# Reviewing Social Discounting within Intergenerational Moral Intuition 

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#### Abstract

This article introduces some of the main arguments of discounting, namely time preference, growth, uncertainty and ethics. We develop the concept of intergenerational moral intuition with the aid of three simple axioms. This concept allows us to draw strong conclusions for various factors (especially time preference and decreasing marginal utility) that influence the choice of the discount rate used by society. We introduce some limitations of the social discount rate as represented by the now standard formula $\rho+\theta g$. These limitations are partly based on our concept of intergenerational moral intuition and partly on the assumption of altruism. We furthermore argue, that discounting should reflect our expectations about the future much more than previously thought. The limitations that we describe lead us to develop a flexible approach to discounting, which is based on the method of intergenerational moral intuition. We call this the intuitive discount rate and show the conditions, under which it is applicable and improves standard discounting practices.


Keywords: Discounting, economic growth, uncertainty, intergenerational equity, environment.
JEL Classification: D6, I3, Q3.

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## 1 Introduction

Close to every model that deals with choice over time, be it theoretical, empirical or a simulation, utilises the discount factor as an evident and eminent part of life. However, do we really know about its components? What are the reasons for and against discounting based on its various elements? How are future generations affected by this? Should the government hold the same preferences as the individuals and if not, are moral reasons sufficient to drive their decisions? Is a policy maker allowed to use the social discount rate (SDR) under all circumstances? These and other questions are treated in this article.

One of the biggest controversies in today's macroeconomics is the problem of how to value future costs and benefits arising from long-run projects. This becomes even more apparent once we consider issues like climate change, nuclear waste, resource depletion, loss of biodiversity and natural habitats. These problems have such an extended time horizon, that we are in the habit of not only talking in years or decades, but centuries. The scope of these problems is of a scale never encountered before. Any decision that places too little weight on future outcomes, can doom later generations to live their lives on a devastated planet with no return ticket. Time has only one direction, especially when irreversibilities are involved like the ones we encounter in environmental problems. This then asks us to review the way we value future generations, and compare it to the way we ought to value them.

Several methods evolved during the past years to understand and measure the discount factor for various projects and situations (Lind, 1982; Portney and Weyant, 1999). Some methods were purely based on economic arguments of opportunity cost, some looked at ethical dimensions of discounting, other methods were trying to implement environmental discount rates, and still others were estimating the best rates for public projects. The large amount of methods reflects the sometimes rather specific circumstances of use. Due to the multitude of arguments available, Moore et al (2003) notice, that "Many governmental CBAs employ SDRs without any well specified rationale". Furthermore, discounting over 100 years at either $5 \%$ or $2 \%$ leads to an 18 -fold difference, whereas the same rates, applied over a period of 200 years, already lead to a 350 -fold difference. Hence, only little differences in discount rates can accumulate to a substantial amount. For problems like climate change or nuclear waste, where we expect benefits now and costs later, this can lead to a neglect of the costs in the distant future, thus not representing the true cost to society.

For these reasons we shall introduce some of the main arguments for discounting here, being especially concerned with the ethical dimension of the arguments under uncertainty. Given that the available methods for discounting are vast we do not pretend to provide a complete survey, but rather attempt to approach the literature from a policy maker's point of view who is caught between efficiency arguments and his moral intuition. The idea in this paper is to show which kind of considerations must go into the discount rate under what circumstances, where these considerations lead to contro-
versies, and how we could deal with some of them. This then leads us to propose what we call the intuitive discount rate.This is a rate that derives from moral intuition and is able to withstand standard moral as well as efficiency criticisms better than most of the current proposals like the opportunity cost view or prescriptive view.

Throughout the paper, population and inflation will be kept constant and the simplest mathematical formulations will be used in order to improve the theoretical presentation. The utility function will be assumed to have the CRRA (Constant Relative Risk Aversion) form, such that $u(c)=\frac{c^{1-\theta}}{1-\theta}$, with $u_{c}(c)>0$ and $u_{c c}(c)<0$. Due to the CRRA form, the inverse of the intertemporal elasticity of substitution, $-\frac{u_{c c}(c) c}{u_{c}(c)}$, will always be equal to $\theta$.

## 2 Where economists agree

We shall consider two main actors, the representative agent and the policy maker, and we shall only treat the infinite horizon case.

We assume that the representative agent has the same utility function each period and believes that the future is less important to him the further it is away. The mathematical way to represent this idea is via the Bergsonian-Samuelson social welfare function

$$
\begin{equation*}
\sum_{t=0}^{\infty} u\left(c_{t}\right) \beta^{t}, \tag{1}
\end{equation*}
$$

where $u\left(c_{t}\right)$ represent the agent's utility of consumption at time $t$ (with $u_{c}>0$ and $u_{c c}<0$ ), and $0<\beta<1$ the value he assigns to the future period.

Another, less abstract way to think about the world is to assume that it is made up of consecutive generations. Each generation consists of several households $i$, with $i=1, \ldots, I$, and each household $i$ has some kind of altruistic relationship, $u^{i}\left(c_{t}^{i}, u_{t+1}^{i}\right)$, with the household's next generation, $u_{t+1}^{i}$. In other words, a father loves his son and therefore is happy if the son is happy. Taking the simple case of a separable utility function and assuming that the father is slightly selfish, $u^{i}\left(c_{t}^{i}\right)+\beta u_{t+1}^{i}$, where $0<\beta<1$ represents the degree of selfishness, then by recursive substitution we can write the welfare of household $i$ in generation $t=0$ as

$$
\begin{equation*}
u^{i}\left(c_{0}^{i}\right)+\beta u^{i}\left(c_{1}^{i}\right)+\beta^{2} u^{i}\left(c_{2}^{i}\right)+\cdots \quad \forall i \tag{2}
\end{equation*}
$$

Now assuming all households have the same utility, such that $u^{i}=u / I$ for all $i=1, \ldots, I$, and summing up over all households in the generation, $\sum_{i=1}^{I} u^{i}=u$, then average each generation's level of consumption ${ }^{1}$, we get

$$
\begin{equation*}
u\left(c_{0}\right)+\beta u\left(c_{1}\right)+\beta^{2} u\left(c_{2}\right)+\cdots . \tag{3}
\end{equation*}
$$

[^1]Writing in a more concise form, this then becomes

$$
\begin{equation*}
\sum_{t=0}^{\infty} u\left(c_{t}\right) \beta^{t} \tag{4}
\end{equation*}
$$

which is the same mathematical representation as the representative agent case ${ }^{2}$. Therefore, the assumption of a representative, infinitely-lived agent can serve as a means of approximation to a dynastic, or altruistic world or vice versa.

The ranking of different utility streams is rather straightforward in this case. The utility stream $W=\left\{u_{1}, u_{2}, \ldots\right\}$ ranks above the utility stream $W^{\prime}=\left\{u_{1}^{\prime}, u_{2}^{\prime}, \ldots\right\}$, if and only if $W>W^{\prime}$. In other words, the overall utility achieved by the consumption stream $C=\left\{c_{1}, c_{2}, \ldots\right\}$ is preferred to the overall utility from the consumption stream $C^{\prime}=$ $\left\{c_{1}^{\prime}, c_{2}^{\prime}, \ldots\right\}$ if $\sum_{0}^{\infty} u\left(c_{t}\right) \beta^{t}>\sum_{0}^{\infty} u\left(c_{t}^{\prime}\right) \beta^{t}$.

The interpretation of the discount factor, in this most simplified setting, is twofold: It can either refer to pure time preference (the infinitely lived agent) or to emphatic distance (the altruistic generation). Both means reach the same ends. In the case of the representative agent, the discount factor is not necessarily introduced due to mathematical convenience, but rather due to overwhelming empirical evidence. The first ones to notice that agents discount future outcomes were Böhm-Bawerk (1889) followed by Irving Fisher (1930).

### 2.1 Pure time preference

Pure time preference was interpreted by Irving Fisher (1930) as consisting of fashion, foresight, self-control, life-expectancy, habit and finally a concern for others. For our purpose, this includes too many concepts in one. Following Diamond's (1965) explanation, pure time preference (or what he called eventual impatience) describes the following situation:
Pure time preference: For $u_{1}>u_{t}$, where $u_{t}$ can represent the period $t$ utility function and $t$ refers to time, the preference ordering of the vectors $\left(u_{1}, u_{2}, u_{3}, \ldots, u_{t} \ldots\right) \succ$ $\left(u_{t}, u_{2}, u_{3}, \ldots, u_{1} \ldots\right)$ implies time preference.
A way to introduce a measure of this pure time preference (or impatience) is to argue that events in the future evaluated now weigh less in the consideration of agents, than if these events would happen now. It is further argued that the more distant these events are, the less agents care about them now. This then allows formalising a function, which is continuous, and for reasons of time-consistency (see Strotz (1956)), must be 'memoryless'. The explicit form used is $e^{-\rho t}$ in continuous time, and $\beta=\left(\frac{1}{1+\rho}\right)^{t}$ in discrete time, with $\rho$ representing the discount rate and $0<\beta<1$ the discount factor.

[^2]Empirically, the existence of a positive rate of risk-less government bonds is generally believed to reflect agent's time preference. However, as risk-less government bonds usually span for a fairly limited amount of time (about 30 years), they only reflect the time preference of the current generation with regards to the next couple of years. If this is the case, it is doubtful whether it is meaningful to extrapolate the agents' preferences for the deep future. As such, it is important to note a potential difference between short-run and long-run time preference arguments.

Recently, there has been some empirical evidence that pure time preference might not be 'memoryless', but rather that it may vary over time. The empirical evidence is growing fairly rapidly as a recent paper by Frederick et al (2002) shows. This new way to discount future outcomes has been called hyperbolic discounting.

The most general function for introducing hyperbolic discounting seems to be due to Loewenstein and Prelec (1992), who use

$$
\begin{equation*}
\beta=(1+\alpha t)^{-\frac{\rho}{\alpha}}, \tag{5}
\end{equation*}
$$

for $\rho, \alpha>0$, and where $\alpha$ describes the strength of deviation from the exponential discount factor, and $\rho$ represents the constant discount rate. For $\alpha$ tending to zero we obtain the exponential discount factor, $\beta=e^{-\rho t}$.

The main problem with hyperbolic discounting is time-inconsistency. This timeinconsistency (see e.g. Strotz (1956)) refers to an agent who will revise his decisions over time, even though he has perfect foresight and has chosen optimally before. If we use the Bergsonian welfare function augmented by a hyperbolic discount rate, as developed by Phelps and Pollack (1968), where $W=u\left(c_{t}\right)+\beta \sum_{t=1}^{T-t} \rho^{\tau} u\left(c_{t+\tau}\right)$, then preferences viewed from date $t, \frac{u^{\prime}\left(c_{t+1}\right)}{\beta u^{\prime}\left(c_{t+2}\right)}$, are different from the preferences viewed from time $t+1$, $\frac{u^{\prime}\left(c_{t+1}\right)}{\beta \rho u^{\prime}\left(c_{t+2}\right)}$. Therefore we can expect the representative agent to review his decisions over and over again. This has been shown and applied to explain various anomalies such as procrastination (Akerlof (1991), O'Donoghue and Rabin (2001)), addiction (Becker and Murphy (1988), O'Donoghue and Rabin (2000)), and habit formation (Fuhrer (2000)). Obviously, a preference relationship that is subject to such harmful behaviour as addiction is truly something we should avoid basing our decisions on. This observation is especially of concern for the choice of specific social welfare functions. Another result due to Hepburn (2003) is that ignoring the problem of time-inconsistency can imply the depletion of a renewable resource. This depletion seems always optimal from the current perspective, but will obviously prove to be rather sub-optimal in the retrospective.

### 2.2 Individual versus social welfare

We noticed that the homo oeconomicus, our representative agent, will always act in his own selfish interest (as represented by the degree of selfishness). We will now show that either this selfish interest might not be compatible with, or not equivalent to, the interests of society.

In two early papers, Marglin (1963) and Sen (1967) showed how individuals, acting collectively, can achieve a pareto-improvement over individuals acting alone. This has obviously some relation to the famous pins-and-needles example by Adam Smith and to the freerider problems. Even in Biology do we observe that bacteria which live and 'work' together in a society are able to survive and grow, whereas alone they couldn't. Hence, if society can improve on the original outcome, then there should be some kind of social enforcement to achieve this improvement. This especially concerns externalities. If we interpret environmental degradation as an externality imposed by the current generation upon later generations, then somebody must step in to protect the rights of the future.

Another reason why society's welfare could be different from the individual's welfare is given by the so-called dichotomy of personality. This is based on the idea that the selfish agent has a heart somewhere inside his body which is beating for society's concern. For example, watching a child starve to death on television will surely move us, but not as far as to the next bank to send money to this child. If, however, we were asked directly by a person standing in front of us to save the life of this starving child by giving an incredible sum of money, then our moral guardian will - very likely - make us save this life. A different example could be that we would like to vote for the implementation of a speed limit even though we enjoy travelling as fast as possible. We therefore observe that our self-centered mind overweights this cautious voice in our heads, leading us to place our own interests above those of society.

An argument somewhat along similar lines comes from the altruistic agent. The discounting of our altruistic agent is a result of his emphatic distance to the next and later generations. However, as is widely argued, a good society should not have emphatic distance to any of its members.

Yet another reason in favour of using a different welfare function than the individual results from relaxing the assumption of the infinitely lived agent. Then time preference is often interpreted to be due to a limited life-span. Once you die the things that happen after you do not touch you as much as the ones that happen during your lifetime. But while an individual's life span is limited, society is ongoing.

Finally, time has only one direction. The choices of the future are hardly heard today. Protecting the future generations' rights and interests as well as their freedom of choice would be the basic requirement of any well-functioning society.

All these prior points direct us towards the importance of a social welfare function different from the discounted utilitarian one. In other words, a policy maker, who must decide upon which project to implement, must also choose criteria to evaluate which project is superior to another one. These criteria will decide upon the way he weighs the welfare of different agents at different points in time.

As before, we shall only be concerned with our representative (or altruistic) agent, which means we shall analyse how the policy maker ought to weigh the welfare of consecutive generations. This requires us to look closer at the ethical arguments for and
against time preference.

### 2.3 Ethical dimension of pure time preference

Prescriptive or ethical arguments have been given by economists for some while, e.g. by Marshall (1890), Pigou (1920) or Ramsey (1928), who suggest that time preference is morally unacceptable, or due to intellectual flaws, defective reasoning, or simply, as Harrod (1948, p.40) puts it, the 'conquest of reason by passion'. This conquest of reason by passion could for example be summarised by a person eating a whole chocolate bar in one go and saving nothing for later. Acting alone, our representative agent will always act in his own best interests. If we perceive that his interests are not in the best interest for the course of society, then we must attempt to modify his behaviour. The ethical arguments then naturally extend to the assumption that if agents possess time preference, it should be the role of the government to rectify this 'mistake' (see also Bentham and Hume). Utilising this argument, a policy maker should not show preference for current generations (see also Parfit (1984), Rawls (1999)).

However, we noticed above that discounting for time preference is a necessary component of any decision process for an efficient outcome. Nevertheless, under discounting some results become morally dubious once we consider introducing further criteria in addition to the efficiency criterion. In the literature on climate change, for example, efficiency could require us to completely neglect our effect on future generations. It is a well-known fact that present generations who behave optimally with too high a discount rate can reduce environmental quality to such extent that future generations will be living on a devastated planet. In the intratemporal economics, this can be visualised within the Edgeworth Box. An efficient outcome could imply one where a single person has nearly everything while another has barely nothing. In order for this not to happen in the intertemporal decision process, a useful criterion has been suggested, namely the equity criterion. Arrow et al (1995) called this the anonymity principle and in philosophy this is generally referred to as universalizability of preferences. We shall call this concept time neutrality.
Time neutrality: In line with the argument above, we can write for $u_{1}>u_{t}$, where $u_{t}$ can represent the utility function and $t$ refers to time, the preference ordering of the vectors $\left(u_{1}, u_{2}, u_{3}, \ldots, u_{t} \ldots\right)=\left(u_{t}, u_{2}, u_{3}, \ldots, u_{1} \ldots\right)$ implies time neutrality.
This criterion suggests that if we give equal weight to all generations, then this should result in an ethically more acceptable outcome, than if we only base our decisions on the efficiency criterion. Hence, it is believed, that if we do not discriminate between generations, we will achieve a morally more acceptable outcome. That this is not necessarily the case will be shown by the discounting paradox ${ }^{3}$.

[^3]
### 2.4 A discounting paradox

We develop upon a standard result in order to show that focusing solely on discounting might not catch the full problem and lead to a rather counterintuitive paradox. Let us imagine a society attempting to maximise its discounted sum of utility subject to a capital accumulation (with depreciation) and an essential, but nonrenewable resource (see Dasgupta and Heal, 1974). After maximisation, we obtain the modified Ramsey rule

$$
\begin{equation*}
\hat{c}=\frac{\alpha K^{\alpha-1} R^{\beta}-\rho-\delta}{\theta}, \tag{6}
\end{equation*}
$$

where $\hat{c}$ is the rate of consumption growth, $K$ is capital, $\alpha$ is capital share of production, $R$ is the non-renewable resource flow, $\beta$ is the resource share of production, $\rho$ refers to the discount rate, $\delta$ is the depreciation rate and $\theta$ is the elasticity of substitution.

We know that the limited non-renewable resource $R$ will tend to zero over time and hence the marginal product of capital will tend to zero, too ${ }^{4}$. Introducing nonrenewable resources into the growth framework suggests that the link between current and future generations is far more fragile than previously thought. The paradox is then that with any non-negative discount rate, $\rho \geq 0$, optimised consumption will fall to zero over time ${ }^{5}$. Thus maximising, even with a zero rate of discount, implies doom for later generations. This suggests that the discount factor is not necessarily the source of evil, as we impose zero consumption on later generations even though we treat each generation time neutral. Furthermore, there exists a welfare function (the maximin criterion) which not only treats each generation the same but, in this case, is also able to provide equality in utility for all generations - from the first to the last. We must thus be reminded that the discussion on the discount rate covers only a part of the actual problem, namely to decide upon a morally and intellectually acceptable outcome first, and then to examine the most efficient way to achieve it. In the next section we shall propose a way how to deal with this issue.

## 3 The concept of intergenerational moral intuition

In the previous sections we saw that economists and philosophers alike argue that equity reasons demand society to treat generations time neutral. However, we presented a discounting paradox, which suggests otherwise. Even though all generations are treated equally, this does not necessarily lead to an equitable outcome. In effect, we do not have a single concept that is able to deal with this problem. For this reason, we shall from now on use the concept of intergenerational moral intuition when we speak about moral arguments. Being a rather vague principle with seemingly no prior application in the economical literature on intergenerational equity, we here interpret it as a

[^4]mix of Welfarism and Egalitarianism. Here, Welfarism is used in the sense that if agents want to do something good for later generations, they should be allowed to, and Egalitarianism is used in the sense that if agents do something bad to later generations, the policy maker should prevent this from happening. We shall now formalise this concept with three simple axioms.
Axiom 1: If agents want to do something good ${ }^{6}$ for consecutive generations, they should be allowed to.
Axiom 2: If agents' actions induce an adverse outcome ${ }^{7}$ on any of its subsequent generations, they should be prevented from doing so.
Axiom 3: Axiom 2 always has priority over Axiom 1.
By resorting to intergenerational moral intuition we are thus able to pick the preferred concept of either theory and can thereby avoid some of the common controversies and criticisms that make us move away from either theory. We now apply this concept to some examples in order to understand its implications.

Returning to Harrod's criticism of time preference, 'conquest of reason by passion' could refer to the worker who obtains his weekly pay, but instead of going home to feed his hungry family he squanders all the pay on cheap drinks. It appears that even though his behaviour is rational from his point of view at the time of decision, he completely neglects the needs and desires of his family. A different example from the environmental economic literature is given by Dasgupta and Heal (1979). For any positive rate of time preference, the efficient, optimal consumption path will be such that the nonrenewable resource stock will be depleted in finite time and consumption will go zero ${ }^{8}$. Maximisation of discounted utility can thus lead to current generations living a happy life, but thereby dooming future generations. Similarly, Hepburn (2003) shows that a hyperbolic discount factor can lead to the depletion of a renewable resource in finite time and hence to zero consumption for later generations. Hence all examples, from the 'conquest of reason by passion' example to the depletion of a renewable resource, result in a distributive or moral problem - not an efficiency problem. More importantly, from the point of view of efficiency, all criteria for an optimal outcome are fulfilled. Given our concept of intergenerational moral intuition, we however notice that Axiom 2 is violated.

On the other hand, if our worker values the future less but still not less enough to further his knowledge and to increase the amount of wealth he holds, and if this in turn does not bear negative consequences for later generations, then it seems that time preference is not a problem per se. This can even lead to the outcome of a richer future generation. Hence our worker is happy to further the wealth of nations, which in turn

[^5]helps to enrich the strength of institutions (see e.g. Rawls (1971)) and to increase the happiness of future generations. Here our moral intuition would be that if the father wants his son to be richer, then he should be allowed to make his son richer. Today's maximin literature as well as the literature on the social discount rate (henceforth SDR) will however not perceive this as a morally acceptable behaviour. This literature strictly violates Axiom $1^{9}$. We shall discuss this under the section on growth.

## 4 Where economists disagree

The previous section gave fairly strong reasons to believe that a policy maker should not only represent the preferences of the representative agent (that means strive for efficiency), but should also care about the social dimension of choices, which could substantially differ from the private dimension. One way of introducing a social or moral dimension is to argue, that the rate of time preference should be set to zero, as Ramsey (1928) and others did. Hence the utility of each generation, from now until the infinite future, receives the same weight in social decision taking, such that the social welfare function, W , will be of the following form: $\mathrm{W}=\sum_{t=0}^{\infty} u\left(c_{t}\right)$, where the discount factor, $\beta$, from before is set equal to one.

Our concept of intergenerational moral intuition will only allow for a time preference of zero, if otherwise current generations impose harm on any future generation. Apart from therefore conflicting with our concept of intergenerational moral intuition (if Axiom 2 is violated), this is where we also face problems of a more mathematical nature.

### 4.1 The impossibility of ranking

A social welfare function (SWF) based on moral concerns ought to rid itself of pure time preference. This, however, leads to a problem of ranking. A SWF without pure time preference, e.g. $\mathrm{SWF}=\sum_{t=0}^{\infty} u\left(c_{t}\right)$, with utility bounded away from zero and infinite horizon will have an infinite sum. Hence, it is not directly possible to compare two utility streams unless the level of utility converges to zero over time. Or, put differently, the Weierstrass theorem ${ }^{10}$ fails to hold, which makes it impossible to find a maximum. But how can a policy maker choose between two infinite social welfare levels? Some economists therefore suggest that it is necessary to impose a discount rate in order to avoid the problem of ranking. We argue against this behaviour with the subsequent reasons.

[^6]The proposition is that the undiscounted utilitarian SWF provides an incomplete ranking and hence it is not possible to find a function with which to choose an optimal path. Even stronger, Koopmans (1960) showed that any SWF which is based on the axioms of stationarity ${ }^{11}$, independence ${ }^{12}$, continuity, sensitivity and boundedness, must possess impatience. The real implication of Koopmans argument is to demonstrate that the choice of the SWF depends on the normative arguments which we use when choosing a SWF. Put differently, the argument raises the question as to what we expect from a SWF, or what we intend to achieve when using a SWF.

In addition, we would like to note that several methods have been developed during the past years ${ }^{13}$ which deal perfectly well with this problem. Thus, the only acceptable reasons for introducing a discount rate are based on empirical evidence or theoretical reasons, but definitely not on convenience reasons.

### 4.2 Prescriptive and opportunity cost discounting

The economic community is split into two camps or approaches. One is the prescriptive approach, whose proponents ask as to how the future should be valued. They (e.g. Cline (1992)) believe that the rate of pure time preference should be set equal to zero and that market rates generally provide unsatisfactory guidance regarding opportunity costs. The proponents of the opportunity cost view ${ }^{14}$ (e.g. Nordhaus (1994)) are much more focused on the efficiency criterion and concerned with the opportunity cost of capital. Hence prescriptive discounting is making use of a social welfare different from the discounted utilitarian one, whereas descriptive discounting believes in the discounted utilitarian criterion. Given our concept of intergenerational moral intuition, we can provide strong reasons against either of the two views under some circumstances, but can also support both views under other circumstances. This is done in the following sections.

### 4.3 Consumer sovereignty

Discount rates of agents are non-unique, a phenomenon which has also been labeled consumer sovereignty. This result derives not only from theoretical arguments (see Henderson and Bateman (1995)), but also from empirical evidence (see Cropper et al (1994), Prelec and Loewenstein (1997)), where it is shown that different discount rates apply to different levels of wealth, location, products, race and age. Other empirical

[^7]evidence regarding discount rates has been obtained by Thaler (1981) from saving behaviour; by Horowitz (1991) from experiments on risk; by Hausman (1979) from actual markets.

A way in which these results have been interpreted is that the appropriate discount rate is always project specific, as already noticed by Lind (1982). This obviously indicates a high level of arbitrariness towards the discount factor. If decision takers arbitrarily tamper with the discount factor in order to make it project specific, this strongly implies that they adjust the values so as to obtain the outcomes they wish to attain. Especially given the various uncertainties surrounding the measurement and elements of the discount rate ${ }^{15}$, values could be arbitrarily chosen to fit desired circumstances. If this is the case, then the discount factor could lose its rather objective feature of being a relative price. Thus, we feel that it is vital to use the theoretical framework as far as possible when taking decisions.

## 5 The case of growth

The foregoing analysis is based on the underlying reasons - economical as well as moral ones - why policy makers should, or should not, follow individual preferences with respect to pure time preference or emphatic distance as well as uncertainty. We shall now extend this to the case of growth, treating growth under certainty and uncertainty separately. We will also attempt to clarify why policy makers wish to adjust the discount rate for growth rates, and provide a theoretical counterexample to this.

### 5.1 Growth with certainty

If we should ever have perfect foresight, then we could predict the rate of growth flawlessly. Let us use this idea to see what is going in general, and then relax the assumption of perfect foresight to incorporate uncertainty. Under the standard assumption of decreasing marginal utility ( $\frac{\partial^{2} u}{\partial c^{2}}=u^{\prime \prime}<0$ ), future increases in consumption will be valued less if agents are richer in the future. Put differently, the richer the future generations are, the lesser they will value additions in consumption.

The strong implication of this assumption is that if we expect continuous growth, we should discount for equity reasons. This is an idea which is supported by economists and Egalitarians alike (see Arrow (1999), Rawls (1999)). A simple Ramsey-type model can show this (see Tullock (1964)). As demonstrated in Appendix 8.1, if we maximise discounted utility subject to a capital accumulation, then the standard Ramsey condition

[^8]can be rewritten as
\[

$$
\begin{equation*}
\hat{c}=\frac{r-\rho}{\theta} \Longleftrightarrow r=\rho+\theta \hat{c}, \tag{7}
\end{equation*}
$$

\]

where $r$ is the market interest rate, $\rho$ represents pure time preference, $\theta$ the inverse of the intertemporal elasticity of substitution, and $\hat{c}$ is the growth rate of consumption or output. The right hand side term, $\rho+\theta \hat{c}$, is the so-called social discount rate.

Barro and Sala-i-Martin (1995) show that if the optimal saving ratio is constant during the transition period, it will equal $\frac{1}{\theta}$. Estimates of $\theta$ are ranging between 0.8 (Pearce and Ulph (1994)) and around 1.5 (Scott (1989), see also Arrow et al (1995)). A value of $\theta=1.5$ implies that a $1 \%$ increase in consumption reduces the marginal utility by $1.5 \%$. Furthermore, the higher the value of $\theta$, the more the social planner will be concerned with equity. With $\theta \rightarrow \infty$, we will obtain a perfectly egalitarian outcome with zero consumption growth. However, a value of $\theta=1.5$ would suggest that the saving ratio should be at least $2 / 3$, a value which is rather high (e.g. Ramsey (1928), Arrow (1999)). It is therefore argued that if we know there will be growth, then equity criteria demand us to reduce our savings in order to obtain a more egalitarian outcome.

In fact, positive long-run consumption growth will be possible in the Ramsey model on the path leading to the steady state, and in the endogenous growth literature (under certain conditions) all the time. Hence, later generations can be much richer than current generations. If we were only concerned with optimality conditions, our problems would stop here. However, when introducing equity considerations, a situation with richer future generations is deemed inequitable (Baumol (1968), Arrow (1999), Rawls (1999)). There are several arguments following this remark.

The first is that the government should rectify this inequality. This observation leads to the implementation of a special discount rate for public projects, the so-called social discount rate, SDR. It adjusts the time preference rate $(\rho)$ for the subjective value of the growth rate ( $\theta \hat{c}$ ). An interesting observation is that the SDR could very well be negative, for example if there happens to be a large enough economic decline ( $-\theta g>\rho$ with $g$ negative) instead of growth. As the first part of appendix 8.1 shows, even though the reason for the use of the SDR is intuitively appealing, there are some theoretical controversies. These are summarised in the following Proposition:

Proposition 1 The concept of the social discount rate leads, assuming perfect markets but allowing for growth, to the same criterion as given by the strict maximin principle.

## Proof 1 See Appendix 1.

We prove in Appendix 1, that a policy maker, who uses the SDR, in effect utilises the maximin criterion. This argument seems to suggest that the two concepts of the SDR and maximin are - at least theoretically - not far apart. Policy makers should therefore reconsider for which exact reasons they utilise the SDR, and whether they do not prescribe themselves to the maximin criterion. This criterion has, however, been weakened
even by Egalitarians like Rawls (1971) for the intergenerational conflict, where positive growth is allowed to a certain extent.

In addition, the altruistic framework provides strong reasons suggesting that the SDR is unnecessary. The SDR has been proposed for equity reasons. However, if agents really deem the situation of richer future generations as unfair, they would adjust their time preference themselves: If I know that my children will be much richer than I am now, why should I excessively care about them in my saving decisions? This leads us to question the necessity of the SDR under positive economic growth. The (altruistic) agent is perfectly happy to save more for his forthcoming generation. Why should a policy maker ever feel obliged to reduce his happiness (violating Axiom 1)? The conditions in which a policy maker who induces the SDR, reduces even the first period's utility, are summarized in the following Proposition.

Proposition 2 Under altruistic preferences, the use of the social discount rate can reduce the welfare of the current and the later generations if the altruism is strong enough in relation to the productivity of capital and the growth of the economy.

## Proof 2 See Appendix 1.

We show in Appendix 8.1 that a policy maker, who uses the SDR, can potentially reduce the father's and the son's utility and thereby violate our Axiom 2. Therefore, the only reason for accepting the need for a maximin outcome, seems to be for the case that one generation (or agent) is worse off than the other generations, but cannot do anything on its own to rectify this issue. This is certainly not the case if an altruistic father wishes to save more for his son (otherwise Axiom 1 is violated). The concept of intergenerational moral intuition would therefore not allow the use of the SDR, as both its axioms would be violated.

For now, this not only suggests that a society should not have time preference towards any generation (utilising the arguments of section 2.2), but furthermore it implies that society should not discount for equity reasons if we expect positive economic growth, as the growth rate derives from the preferences of the altruistic generation. On the other hand, if we expect negative growth, then the current generation's preferences impose harm on the future generations. Equity reasons (or moral intuition) then demand society to induce the current generation to take the effect of their decisions on future generations more seriously.

Conclusively, this reasoning would lead us to only discount if we expect negative growth. The rate of discount, would then be $\theta g$, which would be negative.

We must now check again, whether the use of this discount rate leads to an outcome more in line with our intergenerational moral intuition. The following section shows that this discount rate is not intuitively acceptable for several reasons. We therefore propose the intuitive discount rate, which is able to withstand most of the standard criticism from equity and efficiency arguments.

### 5.1.1 The intuitive discount rate

As we argued in the preceding section, inducing the above discount rate under the expectation of positive economic growth implies discounting with a rate equal to zero. Inducing this rate on our current generation makes them save much more than they would otherwise be willing to. Hence, the above discount rate under positive economic growth is intuitively unfair to the current generation, and favors later generations considerably. This problem can be shown rather neatly with the discounting of lives problem. Morally, we should not discount lives. However, if we measure a life lost as the opportunity cost forgone, then we know that the value of a life will increase over time with increasing productivity. Hence, not discounting for the value of lives implies favoring later lives over current ones. Then equity and fairness reasons would demand us to discount in order to avoid favouring future lives over current ones.

Furthermore, using the above discount rate under economic decline results in a negative discount rate. However, a negative discount rate favors future generations even more than would be fair to the current generation. How can we then be fair to future generations, such that we do not impose economic decline on them, but at the same time be fair to the current generation by not asking to give up too much and allowing the current generation to save more in case it wishes to?

We therefore wish to propose a method, in which fairness and altruistic transfers could, at least partially, coexist. We call the discount rate, which we shall derive from this method, the intuitive discount rate. The methodology is explained below and depicted in table 1.

Assuming positive economic growth, the approach we suggest in this case is to discount with $\theta g$ if $\theta g \leq \rho$, and with $\rho$ if $\theta g \geq \rho$. Discounting with $\theta g$ if $\theta g \leq \rho$ still allows positive transfers between generations, treats the generations time neutral from a pure time preference point of view, but at the same time takes care of the changing values due to growth. On the other hand, if $\theta g \geq \rho$, then discounting with $\rho$ takes fully care of the altruistic preferences and avoids the potential problem of decreasing the current and future generation's utility. More importantly, the current generation sacrifices some of its consumption for its future generation, which is why it would be rather counterintuitive to induce a reduction in the sacrifices, as e.g. the standard SDR would.

Assuming negative economic growth, the approach we suggest is to discount with the standard SDR, as given by $\rho+\theta g$. If the current generation feels such a low concern for future generations that this leads to harm imposed on the future generations, then the policy maker must induce a discount rate, to avoid such harm (by Axiom 2). At the same time, this discount rate should be of such a level that it does not favor future generations either (by Axiom 1). We argue, that the rate $\rho+\theta g$ is just this level.

We feel that intergenerational moral intuition provides enough justification to prefer the intuitive discount rate to opportunity cost rates, the SDR or the prescriptive view. The reason is that we built the intuitive discount rate on a more fundamental value principle of intergenerational equity than either of those theories do. Either theory fails
under specific circumstances. By using intergenerational moral intuition, we are able to avoid just those circumstances, due to which the standard discounting methods have failed.

| Table 1: The intuitive discount rate |  |  |
| :---: | :---: | :---: |
|  | $g>0$ | $g \leq 0$ |
| $\theta g \leq \rho$ | $\theta g$ | $\rho+\theta g$ |
| $\theta g \geq \rho$ | $\rho$ | $\rho+\theta g$ |

### 5.1.2 Some further problems with discounting for decreasing marginal utility

The third observation introduces the 'limits to growth' debate, where it is observed that future growth very strongly depends on highly uncertain factors like the risk of catastrophes, the possibility of ongoing technological progress, or the substitutability between capital and the environment. Given the huge uncertainty with regards to the future growth rate, it seems rather risky, or even controversial, to discount with this rate. This will be further discussed in section 5.2. Also, there is large uncertainty with regards to the elasticity of substitution, $\theta$, with estimates ranging between 0.8 and 1.5. Let us imagine possible long-run growth rates between $0.3 \%$ and $4 \%$, then we obtain a range of $0.24-0.45 \%$ or $3.2-6 \%$ for the subjective value of growth. Without a doubt, discounting with these values makes rather striking differences for the evaluation of long-run projects.

Lastly, discounting for the subjective value of growth is strongly based on the assumption of a concave utility function. What is commonly forgotten is that a lot of goods, which were luxury goods for our parents, evolved to be necessities for us. This can be explained in two ways. Either the preference for those luxury goods changes, or we become used to a certain level of consumption and therefore need ever increasing incentives in order to obtain a certain level of utility. If one accepts these arguments, one could argue strongly that discounting for growth is a rather questionable practice.

### 5.2 Growth with uncertainty

We really do not have perfect foresight, much to our dismay. Especially when talking about long-term horizon problems, our ability to foresee the next steps of our development is very limited. This means we must somehow estimate, approximate, or simply guess the opportunities for growth. We can also attempt to understand what uncertainty does to our models in general terms.

### 5.2.1 Catastrophes and optimal growth

The first observation is that under the possibility of catastrophes, the discounted utilitarian SWF can be used instead of most other SWF. We therefore base the following analysis on the Bergsonian-Samuelson SWF with uncertainty. We assume that the probability of a catastrophic event happening is a constant $1-p$, with $0<p<1$. Then the expected utility derives as follows

$$
\begin{equation*}
V=U\left(c_{0}\right)+p \beta U\left(c_{1}\right)+p^{2} \beta^{2} U\left(c_{2}\right)+\ldots+p^{t} \beta^{t} U\left(c_{t}\right)+\ldots=\sum_{0}^{\infty}(p \beta)^{t} U\left(c_{t}\right) \tag{8}
\end{equation*}
$$

This suggests, that the uncertainty of an event lowers the effective discount factor.
We, however, must remember, that this implies discounting for uncertainty, and not for time. Even this can imply some very strong results, which could be morally unacceptable. For example, if we believe that increased CO 2 emissions result in a flooding of delicate areas in 300 years, which could kill approximately 2.2 million people, then by discounting for this uncertainty with a rate of $5 \%$ we would prefer saving one life today to saving the lives of 2.2 million in 300 years.

What do the above results imply for the optimal level of consumption growth? If we maximise the expected utility ${ }^{16}$ subject to a capital accumulation constraint, and assume that the probability of no catastrophe happening is a constant, $p$, then this reduces the optimal growth rate, $g$, as $g=\frac{F^{\prime}(K)-(\rho+p)}{\theta}$. The intuition is that high savings for generations who might not exist is simply not reasonable from the point of view of the current generation.

We can therefore observe that uncertainty of preferences reduces the optimal growth rate. As we cannot be certain of the existence of future generations, we will save less for them and instead consume more ourselves. This decrease in savings for the future generations is depending on our expectation of their chances of existence.

### 5.2.2 Uncertainty of productivity in the future

Weitzman (1998) noticed that if we use the interest rate as a means of discounting (the descriptive approach), then instead of using the expected discount rate, $e^{-E(r) t}$, we should really use the expected discount factor (which he called certainty equivalence discount rate), $E\left(e^{-r t}\right)$. By Jensen's inequality we get that $E\left(e^{-r t}\right)>e^{-E(r) t}$. For example, let us assume there are two states with equal probability of 0.5 , one with a discount rate of $2 \%$, the other with one of $6 \%$. Hence the expected discount rate amounts to $4 \%$. Then $0.5 e^{-2}+0.5 e^{-6}>e^{-4}$. This result follows from Jensen's inequality, as $e^{-r t}$ is a convex function. This implies that the certainty equivalence discount rate is decreasing the longer the horizon, and approaches a minimum possible discount rate for time go-

[^9]ing to infinity. Newell and Pizer (2001) use a random walk model to analyse certainty equivalence discount rates and come to a similar conclusion as Weitzman.

### 5.2.3 Uncertainty of returns to investment

In reality we observe that interest rates are not unique. Usually, the higher the expected return, the higher the risk to investors. This is obviously because markets (for insurance) are inherently incomplete due to limited foresight or simply because no markets exist for certain goods. Private investors close to always have to bear a premium on the rate of interest they pay in order to compensate for risk or uncertainty from price or performance risks, market frictions, asymmetric information and moral hazards. Thus, the interest rate they face exceeds the uncertainty- free or risk-free rate. Or, in other words, a risk-averse lender will demand a risk premium so as to be compensated for higher risks with a higher expected return. Several authors noticed (Arrow and Kurz, 1970), that public projects should be treated differently from private projects. If a private investment unexpectedly returns no or even negative profits, the investor needs to bear all the losses. In contrast to this, the government is able to spread (or pool) risks to a much higher degree than the individual. This implies that government projects should be evaluated with a lower discount rate, potentially even the risk-free rate.

One must keep in mind, however, that there exist large investors nowadays who can bundle funds from various sources, which even exceed the GDP from many smaller countries. Furthermore, some governments are more prone to risk than others, for example due to the possibility of revolutions, internal political instability and high budget deficits. This suggests, that if the above argument is used, it must be employed with some caution.

### 5.2.4 Other uncertainties

## Potential backstop technology:

Uncertainty with regards to the existence of a backstop technology ${ }^{17}$ has been dealt with by Dasgupta and Heal (1974). They showed that the possibility of a backstop technology can allow us to give less weight to the future in our decision taking. That means, if we are fairly sure that in 100 years we will have the technology to power engines with the virtually infinite resource hydrogen, then we do not need to limit our consumption of oil now. We only need to make sure that there will be enough oil to run our economy until the expected backstop technology arrives. However, we are completely uncertain as to when we will have our backstop technology. If we then take the safe road and behave as if we believe that it doesn't arrive, but still account for the possibility that it could, then the possibility for a backstop technology leads to a lower

[^10]discount factor. What must be remembered (and is usually not discussed) is that this lower discount factor must only be used for the evaluation of projects which are subject to this backstop technology. This implies that if we expect a backstop technology for oil, but the policy maker needs to evaluate a project on education or the establishment of a new park, then the discount factor should not take this backstop technology into account.

If we use a simple example from cost benefit analysis, then we can show, that the possibility of a backstop technology might actually only discount the costs, and not the benefits. The following equation gives the net present value (NPV) of any project, with $B_{t}$ referring to benefits and $C_{t}$ referring to costs such that

$$
\begin{equation*}
\mathrm{NPV}=\sum_{t=0}^{\infty}\left(B_{t}-C_{t}\right) \beta^{t} \tag{9}
\end{equation*}
$$

Let us now assume that there exists a probability of a backstop technology, which reduces our costs to zero. For example, finding a method how to make nuclear waste harmless and reduce the probability for a catastrophe to zero will virtually eliminate all the environmental costs associated with the production of nuclear energy. Let this probability be expressed by $0<1-p<1$. Hence the probability of no backstop technology being found is $p$, and we can rewrite the equation above in order to include the possibility of the backstop technology

$$
\begin{equation*}
\mathrm{E}(\mathrm{NPV})=\sum_{t=0}^{\infty}\left(B_{t}-p C_{t}\right) \beta^{t} \tag{10}
\end{equation*}
$$

Thus we can see, that this backstop technology actually only makes us weigh the total costs by less, but leaves the benefits untouched. Consequently, our example shows, that the way we introduce the backstop technology matters for discounting.

## Substitutability between consumption and the environment in utility:

Let us assume that utility is a function of consumption and the environment, $S_{t}$, such that $U(C, S)$, where time subscripts are omitted. By our usual assumptions, utility increases in consumption, $U_{C}(C, S)>0$, at a decreasing rate, $U_{C C}(C, S)<0$. We furthermore assume that utility increases in the stock of the environment, $U_{S}(C, S)>0$, at a decreasing rate $U_{S S}(C, S)<0$. The substitutability between consumption and the environment is decided by the marginal rate of consumption when the environment changes, $U_{C S}(C, S)$. The separable case, $U_{C S}(C, S)=0$ allows perfect substitution between the two, meaning that even if the stock of the environment is totally depleted, utility can still be obtained ${ }^{18}$. If $U_{C S}(C, S)>0$ (respectively $U_{C S}(C, S)<0$ ), the marginal utility of consumption increases (respectively decreases) with an increase in the stock of the environment ${ }^{19}$. Depending on which case arises, this has strong implications for

[^11]the discount rate a policy maker should use for the evaluation of projects. As demonstrated in Appendix 8.1, the SDR is the rate at which the value of an additional unit of consumption changes when its consumption is postponed. However now, our calculation needs to take account of the changes in the environment as well. Hence our new, environment-augmented $\mathrm{SDR}^{20}$ becomes
\[

$$
\begin{equation*}
\rho+\theta_{C} \hat{C}+\theta_{S} \hat{S} \tag{11}
\end{equation*}
$$

\]

where $\theta_{C}=-\frac{C U_{C C}}{U_{C}}$ as before, and $\theta_{S}=-\frac{S U_{C S}}{U_{C}}$. If we are certain that the separable case, $U(C)+U(S)$ holds, then we can use our standard SDR. For the other cases, where the marginal utility of consumption changes with changes in the environment, this can either increase $\left(U_{C S}<0\right)$ or decrease $\left(U_{C S}>0\right)$ the SDR, given that environmental quality increases. This, and the case for decreasing environmental quality, is depicted in Table 2. These results are delicate, however. It is only possible to utilise the environment aug-

| Table 2: Substitution effects |  |  |
| :---: | :---: | :---: |
|  | Effect on SDR |  |
| Substitution | $\hat{S}>0$ | $\hat{S}<0$ |
| $U_{C S}=0$ | none | none |
| $U_{C S}<0$ | $(+)$ | $(-)$ |
| $U_{C S}>0$ | $(-)$ | $(+)$ |

mented SDR if consumption can substitute for the environment in the utility function. But which case arises? Do we need to increase, decrease or keep our SDR constant? Will environmental quality increase or decrease in the long-run? For example, if mankind can survive without environmental quality (because technology allowed us to replicate the environment), then the separable case seems to be acceptable. Furthermore, why should our marginal utility from consuming an ice cream be reduced if we move from a park into a dark garage? On the contrary, it also seems rather unrealistic to assume that the marginal utility from driving our car increases when the streets are converted back into forests.

One way to approach the substitutability issue is by asking ourselves what kind of world we would like our future generations to live in, and what kind of world our future generations themselves would like to inhabit. Again, a clear-cut answer is rather difficult to give. The only means we have is the belief that their preferences will be similar to our preferences.

Finally, it seems that humans are able to adapt well to changing circumstances. This assumption, called habit formation, implies that we can become used to a deteriorating level of the environment over time. Also, what were luxury goods for our parents evolve to be necessities for us. Hence, increases in consumption only temporarily provide a higher level of utility. It seems that if we discount for growth of consumption

[^12]and the environment under these assumptions, then we falsely weigh future preferences lower.

## 6 Synthesis

We summarized the main effects that are singled out in the prior analysis, in Table 3 below. Without considering the concept of intergenerational moral intuition, we are able to draw several conclusions. If efficiency is society's main objective, then the public discount rate must differ from the private one for the following reasons: risk pooling, dichotomy of personality, and potentially the existence of a backstop technology (if private agents don't take this into account).

Table 3: Rational for private and public discount rate

| Arguments | Private rate | Public rate |
| :--- | :--- | :--- |
| Time preference | risk free rate | efficiency: risk free rate <br> morality: no time preference |
| Emphatic distance | risk free rate (?) | efficiency: risk free rate (?) <br> morality: no emphatic distance |
| Dichotomy of personality | public rate $\neq$ private rate |  |
| Uncertainty (eg. catastrophes) | increase | increase |
| Risk (financial) | increase | no effect (risk pooling) |
| Backstop technology |  | increase |
| Positive economic growth |  | efficiency: no effect <br> morality: $\theta g$ if $\theta g \leq \rho$ or $\rho$ if $\theta g \geq \rho$ |
| Negative economic growth |  | efficiency: no effect <br> morality: $\rho+\theta g$ |
| Substitution effects | efficiency: no effect <br> morality: $\theta_{C} \hat{C}+\theta_{S} \hat{S}$ if $\theta_{C} \hat{C}+\theta_{S} \hat{S} \leq \rho$ <br> or $\rho$ if $\theta_{C} \hat{C}+\theta_{S} \hat{S} \geq \rho$ |  |

On the other hand, if standard moral arguments are at the forefront of public decision taking, then the picture is much different. Society should attempt to treat each of its members, may they exist now or in the future, in the same way (no time preference, no empathic distance). However, taking into account (catastrophic) uncertainty and the potential existence of a backstop technology will result in a positive discount rate. Furthermore, if there is potential negative economic growth or environmental decline, this may increase or decrease the public discount rate. Lastly, uncertainty of productivity in the future should be incorporated by using a decreasing discount rate. This suggests that the discount rate that we use is much more dependent on our expectations of the future than we previously thought, especially if we take into account substitutability, our
expectations about future economic and environmental growth, and uncertainty with regards to factors like productivity or risky returns.

If we now use the concept of intergenerational moral intuition, then we must reconsider the analysis under different scenarios of growth or decline. We will not repeat the arguments here from section 5.1.1.

## 7 Conclusion

In this article we introduce some of the main arguments for discounting, namely the pure rate of time preference, growth, uncertainty and ethics.

Furthermore, we develop the concept of intergenerational moral intuition, which we base on three simple, intuitive axioms. These axioms don't allow the current generation to take choices that impose adverse consequences on future generations, but allow the current generation to increase the future generation's happiness. This concept is thus a mix between Welfarism and Egaliterianism, but taking the preferred concept of each one.

We notice, that the pure rate of time preference is an eminent part of private decision taking. However, a good society should not have empathic distance to any of its members - be they current or future ones. This leads to two approaches. One focuses on equity and moral arguments, and another, which is more concerned with direct tradeoffs and efficiency.

With the aid of a simple paradox we demonstrate the limitations of only basing moral decision taking on the choice of the discount rate. This gives strong support for our new concept of intergenerational moral intuition.

One reason for discounting, which is by now common in the literature, is growth with decreasing marginal utility of consumption. We argue that using this method commonly and under every circumstance could even be counter-intuitive. By using the altruistic framework we prove that if the future generations are richer than the current ones, then adjusting the discount rate upwards by the subjective value of growth can lead to a reduction in the happiness of the current and the future generations. Hence larger discounting, in order to obtain a stricter egalitarian outcome, seems to be rather counter intuitive. On the other hand, if today's consumption decisions reduce the welfare of future generations, then it seems necessary to reduce the discount rate, in order to provide more weight to future's needs and desires.

We then show how the concept of the social discount rate, as represented by the formula $\rho+\delta g$, is mathematically equivalent to the maximin criterion. This suggests that policy makers who are in favour of using the concept of the SDR should reflect, whether they don't prescribe themselves to the maximin criterion, and discuss policies in that light.

We show that fair valuation requires to adjust the discount rate for the changing
valuation due to growth. But in addition we notice that altruistic generations might wish to leave more transfers and hence do not want to adjust the discount rate by this changing valuation. The solution we propose policy makers to take in this case is to discount with $\theta g$ if $\theta g \leq \rho$, and with $\rho$ if $\theta g \geq \rho$. Discounting with $\theta g$ if $\theta g \leq \rho$ still allows positive transfers between generations, but at the same time takes care of the changing values due to growth. On the other hand, if $\theta g \geq \rho$, then discounting with $\rho$ takes fully care of the altruistic preferences and avoids the problem of decreasing the current and future generation's utility. We call this method of discounting the intuitive discount rate, as it is based on the concept of intergenerational moral intuition.

The section on growth with uncertainty introduces the arguments of Weitzman (1998) and Newell and Pizer (2001), who discuss how uncertainty leads to discount rates that decline through time.

We introduce a new problem when discounting for a possible backstop technology and finally show, how substitutability between economic and environmental factors in the utility function can lead to rather strong consequences for the social discount rate.

Overall we believe, that intergenerational moral intuition is a strong concept when reasoning about the appropriate choice of the discount rate, as it provides a minimum acceptable frame, in which any policy should be analysed. Furthermore, the intuitive discount rate, which we derive from intergenerational moral intuition, seems to be a valuable improvement upon the standard social discount rate.

## 8 Appendix

### 8.1 Derivation of the SDR

The social discount rate (SDR) can be derived by two means. One way is by noting the change in the value of an additional unit of consumption obtained in the next period, another means is by maximising the SWF subject to capital accumulation and rewriting the Ramsey condition.

The first way is captured mathematically as follows:

$$
\begin{equation*}
-\frac{\frac{\partial}{\partial t}\left(\frac{\partial u\left(c_{t}\right) e^{-\rho t}}{\partial c_{t}}\right)}{\frac{\partial u\left(c_{t}\right) e^{-\rho t}}{\partial c_{t}}} . \tag{12}
\end{equation*}
$$

This gives the following SDR:

$$
\begin{equation*}
\rho+\theta g \tag{13}
\end{equation*}
$$

where $g=\frac{\dot{c}}{c}$ and $\theta$ is the elasticity of substitution.

The other approach is via maximising the discounted utilitarian SWF subject to a capital accumulation constraint. Mathematically we obtain the following method of derivation:

$$
\begin{equation*}
\max _{\left\{c_{t}\right\}} \int_{t=0}^{\infty} u\left(c_{t}\right) e^{-\rho t} d t \quad \text { subject to } \quad \dot{K}_{t}=F\left(K_{t}\right)-C_{t} \tag{14}
\end{equation*}
$$

Optimising, we obtain the Ramsey rule

$$
\begin{equation*}
\frac{\dot{c}}{c}=\frac{F^{\prime}(K)-\rho}{\theta}, \tag{15}
\end{equation*}
$$

with $\theta=-\frac{c u^{\prime \prime}}{u^{\prime}}$. Let us assume that ongoing technological change and human ingenuity can allow us to grow forever, and let this rate (for simplicity) be constant. This is equivalent to assuming that we are dealing with the AK-case, such that $F(K)=A K$, wherefore the optimal consumption growth rate becomes

$$
\begin{equation*}
\frac{\dot{c}}{c}=\frac{A-\rho}{\theta} \tag{16}
\end{equation*}
$$

Then the current practice is such that the policy maker utilises the SDR, which we can derive by modifying equation (16) which in this case is equal to $A=g \theta+\rho$.

### 8.2 Proof of equivalence between SDR and maximin

A policy maker, who bases his decisions on the social discount rate, uses the following social welfare function

$$
\begin{equation*}
\int_{t=0}^{\infty} u\left(c_{t}\right) e^{-(\rho+\theta g) t} d t \tag{17}
\end{equation*}
$$

For a constant rate of growth $g$ this new SWF has a simple mathematical solution. Our maximising problem becomes

$$
\begin{equation*}
\max _{\left\{c_{t}\right\}} \int_{t=0}^{\infty} u\left(c_{t}\right) e^{-(\rho+\theta g) t} d t \quad \text { subject to } \quad \dot{K}_{t}=F\left(K_{t}\right)-C_{t} \tag{18}
\end{equation*}
$$

The optimal path for consumption then becomes, if we assume that $g=\frac{\dot{c}}{c}$ is constant:

$$
\begin{equation*}
\frac{\dot{c}}{c}=\frac{A-\rho}{2 \theta} . \tag{19}
\end{equation*}
$$

But now the growth rate is halved. So in effect the policy maker must use a different SDR, namely $\rho+2 \theta g$. However, the optimal path which we derive from this then evolves to be

$$
\begin{equation*}
\frac{\dot{c}}{c}=\frac{A-\rho}{4 \theta} \tag{20}
\end{equation*}
$$

and so on. That means that by recursive substitution, a rational forward looking planner will use a SDR which will be equal to infinity, $\operatorname{SDR}=\infty$. But using a SDR which equals infinity implies the Rawlsian outcome. It is optimal to never grow, equivalent to the maximin outcome. Therefore, a policy maker who intends to use a SDR, will not use the preferences of the representative agent, but implicitly utilises the Rawlsian maximin criterion.

### 8.3 Proof, that the SDR can reduce welfare under altruism

Let us assume that our first generation is altruistic, which implies he has a utility function of the form $u\left(c_{t}, u_{t+1}\right)$, where $u_{t+1}$ represents the utility of his subsequent generation. If we assume the separable form $u\left(c_{t}, u_{t+1}\right)=u\left(c_{t}\right)+\omega u_{t+1}$, where $\omega$ is the SDR induced by the policy maker, then by recursive substitution we can obtain the standard discounted utilitarian criterion, $\sum_{t=1}^{\infty} u\left(c_{t}\right) \omega^{t-1}$. We now need to show that a policy maker who induces the SDR can actually reduce and not increase the current generation's welfare. We shall analyse the change of the current generation's utility. Taking total differentials holding the utility from all $t \geq 2$ constant, we get

$$
\begin{equation*}
\Delta W=u^{\prime}\left(c_{1}\right) \Delta c_{1}+\omega u^{\prime}\left(c_{2}\right) \Delta c_{2} \tag{21}
\end{equation*}
$$

This represents the first generation's utility level given some consumption in period $t+1$ is redistributed to period $t$. Now, substitute $F^{\prime}(K) \Delta c_{1}=-\Delta c_{2}$, we get that

$$
\begin{equation*}
\Delta W=\left(u^{\prime}\left(c_{1}\right)-\omega F^{\prime}(K) u^{\prime}\left(c_{2}\right)\right) \Delta c_{1} . \tag{22}
\end{equation*}
$$

Therefore, the change in welfare will be negative, $\Delta W<0$, if and only if the following condition holds, such that

$$
\begin{equation*}
\frac{u^{\prime}\left(c_{1}\right)}{u^{\prime}\left(c_{2}\right)}<\omega F^{\prime}(K) \tag{23}
\end{equation*}
$$

Assuming perfect markets, the social discount rates redistributes such that $u^{\prime}\left(c_{1}\right)=$ $u^{\prime}\left(c_{2}\right)$, wherefore the change in welfare is negative if $\omega>\frac{1}{F^{\prime}(K)}$. As the second generation has less to consume, its utility is reduces as well (as we assume $\Delta c_{2}<0$ due to the redistribution), wherefore overall welfare is diminished.

### 8.4 Expected utility in continuous time

Let us assume there exists the possibility of a catastrophe which, for ease of demonstration, reduces the level of utility to zero. This could be the breach of an unknown pollution threshold or the possibility of nuclear catastrophes, or even the probability of an asteroid colliding with earth. Let this possibility be represented by the hazard function $h_{t}$, which represents the probability that a catastrophes happens in a very
small interval of time given that it didn't happen before. The survivor function $S_{t}$ stands for the probability that no catastrophe has happened in the interval $[0, t]$, and it is related to the hazard function, $S_{t}=e^{-\int_{0}^{\tau} h_{s} d s}$. The uncertainty regarding the actual event is represented with the utility function, $E\left(\int_{0}^{\tau} u\left(c_{t}\right) e^{-\rho t}\right)$. Writing the cumulative density function of $\tau, S_{t}^{C}=1-S_{t}$, we can rewrite the expected utility function as $\int_{0}^{\infty} \int_{0}^{\tau} u\left(c_{t}\right) e^{-\rho t} d t d S_{\tau}^{C}$, where $d S_{\tau}^{C}$ can be rewritten as $S_{\tau} h_{\tau} d \tau$. By changing the order of integration, $\int_{0}^{\infty} u\left(c_{t}\right) e^{-\rho t}\left(\int_{t}^{\infty} S_{\tau} h_{\tau} d \tau\right) d t$, we can get $\int_{0}^{\infty} u\left(c_{\tau}\right) S_{\tau} e^{-\rho t} d \tau$, and performing a change of variable, $y_{t}=-\ln S_{t}$, we obtain the following expression:

$$
\begin{equation*}
\int_{0}^{\infty} u\left(c_{t}\right) e^{-\rho t-y_{t}} d t \tag{24}
\end{equation*}
$$

Hence, risk (or uncertainty) is equivalent to a reduction in the discount factor ${ }^{21}$. As far as empirical evidence goes, there is a larger body of literature available which shows that individuals discount own risks (see Johannesson and Johanson (1996)). For a discussion see Pearce et al (2003).

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## Environmental Economics \& Management Memorandum

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[^1]:    ${ }^{1}$ For example via transfers.

[^2]:    ${ }^{2}$ There are some conditions for this to hold. Firstly, the degree of selfishness must be strong enough, meaning $0<\beta \leq 1$, in order for there to be links between generations. Secondly, this assumes perfect foresight and does not apply to other altruistic frameworks like joy-of-giving bequests.

[^3]:    ${ }^{3}$ This provides the reason why we refrained from calling the above concept the equity criterion.

[^4]:    ${ }^{4}$ An observation is, that if the government bases its decisions on a SDR, this SDR will be hyperbolic.
    ${ }^{5}$ The prior analysis is based on the assumption that at least at time $t=0$ we have $\alpha K^{\alpha-1} R^{\beta}>\rho+\delta$.

[^5]:    ${ }^{6}$ Although loosely defined, this can be interpreted as for example increasing future's utility via savings.
    ${ }^{7}$ An adverse outcome can be interpreted as violating what we call constraint sustainability, as captured by $\frac{d u}{d t} \geq 0$. See Pezzey (1997) for a discussion of this kind of constraint in optimal growth models.
    ${ }^{8}$ Given that we maximise the representative agent's utility function subject to capital accumulation and a finite stock of non-renewable resources, and given that the resource is essential for production.

[^6]:    ${ }^{9}$ However, Rawls (1971) himself did not prescribe himself fully to the maximin principle in the intergenerational conflict. The reason for this - increasing the strength of institutions - is however rather vague and arguably subject to strong debates.
    ${ }^{10}$ The Weierstrass Theorem states that any function $f(x)$ that is continuous on a closed and bounded set has at least one maximum point.

[^7]:    ${ }^{11}$ Stationarity implies that if $\left\{c_{1}, c_{2}, c_{3} \ldots\right\}>\left\{c_{1}, c_{2}^{\prime}, c_{3}^{\prime} \ldots\right\}$, then $\left\{c_{2}, c_{3} \ldots\right\}>\left\{c_{2}^{\prime}, c_{3}^{\prime} \ldots\right\}$. Loewenstein and Prelec (1992) provide empirical evidence against this axiom. The axiom is however normative, and therefore not subject to empirical criticism.
    ${ }^{12}$ Independence means that if $\left\{c_{1}, c_{2}, c_{3} \ldots\right\}>\left\{c_{1}^{\prime}, c_{2}, c_{3} \ldots\right\}$, then $\left\{c_{1}, c_{2}^{\prime}, c_{3}^{\prime} \ldots\right\}>\left\{c_{1}^{\prime}, c_{2}^{\prime}, c_{3}^{\prime} \ldots\right\}$.
    ${ }^{13}$ See e.g. von Weitzäcker's (1965) overtaking criterion.
    ${ }^{14}$ This view has been labeled the descriptive view, although it is as prescriptive as the prescriptive view. We would therefore advocate the use of opportunity cost view instead of descriptive view.

[^8]:    ${ }^{15}$ The rather complete survey by Frederick, Loewenstein and O'Donoghue (2002) summarizes empirical studies. The differences in discount rates obtained range from $-6 \%$ to infinity, with long-run rates (on health and life) of around $0 \%$ to $20 \%$. This obviously does not assist in policy decision-taking, especially on deep future issues.

[^9]:    ${ }^{16}$ See Appendix 8.4 for the derivation of expected utility in continuous time.

[^10]:    ${ }^{17}$ A backstop technology is a technology that makes us independent of essential factors of production. For example, being able to make engines run on hydrogen will relieve us from constraints on limited oil and gas.

[^11]:    ${ }^{18} \mathrm{We}$ are only looking at the effect on utility, and not incorporating catastrophic effects here.
    ${ }^{19}$ The limit case, where $U(C, S)=\min \{C, S\}$ seems rather unrealistic to be even of theoretical interest.

[^12]:    ${ }^{20}$ See also Zhu and Weikard (2003) for a similar but less elaborate treatment of this concept.

[^13]:    ${ }^{21}$ With special thanks to Benteng Zou, see also Clarke and Reed (1994).

