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A comparative study of Energy Saving Technical Progress in a Vintage Capital Model

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A comparative study of Energy Saving Technical Progress in a Vintage Capital Model.*

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Abstract

We analyze the hypothesis about the effectiveness of energy saving technologies to reduce the trade-off between economic growth and energy preservation. In a general equilibrium vintage capital model with embodied energy saving technical progress, we show that positive growth is only possible if the growth rate of the energy saving technical progress exceeds the decreasing rate of the energy supply.

Keywords: Nonrenewable resources, Energy saving technical progress, Vintage capital.

Journal of Economic Literature: C68, O31, O41, Q32, Q43

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1 Introduction

Fossil fuel–more precisely petroleum and its refinery products–is an essential input in all modern economies. It has been argued that the limited availability of this basic input and the stabilization of greenhouse gases concentration call for a reduction of fossil fuel consumption. However, the reduction in petroleum consumption could have a negative impact on economic growth and development through cutbacks in energy use (Smulders and de Nooij (2003)). Therefore, there is a clear trade-off between energy reduction and growth.

Some authors (see for instance Carraro, Gerlagh and van der Zwaan (2003)) suggest that this trade-off could be less severe if energy conservation is raised by energy saving technologies. In this paper, we re-examine the exhaustion problem of fossil fuel. In particular, we study the previous trade-off in a general equilibrium framework with energy saving technical progress. This model, based on Boucekkine, Germain and Licandro (1997), considers an economy with exogenous energy saving technical progress embodied in the new equipment. As Baily (1981) observes, technical advances are typically incorporated to the economy through investment. Therefore, the old capital goods get less and less efficient over time, which might well induce the firms to scrap them (obsolescence). In our economy, we assume that different vintages of capital coexist in each period. Since new vintages are less energy consuming, firms may decide to replace the oldest and less efficient vintage. Indeed, if we model the idea of minimum energy requirement to use a machine by assuming complementarity between capital and energy inputs, finite scrapping time is optimal (Boucekkine and Pommeret (2004)). This idea is implemented in our paper, and it is consisted with the empirical evidence put forward by Hudson and Jorgenson (1974), or Berndt and Wood (1975).

Our model incorporates two new elements with respect to the standard framework. First, we assume embodied technical progress in contrast to the typical neoclassical specification of neutral and disembodied technical progress. Second, we consider a vintage capital model, with endogenous scrapping decision. The standard models consider homogenous and infinitely lived capital stock.

We perform a comparative study to contrast constant and decreasing returns to scale, for two possible scenarios: constant (optimistic) and decreasing (pessimistic) exogenous energy supply. We find that, under the assumption of existence of a balance growth path (BGP) defined by constant growth rate of all the endogenous variables and constant scrapping age, constant returns to scale achieves positive long run growth if the growth rate of the energy saving technical progress exceeds the decreasing rate of the energy supply.

The paper is organized as follows. In section 2, we describe the general case model, with the representative consumer's problem and the rules that depicts both the optimal investment and the scrapping behavior of firms. The BGP is presented in section 3, where we show the necessary conditions for its existence in both constant and decreasing returns to scale. Finally, some concluding remarks are considered in section 4.

2 The Model

Following Boucekkine *et al.* (1997), we consider an economy where the population is constant and there is only one good (the numeraire good), which can be assigned to consumption or investment. The good is produced in a competitive market by mean of a technology defined over vintage capital. Both constant and decreasing returns to scale are considered here. Also, we assume a competitive labor market and exogenously available energy supply.

2.1 Household

Let us assume that the representative household considers the following standard inter-temporal maximization problem with a constant relative risk aversion (CRRA) instantaneous utility function

$$\max_{c(t)} \int_0^\infty \frac{c(t)^{1-\theta}}{1-\theta} e^{-\rho t} dt \tag{1}$$

subject to the budget constraint

$$\begin{aligned} \dot{a}(t) &= r(t)a(t) - c(t) \\ &a(0) \;\text{ given} \\ \lim_{t\to\infty} a(t)e^{-\int_0^t r(z)dz} &= 0 \end{aligned} \tag{2}$$

with initial wealth a_0, where $c(t)$ is per-capita consumption, $a(t)$ is per-capita asset held by the consumer at the interest rate $r(t)$ which is taken as given for the household. θ measures the constant relative risk aversion, and ρ is the time preference parameter (it is assumed to be a positive discount factor). Since our paper does not explicitly treat labour, we assume that it has no value leisure for the consumer. Then, in order to simplify the model, it is considered an inelastic labour supply normalized to one. The corresponding necessary conditions are $r(t) = \rho + \theta \frac{\dot{c}(t)}{c(t)}$, with $\lim_{t\to\infty} \lambda(t)a(t) = 0$, where $\lambda(t)$ is the co-state variable associated with the wealth accumulation equation.

2.2 Firms

The good is produced competitively by a representative firm solving the following optimal profit problem

$$\max_{y(t),i(t),T(t)} \int_0^\infty [y(t) - i(t) - e(t)P_e(t)]\, R(t)dt \tag{3}$$

subject to

$$y(t) = A\left(\int_{t-T(t)}^{t} i(z)dz\right)^{\alpha}, \quad 0 < \alpha \leq 1 \tag{4}$$

$$e(t) = \int_{t-T(t)}^{t} i(z)e^{-\gamma z}dz, \quad 0 < \gamma < \rho \tag{5}$$

with the initial conditions $i(t)$ given for all $t \leq 0$. $e(t)$ is the demand of energy at a given price $P_e(t)$. The firm considers the energy price has given; however, it is endogenously determined in the energy market equalizing the demand and supply of energy. $i(t)$ is the investment of the representative firm, and the output is represented by $y(t)$[1]. Equation (4) is our technology defined over vintage capital. The energy demand is obtained by equation (5). Here $\gamma > 0$ represents the rate of energy saving technical progress and $T(t)$ is the age of the oldest operating machines or scrapping age. The discount factor $R(t)$ takes the form $R(t) = e^{-\int_0^t r(z)dz}$. Finally, we assume that $0 < \gamma < \rho$ to well define our integral[2].

Notice that the new technology is more energy saving. Certainly, each vintage $i(t)$ has an energy requirement $i(t)e^{-\gamma t}$ [3]. Moreover, it is important to observe that we assume complementarity between capital and energy (Leontieff technology), which model the idea of minimum energy requirement to use a machine. Furthermore, this assumption is undeniable from numerous studies; for instance Hudson and Jorgenson (1974), or Berndt and Wood (1975). As we observed in the introduction, this complementarity ensures a

[1] We can consider an alternative technology where decreasing returns to scale only affects new vintages (not all the active vintages) $y(t) = A\int_{t-T(t)}^{t} i(z)^{\alpha}dz$. Similarly to our case, it can be checked that the results remain the same.

[2] It is a standard assumption in the exogenous growth literature to have a bounded objective function.

[3] This assumption is central in the early vintage capital models. See Solow *et al.* (1966).

finite optimal scrapping age[4].

We define the capital stock

$$K(t) = \int_{t-T(t)}^{t} i(z)dz \tag{6}$$

and the optimal life of machines of vintage t [5]

$$J(t) = T(t + J(t)) \tag{7}$$

From the first order condition (FOC)[6] for $i(t)$, we get the *optimal investment rule*

$$\begin{aligned} \int_{t}^{t+J(t)} \alpha A \left(\int_{\tau-T(\tau)}^{\tau} i(z)dz \right)^{\alpha-1} e^{-\int_t^\tau r(z)dz} d\tau = \\ 1 + \int_{t}^{t+J(t)} P_e(\tau) e^{-\gamma t} e^{-\int_t^\tau r(z)dz} d\tau \end{aligned} \tag{8}$$

where the left hand side (LHS) is the discounted marginal productivity during the whole lifetime of the capital acquired in t, 1 is the marginal purchase cost at t normalized to one, and the second term on the right hand side (RHS) is the discounted operation cost at t.

The optimal investment rule establishes that firms should invest at time t until the discounted marginal productivity during the whole lifetime of the capital acquired in t exactly compensates for both its discounted operation cost and its marginal purchase cost at t.

From the FOC for $T(t)$, we have the *optimal scrapping rule*

$$A\alpha \left(\int_{t-T(t)}^{t} i(z)dz \right)^{\alpha-1} = P_e(t) e^{-\gamma(t-T(t))} \tag{9}$$

The optimal scrapping rule states that a machine should be scrapped as soon as its marginal productivity (which is the same for any machine whatever its age) no longer covers its operation cost (which rises with age).

[4] $T(t) < \infty$ is an essential and standard assumption considering the possibility of replacement ($T(t) \longrightarrow \infty$ implies no scrapping). See, for example, d'Autume and Michel (1993) and Bardhan (1969).

[5] Notice that $T(t) = J(t - T(t))$.

[6] Following Boucekkine *et al.* (1997), we can consider an intermediary sector to create intermediate inputs for the final production. It is easy to observe that the case of symmetric equilibrium is exactly equivalent to our model without intermediate good sector. However, as Krusell (1998) observes, if the product-specific returns to R&D are strong enough, there may be asymmetric steady-state equilibria, in which large and small capital firms coexist. Nevertheless, this is not the case in our model with exogenous technical progress.

Here the marginal productivity is given by $\alpha A(\int_{t-T(t)}^{t} i(z)dz)^{\alpha-1}$, and $P_e(t)e^{-\gamma(t-T(t))}$ represents the operation cost.

2.3 Decentralized equilibrium

The (decentralized) equilibrium of our economy is characterized by equation (2), the necessary and transversality condition of the household problem, equations (4)–(7), the optimal investment rule, the optimal scrapping rule, and the following two additional equations to close the model: $c(t) + i(t) = y(t)$ and the equilibrium condition in the energy market $e(t) = e_s(t)$. $e_s(t)$ is the available energy supply[7], which in our model is assumed to be exogenous.

3 Balanced growth path

3.1 Definition of balanced growth path

Let us define our balanced growth path (BGP) equilibrium as the situation where all the endogenous variables grow at a constant rate, with constant and finite scrapping age $T(t) = J(t) = \overline{T}$ (Terborgh-Smith result)[8]. Boucekkine *et al.* (1998) considered a model equivalent to our case with constant returns to scale ($\alpha = 1$). Following Van Hilten (1991), they presented a sufficient condition for the existence of a particular BGP with both constant scrapping age and constant available energy supply.[9] For the case of decreasing returns to scale ($0 < \alpha < 1$), we find that an analytical proof of the existence of such a BGP, using Van Hilten's technique, is not possible[10] (Pérez-Barahona and Zou (2003)). Moreover, it is not difficult to check that an alternative BGP, with not constant scrapping age, is not compatible with constant growth of the other endogenous variables.

As a consequence, in order to compare constant and decreasing returns to scale, we present the necessary conditions of our BGP for both constant and decreasing returns to scale.

[7]The available energy supply is a flow (exogenous) variable; for example, petrol or any petroleum refinery product to generate energy. Here we do not explicitly treat extraction sector either producer countries.

[8]Such an equilibrium is well known in the economic literature; for example, P.K. Bardhan (1969) and Boucekkine *et al.* (1997).

[9]They assume a technology that saves labour instead of energy, with constant (exogenous) labour supply. Also, intermediate good sector and symmetric equilibrium are assumed.

[10]Observe that $T(t)$ is forward-looking, but depends on its own value in a particular and endogenous point of time. This type of variable is not standard in economic models.

3.2 Necessary Conditions

For the general case $0 < \alpha \leq 1$, we get from the necessary condition of the household problem and along the BGP

$$r(t) = \rho + \theta\gamma_c = \text{constant} = r^* \tag{10}$$

and

$$e^{-\int_t^\tau r(z)dz} = e^{-r^*(\tau - t)} \tag{11}$$

where γ_c is the growth rate of consumption.

Taking (9) in (8), differentiating with respect t and rearranging terms, we obtain:

$$(e^{\gamma T} - 1) - \frac{\gamma}{\gamma_{P_e} - r^*}(e^{(\gamma_{P_e} - r^*)J} - 1) = \frac{r^*}{\overline{P}_e} e^{(\gamma - \gamma_{P_e})t} \tag{12}$$

where γ_{P_e} and $\overline{P}_e$ are, respectively, the growth rate and the level of the energy prices. The LHS is constant for any t in the BGP, and the RHS is a function of t. So the equality holds if and only if $\gamma = \gamma_{P_e}$. As in the standard growth model, this result states that, in terms of energy saving, energy prices grow at the same rate as productivity. Moreover, Boucekkine and Pommeret (2004)[11] observe that this result can be justified in the context of intertemporal equilibrium model of optimal extraction of a non-renewable resources.

By definition of $K(t)$, we have along the BGP that

$$K(t) = \begin{cases} \frac{\bar{i}}{\gamma_i}(1 - e^{-\gamma_i \overline{T}})e^{\gamma_i t} & \text{if } \gamma_i > 0 \\ i^* \overline{T} & \text{if } \gamma_i = 0 \end{cases} \tag{13}$$

where $i(t) = \bar{i}e^{\gamma_i t}$. Then, the growth rate of investment (γ_i) and the growth rate of capital stock (γ_K) are equal.

Moreover, by (9) and $\gamma = \gamma_{P_e}$

$$A\alpha K(t)^{\alpha - 1} = \overline{P}_e e^{\gamma \overline{T}} \tag{14}$$

[11] Our result $\gamma_{P_e} (= \gamma) < r^*$ is also consistent with the assumption made in Boucekkine and Pommeret (2004) about the growth rate of energy prices lower than the interest rate. Indeed, they point out that if $\gamma_{P_e} > r^*$ the firm would have an incentive to infinitely get in debt to buy an infinite amount of energy.

where $P_e(t) = \overline{P}_e e^{\gamma\overline{T}}$. Substituting (13) into (14) yields

$$e^{\gamma_i(\alpha-1)t} = \frac{\overline{P}_e e^{\gamma\overline{T}}}{A\alpha\overline{K}^{\alpha-1}} \tag{15}$$

It is easy to see that (15) holds if and only if $\alpha = 1$ and/or $\gamma_i = 0$. Then, at this point, we have to distinguish between constant and decreasing returns to scale. If we have decreasing returns to scale ($0 < \alpha < 1$) then $\gamma_i = 0$. However, for the case of constant returns to scale ($\alpha = 1$) γ_i is undetermined *a priori*.

3.2.1 Constant returns to scale

$\alpha = 1$ can not ensure long run growth in our model because of the endogenous scrapping decision and the minimum energy requirement to use a machine. Now, we are going to study the necessary conditions for our BGP according to the different values of γ_i.

Let us assume that the energy market is in equilibrium along the BGP, energy demand equals energy supply ($e_s(t)$). We make a distinction between two possible scenarios. There is an optimistic scenario with constant available energy supply. However, there is a gloomy situation where the available energy supply is decreasing[12].

Case A: $\gamma_i = 0$

From equation (5) we get the energy demand along the BGP

$$e(t) = \frac{\overline{i}}{\gamma}(e^{\gamma\overline{T}} - 1)e^{-\gamma t} \tag{16}$$

A.1 Constant available energy supply $e_s(t) = \overline{e}_s$

Equalizing $e(t) = \overline{e}_s$ in equation (16), we get

$$\frac{\overline{i}}{\gamma}(e^{\gamma\overline{T}} - 1)e^{-\gamma t} = \overline{e}_s \tag{17}$$

Since RHS is constant, LHS has to be constant. This is impossible because $\gamma > 0$ and $\overline{T}$ is finite. Then, we get a contradiction. In case B we are going

[12]There is no point in assuming increasing available energy supply, since we are considering resources subject to exhaustion. The most optimistic scenario is constant available energy supply.

to study $\gamma_i \neq 0$

A.2 Decreasing available energy supply $e_s(t) = \overline{e}_s e^{-\gamma_{e_s} t}$, with $\gamma_{e_s} > 0$

Equalizing $e(t) = \overline{e}_s e^{-\gamma_{e_s} t}$ in equation (16) yields

$$\frac{\overline{i}}{\gamma}(e^{\gamma\overline{T}} - 1)e^{-\gamma t} = \overline{e}_s e^{-\gamma_{e_s} t} \tag{18}$$

Then, γ has to equal γ_{e_s} to have BGP. This means that the economy chooses a growth rate of energy saving technical progress equal to the decrease rate of energy supply. As a consequence:

$$i(t) = i^* = \frac{\overline{e}_s \gamma}{e^{\gamma\overline{T}} - 1} \tag{19}$$

Since $\gamma_i (= \gamma_K) = 0$, from the production function $y(t) = AK(t)$, $\gamma_y = \gamma_K = 0$.

From the budget constraint $y(t) = c(t) + i(t)$, $\gamma_c = 0$.

Case B: $\gamma_i \neq 0$

Taking $i(t) = \overline{i} e^{\gamma_i t}$ in equation (5), the energy demand along the BGP is given by

$$e(t) = \begin{cases} \frac{\overline{i}}{\gamma_i - \gamma}(1 - e^{-(\gamma_i - \gamma)\overline{T}})e^{(\gamma_i - \gamma)t} & \text{if } \gamma_i \neq \gamma \\ \overline{i}\overline{T} & \text{if } \gamma_i = \gamma \end{cases} \tag{20}$$

B.1 Constant available energy supply $e_s(t) = \overline{e}_s$

If $\gamma_i \neq \gamma$, equalizing $e(t) = \overline{e}_s$ in equation (20), we obtain

$$\overline{e}_s = \frac{\overline{i}}{\gamma_i - \gamma}(1 - e^{-(\gamma_i - \gamma)\overline{T}})e^{(\gamma_i - \gamma)t} \tag{21}$$

Similar to the case A.1, we get a contradiction because $\gamma > 0$ and $\overline{T}$ is finite.

If $\gamma_i = \gamma$, equation (20) yields

$$\overline{e}_s = \overline{i}\overline{T} \tag{22}$$

Hence, in this case we can have BGP with $\gamma_i (= \gamma_K) = \gamma$ Moreover, as $y(t) = AK(t)$ then $\gamma_y = \gamma_K (= \gamma)$. From the budget constraint $y(t) = c(t) + i(t)$, we also achieve that $\gamma_c = \gamma$.

B.2 Decreasing available energy supply $e_s(t) = \overline{e}_s e^{-\gamma_{e_s} t}$, with $\gamma_{e_s} > 0$

If $\gamma_i \neq \gamma$, equalizing $e_s(t) = \overline{e}_s e^{-\gamma_{e_s} t}$ in equation (20), we find

$$\overline{e}_s e^{-\gamma_{e_s} t} = \frac{\overline{i}}{\gamma_i - \gamma}(1 - e^{-(\gamma_i - \gamma)\overline{T}})e^{(\gamma_i - \gamma)t} \tag{23}$$

It is straightforward to see that a BGP is possible if $\gamma_i = \gamma - \gamma_{e_s}$ Furthermore, if $\gamma > \gamma_{e_s}$ our economy has long run growth because $\gamma_i = \gamma - \gamma_{e_s} > 0$[13]. However, if $\gamma = \gamma_{e_s}$ we get that $\gamma_i = 0$ which contradicts our initial statement. Finally, for the case $\gamma < \gamma_{e_s}$ our economy has no positive long run growth; indeed, $\gamma_i = \gamma - \gamma_{e_s} < 0$

If $\gamma_i = \gamma$, from equation (20) we get

$$\overline{e}_s e^{\gamma_{e_s} t} = \overline{i}\overline{T} \tag{24}$$

However, this is impossible because $\gamma_{e_s} > 0$ and $\overline{T}$ is finite.

To sum up our results, we establish the two following propositions for constant returns to scale:

Proposition 1 *Along the balanced growth path, assuming $\alpha = 1$, $e_s(t) = \overline{e}_s$ and $\gamma < \rho$,*

1. *the interest rate $r(t) = r^* = \rho + \theta\gamma$;*
2. *the growth rate of energy prices equals the growth rate of energy saving technical progress ($\gamma_{P_e} = \gamma$);*
3. *the growth rate of investment and capital stock are equal to the growth rate of energy saving technical progress ($\gamma_i = \gamma_K = \gamma$);*
4. *the growth rate of final good output equals the growth rate of energy saving technical progress ($\gamma_y = \gamma$);*
5. *the growth rate of consumption equals the growth rate of energy saving technical progress ($\gamma_c = \gamma$).*

[13] $\gamma_K = \gamma_i = \gamma - \gamma_{e_s} > 0$ Since $y(t) = AK(t)$, then $\gamma_y = \gamma_K = \gamma - \gamma_{e_s}$. From the budget constraint $y(t) = c(t) + i(t)$, we get that $\gamma_c = \gamma - \gamma_{e_s}$

Proposition 2 *Along the balanced growth path, assuming* $\alpha = 1$, $e_s(t) = \overline{e}_s e^{-\gamma_{e_s} t}$ *and* $\gamma < \rho$,

1. *the interest rate* $r(t) = r^* = \rho$*;*
2. *the growth rate of energy prices equals the growth rate of energy saving technical progress* ($\gamma_{P_e} = \gamma$)*;*
3. *if* $\gamma = \gamma_{e_s}$*, then*
   - *(3.1) there is no growth in the investment and the capital stock* ($\gamma_i = \gamma_K = 0$)*,*
   - *(3.2) the growth rate of final good output is zero* ($\gamma_y = 0$)*,*
   - *(3.3) there is no growth in the consumption* ($\gamma_c = 0$)*;*
4. *if* $\gamma > \gamma_{e_s}$*, then*
   - *(4.1) there is positive growth in the investment and the capital stock* ($\gamma_i = \gamma_K = \gamma - \gamma_{e_s} > 0$)*,*
   - *(4.2) the growth rate of final good output is zero* ($\gamma_y = \gamma - \gamma_{e_s}$)*,*
   - *(4.3) there is no growth in the consumption* ($\gamma_c = \gamma - \gamma_{e_s}$)*;*
5. *if* $\gamma < \gamma_{e_s}$*, then the economy decreases is in the rate* $\gamma_{e_s} - \gamma$*.*

We have to point out that constant returns to scale, in an optimistic scenario (*i.e.*, $e_s(t) = \overline{e}_s$), generates exogenous growth with the same rate as the growth of (exogenous) energy saving technical progress (γ). However, if a gloomy scenario is assumed (*i.e.*, $e_s(t) = \overline{e}_s e^{-\gamma_{e_s} t}$) our model can achieves long run growth with rate $\gamma - \gamma_{e_s}$. If the growth rate of the energy saving technical progress is greater than the decreasing rate of energy supply, the growth is positive. However, if the growth rate of the energy saving technical progress is equal or lower than the decreasing rate of energy supply, we get no or negative growth respectively. The reason is the following. First of all, we consider a BGP with constant and finite scrapping age following the Terborgh-Smith result. Second, energy supply has two effects over the economy. On the one hand, through the energy prices; and on the other, through the Leontieff production function, which model the idea of minimum energy requirement to use a machine. When we consider constant available energy supply (optimistic scenario), following our definition of BGP, we are going to have long run growth because of the exogenous energy saving technical progress (in this case energy prices increases since the economy grows at the growth rate of the exogenous energy saving technical progress). However, in the case of decreasing available energy supply (gloomy scenario) the situation is quite different. Energy prices increase because of the decreasing available energy supply. Now, if the energy saving technical progress exactly compensates the decreasing available energy supply ($\gamma = \gamma_{e_s}$), then there is no long run growth, despite of having constant returns to scale (see equation (18)), because there is constant replacement and the energy supply is going to affect growth negatively through the minimum energy requirement (Leontieff technology). However, if the energy saving technical progress exceeds the decreasing energy supply, the second effect is also avoided and our economy can depict growth at a rate that is the difference between the growth rate of energy saving technical progress and the decreasing rate of energy supply, $\gamma - \gamma_{e_s}$. Finally, if the energy saving technical progress is not stronger enough to offset the decreasing energy supply, our economy decreases.

3.2.2 Decreasing returns to scale

As before, equation (15) holds if and only if $\alpha = 1$ and/or $\gamma_i = 0$. Since now we consider decreasing returns to scale ($0 < \alpha < 1$), the growth rate of investment has to be zero. As a consequence, $\gamma_K = 0$ because $\gamma_i = \gamma_K$.

Considering the energy market in equilibrium along the BGP, from equation (5) we conclude that a BGP is only possible under a gloomy scenario (*i.e.*, $e_s(t) = \overline{e}_s e^{-\gamma_{e_s} t}$) with $\gamma = \gamma_{e_s}$ Then

$$i(t) = i^* = \frac{1}{\overline{\overline{T}}} \left(\frac{\overline{P_e} e^{\gamma_{e_s} \overline{T}}}{A\alpha} \right)^{\frac{1}{\alpha - 1}} \tag{25}$$

Since $y(t) = AK(t)^\alpha$ and $\gamma_i = 0$, from equation (13) we get $y(t) = y^* = A(i^*\overline{T})^\alpha$. Hence, $\gamma_y = 0$. Considering the budget constraint along the BGP, it is straightforward to show that $\gamma_c = 0$.

Then, we have the following proposition:

Proposition 3 *Along the balanced growth path, assuming* $0 < \alpha < 1$, $e_s(t) = \overline{e}_s e^{-\gamma t}$ *and* $\gamma < \rho$,

1. *the interest rate* $r(t) = r^* = \rho$;

2. *the growth rate of energy prices equals the growth rate of energy saving technical progress* ($\gamma_{P_e} = \gamma$);

3. *there is no growth in investment and the capital stock* ($\gamma_i = \gamma_K = 0$);

4. the growth rate of final good output is zero ($\gamma_y = 0$);

5. there is no growth in consumption ($\gamma_c = 0$).

We have to remark that this case has no growth in the long run. This behavior is explained, on the one hand, by the assumption of decreasing returns to scale and, on the other hand, because here we get that both the scrapping age and the exogenous energy saving technical progress are not strong enough to overcome these decreasing returns. The reason is the following. Our framework considers a CRRA instantaneous utility function, and as a consequence, the interest rate is constant in the long run. Then, consistent with the Terborgh-Smith result, the scrapping age is also constant along the BGP. Taking the optimal investment rule in the long run

$$\begin{gathered}\alpha A e^{\rho t} \int_t^{t+\overline{T}} \left(\int_{\tau-\overline{T}}^{\tau} i(z) dz \right)^{\alpha-1} e^{-\rho \tau} d\tau = \\ 1 + \overline{P}_e \frac{1}{\rho - \gamma} (1 - e^{-(\rho-\gamma)\overline{T}})\end{gathered} \tag{26}$$

it is straightforward to show that the discounted operation cost is constant because the effect of the energy saving technical progress (γ) is offset by the decreasing available energy supply. Hence, as the marginal purchase cost(=1) remains constant, the investment has also to be constant along the BGP. The economic interpretation of this result is similar to the case of constant returns to scale. However, in the case of decreasing returns to scale things are much worse. Our result is consistent with the partial equilibrium model of Boucekkine and Pommeret (2004), which also depicts no growth along the BGP. Furthermore, considering a canonical vintage capital model with arrowian learning by doing technical progress, d'Autume and Michel (1993) get that decreasing returns to scale "kills" growth in the long run. Our model is mathematically close to their economy taking energy instead of labor.

4 Concluding remarks

We analyzed the hypothesis about the effectiveness of energy saving technologies to reduce the trade-off between economic growth and energy preservation. In order to incorporate the role of technology replacement, we developed a general equilibrium model, where the output is produced by a vintage capital technology with endogenous scrapping rule. New vintages obsolete old machines because of their lower energy requirements. Constant and decreasing returns to scale are distinguished to develop a comparative study.

Under constant returns to scale and optimistic context (constant available energy supply), long run growth is possible (Proposition 1). In this case, the (exogenous) growth rate of the economy equals the (exogenous) growth rate of energy saving technical progress. However, considering a more realistic situation of gloomy scenario (decreasing available energy supply), our economy only can achieve long run growth if the growth rate of the energy saving technical progress excess the decreasing rate of the energy supply (Proposition 2). Here, the growth rate of our economy is given by the difference between the growth rate of energy saving technical progress and the decreasing rate of the energy supply ($i.e.$, $\gamma - \gamma_{e_s}$). Furthermore, when we assume decreasing returns to scale, the economy achieves BGP only for the gloomy case; nevertheless, our economy does not exhibit growth in the long run (Proposition 3). However, we could escape from the decreasing returns to scale incorporating additional elements such as learning-by-doing, human capital, subsidies, etc[14]. The constant scrapping age and the reduced availability of energy (which affects the economy through the energy prices and the minimum energy requirements) explain our results in both constant and decreasing returns to scale.

In conclusion, we could have compatibility between economic growth and energy preservation adopting energy saving technologies. However, the growth rate of the energy saving technical progress has to be greater than the decreasing rate of the energy supply to ensure a positive long run growth. Since energy saving technical progress is exogenous in our model, an interesting extension is considering that the economy endogenously decides the growth rate of energy saving technical progress. An R&D sector of energy saving technologies could be a good way to study this problem.

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[14]Notice that, in our model, the investment only considers energy saving issues.

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