growiec@core.ucl.ac.be

‡Corresponding author. CORE, Université Catholique de Louvain, 34 Voie du Roman Pays, 1348 Louvain-la-Neuve, Belgium. E-mail address: schumacher@core.ucl.ac.be

1 Introduction

The sheer size of today's population coupled with its endless desires imposes a never known demand on the production of goods and services. The world's GDP has never been as high as now, and with the advent of China's and India's economic expansions, it seems clear that world production is bound to increase substantially. A noticeable problem is, however, that production depends on various inputs, some of which are known to be limited as well as non-renewable. This has raised concerns (also by major institutions such as RFF, World Bank) for the possibility of continuing production at today's levels to the further future. As pointed out in a special issue of the Review of Economic Studies already in 1974, the ability to extend our current opportunities to future generations when production utilizes limited non-renewable resources depends on a variety of factors, of which two have been singled out as the most crucial ones: substitutability and technical change.

Substitutability is vital because, as Dasgupta and Heal (1974) show, non-renewable resources are essential for production if other inputs are poor substitutes for them. For example, insulation can reduce the amount of oil necessary for heating, but cannot be a perfect substitute for oil. Empirical evidence for low substitution possibilities is provided by Cleveland and Ruth (1997). If this is actually the case, this strand of research predicts a bleak future for generations to come.

Technical change, either exogenous as in Solow (1956) or endogenous as in Romer (1990), is another factor vital for further understanding of production processes. As an example, one could think of cars which are developed such that they can do the same mileage with less and less petrol. Equipped with these new growth-theoretic tools, resource economists (e.g. Scholz and Ziemes, 1999; Schou, 2000; Bretschger, 2005; Grimaud and Rouge, 2005) return with the hope that, even under initially unfavorable circumstances, production possibilities would not decline due to human ingenuity. These theories teach us that the general requirement for non-declining production is a fast enough technical change. However, it is again Cleveland and Ruth (1997, p. 217) who forcefully argue that "...technology and substitution have not been sufficiently strong to offset the effects of depletion at the macroeconomic scale in some nations."

Given the extensive theoretical research on a time-invariant elasticity of substitution as well as on factor-neutral technical change on the one side, but the rather bleak outlook from the empirical literature on the other side, we shall consider a different way of investigating the potential for sustainability. This we will do in the following way.

Unlike the classic works (Solow, 1974; Stiglitz, 1974; Dasgupta and Heal, 1974) which mainly analyze the relationship between capital and non-renewable resources, this article concentrates on the relationship between non-renewable and renewable resources in production. We use a production function of the constant elasticity of substitution (CES) type to allow for various degrees of substitutability between these two inputs. Thus, our paper is closely related to recent articles by André and Cerdá (2005) as well as Grimaud and Rouge (2005). Our analysis, though, unlike these contributions, allows for technical change in the actual elasticity of substitution (EoS hereafter). This permits us to gain new insights into short-run and long-run dynamics. To our knowledge, this approach is novel to the literature.[1] In addition, we will analyze biased technical change in the partial elasticities.

The idea of technical change in the EoS does not just come out of nowhere. Demands to analyze the effect of technical change on the elasticity of substitution have been raised time and again during the past 70 years, with stronger and clearer requests. Already Hicks, who invented the concept of the EoS in the 30's, points out that the elasticity of substitution might change because "(...) methods of production already known, but which did not pay previously, may come into use." Even more precise, he suggests that the increase in substitutability "(...) partly takes place by affording a stimulus to the invention of new types." (1932, p. 120) De La Grandville then proposes to think about the EoS as "a measure of the efficiency of the productive system" (1989, p. 479), which is something that is going to become clear during the following sections. Yuhn, who tests de La Grandville's proposition, suggests to view the elasticity of substitution as "a 'menu of choice' available to entrepreneurs." (1991, p. 344) All of these researchers believe the EoS to be a decisive component, if not a determinant, of growth. They furthermore suggest that the elasticity is by no means invariable over time. That there might exist technical innovations driving changes in the elasticity of substitution has recently been put forward by Klump and Preissler (2000, p. 52) who suggest that "[a]s far as invention of new methods of production is concerned, the elasticity of substitution as a measure of economic progress can, of course, be related to a society's capability to create and maintain a high rate of innovative activities." Finally, and most precisely, Bretschger (2005, p. 150) suggests that "all possibilities of substitution and, specifically, the effects technology exerts on promoting substitution have to be studied." Among others, these points quoted here provide a foundation for introducing technical change into the EoS.

In the subsequent section, we introduce technical change in the distribution parameters (and thus, unit productivities) of the renewable and non-renewable resources. We show how this can be linked to the literature on biased technical change (e.g. Acemoglu, 2003).

[1]There are papers which consider an EoS which changes over time, but they do not refer to non-renewable and renewable resources and also, the mechanisms therein do not have the "technological progress" flavour. These contributions are Miyagiwa and Papageorgiou (2005) – with a changing EoS between capital and labour, and Petith (2001) – with (broadly defined) land and labour.

We assume that both resource inputs are subject to technical change and therefore increase their productivity, but one of the resources is subject to quicker technical change than the other resource. This allows us to compare our results to those present in the literature, notably in the papers which adapt Acemoglu's (2003) framework to renewable and non-renewable resources (e.g. Grimaud and Rouge, 2005) and in other related contributions (e.g. Amigues *et al.*, 2006). To provide some slightly more intuitive perspective, if we take the Cobb-Douglas function as an example, then our extension permits to analyze the effect of changing Cobb-Douglas shares.

There are several reasons for which we focus on exogenous technical change here. Firstly, before trying to investigate under which circumstances a policy maker should invest more in the one type of technical change or the other, one has first to understand the generic dynamic effects of these changes. As we shall show, in many cases these generic effects are far from obvious. Secondly, the exogenous technical change approach enables one to implicitly recover the value a policy maker would attach to the increases in the EoS and other technological parameters. This helps predict how much should be invested in e.g. R&D aimed at increasing the EoS in more sophisticated environments. For these reasons we believe that our simplified approach is not only justifiable, but also sufficient as a good first step.

The article is structured as follows. Section 2 lays out the simple Ramsey–Hotelling framework which we use for studying various types of technical change. Section 3 presents the benchmark case of no technical change. In section 4 we introduce an increasing elasticity of substitution. In section 5 we compare these results to the results obtained when biased technological change is allowed for. Section 6 discusses the way in which the renewable resource works as a "potential backstop technology" in the model. Section 7 concludes.

2 The model

This section introduces the implications of a changing elasticity of substitution as well as technical change in the partial elasticities of a CES bundle consisting of a non-renewable and a renewable resource.

2.1 Technical change in the CES function

Throughout the analysis, we shall be using the standard constant-returns-to-scale CES production function, as derived in the seminal article by Arrow *et al.* (1961). We shall allow for technical change in its EoS, or alternatively, in its distribution parameter. The intermediate resource input R is produced with flows of non-renewable resource R_N and renewable resource R_R. They are combined according to:

$$R(t) = [\psi(t) R_N(t)^{\theta(t)} + (1-\psi(t)) R_R^{\theta(t)}]^{\frac{1}{\theta(t)}}, \tag{1}$$

where the distribution factor is given by $\psi(t) \in [0,1]$, and the elasticity of substitution, $\sigma(t) \in (0, +\infty)$, is related to the elasticity parameter $\theta(t)$ via

$$\theta(t) = \frac{\sigma(t) - 1}{\sigma(t)}. \tag{2}$$

The EoS $\sigma(t)$ and the distribution parameter $\psi(t) \in [0,1]$ are allowed to change over time.

The non-renewable resource stock $0 < S_N(0) < \infty$ is extracted according to $\dot{S}_N(t) = -R(t) \leq 0$.[2]

For the renewable resource R_R, we assume that it arrives in *constant flows* over time, $R_R \equiv$ const. This is a strong assumption which helps simplify the subsequent analysis. Yet, it is excusable: Dasgupta and Heal (1974) consider the renewable resource to be "a perfectly durable commodity which provides a flow of services at [a] constant rate" (Dasgupta and Heal, 1974, p. 19). Moreover, the renewable resource is a potential backstop technology: it becomes a backstop technology if the economy succeeds in shifting σ above unity.

Taking the growth rate of R and dropping time subscripts yields

$$\hat{R} = \epsilon_N \hat{R}_N + \epsilon_R \hat{R}_R + \epsilon_\sigma \hat{\sigma} + \epsilon_\psi \hat{\psi}, \tag{3}$$

where the partial elasticities, $\epsilon_i = \frac{\partial R}{\partial i}\frac{i}{R}$, for $i = R_N, R_R, \sigma, \psi$, are given by

$$\begin{aligned}
\epsilon_N &= \frac{\psi R_N^\theta}{R^\theta}, \\
\epsilon_R &= \frac{(1-\psi) R_R^\theta}{R^\theta}, \\
\epsilon_\sigma &= \frac{\psi R_N^\theta \ln R_N + (1-\psi) R_R^\theta \ln R_R - R^\theta \ln R}{(\sigma - 1) R^\theta}, \\
\epsilon_\psi &= \frac{\psi (R_N^\theta - R_R^\theta)}{\theta R^\theta}.
\end{aligned}$$

Both $\epsilon_N \in [0,1]$ and $\epsilon_R \in [0,1]$ function as shares in the traditional sense. This is due to the assumption of constant returns to scale in the CES function. $\epsilon_\sigma > 0$ implies that

[2] Throughout the article we shall use $\dot{B}(t)$ to denote the time derivative of $B(t)$ and $\hat{B}(t)$ to denote its growth rate. For any function $G(B)$, $G'(B)$ denotes its first derivative, and $G''(B)$ its second derivative with respect to B.

improvements in the EoS always increase output. We also have that $\epsilon_\psi > 0$ if $R_N > R_R$ ($\epsilon_\psi < 0$ if $R_N < R_R$) which suggests that technical change in the distribution factor should always go in the direction of the relatively abundant resource input.

This result is complementary to the one obtained by André and Cerdá (2005): optimizing subject to both resource inputs allows them to find a corner solution where it might be useful to delay the exploitation of the renewable resource in order to allow for its stock to grow towards the maximum sustainable yield. Such result is not possible here, as we concentrate on the case with constant renewable resource flows. However, we are more precise in defining under which circumstances it is more profitable (in terms of total output) to increase the relative productivity of a given type of resource: it should be relatively abundant.

2.1.1 CES and changing elasticity of substitution

The EoS σ gives the percentage change in relative quantities of used resources given a one percent change in their relative prices. If $\sigma = 0$ then the function is Leontief so the inputs are perfect complements, for $\sigma = 1$ we obtain the standard Cobb-Douglas form (as a limiting case), and if $\sigma = +\infty$ then the function is linear so the inputs are perfect substitutes. The EoS therefore gives information on the ease with which one can move along a given isoquant, and in that way it can either be understood as a measure of flexibility, efficiency (de La Grandville, 1989), or 'menu of choice' (Yuhn, 1991). We shall view the EoS as a measure of technical efficiency.

As can be seen in Figure 1, the higher the EoS, the larger the amount of output R for *any* given inputs R_N and R_R. The intermediate resource input production is an increasing function of the EoS ($\partial R/\partial\sigma > 0$), and follows a convex-concave shape.

The EoS pulls double duty: on the one hand it is a measure of efficiency, and on the other it determines the essentiality of the inputs. If $\sigma \leq 1$, then $R \to 0$ if any of the two inputs goes to zero. On the contrary, if $\sigma > 1$, neither of the inputs is essential any longer.

We shall assume exogenous technical change in the EoS, as summarized by the following reference formula:

$$\sigma(t) = \sigma(0)e^{st}, \qquad \sigma(0) > 0, \tag{4}$$

which implies that the growth rate of σ is constant. Moreover, whatever the initial relationship between our inputs, they will become perfect substitutes in the limit. This assumption simplifies the subsequent analysis a lot but is admittedly strong, and in fact not necessary for at least one crucial qualitative results: If we take the EoS as a "dummy" for the essentiality of inputs then we only require σ to exceed unity from some point in time on. But, as is obvious from Figure 1, the consecutive efficiency gains from improve-

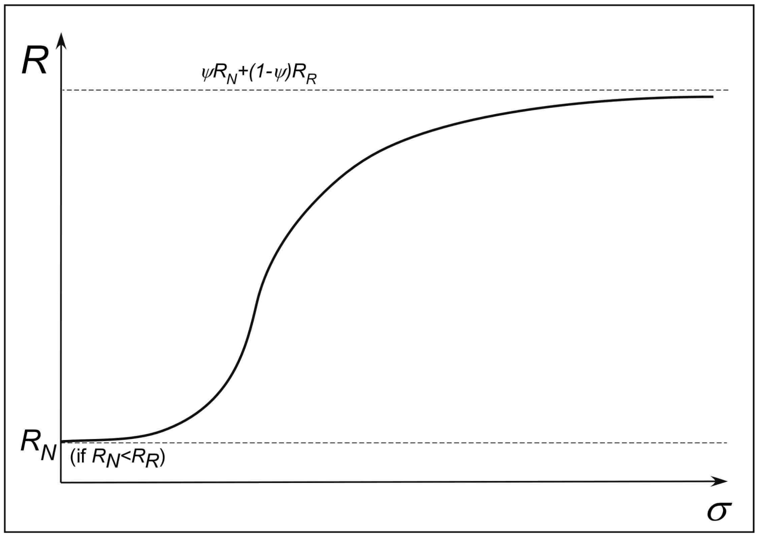

Figure 1: R as a function of σ

ments in the EoS should not be neglected and, as we shall show, provide new qualitative insights for the short-run dynamics. Finally, allowing for the EoS to approach infinity allows our article to serve as a bridge to Tahvonen and Salo (2001) who assume perfect substitutability between non-renewable and renewable resources.[3]

2.1.2 CES and biased technical change

As an alternative to a changing elasticity of substitution, we shall introduce technical change in the distribution parameter ψ which proxies the bias in factor-augmenting technical change.

The standard literature focuses mainly on efficiency improvements in the usage of the non-renewable resource (e.g. Scholz and Ziemes, 1999, Amigues et al., 2006). We are, however, going to focus on faster improvements in the usage of the renewable resource. Following this idea, we shall assume that technical change affects the relative share of the two inputs by changing the distribution factor as follows:

$$\psi(t) = \psi(0)e^{zt}, \qquad \psi(t) \in [0,1], \quad \psi(0) \in (0,1), \tag{5}$$

[3]We have omitted the important issues of normalization here (cf. de La Grandville, 1989, Klump and Preissler, 2000). These issues have been neglected in most prior literature but are clearly important if one does not want to do inter-family comparisons of CES functions. A proof that our approach is in fact consistent with normalization is available from the authors upon request.

where the growth rate of ψ is a constant $z \leq 0$.

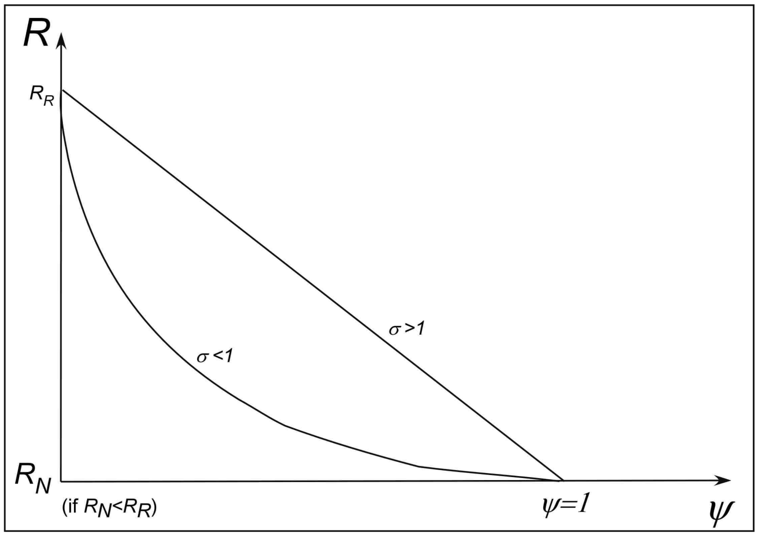

Figure 2: R as a function of ψ

The effect of changing ψ on R is

$$\frac{\partial R}{\partial \psi} = \frac{1}{\theta}\left(R_N^\theta - R_R^\theta\right)\left[\psi R_N^\theta + (1-\psi)R_R^\theta\right]^{\frac{1-\theta}{\theta}}, \tag{6}$$

implying that $\partial R/\partial\psi > 0$ if $R_N > R_R$, and $\partial R/\partial\psi < 0$ if $R_N < R_R$. Figure 2 illustrates the effect of changing ψ for the case of $R_N < R_R$ and two different values of σ. The lower the possibility to substitute the stronger the initial effect of changes in ψ. In the extreme, for Leontief inputs, a marginal change from $\psi = 0$ to $\psi > 0$ will result in a reduction in the intermediate output from R_R to R_N (given $R_N < R_R$). Biased technical change can thus be viewed as varying the relative productivity of R_R and R_N. The marginal rate of technical substitution is given by $MRTS = \frac{\psi}{1-\psi}\left(\frac{R_N}{R_R}\right)^{-1/\sigma}$: changes in ψ affect the slope of the isoquants of R leaving their curvature intact.

2.2 The Ramsey–Hotelling framework

We shall now embed our intermediate resource input R with technical change in σ or ψ in a simple Ramsey-type model with an infinite planning horizon, where a representative agent maximizes discounted utility subject to an equation of motion of the non-renewable resource stock.

Formally, this means that our infinitely-lived representative agent maximizes

$$\begin{aligned}
\max_{\{R_N(t)\}_{t=0}^{\infty}} & \int_0^{\infty} U(Y(t))e^{-\rho t}dt, \qquad \text{subject to:} \\
\dot{S}_N(t) &= -R_N(t), \\
R_R &\equiv \text{const}, \\
Y(t) &= F(A(t), L, R_N(t), R_R) = A(t)L^{1-\beta}R(t)^{\beta}, \\
R(t) &= [\psi(t)R_N(t)^{\theta(t)} + (1-\psi(t))R_R^{\theta(t)}]^{\frac{1}{\theta(t)}}.
\end{aligned}$$

At this point we assume that $U(Y) = Y^{1-\gamma}/(1-\gamma)$, where $\gamma = -YU''(Y)/U'(Y) \in (0,\infty)$[4] is the inverse of the intertemporal elasticity of substitution. We abstract from population growth and normalize $L \equiv 1$.

Technical change is decomposed into three components: factor-neutral technical change $A(t) = A(0)e^{gt}$ with $g \geq 0$, technical change in the EoS, $\sigma(t) = \sigma(0)e^{st}$ with $s \geq 0$, and the bias in technical change due to $\psi(t) = \psi(0)e^{zt}$ with $z \leq 0$. On top of this, we assume that either $s = 0$ with $z < 0$ or $z = 0$ with $s > 0$ for the sakes of normalization and the transparency of results.

Since no savings decision is allowed for, all output is immediately consumed and thus $C(t) = Y(t)$ for all t.

The simple, "bare-bones" model which we use features a number of strong simplifying assumptions, such as constancy of the renewable resource flow over time, exogenous technical change, and the neglect of capital accumulation. On the other hand, we agree to pay such a price, because in return we get a clear understanding of the dynamics in all, even the most complex cases. This makes our article a good starting point for further analyses: relaxing several of the assumptions would provide interesting questions for future research. In particular, we point out that our results should be verified in an optimal growth framework which fully endogenizes research in both resource inputs. Also the mechanisms behind technical change in our basic model should be provided with a formal treatment.

The first-order conditions give us (see Appendix A.1 for more details):

$$\left(\widehat{\frac{\partial U(Y)}{\partial R_N}}\right) = \rho. \tag{7}$$

[4] $\gamma > 1$ implies that utility is bounded from above. $\gamma \geq 1$ implies that the utility of zero consumption is minus infinity, enforcing positive consumption at all times. If $\gamma < 1$ then zero consumption at some (later) t is not automatically ruled out.

This is the Ramsey-Hotelling condition that characterizes the interior solution of the optimization problem. Rewriting, we obtain that

$$\hat{Y} = \frac{\hat{F}_N - \rho}{\gamma}, \tag{8}$$

with $F_N = \partial F / \partial R_N$, which states that the growth rate of income, $\hat{Y}$, is positive if the growth rate of the marginal product of the non-renewable resource, $\hat{F}_N$, exceeds the discount rate.

In its core, this result is similar to the outcome of the "cake-eating" model (see e.g. Dasgupta and Heal, 1974). In the cake-eating model, the marginal product of the non-renewable resource is a function of two variables only: the rate of depletion and the rate of factor-neutral technical change. Hence, without large enough factor-neutral technical change, the growth rate of the marginal product of the non-renewable resource will be negative for all times and income will decline continuously. Analogously, income growth can be positive in the Ramsey–Hotelling model only if technical change is fast enough to keep the growth rate of the marginal product of the non-renewable resource above the discount rate. However, as we decompose technical change into three different kinds, we obtain more possibilities to achieve sustainable production. The way in which the model's results are changed due to the introduction of technical change in the elasticity of substitution and biased technical change through the distribution parameter will become clear in the subsequent sections.

Solving equation (8) for $\hat{R}_N$, we obtain the optimal growth rate of the non-renewable resource extraction in terms of other variables:

$$\hat{R}_N = \frac{(1-\gamma)g - \rho + [((1-\gamma)\beta - \theta)\epsilon_\sigma - \frac{1}{\sigma}(\ln R - \ln R_N)]s + [((1-\gamma)\beta - \theta)\epsilon_\psi + 1]z}{(1-\theta) - ((1-\gamma)\beta - \theta)\epsilon_N}. \tag{9}$$

As is proved in Appendix A.2, the denominator of the above expression is always positive. We also notice that all three kinds of technical change affect the dynamic path of the optimal resource extraction rate R_N.

2.3 Comparative statics

We shall now turn to the comparative statics which provide further insights into the impact of certain parameters on the growth rate of income. Along the optimal growth path we obtain the following comparative statics:

$$
\begin{aligned}
\frac{\partial \hat{Y}}{\partial \rho} &= -\frac{\beta \epsilon_N}{(1-\theta)-((1-\gamma)\beta-\theta)\epsilon_N} < 0, &(10)\\
\frac{\partial \hat{Y}}{\partial g} &= \frac{(1-\theta)+\theta\epsilon_N}{(1-\theta)-((1-\gamma)\beta-\theta)\epsilon_N} > 0, &(11)\\
\frac{\partial \hat{Y}}{\partial s} &= \frac{\beta(1-\theta)[\epsilon_\sigma-\epsilon_N(\ln R-\ln R_N)]}{(1-\theta)-((1-\gamma)\beta-\theta)\epsilon_N}, &(12)\\
\frac{\partial \hat{Y}}{\partial z} &= \frac{\beta[\epsilon_N+\epsilon_\psi(1-\theta)]}{(1-\theta)-((1-\gamma)\beta-\theta)\epsilon_N}. &(13)
\end{aligned}
$$

Along the optimal path, the discount rate affects the growth rate of income always negatively (Eq. (10)). Clearly, the less we care about the future the more resources we use up now. This brings about a larger current level of income but less growth potential since less resources are left for use later on.

The growth rate of income is unambiguously positively related to the rate of factor-neutral technical change g (Eq. (11)). This result also carries forward from the standard literature.

We have already shown that changes in the distribution factor should always go towards the kind of resource which is more abundant and thus more important for production. Equation (13) confirms it by saying that increases in z have a positive effect of income growth if $\epsilon_\psi > -\frac{\epsilon_N}{1-\theta}$. Hence, a sufficient condition for a positive effect of z on income growth is $\epsilon_\psi > 0$, i.e. if the more abundant resource is subject to relatively faster technical change.

The effect of s on income growth depends on whether more renewable or more non-renewable inputs are used in production. If more non-renewable inputs are used then the effect of s on income growth is unambiguously positive. If more renewable resources are used, then the effect of s on income growth depends on the importance of the non-renewable resource for production relative to the partial elasticity of σ and cannot be unambiguously signed. This observation seems then crucial for the transition period. This gives us another reason for not only looking at a fixed EoS, but especially points at the significance of looking at a changing EoS in the short-run.

3 The benchmark case

3.1 The long run

The properties of the benchmark model with no technical change in σ ($s = 0$) and ψ ($z = 0$) are similar to the ones established in previous literature (see e.g. Dasgupta and Heal, 1974). We distinguish three important sub-cases here: $\sigma > 1$, $\sigma = 1$, and $\sigma < 1$.

Case $\sigma > 1$. If non-renewable and renewable resources are gross substitutes, then the following asymptotic results hold:

$$\begin{aligned} Y = F(A, L, R_N, R_R)|_{R_N=0} &> 0 \\ \lim_{t\to\infty} \epsilon_N = 0 \qquad & \lim_{t\to\infty} \epsilon_R = 1 \\ \Rightarrow \qquad & \lim_{t\to\infty} \hat{Y} = g. \end{aligned}$$

As the non-renewable resource is not essential for production, it becomes a virtually negligible input when it approaches zero. Hence, income growth can stay positive forever if only there is factor-neutral technical change, $g > 0$.

Case $\sigma = 1$. In the Cobb-Douglas case the non-renewable resource is essential for production and

$$\begin{aligned} Y = F(A, L, R_N, R_R)|_{R_N=0} &= 0 \\ \epsilon_N = \psi \qquad \epsilon_R &= 1 - \psi \\ \lim_{t\to\infty} \hat{R}_N &= \frac{(1-\gamma)g - \rho}{1 - (1-\gamma)\beta\psi} \\ \Rightarrow \qquad \lim_{t\to\infty} \hat{Y} = \frac{g}{1 - (1-\gamma)\beta\psi} &- \frac{\beta\psi\rho}{1 - (1-\gamma)\beta\psi}. \end{aligned}$$

We see that in this case, the gradual depletion of the non-renewable resource pulls down the long-run growth rate of the economy. This is especially apparent when one compares this case to the $\sigma > 1$ case.

Case $\sigma < 1$. If the two types of resources are gross complements, we have that

$$\begin{aligned} Y = F(A, L, R_N, R_R)|_{R_N=0} &= 0 \\ \lim_{t\to\infty} \epsilon_N = 1 \qquad \lim_{t\to\infty} \epsilon_R &= 0 \\ \lim_{t\to\infty} \hat{R}_N &= \frac{(1-\gamma)g - \rho}{1-(1-\gamma)\beta} \\ \Rightarrow \quad \lim_{t\to\infty} \hat{Y} = \frac{g}{1-(1-\gamma)\beta} &- \frac{\beta\rho}{1-(1-\gamma)\beta}. \end{aligned}$$

The partial elasticity of the non-renewable resource tends to one because R_N,as it approaches zero, drives the output if it is a gross complement to R_R. In this case the rate of resource depletion provides an even stronger drag on per capita income than in the Cobb-Douglas case.

The important transversality condition, guaranteeing finiteness of the objective integral, is for all three cases $(1-\gamma)g < \rho$. It is automatically satisfied if $\gamma > 1$ and $g \geq 0$.

As far as long-run results are concerned, the benchmark case confirms the decisive importance of the EoS in the production process, a result that carries forward from the classical literature.

The more patient the representative agent, i.e. the smaller is ρ, the more important is the future for her and therefore the larger the growth rate of resource extraction. If we assume away the possibility of depletion in finite time, then this implies that initially smaller amounts of resources will be utilized.

If zero consumption is not penalized by infinite disutility (which is the case if $\gamma < 1$), finite-time depletion of the non-renewable resource is possible irrespectively of whether the resource inputs are gross substitutes or gross complements. This case is quite counterintuitive and empirically implausible. If $\gamma \geq 1$, on the other hand, then finite-time depletion will be the case only if $\sigma > 1$.

3.2 The short run

We study the optimal path of income in the benchmark case using numerical simulations. The results are presented in Figure 3.[5]

[5]The simulation procedure is described in the Appendix A.3. Unless stated otherwise, we use the following parameter choices for all simulations: $\beta = 1/3$, $\gamma = 2$, $R_R = 2$, $\rho = 0.05$, $\sigma(0) = 0.1$, $\psi(0) = 0.9$, $S(0) = 300$. Some of these parameters can be easily verified (γ, β, ρ), others have been chosen freely.

< Figure 3 here >

In the case $\sigma > 1$ or $\gamma < 1$, there is finite-time depletion of R_N but it happens late enough not to affect the short-run results substantially.

The interpretation of the above result is as follows. The discounted utilitarian criterion regards future consumption as less important than current consumption, wherefore most non-renewable resources will be extracted when utility seems most valuable. The simulations show that in case the non-renewable resource is not essential for production ($\sigma = 2$), the share of R_N in production is initially very large, suggesting that most non-renewable resources are used initially, and then declines to zero over time. Thus the level of GDP is initially larger for the case of high EoS in comparison to the case of $\sigma \leq 1$, as technical efficiency does not constrain the intermediate input R by complementarity problems. The GDP growth rate is initially higher in the case of complementarity as at first more non-renewable resources are conserved as production is depending on those resources later. Moreover, the amount of the non-renewable resource used in production is divided more equally over time the lower is σ, due to its essentiality for production.[6] Nevertheless though, if the non-renewable resource and the renewable one are gross complements in production, the decrease in the amount of non-renewable resources available decreases the overall resource bundle that can finally be used in production. In such case, the non-renewable resource is driving the size of the resource bundle R, implying that the share of the non-renewable resource in the resource bundle will go to one. Hence, the (negative) GDP growth rate will become smaller and smaller. In contrast to this, if the non-renewable resource is inessential ($\sigma > 1$) then, as less and less of it remains available, its share in the resource bundle will drop to zero. In this sense, the non-renewable resource allows greater GDP levels as long as it is available, but as soon as it gets depleted, GDP goes to the level that it would have had, had it been produced without the non-renewable resource.

3.3 The possibility of finite-time depletion

If the non-renewable resource is not essential for production ($\sigma > 1$), it will be depleted in finite time. Formally, one could write the finite-time depletion problem as a problem

[6] For the case $\sigma = 2$, R_N gets depleted around $T^* = 160$ which is not visible to the naked eye.

of choosing an optimal T in:

$$
\begin{aligned}
\max_{\{R_N(t)\}_{t=0}^T, T} \quad \mathcal{U}_0 &= \int_0^T U(Y(t))e^{-\rho t}dt + \int_T^\infty U(\bar{Y}(t))e^{-\rho t}dt, \qquad \text{subject to:} \\
\dot{S}_N(t) &= -R_N(t), \\
R_R &\equiv \text{const}, \\
Y(t) &= F(A(t), L, R_N(t), R_R) = A(t)L^{1-\beta}R(t)^\beta, \\
R(t) &= [\psi(t)R_N(t)^{\theta(t)} + (1-\psi(t))R_R^{\theta(t)}]^{\frac{1}{\theta(t)}}, \\
S_N(T) &= 0,
\end{aligned}
$$

where $\theta > 0$, and $\bar{Y}$ is the exogenously given amount of output produced with renewable resources only: $\bar{Y}(t) = (1-\psi)^{\beta/\theta} R_R^\beta A(0)e^{gt}$.

The problem is solved in two steps: first, one chooses the optimal path of non-renewable resource extraction given T, and then one chooses T knowing these "response functions" such that the sum of both integrals, $\mathcal{U}_0$, is maximized. The first step gives the already discussed condition (9) subject to a boundary condition $S_N(T) = 0$. Because the dynamic equation (9) is not solvable, we shall again resort to numerical simulations here to approximate the optimal depletion time T^*. The results of the numerical exercise, which takes $\gamma = 2$ and $g = 0$, are summarized in Table 1. In particular, for the case $\sigma > 10$, the optimal depletion time is $T^* = 110$ as presented in Figure 4.

σ	1	1.1	1.5	2	10	40	100	$+\infty$
T^*	$+\infty$	>300	180	160	110	≈ 100	≈ 100	0

Table 1: Optimal exhaustion time T^* as a function of σ

< Figure 4 here >

4 Increasing flexibility

4.1 The long run

We shall now analyze the effect of allowing for technical change in the EoS σ, but not in the distribution parameter ψ, such that $s > 0$, $z = 0$. The long-run asymptotics are straightforward:

$$Y = F(A, L, R_N, R_R)|_{R_N=0} > 0$$
$$\lim_{t\to\infty} \sigma = \infty \Rightarrow \lim_{t\to\infty} \theta = 1$$
$$\lim_{t\to\infty} \epsilon_N = 0 \qquad \lim_{t\to\infty} \epsilon_R = 1 \qquad \lim_{t\to\infty} \epsilon_\sigma = 0$$
$$\Rightarrow \quad \lim_{t\to\infty} \hat{Y} = g$$

Furthermore, there is no need to assume unbounded growth in the EoS. We do this for simplicity here but we would have obtained the same asymptotic growth rate of income if we had assumed that σ grew until some time t_0, after which $\sigma(t) > 1$, i.e. non-renewable resources were not essential for production any more. Hence, these asymptotic properties hold as long as the EoS manages to cross the "magical" barrier of one.

The required transversality condition is again $(1-\gamma)g < \rho$ which is automatically satisfied if $\gamma > 1$ and $g \geq 0$.

4.2 The short run

In the case of increasing EoS, we know that from a certain time $t_0 \geq 0$ onwards, given the EoS improved to exceed one, the resource inputs will be gross substitutes. Thus, the non-renewable resource will necessarily be depleted in finite time. This result is intuitive because perfect substitutability between non-renewable and renewable resources allows us to utilize the non-renewable resource without compromising the productivity of the renewable one in any way.

The results of the simulative exercise are presented in Figure 5. Vertical dashed lines indicate the moment in which the elasticity of substitution crosses one.

< Figure 5 here >

As the elasticity of substitution increases it is at first optimal to utilize non-renewable resources at an increasing rate. This is because initially, the improvements in technical efficiency allow for large increases in the intermediate input R.[7] Hence, this allows for an increasing growth rate of income. However, these initial exponential-like increases in R, due to the increasing EoS, level off quickly, such that consecutive improvements in σ increase the overall resource bundle only slightly. It is now optimal to slow down

[7]The initial level of $\sigma(t)$ is set at $\sigma(0) = 0.1$, which corresponds to the strongly increasing part of Figure 1.

the extraction of the non-renewable resource, as it is still essential for production. At this time, the model is rather similar to the benchmark case with an essential resource and produces a negative growth rate of income. At one point in time, the technical efficiency will have improved substantially enough, so that the non-renewable resource will not be essential for production any more. The model then traces the benchmark case again, but with a non-essential non-renewable resource. However, continued improvements in the technical efficiency parameter result in further improvements in the intermediate good production process. Therefore, the economic growth rate can be positive. As the consecutive improvements in efficiency increase the intermediate resource by less and less, and as the non-renewable resource stock is depleted in some finite time after σ crosses unity ($T^* > t_0$; in the simulated case, we obtain $T^* = 170$),[8] the growth rate of income tends to zero from above.

Generally speaking, one can notice that the changing EoS pulls double-duty. On the one hand, it reflects technical efficiency, and on the other hand, it reflects the essentiality of the resource. We find that improvements in technical efficiency through increases in the EoS are more relevant for the short-run, whereas essentiality determines long-run behaviour.

4.3 Finite-time depletion

As a consequence of the fact that R_N becomes eventually inessential, we obtain that finite-time depletion is certain. Numerical computations of the optimal depletion time T^* have been summarized in Table 2. We emphasize that all optimal depletion times are strictly (and significantly) greater than the time in which σ crosses unity, which is in our case $t_0 = 115$.

s	0	0.005	0.01	0.02	0.03	0.04
T^*	$+\infty$	>300	260	170	≈ 160	≈ 160

Table 2: Optimal exhaustion time T^* as a function of s

[8]Again, this fact is not visible to the naked eye in the figure. R_N approaches zero smoothly and approaches the vicinity of zero much earlier than it actually takes the zero value.

5 Biased technical change

5.1 The long run

We shall now compare the case of technical change in the EoS with the biased technical change case. We assume $s = 0$, $z < 0$, and $g \geq 0$ which implies that technical change improves the efficiency of the renewable resource more quickly than it improves the efficiency of the non-renewable resource. A priori, it may seem counterintuitive to suggest this as a way out of the long-run constraint imposed by the essentiality of a non-renewable resource. However, having Cleveland and Ruth's (1997) observation in mind, namely that technical change has not been sufficiently strong to offset the effects of depletion, we believe that looking at technical change from a different angle, through its effect on the distribution parameters, deserves some scrutinization. We shall show and explain why and when this kind of biased technical change is useful.

We shall be dealing with three cases: $\sigma > 1$, $\sigma = 1$, and $\sigma < 1$, where the last case is further divided into three sub-cases of fast, medium, and slow biased technical change, delineated by the expression $z - \theta(\rho - (1-\gamma)g)$ being negative, zero, and positive, respectively.

Case $\sigma > 1$. If the resource inputs are gross substitutes, then the following asymptotic properties hold:

$$Y = F(A, L, R_N, R_R)|_{R_N=0} > 0$$

$$\lim_{t\to\infty} \epsilon_N = 0 \qquad \lim_{t\to\infty} \epsilon_R = 1 \qquad \lim_{t\to\infty} \epsilon_\psi = 0$$

$$\Rightarrow \quad \lim_{t\to\infty} \hat{Y} = g.$$

As the renewable resource can be easily substituted for the non-renewable resource, its flow into the intermediate resource input alone can guarantee positive long-run output. In this case, biased technical change has no qualitative effect on the asymptotic results and the non-renewable resource will be depleted in finite time just as in the benchmark case.

The required transversality condition is again $(1-\gamma)g < \rho$ which is automatically satisfied if $\gamma > 1$ and $g \geq 0$.

Case $\sigma = 1$. For the Cobb-Douglas case, it turns out that biased technical change with the share of the non-renewable resource $\epsilon_N = \psi \to 0$ is enough to guarantee positive output forever even in the absence of factor-neutral technical change ($g = 0$) but in contrast to the $\sigma > 1$ case, R_N will be depleted only in infinite time:

$$Y = F(A, L, R_N, R_R)|_{R_N=0} = 0$$
$$\lim_{t\to\infty} \epsilon_N = 0 \qquad \lim_{t\to\infty} \epsilon_R = 1 \qquad \lim_{t\to\infty} \epsilon_\psi = 0$$
$$\lim_{t\to\infty} \hat{R}_N = (1-\gamma)g + z - \rho \qquad \lim_{t\to\infty} \hat{Y} = g$$

The transversality condition is again $(1-\gamma)g < \rho$ which is automatically satisfied if $\gamma > 1$ and $g \geq 0$.

Case $\sigma < 1$. If the resource inputs are gross complements, the speed of technical change is crucial for the long-run results. We find that three distinct regimes may emerge. In all of them, though, only infinite-time depletion is possible.

FAST TECHNICAL CHANGE: $z < \theta(\rho - (1-\gamma)g)$. This assumption implies that the technical change is quick enough to fully compensate for the declining flow of non-renewable resources. This condition is more likely to be satisfied the weaker the complementarity of resource inputs (the greater the negative θ). Analogously, the greater the degree of complementarity between the resource inputs, the faster must be the increase in the relative productivity of the renewable resource. We obtain:

$$Y = F(A, L, R_N, R_R)|_{R_N=0} = 0$$
$$\lim_{t\to\infty} \epsilon_N = 0 \qquad \lim_{t\to\infty} \epsilon_R = 1 \qquad \lim_{t\to\infty} \epsilon_\psi = 0$$
$$\lim_{t\to\infty} \hat{R}_N = \frac{(1-\gamma)g + z - \rho}{1-\theta} \qquad \lim_{t\to\infty} \hat{Y} = g$$

MEDIUM TECHNICAL CHANGE: $z = \theta(\rho - (1-\gamma)g)$. In this knife-edge case, the speed of technical change is just enough to guarantee that the share of the non-renewable resource, $\epsilon_N \to \bar{c}$ where $\bar{c} \in (0, 1)$. The depletion of R_N provides a drag on the long-run growth rate of the economy. These results are similar to the less parsimonious case of slow technical change, described below. The details are available from the authors upon request.

SLOW TECHNICAL CHANGE: $z > \theta(\rho - (1-\gamma)g)$. If technical change is too slow to fully compensate for the shrinking flow of R_N, then its depletion exerts an unambiguously harmful effect on the long-run growth rate of the economy. The asymptotic results for this case are given by:

$$Y = F(A, L, R_N, R_R)|_{R_N=0} = 0$$
$$\lim_{t\to\infty} \epsilon_N = 1 \qquad \lim_{t\to\infty} \epsilon_R = 0 \qquad \lim_{t\to\infty} \epsilon_\psi = \frac{1}{\theta} < 0$$
$$\lim_{t\to\infty} \hat{R}_N = \frac{(1-\gamma)g + \frac{((1-\gamma)\beta)}{\theta}z - \rho}{1-(1-\gamma)\beta}$$
$$\Rightarrow \quad \lim_{t\to\infty} \hat{Y} = \frac{g - \beta\rho + \frac{\beta}{\theta}z}{1-(1-\gamma)\beta}.$$

For this sub-case, the transversality condition is different: the long-run impact of the bias in technical change coupled with the gradual depletion of the non-renewable resource has to be accounted for. It is now $(1-\gamma)(g + \frac{\beta}{\theta}z) < \rho$. However, it is still automatically satisfied if $\gamma > 1$ and $g \geq 0$.

These results allow us to draw conclusions which are a little more precise than those of recent research. For example, André and Cerdá (2005) derive equations describing the optimal dynamic evolution of the resource input ratio and the optimal output path, and then conclude from these that the "equations (...) express, in a mathematical way, the interest (and, in the long run, the need) to promote the research and use of renewable energy sources (...) to substitute nonrenewable energies (...) from a sustainability perspective." Our analysis allows to find that biased technical change has its merits when it is directed to the *more abundant resource input*. This will certainly imply that in the short run it would be optimal to invest more in research promoting the non-renewable resource rather than the renewable one, and only much later should the bias be reversed.

One more finding should be emphasized here. In the case $\sigma < 1$, there is one more "bifurcation" set of parameter values, which bounds away from the other cases of qualitatively different dynamic behaviour of the model. Indeed, authors working within the Acemoglu (2003) framework deal with (endogenously determined) biased technical change of the type which we call "slow"; their models typically do not allow for a jump to the regime where biased technical change is "fast".[9]

Summarizing, the long-run results for the biased technical change case stand in stark contrast to the results for the increasing flexibility case where technical change was unambiguously good and its desirable direction could be only towards higher substitutability. Moreover, we find here that the actual *value* of substitutability σ plays a more important role for the long-run dynamics even than the *growth rate* of the distribution parameter

[9] To keep our results closer to this strand of literature, one could redefine $\psi \equiv a^\theta$. Then, $z = \theta\hat{a}$ with $\hat{a} > 0$, and the condition for "fast" biased technical change becomes $\hat{a} > \rho - (1-\gamma)g$.

ψ, denoted as z. A one-shot improvement in the EoS could be more beneficient for the economy than perpetual growth in the productivities of resource inputs.

5.2 The short run

The results of the simulative exercise for the case of biased technical change are presented in Figures 6 and 7. In the first of these figures, we distinguish between the three major cases of $\sigma > 1$, $\sigma = 1$ and $\sigma < 1$. In the second one, we analyze the difference in dynamics stemming from the fact that the speed of biased technical change z may be either slow or fast (in figure 6, it is slow).

< Figure 6 here >

We can interpret these simulation results as follows. The discounted utilitarian criterion leads the infinitely-lived agent to perceive her current income as more important for the creation of utility, so she initially utilizes most non-renewable resources. As in all previous cases, we observe the crucial role played by the EoS. If $\sigma < 1$ then the relatively scarce resource drives production. Conversely, if $\sigma > 1$, then the relatively more abundant resource will be decisive for the amount of the intermediate good. We see this in the picture of ϵ_N. Initially, more non-renewable resources are utilized in production. For the case of $\sigma < 1$, this implies that the renewable resource constrains the amount of intermediate output, and therefore relatively quicker improvements in the marginal product of the renewable resource (as captured by $z < 0$) reduce the drag on intermediate good output imposed by the renewable resource. In contrast to this, if the non-renewable resource is the largest input in the intermediate good and $\sigma > 1$, then the non-renewable resource drives the intermediate good output. Hence, if the importance of the renewable resource now increases relatively to the non-renewable resource, then the effect on GDP growth is negative. These initial effects are reversed once the relative importance of the resource inputs changes due to the depletion of the non-renewable resource stock. We also see that ϵ_σ explodes to infinity in the $\sigma < 1$ case, indicating that improvements in the EoS would be increasingly valued as R_N gets depleted.

< Figure 7 here >

Further simulations show that in case the resources are gross complements ($\sigma < 1$), the speed of z is vital in order to allow for a non-decreasing long-run income. For the case of $\sigma = 1/2$, our simulations demonstrate that the assumed speed of biased technical change, $z = -0.02$, is not sufficient to outweigh the decrease in the non-renewable resource flow.

Hence, the GDP level will tend to zero, a result in line with the benchmark case. However, if the speed of z outweighs the reduction in the flow of non-renewable resources (in our case, $z = -0.06$), then a positive constant GDP level can be attainable. The speed of z must be such that the renewable resource becomes quickly enough more and more important to production in order to compensate for the decreasing quantities of the non-renewable resource available, $z < \theta(\rho - (1-\gamma)g)$. Only then is it guaranteed that technical change wins the race against the gradual disappearance of the essential production input.

6 Renewable resource as a potential backstop technology

One further corollary may be drawn from our analysis. We see that the renewable resource works as a *potential* (or *conditional*) backstop technology in this model. It is a backstop technology (cf. Nordhaus, 1973)[10] because it is a technology which allows to produce positive output forever without the need to use non-renewable resources as well. But it is only potential, because the economy has first to assure that the EoS between the two resource inputs exceeds one. Of course, this need not be the case: even if we allow for factor-augmenting technical change in both types of resources, σ may well stay below unity. The benchmark case with $\sigma \leq 1$ and the biased technical change case with $\sigma < 1$ illustrate the trouble with such a situation.

Whereas in the benchmark case and in the biased technical change case, the renewable resource works as a conditional backstop technology (the condition being that $\sigma > 1$), an EoS increasing over time in a way which ensures crossing unity gives it the characteristic of a *dynamically emerging* backstop technology. In such case, this backstop technology is known from the start, the "application" of this technology is certain, but the timing depends crucially on the growth rate of EoS.

We also find that once the conditions for the usage of renewable resources as a backstop technology are satisfied, it becomes profitable to deplete the non-renewable resource in finite time.

[10] Nordhaus (1973), pp. 547-548, writes: "The concept (...) is the backstop technology, a set of processes that (1) is capable of meeting the demand requirements and (2) has a virtually infinite resource base".

7 Conclusion

This article investigates two non-standard ways of looking at technical change: technical change in the elasticity of substitution as well as technical change in the partial elasticities. We use a simple Ramsey–Hotelling model to compare the long- and short-run implications of the two types of technical change.

These approaches follow recent suggestions found in the literature, i.e. Bretschger (2005, p. 150) suggests that "all possibilities of substitution and, specifically, the effects technology exerts on promoting substitution have to be studied".

Continued substitutability improvements have two effects. One, they improve upon the technical efficiency of the overall resource bundle. Two, they help render the non-renewable resource inessential for production from some time on. Our analysis shows that a policy maker will take the changes in the elasticity of substitution into account when choosing the optimal time path of non-renewable resource extraction. Here we focus on two contributions: the effect of an increasing EoS on the short-run where the main driving force are continuous improvements in the EoS, and on the long-run, where the most important factor is the EoS as a "dummy" for essentiality.

Biased technical change means that both resource inputs are subject to technical change, but that the productivity of one of the resources improves quicker than the other resource. We find that technical change is most useful when it is directed to the more abundant resource, and in the long run, is especially useful if it is quick enough to compensate for reductions in the flows of extracted non-renewable resources. Quantitative changes in the speed of biased technical change can bring about qualitative differences in the long-run outcome of the model.

Our strongest message is however that the elasticity of substitution plays a more important role for the long-run dynamics even than the *growth rate* of the distribution parameter between the resource inputs. In an extreme case, a one-shot improvement in the substitutability is more beneficial for the economy than perpetual factor-augmenting technical change.

In the light of these findings, consecutive research should address the following points. Firstly, exogenous technical change in the substitutability should be endogenized. It is one of the most important parameters, if not the most important, in models with non-renewable resources. This is by no means a simple task and valuable results might only be attainable for special cases. Secondly, one should attempt to compare the outcome of a model with endogenous technical change in the EoS with the outcomes of models featuring endogenous biased technical change in both resource inputs, already present in literature. Such results would help deepening our understanding as to which kind of technical change should be promoted by policy makers. Clearly, it is difficult to use the standard balanced growth approach to capture an optimal shift in research towards to more abundant resource. Therefore, how can one model the possibility of an optimal shift of R&D effort from one sector to the other.

Thirdly and finally, our analysis has (partly) been conducted in response to the bleak outlook painted by Cleveland and Ruth (1997), who suggest that the "traditional" types of technical change do not seem to be fast enough. Knowing the way in which technical change in the EoS as well as in the distribution parameters updates the existing results, it now remains to be asked whether empirical evidence still gives the same bleak outlook as before.

A Appendix

A.1 Optimization of the Hotelling model

We can write the Hamiltonian as follows (omitting time subscripts):

$$H(R_N, S_N, \lambda) = U\left(A[\psi R_N^\theta + (1-\psi)R_R^\theta]^{\frac{\beta}{\theta}}\right)e^{-\rho t} - \lambda R_N. \tag{14}$$

The Pontryagin maximum conditions are:

$$\frac{\partial H}{\partial R_N} = 0 \quad \Rightarrow \quad \lambda = \frac{\partial U}{\partial R_N}e^{-\rho t}, \tag{15}$$

$$\frac{\partial H}{\partial S_N} = -\dot{\lambda} \quad \Rightarrow \quad -\dot{\lambda} = 0. \tag{16}$$

Differentiating equation (15) with respect to time and substituting into equation (16) gives

$$\hat{Y} = \frac{\hat{F}_N - \rho}{\gamma}, \tag{17}$$

which is equation (8) in the main text. The marginal product of the non-renewable resource R_N is given by $F_N = A\beta\psi R_N^{\theta-1}R^{\beta-\theta}$. Its growth rate is given by

$$\hat{F}_N = g + z + (\theta - 1)\hat{R}_N + (\beta - \theta)(\epsilon_N\hat{R}_N + \epsilon_\psi z + \epsilon_\sigma s) + \frac{1}{\sigma}[\log(R_N) - \log(R)]s. \tag{18}$$

As the growth rate of income is $\hat{Y} = g + \beta(\epsilon_N\hat{R}_N + \epsilon_\psi z + \epsilon_\sigma s)$, we can substitute these two growth terms into equation (17) to get

$$(1-\gamma)g + [1 + (\beta-\theta)\epsilon_\psi - \gamma\beta\epsilon_\psi]z + [(\beta-\theta)\epsilon_\sigma - \gamma\beta\epsilon_\sigma]s - \rho + \frac{1}{\sigma}[\log(R_N) - \log(R)]s$$
$$= [\gamma\beta\epsilon_N + 1 - \theta - (\beta-\theta)\epsilon_N]\hat{R}_N.$$

Solving for $\hat{R}_N$ gives us the optimal growth rate of the non-renewable resource extraction, equation (9).

A.2 Proof that the denominator of $\hat{R}_N$ is always positive

The denominator of $\hat{R}_N$ is given by $(1-\theta) - ((1-\gamma)\beta - \theta)\epsilon_N$. Rewriting this gives $(1-\theta) - (1-\gamma)\beta\epsilon_N + \theta\epsilon_N + \epsilon_N - \epsilon_N = (1-\epsilon_N)(1-\theta) - [(1-\gamma)\beta - 1]\epsilon_N$. As we know that $\epsilon_N \in [0,1]$ and $\theta \in (-\infty, 1]$, we also know that the first term is greater or equal to zero. As $\gamma \in (0,\infty)$, $0 < \beta < 1$, then $(1-\gamma)\beta - 1 < 0$, so the denominator turns out to be a sum of two non-negative expressions, one of them being strictly positive. This implies that $(1-\theta) - ((1-\gamma)\beta - \theta)\epsilon_N > 0$. ■

A.3 The simulation procedure

The simulations have been run using the Matlab routine ODE45. They have been done in two steps. In the first step, we implement the "shooting" method to find the initial amount of extracted resource $R_N(0)$, i.e. we run the routine for a range of initial values and pick the one that guarantees $S_N(t) \to 0$. Since of course, the simulations have to be stopped in finite time, we approximate this limit value by $S_N(300)$. This approximation is valid in the case of infinite-time depletion, because with our parameter choices, the remaining stock of non-renewable resource becomes negligible by the time $t = 300$. In the second step, we re-run the routine with the correct value of $R_N(0)$ as an initial condition. Having obtained the time path of R_N, we calculate the time paths of other variables by straightforward inserting.

In the case finite-time depletion is possible, we run the simulations for different values of T (the moment in time when the resource is exhausted) separately. The FOC remains the same, but we have to take into account the boundary condition $S_N(T) = 0$ (fixed terminal point). After T, the economy uses for production the renewable resource flow R_R only. Other variables of the model continue their dynamic evolution without any interruption.

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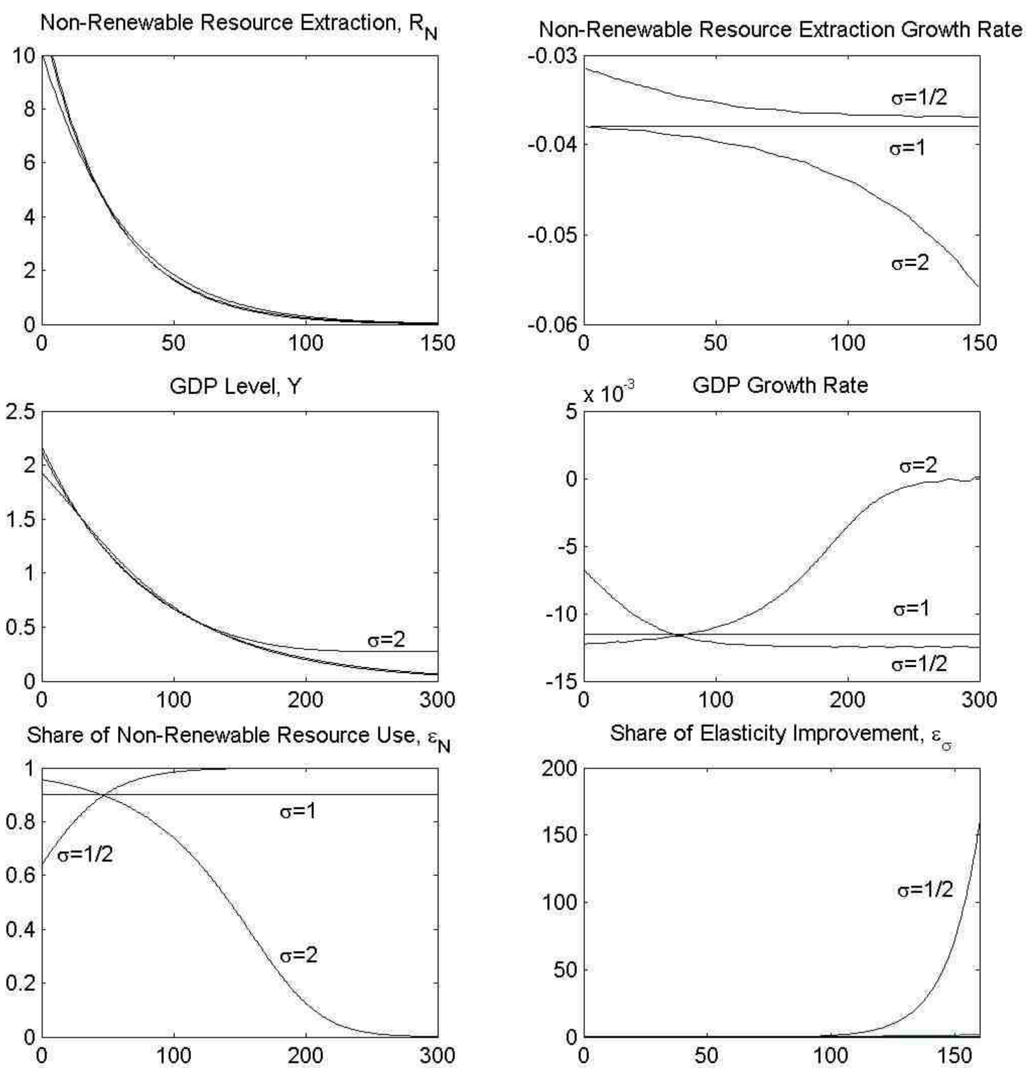

Figure 3: Benchmark case, $s = 0$, $z = 0$, $g = 0$

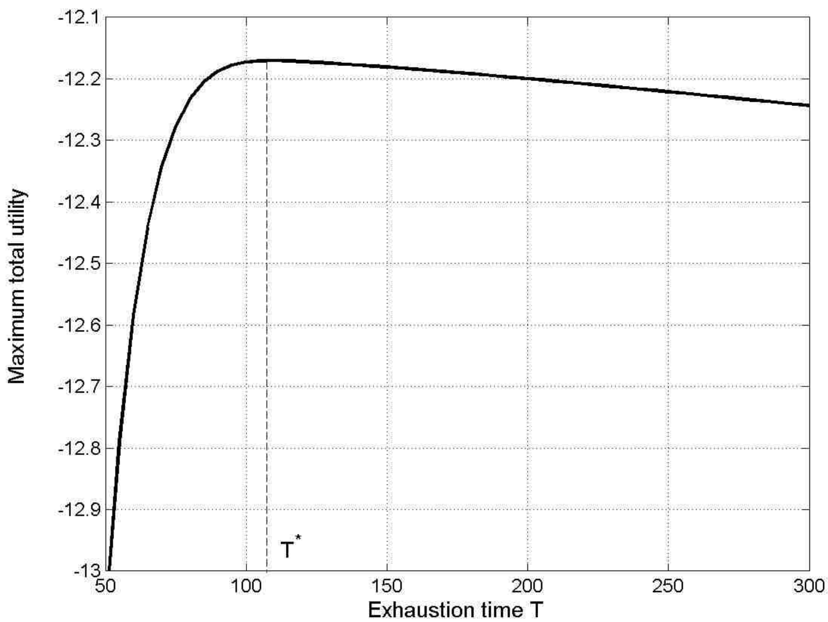

Figure 4: Exhaustion time in the case $\sigma = 10$

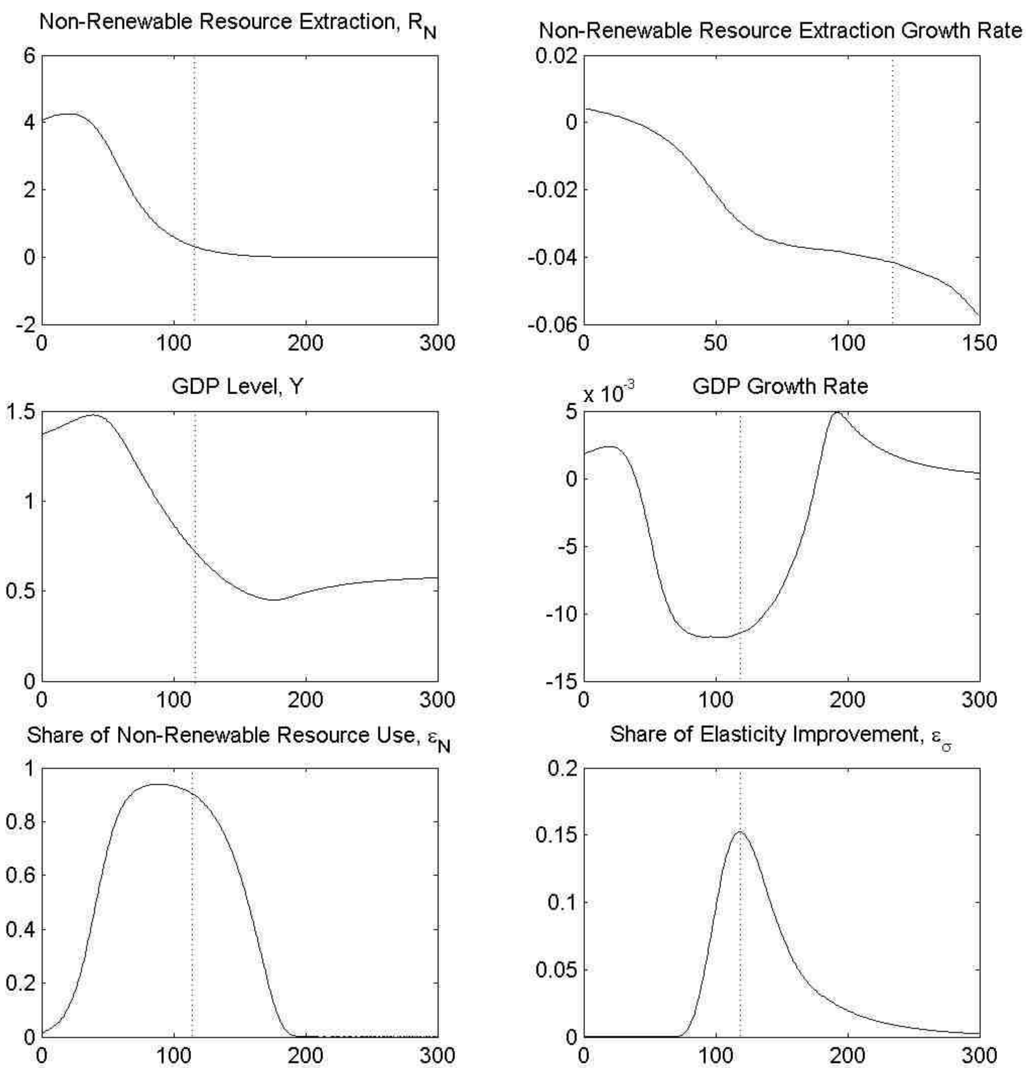

Figure 5: Increasing flexibility case, $s = 0.02$, $z = 0$, $g = 0$

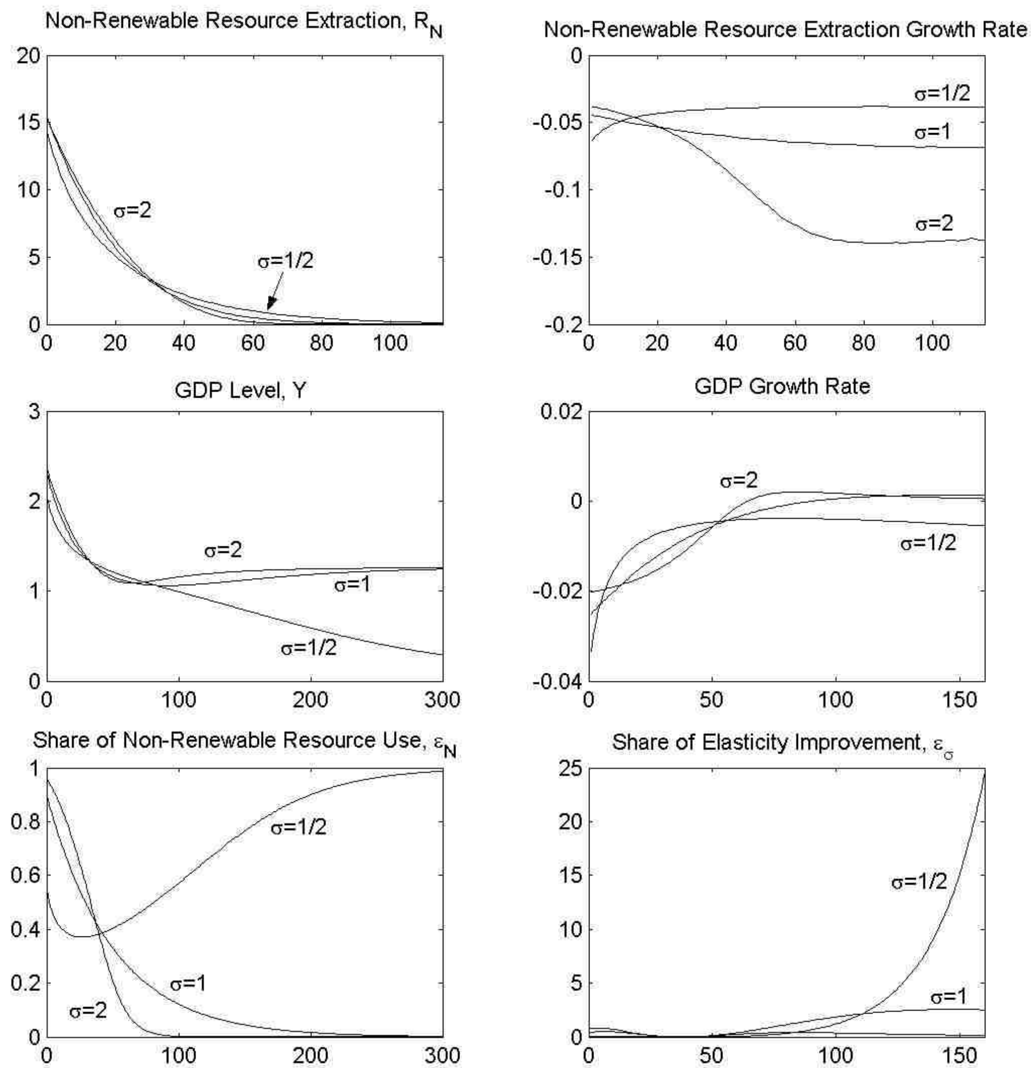

Figure 6: Biased technical change case, $s = 0$, $z = -0.02$, $g = 0$

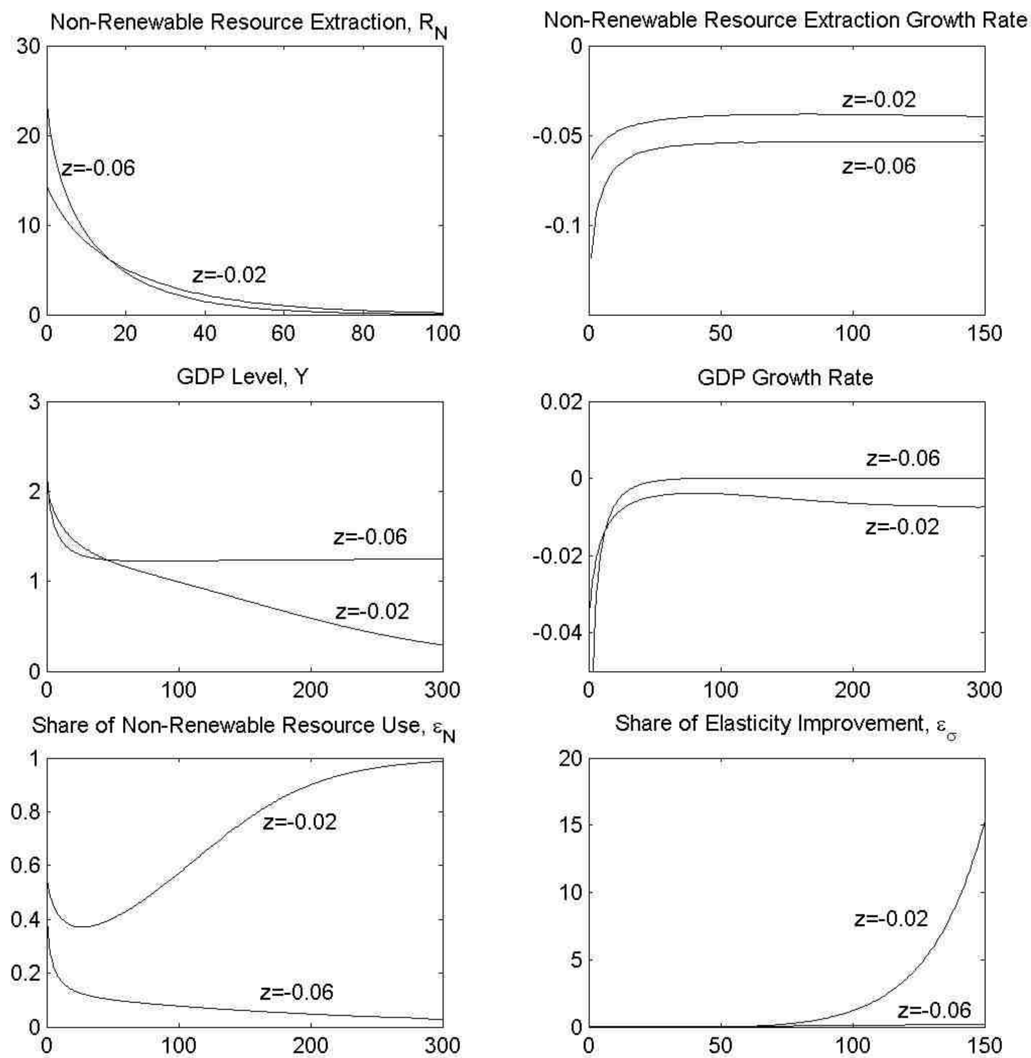

Figure 7: Biased technical change case, $s = 0$, $g = 0$. Dependence on the magnitude of the bias in technical change, z.

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Chair Lhoist Berghmans in Environmental Economics and Management
Center for Operations Research & Econometrics (CORE)
Université catholique de Louvain (UCL)
Voie du Roman Pays 34
B-1348 Louvain-la-Neuve, Belgium

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