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A dynamic quantitative macroeconomic model of bank runs

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DISCUSSION PAPER

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A dynamic quantitative macroeconomic model of bank runs[1]

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Abstract

We study the macroeconomic effects of systemic bank runs in a neoclassical model with a microfounded banking system. In every period, the banks provide insurance against some idiosyncratic liquidity shocks, but the possibility of sunspot-driven bank runs distorts the equilibrium allocation. In a quantitative exercise, we find that the banks, when the probability of a run is sufficiently low, choose a contract that is not run-proof, and satisfy an equal service constraint. In equilibrium, a shock to the probability of a run leads to a maximum drop in GDP of 5.6 percent, and a maximum welfare loss of 0.17 percent.

Keywords: financial intermediation, bank runs, welfare costs, calibration
JEL Classiffication: E21, E44, G01, G20

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1 Introduction

What are the macroeconomic effects of systemic bank runs, and how big are they? The interest in this question comes from the observation that systemic bank runs are not a phenomenon of the past: in fact, they may occur whenever long-term illiquid assets are financed by short-term liquid liabilities, and the providers of short-term funds all lose confidence in the borrower's ability to repay, or are afraid that other lenders are losing their confidence. The Argentinian bank run of 2001-2002 is a typical example of one of those episodes, and led to a massive drop in GDP of around 11 percentage points. Furthermore, there exists a wide consensus that the U.S. money market funds have experienced a run-like episode after the bankruptcy of Lehman Brothers in 2008[1] and, more generally, that the financial crisis of 2007-2009 can be interpreted as a systemic run of financial intermediaries on other financial intermediaries.[2] The empirical literature has showed how big the costs of the last crisis were, with a drop in GDP of around 4.8 percentage points, followed by a recovery of around 6 years (Reinhart and Rogoff, 2014). Yet, these numbers do not allow us to identify the mechanism through which the financial system affects the real economy, and this is the reason why we need a formal theory of systemic bank runs. Accordingly, the literature on the economics of banking has extensively analyzed bank runs, but mainly in static partial-equilibrium frameworks, that are not suitable for any formal quantitative analysis. Thus, in the present work, we overcome the limitations of both the empirical and the theoretical literature, and study the macroeconomic effects of systemic bank runs in a dynamic quantitative model with a fully microfounded banking system.

Our model is based on three building blocks, all considered standard workhorses in their own field. The first one is the neoclassical growth model: an infinite horizon, general equilibrium, dynamic model populated by households and firms. As generally known, this model lacks a proper role for a banking system, as the households provide capital to the firms without any intermediation. This leads to our second building block: the theory of banking proposed by Diamond and Dybvig (1983). In this environment, banks provide insurance to their depositors against the realization of an idiosyncratic private liquidity shock, that makes them willing to consume before the maturity of their investment. We choose this environment for two reasons: first, because it provides a rationale for the existence of the banking system, as a mechanism to decentralize the efficient provision of insurance against idiosyncratic risk; second, because it also provides a formal characterization of bank runs. In that respect, our third building block is the seminal paper by Cooper and Ross (1998) – and its refinements by Ennis and Keister (2006) – where bank runs are modeled as self-fulfilling crises, during which all depositors close down their accounts, because they expect everybody else to do the same, forcing the banks to

[1] In just two days, from September 15 to September 17, their total retail funds dropped by more than 4.5 percentage points (source: ICI).

[2] In a recent op-ed on the *New York Times* (March 24, 2012), Tyler Cowen states that "[...] the single most important development to emerge from America's financial crisis [is the fact that] the age of the bank run has returned". See Gorton and Metrick (2012) for a further analysis of this argument.

go bankrupt.[3]

More formally, our economy is populated by a continuum of infinitely-lived identical agents who, at each point in time, are hit by an idiosyncratic private liquidity shock, that makes them willing to consume either in an intermediate period, that we call "night", or when production takes place, in the "morning". To hedge against these shocks, the agents form financial coalitions – or, more commonly, "banks" – that operate in a competitive environment, and allow them to pool the idiosyncratic risk among themselves: the agents make a deposit, and the banks offer them a contract, stating how much they can withdraw each morning and each night. The banks, in turn, finance the contract by investing in liquidity, that they store for the night withdrawals, and in credit, that they provide to a perfectly competitive neoclassical production sector. In this environment, the banks, in equilibrium, offer a contract providing perfect intratemporal risk sharing, i.e. they equalize morning and night consumption in every period, and choose a portfolio of liquidity and credit satisfying an Euler equation, i.e. such that the marginal rate of substitution between consumption in the night of date t and in the morning of date $t+1$ is equal to the marginal rate of transformation of the production technology.

We extend this environment with the introduction of bank runs. To this end, we show that this economy also exhibits an equilibrium where all agents, regardless of the realization of their idiosyncratic shocks, withdraw at night, and that this equilibrium exists if the agents know that the banks do not hold a sufficient amount of liquidity to honor the contract with all of them. Whenever a run equilibrium and a no-run equilibrium coexist, the agents coordinate among them, and choose to run in accordance with the realization of a "sunspot", i.e. an extrinsic event happening with some given probability. This is an assumption that, at the cost of introducing an exogenous parameter, completely uncorrelated with any fundamental of the economy, allows us to model bank runs as self-fulfilling prophecies. Arguably, this is an important point, as many bank runs in the past – and, as we mentioned above, also in the recent financial crisis – arose as a consequence of a coordination failure that led to a generalized panic.[4]

The banks, in turn, take into account the sunspot selection mechanism, and choose, fol-

[3] Our focus is on one specific role (which we believe is key) played by the banking system in the real economy: the management of risk, to hedge against idiosyncratic uncertainty. In this sense, here we do not consider the banks as monitors of investments or producers of information, as in Holmstrom and Tirole (1998) and Diamond and Rajan (2001). Similarly, in modeling financial crises as bank runs, we do not take into account crises arising from shocks to the fundamentals of the economy, like in Allen and Gale (2004).

[4] An alternative to the equilibrium-selection mechanism based on sunspots would be a dynamic version of the "global game" approach of Carlsson and van Damme (1993), according to which the agents receive a noisy signal about the state of the economy, and run if the signal is lower than an endogenous threshold. Goldstein and Pauzner (2005) successfully apply this approach to a static Diamond-Dybvig model, to guarantee the uniqueness of its equilibrium. However, Angeletos et al. (2007) show that a dynamic global game would exhibit a unique equilibrium only if the agents do not learn from their signals: in fact, a global game with learning would still exhibit arbitrarily many equilibria, under the very same conditions that guarantee the uniqueness of the equilibrium in a static environment.

lowing an expected-welfare criterion, between a "run-proof" contract, according to which they hold a sufficient amount of liquidity to pay the night withdrawals of every agent even in the case of a run, and a contract that allows for a potential run. In this second case, the banks also choose how to serve their depositors. In particular, we distinguish between two service protocols: one is the "sequential service constraint" (Wallace, 1988), according to which the banks pay the depositors on a first-come-first-served basis, until the resources are completely exhausted; the other is, instead, the "equal service constraint" (Allen and Gale, 1998), according to which all depositors who participate in the run receive the same share of the available resources. We choose to analyze both cases because there is evidence of both behaviors in the real world: while the sequential service constraint is generally introduced to model the fact that withdrawals happens sequentially, the equal service constraint takes into account that, at a run, the agents try to withdraw all at the same time, in which case they are paid pro-rata.[5]

We characterize the run-proof equilibrium and the run equilibria, both with sequential service and equal service. In the run-proof equilibrium, the banks, in order to be able to serve all depositors, react to a positive probability of a run by lowering the amount of night consumption offered in the contract and by increasing their holding of liquidity, that they partially roll over to the following period. Thus, a positive probability of a run gives rise to a credit tightening, with a corresponding drop in GDP, followed by a recovery.

In the run equilibrium with sequential service, instead, the banks react to a positive probability of a run by increasing night consumption and, as a consequence, the amount of liquidity in portfolio: this happens because, at a run, the objective of increasing the welfare of the depositors, that would motivate the banks to increase night consumption, dominates the objective of serving the highest possible fraction of them, that would instead motivate them to lower night consumption. Thus, a run gives rise to a credit tightening, that leads, in turn, to a drop in GDP. Importantly, the banks do not tighten credit ex post, after the realization of a run, but ex ante, in expectation of a run. Thus, it is the *possibility* of a run that pushes the banks to tighten credit. This also means that the fraction of depositors served when the run equilibrium is selected is equal to the fraction served when the no-run equilibrium is selected. Therefore, the ex-post realization of a run does not affect the amount of credit provided to the production sector, but only the fraction of depositors that are not served and consume zero, with a substantial effect on total welfare.

Finally, in the run equilibrium with equal service, the banks, at least qualitatively, behave in a similar way as in the case with sequential service: they react to a positive probability of a run by increasing night consumption and tightening credit. However, there is a key difference in the way the banks hold excess liquidity, i.e. an amount of liquidity higher than what necessary to finance night consumption. In equilibrium, the excess liquidity is positive whenever its marginal costs are equal to its marginal benefits. On one side, no matter what service protocol

[5]Gaytan and Ranciere (2006) mention that this was the way in which the Argentinian banks, for example, paid their depositors during the run of 2002.

the banks choose, the marginal costs of excess liquidity arise from a tightening of the budget constraint when the no-run equilibrium is selected. This means that the marginal costs are increasing in (i) the *marginal* utility of night consumption, (ii) in the probability that the no-run equilibrium is selected, and (iii) in the opportunity cost of holding excess liquidity, in terms of the forgone return on capital. On the other side, the marginal benefits of excess liquidity arise at a run, so they are always increasing in the probability that the run equilibrium is selected. However, these benefits change with respect to the service protocol that the banks choose. With sequential service, the excess liquidity allows the banks to serve more depositors during a run, so its marginal benefits are increasing in the *average* utility of night consumption. With equal service, instead, the excess liquidity allows the banks to provide higher consumption during a run, so its marginal benefits are increasing in the *marginal* utility of night consumption. If the utility function is sufficiently concave (i.e. the depositors are sufficiently risk averse), the marginal utility of night consumption is higher than the average utility with sequential service, but lower than the marginal utility with equal service. Thus, in order to have positive excess liquidity with sequential service, the probability that the no-run equilibrium is selected and the return on capital must both be sufficiently *low*. However, in order to have positive excess liquidity with equal service, the opposite must be true: the probability that the no-run equilibrium is selected and the return on capital must both be sufficiently *high*.

As we cannot derive a closed-form solution for the expected welfare, we rely on a numerical analysis to characterize the complete equilibrium. We assume that the economy starts in the steady state of the no-run equilibrium (where runs are ruled out by assumption), and is hit by a one-period shock: the probability of the realization of the sunspot spikes to a positive number, and then goes back to zero. Then, we calculate the welfare difference (in consumption equivalents) between the banking equilibria and the no-run equilibrium. To this end, we calibrate the parameters of the model to the U.S. economy. This is an important and interesting task per se, since, to the best of our knowledge, we are the first ones to calibrate the probability of the idiosyncratic shock of a Diamond-Dybvig economy using the U.S. financial accounts.

With these numbers in hand, we quantitatively evaluate how a run affects the real economy in the three banking equilibria. The welfare costs of a run are always increasing in the probability of the realization of the sunspot, as the larger the probability of a run is, the more the banks distort the intertemporal allocation of capital. Moreover, the welfare costs of the equilibrium with sequential service are always higher than those of the equilibrium with equal service, and lower than those of the run-proof equilibrium only for extremely low values of the probability of the realization of the sunspot. Hence, the banks will always prefer equal service over sequential service. These results are based on two features of the equilibrium with sequential service: first, when the economy is hit by a shock to the probability of the realization of the sunspot, a non-negligible fraction of agents consume zero, and have zero utility; second, although the equilibrium credit tightening, and the corresponding drop in GDP, are lower in

this case than in the other two, the transition back to the steady state is slower, because in the run-proof equilibrium, as well as in the equilibrium with equal service, the banks roll over some excess liquidity, which allows for a faster recovery.

Our findings show that the equilibrium with equal service is preferred to the run-proof equilibrium when the shock to the probability that the run equilibrium is selected is below 5 per cent. In that sense, we provide a quantitative characterization of the threshold, whose existence was proved by Cooper and Ross (1998), beyond which the baks avoid runs altogether. Moreover, we find that the welfare costs of a positive probability of a run, depending on the magnitude of the shock to the probability of the realization of the sunspot, are of between 0.0006 and 0.1744 percentage points, and are increasing in the degree of relative risk aversion of the depositors. This happens because the higher the relative risk aversion is, the more the banks are willing to distort the allocation of resources, and smooth the ex-post consumption profiles of the agents between a realized and an unrealized run. Finally, a positive shock to the probability that the run equilibrium is selected leads to a drop in GDP of between 0.0011 and 5.64 percentage points. Again, this effect is increasing in the degree of relative risk aversion of the agents.

The rest of the paper is organized as follows: in section 2, we relate our work to the existing literature; in section 3, we define the environment, and characterize, as a benchmark, the no-run banking equilibrium; in section 4, we introduce bank runs, and characterize the three equilibria; in section 5, we calibrate the model to the U.S. economy, and report the results of our quantitative exercises; finally, we conclude in section 6.

2 Related Literature

In the last years, macroeconomics has shifted its attention to study the role of financial frictions as amplifiers of shocks to the real economy. In that respect, many economists are extending their agendas to introduce some microfoundations for the financial system in otherwise standard macroeconomic models, that could be used for policy analysis. Gertler and Karadi (2011) and Gertler and Kiyotaki (2013) are two notable examples of this line of research, where the banks operate as intermediaries between the households (the lenders of capital) and the firms (the borrowers of capital) and suffer from some kind of moral hazard that gives space to the intervention of the central bank. We see our contribution as a further extension of this class of models to formally assess the role of the financial system in a fully microfounded environment.

Starting from the seminal work of Diamond and Dybvig (1983), a series of papers provides the microfoundations for the process of financial intermediation, based on the ability of the intermediaries (i.e., the banks) to pool risk and offer the efficient level of insurance against idiosyncratic shocks. However, already in their original contribution, Diamond and Dybvig notice that this environment exhibits a multiplicity of equilibria, i.e. that there also exists a run equilibrium, where every depositor withdraws from her account because she expects

everybody else to do the same. Cooper and Ross (1998), in the same static environment, offer a more sophisticated characterization of the run equilibrium, and show how the banks change their portfolio allocation in reaction to the possibility of this event.

The Diamond-Dybvig model has been extensively used in dynamic environments. Bencivenga and Smith (1991) develop a growth model with a banking system and overlapping generations of two-period lived depositors, that has been the base for further analysis of the provision of intertemporal risk sharing (Qi, 1994; Allen and Gale, 1997). Ennis and Keister (2003) and Gaytan and Ranciere (2006) provide two recent examples of models where financial crises, in the form of bank runs, affect the intertemporal allocation of resources and harm long-run growth. With respect to these, our work provides some similar theoretical results, but in a representative-agent setting, that we choose for three reasons: first, because, in a dynamic OLG model, the banks provide insurance against the risk of consuming when young versus when old, while we believe that their activity is more targeted towards short-term shocks; second, because, as showed by Qi (1994), in an OLG model an infinitely-lived bank would invest only in capital, while we are interested in the portfolio composition between capital and liquidity; third, because, as in Ennis and Keister (2003), we cannot get to a closed-form characterization of the equilibrium allocation, but, instead of providing a numerical solution with some a priori parameters, we prefer a formal calibration, that the representative-agent structure of the model makes possible.

3 A Dynamic Model of Banks

3.1 Environment

Time is infinite and discrete, and every period is divided into two sub-periods, that we label 1 and 2, and call for simplicity "day" and "night". The economy is populated by a unitary continuum of infinitely-lived agents, who are endowed with one unit of time in each period, and want to maximize the present discounted value of an infinite stream of utility:

$$W = E \sum_{t=0}^{\infty} \beta^t U(c_{2t}, c_{1t+1}, \theta_t), \tag{1}$$

where $\beta < 1$ is an intertemporal discount factor.

All agents in the economy are affected, at every point in time, by some idiosyncratic uncertainty, that hits them in the form of a preference shock. Being ex-ante equal, every agent draws a type $\theta_t \in \{0, 1\}$, where $0 < \pi < 1$ is the probability of being of type 1, and $1 - \pi$ is the probability of being of type 0. The preference shocks are private information, and are independent and identically distributed across the agents. Therefore, by the law of large numbers, the cross-sectional distribution of the types is equivalent to their probability distribution: π is the fraction of agents who turn out to be of type 1, and $1 - \pi$ is the fraction of agents who turn out to be of type 0. The role of the idiosyncratic shocks is to affect the

sub-period when the agents enjoy consumption. This happens according to the utility function:

$$U(c_{2t}, c_{1t+1}, \theta_t) = \theta_t u(c_{2t}) + \beta(1-\theta_t)u(c_{1t+1}). \tag{2}$$

Thus, depending on the realization of the shock, each agent either consumes during the night of the same period (when $\theta_t = 1$) or in the day of the following one (when $\theta_t = 0$) and, in that respect, we talk about "night consumers" and "day consumers", respectively. The felicity function $u(c)$ is increasing, strictly concave, with $u(0) = 0$, and has a degree of relative risk aversion strictly larger than 1.

In order to hedge against the idiosyncratic shocks, the agents have access to two real technologies. The first is a storage technology, that we call "liquidity", yielding one unit of consumption in every sub-period for each unit invested in the previous one. The second technology is a neoclassical production function $Y_t = F(K_t, A_t L_t)$: it is employed by a large number of competitive firms, and features a labor-augmenting technological process A_t, that grows exponentially at the exogenous rate γ, with A_0 given. The amount of labor L_t is inelastically supplied by the agents at each point in time, and yields a return $w_t = A_t F_2(K_t, A_t L_t)$, i.e. equal to its marginal productivity. We assume that physical capital needs "time to build": the amount invested at time t matures in the morning of $t+1$, yields a return equal to its marginal productivity, and then depreciates at rate d, hence $R_t = F_1(K_t, A_t L_t) + 1 - d$.

To access the two real technologies, in every period the agents form financial coalitions, that we call "banks", to pool risk among themselves.[6] A profit-maximizer bank collects the deposits, and performs two tasks: (i) it invests in liquidity on behalf of their depositors; (ii) it provides lines of credit to the firms, for them to get the physical capital that they need for the production. The lines of credit can be liquidated at night (i.e., before being invested), using a "liquidation technology" that allows the banks to recover $r < 1$ units of consumption for each unit liquidated.[7] Perfect competition and free entry ensure that the bank profits are zero in equilibrium, hence the banks solve the dual problem of maximizing the expected welfare of their depositors, subject to the budget constraints.

In what follows, we study symmetric pure-strategy equilibria, so that, without loss of generality, we can focus our attention on the behavior of a representative bank. The bank exploits the law of large numbers, and solves the following problem:

$$\max_{\{c_{2t}, c_{1t+1}, \ell_t, f_{t+1}, k_{t+1}, z_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \big[\pi u(c_{2t}) + \beta(1-\pi)u(c_{1t+1})\big], \tag{3}$$

[6] We assume that the agents are "isolated", i.e. they do not interact one with each other, once they participate in a banking coalition. In fact, Jacklin (1987) shows that, if we relax this hypothesis, the banks turn out to be redundant, in the sense that the banking equilibrium does not provide more welfare than the one that the agents get in autarky. We compare the banking equilibrium with isolated agents to the autarkic equilibrium in Appendix A, and show that, in contrast to a well-known result of the static Diamond-Dybvig literature, the former Pareto-dominates the latter, even under the assumption of log-utility.

[7] Mattana and Panetti (2014) show that, in an environment where the banks can sell the claims to future capital to the depositors in a Walrasian asset market, a run equilibrium does not exist. The inclusion of a capital-good-producing sector would not significantly alter this result.

subject to the budget constraints:

$$(1-\pi)c_{1t} + \ell_t + f_{t+1} \leq w_t + R_t k_t + z_t, \tag{4}$$

$$\pi c_{2t} \leq \ell_t, \tag{5}$$

$$k_{t+1} \leq f_{t+1}, \tag{6}$$

$$0 \leq z_{t+1} \leq \ell_t - \pi c_{2t}, \tag{7}$$

and to the incentive-compatibility constraint:

$$c_{2t} \leq c_{1t+1}. \tag{8}$$

The bank maximizes the expected welfare of its depositors, by choosing an incentive-compatible deposit contract $\{c_{2t}, c_{1t+1}\}$, a portfolio allocation between liquidity ℓ_t and lines of credit f_{t+1}, the amount of credit k_{t+1} provided to the firms, and the amount of excess liquidity z_{t+1} to roll over to the following period. The budget constraint in (4) states that the bank collects as deposits the net return R_t on the credit k_t provided to the production sector in the previous period, the current salary w_t of its depositors, and the excess liquidity z_t rolled over from the previous period, and allocates them between the day consumption c_{1t} (that was chosen in period $t-1$) of $(1-\pi)$ day consumers, and the financial investment in liquidity and lines of credit. Liquidity must be sufficient to pay an amount of night consumption c_{2t} to π night consumers, and whatever amount is left goes to the following period, according to (7). To clearly point out the difference between the lines of credit and actual credit provided to the firms (which is going to become important when we introduce bank runs), we explicitly state, in equation (6), that k_{t+1} cannot be larger than f_{t+1}. Finally, since the realization of the idiosyncratic types are private information, by the Revelation Principle, the bank needs to impose the incentive-compatibility constraint (8): day consumption must be at least as large as night consumption, in order to induce truth-telling. In fact, if that was not the case, the day consumers would pretend to be night consumers, withdraw from their bank accounts, and store until the following morning.

The definition of the banking equilibrium is as follows:

Definition 1. *Given an initial amount of physical capital K_0, liquidity z_0, and day consumption c_{10}, a no-run banking equilibrium is a price vector $\{R_t, w_t\}$, a bank portfolio strategy $\{\ell_t, f_{t+1}, k_{t+1}, z_{t+1}\}$, a deposit contract $\{c_{2t}, c_{1t+1}\}$, and production inputs $\{K_t, L_t\}$ for every $t = 0, 1, \ldots$ such that, for given prices:*

- *The deposit contract and the bank portfolio strategy solve the banking problem in* (3)*;*
- *The production inputs maximize firm profits;*
- $w_t = F_2(K_t, A_t L_t)$ *and* $R_t = F_1(K_t, A_t L_t) + 1 - d$*;*
- *The credit market and the labor market clear:*

$$K_t = k_t, \tag{9}$$

$$L_t = 1. \tag{10}$$

We conclude the description of the environment by summarizing the timing of the events. In every period t: (i) during the morning, production takes place, and the firms pay out the salary w_t and the return on capital R_t; (ii) at noon, the representative bank pays the day consumption c_{1t}, collects the deposits, and decides the terms of the portfolio strategy $\{\ell_t, f_{t+1}, k_{t+1}, z_{t+1}\}$ and the banking contract $\{c_{2t}, c_{1t+1}\}$; (iii) at night, the idiosyncratic shocks are revealed, and night consumption c_{2t} takes place according to the contract.

3.2 The No-Run Banking Equilibrium

We start our analysis with the characterization of an equilibrium where bank runs are ruled out by assumption. This is the benchmark against which we compare the banking equilibrium with runs, and that allows us to formally characterize the differences between the two.[8]

We assume that $\beta R_{t+1} \geq 1$: this is necessary for the model to exhibit an equilibrium that satisfies the incentive compatibility constraint, as it will become clear when we characterize the equilibrium. It is trivial to argue that the budget constraints all hold with equality, hence we can substitute (5), (6) and (7) into (4) and get:

$$(1-\pi)c_{1t} + \pi c_{2t} + k_{t+1} + z_{t+1} \leq w_t + R_t k_t + z_t, \tag{11}$$

$$z_{t+1} \geq 0. \tag{12}$$

We attach the multipliers $\beta^t \lambda_t$ and $\beta^t \mu_t$ to (11) and (12), respectively. The first-order conditions of the program read:

$$u'(c_{2t}) = \lambda_t, \tag{13}$$

$$u'(c_{1t+1}) = \lambda_{t+1}, \tag{14}$$

$$\lambda_t = \beta R_{t+1} \lambda_{t+1}, \tag{15}$$

$$\lambda_t = \beta \lambda_{t+1} + \mu_t. \tag{16}$$

In equilibrium, the marginal benefit from increasing the current night consumption c_{2t} and the future day consumption c_{1t+1} must be equal to their shadow values, i.e. the amount λ_t and λ_{t+1} by which such increases tighten the budget constraint in t and $t+1$. Since, at each point in time, the shadow values are the same for both day and night consumption, the bank optimally provides perfect intratemporal insurance: the agents who find themselves in the condition of consuming at night receive the exact amount of consumption goods that she would have consumed during the morning of the same day, or $c_{2t} = c_{1t}$ for every t. Furthermore, the bank chooses the amount of credit so as to equalize its marginal costs, in

[8] All competitive equilibria that we characterize here, including the no-run equilibrium, are constrained efficient, in the sense that the first theorem of welfare economics always holds. However, we take the no-run equilibrium as our benchmark, as a proxy for the fact that a public authority, in full control of the money supply, can create liquidity at (almost) zero costs, and therefore avoid bank runs altogether.

terms of a tighter budget constraint at time t, to its marginal benefits, in terms of a slacker budget constraint in $t+1$. Thus, in equilibrium, it allocates its portfolio between liquidity and credit in accordance with an Euler equation, so that the marginal rate of substitution between current night consumption and future day consumption is equal to the marginal rate of transformation of the production technology:

$$u'(c_{2t}) = \beta R_{t+1} u'(c_{1t+1}). \tag{17}$$

As the felicity function $u(c)$ is strictly concave, the contract satisfying the Euler equation is incentive-compatible only if $\beta R_{t+1} \geq 1$, as we argued above. In fact, it is easy to see that, if $\beta R_{t+1} < 1$, we have that $c_{2t} > c_{1t+1}$, and the contract violates the incentive compatibility constraint (8). When $\beta R_{t+1} = 1$, instead, we have that $c_{1t} = c_{2t} = c_{1t+1}$, and consumption does not grow from period t to period $t+1$. However, for the sake of clarity, we assume that, in the rest of the paper, βR_{t+1} is strictly larger than 1, so that the incentive compatibility constraint is slack, and check whether this is true in every numerical exercise that we run in section 5. As the discount factor β is smaller than 1, this also means that the net return on capital R_{t+1} is strictly larger than 1: capital is a technology that provides a higher yield than liquidity, and is the only one that is employed to transfer resources from one period to the following. In other words, providing credit to the production sector always dominates the roll-over of excess liquidity and, in equilibrium, we have that $z_{t+1} = 0$ for every t. We summarize our findings in the following proposition:

Proposition 1. *The banking equilibrium with no runs is characterized by:*

$$u'(c_{2t}) = u'(c_{1t}), \tag{18}$$

$$u'(c_{2t}) = \beta R_{t+1} u'(c_{1t+1}). \tag{19}$$

In equilibrium, the bank does not roll over any excess liquidity, i.e., $z_{t+1} = 0$ *for every* $t = 0, 1, \ldots$.

Two things are worth noticing from this result. First, the classic result of Diamond and Dybvig (1983) applies here: the deposit contract is a mechanism to decentralize the efficient allocation of resources, in the presence of idiosyncratic private liquidity shocks. In other words, the no-run banking equilibrium is constrained-efficient. Second, the agents, at any point in time, consume exactly the same amount of the consumption good, no matter if they consume during the day or during the night, and allocate their savings in accordance with an Euler equation: in that sense, the equilibrium allocation of the no-run banking equilibrium is observationally equivalent to that of a standard neoclassical growth model. We exploit this equivalence to simplify the numerical analysis of section 5. Moreover, this result highlights the fact that adding a microfounded banking system to a general equilibrium model is a meaningful exercise, in these sense of providing some more results than a standard model without

banks, only to the extent that we introduce some financial distortion in the system, too. That is the topic of the incoming sections.

4 Bank Runs

4.1 Environment

The assumption that the realizations of the idiosyncratic shocks are private information provides a rationale for the further existence, in this environment, of a run equilibrium, where all agents withdraw c_{2t} at night, regardless of the actual realization of their idiosyncratic types. The run happens whenever all agents expect that every other agent is going to run, and know that the bank is not able to fulfill its contractual obligations with all of them. In this case, the bank, in addition to the portfolio allocation between liquidity and lines of credit, also chooses an amount D_t of lines of credit to liquidate, in order to accommodate the extra liquidity demand from the depositors. We further assume that the bank "anticipates" the run, as it modifies its investment strategy ex ante to accommodate for the possibility that the bad equilibrium is realized. As a consequence, bank runs will affect how depositors are served, and the credit provided to the firms. In other words, the possibility of a bank run distorts both the financial side and the real side of the economy.

At a run, the bank can choose to serve its depositors in accordance with two different protocols: the "sequential service constraint" and the "equal service constraint". With sequential service, the agents who withdraw at night are served on a first-come-first-served basis. Importantly, the agents can observe neither their position in line nor whether a run is under way. As a consequence, the suspension of convertibility does not prevent the existence of a run equilibrium, and the agents only accept a contract that is independent of the realization of a run.[9] With equal service, instead, the bank publicly announces that a run is under way, and all agents who are in line at the time of the announcement receive an equal share of the liquidity available at that point in time.

More formally, let $\delta_t(r)$ be the fraction of depositors served during a run. With sequential service, the budget constraint at a run reads:

$$k_{t+1} = f_{t+1} - D_t, \tag{20}$$

$$z_{t+1} + \pi c_{2t} + rD_t = \delta_t(r)c_{2t}. \tag{21}$$

The expression in (20) shows how the bank tightens credit via the liquidation technology: the amount of capital k_{t+1} lent to the firms must be equal to the available credit lines f_{t+1} minus the amount D_t liquidated. The expression in (21), instead, shows the bank budget constraint at a run: the bank uses the available liquidity ℓ_t (the sum of what is set aside

[9] This hypothesis is different from the one proposed by Green and Lin (2003), who assume that the agents can observe their positions in line, and show that the banks can offer a contract contingent on that, thus ruling out the existence of the run equilibrium.

for night consumption πc_{2t} and of the excess liquidity z_{t+1}) and the liquidation technology (which allows the bank to recover rD_t units of consumption for each D_t units liquidated) to pay an uncontingent amount of night consumption c_{2t} to $\delta_t(r)$ depositors. Rearranging (21), we obtain:

$$\delta_t(r) = \frac{z_{t+1} + \pi c_{2t} + rD_t}{c_{2t}}. \tag{22}$$

Cooper and Ross (1998) show that, by definition, a run equilibrium exists if and only if $\delta_t(r) < 1$, or $c_{2t} > (z_{t+1} + rD_t)/(1 - \pi)$. Put differently, if the depositors know that the fraction of them that can be served during a run is lower than 1, they will run.

In a similar way, the budget constraint at a run, with equal service, reads:

$$z_{t+1} + \pi c_{2t} + rD_t = c_{2t}^R, \tag{23}$$

meaning that the amount of night consumption c_{2t}^R that the agents receive at a run is equal to the available liquidity, split pro-rata among all of them. Then, a run equilibrium exists if and only if the agents know that the bank does not hold a sufficient amount of liquid assets to pay the night consumption in the case of a run, or $c_{2t} > c_{2t}^R = z_{t+1} + \pi c_{2t} + rD_t$. Notice that this is the same condition that we find when sequential service is into place. Therefore, to simplify the notation, we say that a run equilibrium exists if and only if $\delta_t(r) < 1$, regardless of the service protocol chosen.

In any case when the banking problem exhibits a run equilibrium and a no-run equilibrium at the same time, the depositors coordinate over which one to select in accordance with the realization of an extrinsic event, called "sunspot". If the sunspot is realized, which happens with an exogenous probability q, the agents choose to run, while, if the sunspot is not realized, with probability $1 - q$, they choose not to run.[10] The representative bank, in turn, knows this equilibrium-selection mechanism, and adjusts ex ante its portfolio strategy and the banking contract, so as to maximize the expected welfare of its depositors. In doing so, the bank also indirectly affects the fraction $\delta_t(r)$ of agents that are served during a run, in the case of sequential service, and the amount of consumption c_{2t}^R that the agents receive, in the case of equal service. Thus, it can effectively choose whether a run equilibrium exists or not.

Before going into the details of the modified banking problem, it is useful to recap the timing of actions. In every period t: (i) during the morning, production takes place, and firms pay out the salary w_t and the return on capital R_t; (ii) at noon, the representative bank pays the day consumption c_{1t}, collects the deposits, and decides the terms of the portfolio strategy $\{\ell_t, f_{t+1}, k_{t+1}, z_{t+1}, D_t\}$, the banking contract $\{c_{2t}, c_{2t}^R, c_{1t+1}\}$, and the service protocol in the case of a run; (iii) at night, the idiosyncratic shocks are privately revealed, the agents decide whether to run or not depending on the realization of the sunspot,[11] and the night consumption

[10] Accordingly, in what follows, we interchangeably refer to q as the probability of a run, or of the realization of the sunspot, or that the run equilibrium is selected.

[11] Despite the indisputable darkness, we allow the sunspots to be visible at night. The reader who does not feel comfortable with this assumption can relabel the sunspots "falling stars".

c_{2t} takes place according to the contract.

4.2 The Banking Problem with Runs

If we plug the definition of liquidity, $\ell_t = z_{t+1} + \pi c_{2t}$, the fraction $\delta_t(r)$ of agents served at a run from (22), and c_{2t}^R from (23), respectively, the problem of the representative bank reads:

$$\max_{\substack{\{c_{2t}, c_{1t+1}, \\ k_{t+1}, D_t, z_{t+1}\}}} \sum_{t=0}^{\infty} \beta^t \Big[(1 - q\mathcal{I}_{\delta_t(r)<1}) \big(\pi u(c_{2t}) + \beta(1-\pi) u(c_{1t+1}) \big) + \\ + q\mathcal{I}_{\delta_t(r)<1} \big[\pi + \beta(1-\pi) \big] \max \left\{ \frac{z_{t+1} + \pi c_{2t} + rD_t}{c_{2t}} u(c_{2t}), u(z_{t+1} + \pi c_{2t} + rD_t) \right\} \Big], \tag{24}$$

subject to the budget constraint:

$$(1-\pi)c_{1t} + \pi c_{2t} + k_{t+1} + z_{t+1} + D_t \leq w_t + R_t k_t + z_t, \tag{25}$$

and the non-negativity constraints $D_t \geq 0$, $k_{t+1} \geq 0$ and $z_{t+1} \geq 0$. The indicator function $\mathcal{I}_{\delta_t(r)<1}$ is a dummy variable, that takes the value 1 when the bank chooses a contract such that a run equilibrium exists, and zero otherwise. With probability $(1 - q\mathcal{I}_{\delta_t(r)<1})$, the no-run equilibrium is selected, and the agents' expected welfare can be written as before. With probability $q\mathcal{I}_{\delta_t(r)<1}$, instead, the run-equilibrium is selected: at a run, π agents are night consumers, and consume right away, and $(1-\pi)$ agents are day consumers, who store and consume in the morning of the following period. The agents' welfare depends on the service protocol: with sequential service, only $\delta_t(r)$ agents are served, each getting an amount of night consumption equal to c_{2t}; with equal service, instead, every agent is served, and get c_{2t}^R, depending on the contract. As we mentioned above, the fact that the representative bank chooses the portfolio allocation and the banking contract implicitly gives it the possibility to choose whether the run equilibrium exists. Consequently, the representative bank solves both a "run" equilibrium, where $\mathcal{I}_{\delta_t(r)<1} = 1$, and a "run-proof" equilibrium, where $\delta_t(r) = 1$ and $\mathcal{I}_{\delta_t(r)<1} = 0$. The definition of the equilibrium is as follows:

Definition 2. *Given an initial amount of physical capital K_0, liquidity z_0, and day consumption c_{10}, a banking equilibrium with runs is a price vector $\{R_t, w_t\}$, a bank portfolio strategy $\{\ell_t, f_{t+1}, k_{t+1}, z_{t+1}, D_t\}$, a deposit contract $\{c_{2t}, c_{2t}^R, c_{1t+1}\}$, a service protocol, and production inputs $\{K_t, L_t\}$ for every $t = 0, 1, \ldots$ such that, for given prices:*

- *The bank portfolio strategy, the deposit contract, and the service protocol solve the banking problem in (24);*
- *The production inputs maximize firm profits;*
- *$w_t = F_2(K_t, A_t L_t)$ and $R_t = F_1(K_t, A_t L_t) + 1 - d$;*
- *The credit market and the labor market clear:*

$$K_t = k_t, \tag{26}$$

$$L_t = 1. \tag{27}$$

4.3 The Run-Proof Equilibrium

In the run-proof equilibrium, the representative bank holds a sufficient amount of liquidity to pay all depositors in the case of a run, so that $\delta_t(r) \geq 1$ and $\mathcal{I}_{\delta_t(r)<1} = 0$. Thus, the condition that $\delta_t \geq 1$ is equivalent to imposing onto the no-run problem a liquidity requirement of the form:

$$z_{t+1} + \pi c_{2t} + rD_t \geq c_{2t}, \tag{28}$$

which is more stringent than the liquidity requirement of the no-run equilibrium. Intuitively, this is the reason why, to make the contract run-proof, the bank is forced to hold more liquidity than necessary. More formally, the bank solves the no-run banking problem in (3), subject to the liquidity requirement (28) and the budget constraints:

$$(1-\pi)c_{1t} + \pi c_{2t} + k_{t+1} + z_{t+1} + D_t \leq w_t + R_t k_t + z_t, \tag{29}$$

$$D_t \geq 0, \tag{30}$$

$$k_{t+1} \geq 0, \tag{31}$$

$$z_{t+1} \geq 0. \tag{32}$$

We attach to these constraints the Lagrange multipliers $\beta^t \lambda_t$, $\beta^t \eta_t$, $\beta^t \phi_t$, and $\beta^t \xi_t$, respectively. Moreover, we attach the Lagrange multiplier $\beta^t \mu_t$ to the liquidity requirement (28). The first-order conditions of the program are:

$$c_{2t}: \quad \pi u'(c_{2t}) = \pi \lambda_t + (1-\pi)\mu_t, \tag{33}$$

$$c_{1t+1}: \quad u'(c_{1t+1}) = \lambda_{t+1}, \tag{34}$$

$$k_{t+1}: \quad \lambda_t = \beta R_{t+1} \lambda_{t+1}, \tag{35}$$

$$D_t: \quad r\mu_t + \eta_t = \lambda_t, \tag{36}$$

$$z_{t+1}: \quad \mu_t + \xi_t + \beta \lambda_{t+1} = \lambda_t. \tag{37}$$

We start the characterization of the equilibrium with a preliminary result:

Lemma 1. *In the run-proof equilibrium, $D_t = 0$ and $z_{t+1} > 0$.*

Proof. To see that $D_t = 0$, notice that it is always dominated by z_{t+1}: in fact, the two technologies have the same marginal cost, in terms of tightening the current budget constraint by λ_t, but D_t has a lower marginal benefit, in terms of extra liquidity created ($r\mu_t < \mu_t$). From (28), then, $z_{t+1} \geq (1-\pi)c_{2t}$, and, since c_{2t} must be positive in equilibrium, we also have that $z_{t+1} > 0$. ■

The rationale for this result is straightforward. The representative bank does not use the liquidation technology to create the extra liquidity that makes the contract run-proof because, for each unit of credit lines liquidated, that would yield $R_{t+1} > 1$ units of day consumption

in the following period, it only generates $r < 1$ units of current night consumption. In fact, liquidity is a better alternative than liquidation, as giving up on $R_{t+1} > 1$ units of day consumption in the following period would generate 1 unit of current night consumption. However, this means that the bank, in order to rule out the run, is willing to hold some excess liquidity. By (35) and (37), the Lagrange multiplier on the liquidity requirement $\mu_t = \beta(R_{t+1} - 1)u'(c_{1t+1})$ is strictly positive, as $R_{t+1} > 1$ and $u(c)$ is increasing. So, the liquidity requirement holds with equality, and $z_{t+1} = (1 - \pi)c_{2t}$. In other words, the bank chooses $\ell_t = c_{2t}$, but only a fraction of this liquidity is actually consumed, because, being no run in equilibrium, only the π night consumers withdraw, while the remaining liquidity in excess is rolled over to the following period. This result, together with the first-order conditions, allows us to characterize the equilibrium in the following proposition:

Proposition 2. *The run-proof equilibrium is characterized by:*

$$\pi u'(c_{2t}) + (1 - \pi)\beta u'(c_{1t+1}) = u'(c_{1t}), \tag{38}$$

$$u'(c_{1t}) = \beta R_{t+1} u'(c_{1t+1}). \tag{39}$$

The expression in (38) pins down the intratemporal insurance in the run-proof equilibrium: at the optimum, the marginal cost of increasing c_{2t} (the right-hand side of (38)), in terms of tightening the current budget constraint by an amount equal to $u'(c_{1t})$, must be equal to the marginal benefit of increasing c_{2t}, in terms of higher marginal utility in the current period, that affects only π night consumers (the first element on the left-hand side of (38)), and of the higher excess liquidity that is rolled over to the next period, that relaxes the future budget constraint (the second element on the left-hand side of (38)). The expression in (39), instead, is an Euler equation, regulating the bank portfolio allocation between liquidity and credit to the production sector: as in the no-run equilibrium, the bank chooses a portfolio such that the marginal cost of holding liquidity, in terms of a tighter future budget constraint due to the missed investment opportunity, is equal to its marginal benefit, in terms of higher night consumption. The two expressions together allow us to further characterize the amount of intratemporal insurance that the bank offers to the depositors in equilibrium. To see this, merge (39) into (38), and obtain:

$$\pi u'(c_{2t}) + (1 - \pi)\frac{1}{R_{t+1}} u'(c_{1t}) = u'(c_{1t}). \tag{40}$$

The left-hand side of this expression is a linear combination of two elements, and since $R_{t+1} > 1$, we have:

$$\frac{1}{R_{t+1}} u'(c_{1t}) < u'(c_{1t}) < u'(c_{2t}). \tag{41}$$

Therefore, but the concavity of $u(c)$, we get the following result:

Corollary 1. *In the run-proof equilibrium, $c_{2t} < c_{1t}$.*

The intuition for this corollary lies in the observation that the marginal benefit of increasing night consumption, in terms of the higher excess liquidity that is rolled over to the following period, comes at the opportunity cost of a credit tightening, i.e. a lower investment in capital, that yields a higher return than liquidity (as $R_{t+1} > 1$). For this to be an equilibrium, the marginal utility of night consumption must be higher than the marginal utility of current day consumption, meaning that night consumption must be lower than current day consumption. Thus, the willingness of the bank to hold more liquidity, and make the equilibrium run-proof, comes at two costs: first, the bank provides less intratemporal insurance than in the no-run equilibrium; second, the portfolio allocation is distorted, as the bank is forced to roll over the excess liquidity to the following period and tighten the credit to the production sector, despite it being a more profitable investment opportunity.

4.4 The Run Equilibrium with Sequential Service

When $\mathcal{I}_{\delta_t(r)<1} = 1$, the representative bank holds a portfolio that does not prevent the emergence of potential runs, but chooses how to serve the depositors in accordance with the two protocols described above. When it selects sequential service, the banking problem reads:

$$\max_{\substack{\{c_{2t},c_{1t+1},\\ k_{t+1},D_t,z_{t+1}\}}} \sum_{t=0}^{\infty} \beta^t \Big[(1-q)\big(\pi u(c_{2t}) + \beta(1-\pi)u(c_{1t+1})\big) + \\ +q\big[\pi + \beta(1-\pi)\big]\frac{z_{t+1} + \pi c_{2t} + rD_t}{c_{2t}} u(c_{2t})\Big]. \tag{42}$$

subject to the budget constraints:

$$(1-\pi)c_{1t} + \pi c_{2t} + k_{t+1} + z_{t+1} + D_t \leq w_t + R_t k_t + z_t, \tag{43}$$
$$D_t \geq 0, \tag{44}$$
$$k_{t+1} \geq 0, \tag{45}$$
$$z_{t+1} \geq 0. \tag{46}$$

We attach to these constraints the Lagrange multipliers $\beta^t\lambda_t$, $\beta^t\eta_t$, $\beta^t\phi_t$, and $\beta^t\xi_t$, respectively. The first-order conditions of the program are:

$$c_{2t}: \quad (1-q)\pi u'(c_{2t}) + q\big[\pi + \beta(1-\pi)\big]\left[\delta_t(r)u'(c_{2t}) + [\pi - \delta_t(r)]\frac{u(c_{2t})}{c_{2t}}\right] = \pi\lambda_t, \tag{47}$$
$$c_{1t+1}: \quad (1-q)u'(c_{1t+1}) = \lambda_{t+1}, \tag{48}$$
$$k_{t+1}: \quad \lambda_t = \beta\lambda_{t+1}R_{t+1} + \phi_t, \tag{49}$$
$$D_t: \quad q\big[\pi + \beta(1-\pi)\big]r\frac{u(c_{2t})}{c_{2t}} + \eta_t = \lambda_t, \tag{50}$$
$$z_{t+1}: \quad q\big[\pi + \beta(1-\pi)\big]\frac{u(c_{2t})}{c_{2t}} + \beta\lambda_{t+1} + \xi_t = \lambda_t. \tag{51}$$

We use (47) and (48) to derive an expression that regulates the amount of intratemporal insurance that the bank provides in equilibrium:

$$u'(c_{2t}) - u'(c_{1t}) = \frac{q}{1-q}\big[\pi + \beta(1-\pi)\big]\left[\frac{\delta_t(r)}{\pi}\left(\frac{u(c_{2t})}{c_{2t}} - u'(c_{2t})\right) - \frac{u(c_{2t})}{c_{2t}}\right]. \tag{52}$$

Since $u'(c_{2t})c_{2t} > u(c_{2t})$, by the strict concavity of the utility function and the fact that the coefficient of relative risk aversion is larger than 1, we find:

Lemma 2. *In the run equilibrium with sequential service, $c_{2t} > c_{1t}$.*

This is a key result, because it states that, contrary to the run-proof equilibrium, the bank reacts to a positive probability of a run by increasing the amount of night consumption offered to their depositors above the level of current night consumption. In other words, the bank provides more intratemporal insurance than in the no-run equilibrium. The intuition for this result is the following: when fixing the night consumption c_{2t}, the bank takes into account the threefold effect that it might have on the welfare of the agents if the run equilibrium is selected. In fact, by increasing c_{2t} by one marginal unit, every agent at a run is better off, because she receives a higher amount of consumption. Moreover, higher c_{2t} implies higher liquidity, which means that the fraction $\delta_t(r)$ of agents that are served at a run increases. However, at the same time, night consumption is at the denominator of $\delta_t(r)$, so a higher c_{2t} makes $\delta_t(r)$ go down. What this result shows is that the first two effects dominates the third one, if the agents are sufficiently risk averse.

We use this result to further characterize the equilibrium portfolio strategy in the following lemma:

Lemma 3. *In the run equilibrium with sequential service, $D_t = 0$. Moreover, $z_{t+1} = 0$, provided that:*

$$(1-q)(R_{t+1} - 1) \geq q\big[\pi + \beta(1-\pi)\big]. \tag{53}$$

Proof. For the statement that $D_t = 0$, take (47) and (50), and derive:

$$\eta_t = (1-q)u'(c_{2t}) + q\big[\pi + \beta(1-\pi)\big]\left[\frac{\delta_t(r)}{\pi}\left(u'(c_{2t}) - \frac{u(c_{2t})}{c_{2t}}\right) + (1-r)\frac{u(c_{2t})}{c_{2t}}\right]. \tag{54}$$

This sum is strictly positive, since $u(c_{2t})$ and $u'(c_{2t})$ are both strictly positive, the felicity function $u(c)$ is strictly concave, and the coefficient of relative risk aversion is larger than 1. Hence, $D_t = 0$ by complementary slackness. This ends the first part of the proof. We prove the second part of the lemma by contradiction. Assume that the condition (53) holds, while $z_{t+1} > 0$. By complementary slackness, $z_{t+1} > 0$ implies that $\xi_t = 0$. By (51), this is true only if:

$$\lambda_t - \beta\lambda_{t+1} = q\big[\pi + \beta(1-\pi)\big]\frac{u(c_{2t})}{c_{2t}}. \tag{55}$$

Using (47) and (49) (and the fact that, in equilibrium, it must be the case that $k_{t+1} > 0$,

implying that $\phi_t = 0$), we can rewrite the latter expression as:

$$\left(1 - \frac{1}{R_{t+1}}\right)\left[(1-q) + q[\pi + \beta(1-\pi)]\frac{\delta_t(r)}{\pi}\right]u'(c_{2t}) =$$
$$= q[\pi + \beta(1-\pi)]\left[\frac{1}{R_{t+1}} + \frac{\delta_t(r)}{\pi}\left(1 - \frac{1}{R_{t+1}}\right)\right]\frac{u(c_{2t})}{c_{2t}}. \tag{56}$$

Since, $u'(c_{2t})c_{2t} > u(c_{2t})$, this expression holds only if:

$$(1-q)\left(1 - \frac{1}{R_{t+1}}\right) < q\big[\pi + \beta(1-\pi)\big]\frac{1}{R_{t+1}}, \tag{57}$$

which contradicts (53). This ends the proof. ■

The intuition for this result is the following. Whenever bank runs are possible, and sequential service is into place, there can be a reason to liquidate the credit lines: in order to serve more depositors in the case of a run. The first part of the lemma shows that this can never be the case, if the recovery rate of the liquidation technology is sufficiently low. A second way for the bank to create extra resources is to hold a positive excess liquidity z_{t+1}. For the representative bank to do so in equilibrium, it must be the case that the marginal costs are equal to the marginal benefits. On one side, the marginal costs of excess liquidity come from a tightening of the budget constraint when the no-run equilibrium is selected. Hence, they depend on the shadow value of consumption, that, in equilibrium, is a function of the marginal utility $u'(c_{2t})$. This also means that the marginal costs increase (i) with the probability that the run equilibrium is not selected $(1-q)$, and (ii) with the opportunity cost of investing in excess liquidity, in terms of the forgone return on capital $(R_{t+1} - 1)$. On the other side, the marginal benefits of excess liquidity come when the run equilibrium is selected, as a consequence of the fact that $1/c_2$ more depositors can be served during a run. So, a positive z_{t+1} affects the average utility $u(c_2)/c_2$. The lemma shows that, under the condition that the depositors are sufficiently risk averse, so that the marginal utility of consumption is higher than the average utility of consumption, the representative bank holds a positive excess liquidity if the probability that the run equilibrium is selected is sufficiently high, and if the opportunity cost of excess liquidity is sufficiently low.

Assume from now on that the condition (53) is satisfied, so that $z_{t+1} = 0$. Therefore, we have that $\delta_t(r) = \pi$, as both z_{t+1} and D_t are equal to zero. This means that the bank, in equilibrium, does not serve more agents than in the no-run equilibrium, no matter whether a run is realized or not. With this result in hand, we can simplify the expression in (52), and derive the following proposition:

Proposition 3. *The run equilibrium with sequential service is characterized by:*

$$u'(c_{2t})\left[(1-q) + q\big[\pi + \beta(1-\pi)\big]\right] = (1-q)u'(c_{1t}), \tag{58}$$
$$u'(c_{1t}) = \beta R_{t+1}u'(c_{1t+1}). \tag{59}$$

As before, the Euler equation determines the portfolio allocation between liquidity and credit. Merging these two expressions, we derive:

$$u'(c_{2t})\big((1-q)+q\big[\pi+\beta(1-\pi)\big]\big)=(1-q)\beta R_{t+1}u'(c_{1t+1}), \tag{60}$$

which points out that the probability of a bank run skews the equilibrium portfolio towards liquidity and away from capital: a positive probability of a bank run gives rise to a credit tightening.

4.5 The Run Equilibrium with Equal Service

When the equilibrium contract is not run-proof, the choice between sequential service and equal service substantially modifies the properties of the equilibrium. Intuitively, this happens because, with sequential service, the realization of a bank run affects only the consumption bundle of a fraction of depositors (i.e. those who run, but are not served, and consume zero), while, with equal service, all agents in the economy are served, even if they receive a different amount of consumption in the case that the run equilibrium is selected. More formally, the representative bank maximizes:

$$\max_{\substack{\{c_{2t},c_{1t+1},\\ k_{t+1},D_t,z_{t+1}\}}} \sum_{t=0}^{\infty}\beta^t\left[(1-q)\big(\pi u(c_{2t})+\beta(1-\pi)u(c_{1t+1})\big)+\right.$$
$$\left.+q\big[\pi+\beta(1-\pi)\big]u(z_{t+1}+\pi c_{2t}+rD_t)\right]. \tag{61}$$

subject to the budget constraints:

$$(1-\pi)c_{1t}+\pi c_{2t}+k_{t+1}+z_{t+1}+D_t\leq w_t+R_tk_t+z_t, \tag{62}$$
$$D_t\geq 0, \tag{63}$$
$$k_{t+1}\geq 0, \tag{64}$$
$$z_{t+1}\geq 0, \tag{65}$$

where we used the definition of c_{2t}^R in (23) into the objective function. We attach to these constraints the Lagrange multipliers $\beta^t\lambda_t$, $\beta^t\eta_t$, $\beta^t\phi_t$, and $\beta^t\xi_t$, respectively, and derive the first-order conditions:

$$c_{2t}:\quad (1-q)u'(c_{2t})+q[\pi+\beta(1-\pi)]u'(c_{2t}^R)=\lambda_t, \tag{66}$$
$$c_{1t+1}:\quad (1-q)u'(c_{1t+1})=\lambda_{t+1}, \tag{67}$$
$$k_{t+1}:\quad \lambda_t=\beta\lambda_{t+1}R_{t+1}+\phi_t, \tag{68}$$
$$D_t:\quad q\big[\pi+\beta(1-\pi)\big]ru'(c_{2t}^R)+\eta_t=\lambda_t, \tag{69}$$
$$z_{t+1}:\quad q\big[\pi+\beta(1-\pi)\big]u'(c_{2t}^R)+\beta\lambda_{t+1}+\xi_t=\lambda_t. \tag{70}$$

We use (66) and (67) to derive the intratemporal condition:

$$u'(c_{1t})-u'(c_{2t})=\frac{q}{1-q}\big[\pi+\beta(1-\pi)\big]u'(c_{2t}^R). \tag{71}$$

As the right-hand side of this expression is always positive, because the felicity function $u(c)$ is increasing, we find that, similarly to the run equilibrium with sequential service, the bank provides more intratemporal risk sharing than in the no-run equilibrium:

Lemma 4. *In the run equilibrium with equal service, $c_{2t} > c_{1t}$.*

The rationale for this result lies in the fact that, with equal service, there are two reasons to offer a positive amount of night consumption: (i) to provide consumption to the night consumers, when the no-run equilibrium is selected, and (ii) to increase liquidity, that goes to all depositors, when instead the run equilibrium is selected. This means that, for a given day consumption c_{1t}, the marginal utility of night consumption has to go down, with respect to what happens in the no-run equilibrium. Hence, night consumption has to increase.

From the first-order conditions, we further characterize the equilibrium liquidation policy and the holding of excess liquidity:

Lemma 5. *In the run equilibrium with equal service, $D_t = 0$. Moreover, $z_{t+1} = 0$ provided that:*

$$(1-q)(R_{t+1}-1) \leq q\big[\pi + \beta(1-\pi)\big]. \tag{72}$$

Proof. For the first part of the proof, take (66) and (69), and derive:

$$\eta_t = (1-r)q\big[\pi + \beta(1-\pi)\big]u'(c_{2t}^R) + (1-q)u'(c_{2t}). \tag{73}$$

This is strictly positive, since $u'(c_{2t}^R)$ and $u'(c_{2t})$ are both strictly positive, hence $D_t = 0$ by complementary slackness. This ends the first part of the proof. We prove the second part by contradiction. Assume that $z_{t+1} > 0$ and that the condition (72) is satisfied. By complementary slackness, this is an equilibrium only if $\xi_t = 0$. By (70), this is true if:

$$\lambda_t - \beta\lambda_{t+1} = q[\pi + \beta(1-\pi)]u'(c_{2t}^R). \tag{74}$$

Using (66) and (68), we can rewrite the latter expression as:

$$(1-q)\left(1 - \frac{1}{R_{t+1}}\right)u'(c_{2t}) = \frac{1}{R_{t+1}}q\big[\pi + \beta(1-\pi)\big]u'(c_{2t}^R). \tag{75}$$

Remember that, in the run equilibrium with equal service, it must be the case that $c_{2t} > c_{2t}^R$. Hence, by the strict concavity of the felicity function, $u'(c_{2t}) < u'(c_{2t}^R)$, and $z_{t+1} > 0$ is an equilibrium only if:

$$(1-q)\left(1 - \frac{1}{R_{t+1}}\right) > q\big[\pi + \beta(1-\pi)\big]\frac{1}{R_{t+1}}, \tag{76}$$

which contradicts (72). This ends the proof. ■

The intuition for these results echoes the one of lemma 3. Whenever bank runs are possible, and equal service is into place, there can be a reason to liquidate the available credit lines: in

order to avoid storing liquidity, and to allow the bank to have more resources in the case when the run equilibrium is selected. The first part of the lemma shows that this strategy is never pursued, because the opportunity costs of liquidation, in terms of lower credit provided to the firms, is too high compared to the low recovery rate that the liquidation technology ensures. A second alternative to create some extra resources, then, is to hold some excess liquidity z_{t+1}. As with sequential service, the marginal costs of this portfolio strategy come from a tightening of the budget constraint, and are a function of the marginal utility of consumption $u'(c_{2t})$ when the run equilibrium is not selected. However, the marginal benefits of a positive z_{t+1} affect directly the marginal utility of consumption $u'(c_{2t}^R)$, when the run equilibrium is selected. Given that, for a run equilibrium to exist, we need $c_{2t} > c_{2t}^R$, the strict concavity of the felicity function implies that, contrary to the result in lemma 3, a positive excess liquidity requires that the probability that the run equilibrium is selected is sufficiently low, and that the opportunity cost of excess liquidity is sufficiently high.

Assume now that the amount of excess liquidity z_{t+1} is positive. The combination of (66) and (70), together with the fact that $\xi_t = 0$ by complementary slackness, allows us to complete the characterization of the equilibrium. Imagine that the bank wants to invest a positive marginal amount of resources in excess liquidity, while keeping the total liquidity in portfolio constant. On one side, the benefit of such a strategy comes in terms of higher expected welfare, if the run equilibrium is selected, and of a slacker future budget constraint, in terms of future shadow value of consumption, if the no-run equilibrium is selected:

$$q[\pi + \beta(1-\pi)]u'(c_{2t}^R) + \beta\lambda_{t+1}. \tag{77}$$

On the other side, the cost of such a strategy comes as the bank has to tighten the current budget constraint, by an amount equal to the current shadow value of consumption. In equilibrium, this must be equal to the right-hand side of (66), hence the positive effect of excess liquidity on the expected welfare (the first term of (77)) is exactly compensated by the negative effect of lower night consumption (the second term of the right-hand side of (66)), and the equilibrium amount of excess liquidity is only characterized by:

$$u'(c_{2t}) = \beta u'(c_{1t+1}). \tag{78}$$

Finally, it is easy to see that the bank chooses the portfolio allocation between liquidity and credit so as to satisfy an Euler equation, as in the other cases. We can summarize the characterization of the equilibrium as follows:

Proposition 4. *The run equilibrium with equal service is characterized by:*

$$u'(c_{1t}) - u'(c_{2t}) = \frac{q}{1-q}\big[\pi + \beta(1-\pi)\big]u'(c_{2t}^R), \tag{79}$$

$$u'(c_{1t}) = \beta R_{t+1} u'(c_{1t+1}), \tag{80}$$

$$u'(c_{2t}) = \beta u'(c_{1t+1}). \tag{81}$$

By merging (79) into (80), we get:

$$u'(c_{2t}) + \frac{q}{1-q}\big[\pi + \beta(1-\pi)\big] = \beta R_{t+1} u'(c_{1t+1}), \tag{82}$$

which makes clear that, as in the equilibrium with sequential service, a positive probability of a run skews the equilibrium portfolio towards liquidity, and generates a credit tightening.

5 Quantitative Analysis

The characterization of the banking equilibrium with runs in definition 2 refers to the choice between the run-proof equilibrium and the two run equilibria. In order to compare them, the bank needs to calculate the expected welfare of the depositors in all cases. Given the impossibility to derive them in a closed-form solution, we rely on a numerical analysis.

To this end, we calibrate the parameters of the model to the U.S. economy, and run the following experiment: we let the economy reach the steady state of the no-run equilibrium of section 3, and hit it with a one-period shock to the probability of the realization of a the sunspot q. We evaluate the evolution of the economy on impact, as well as on its trajectory back to the steady state.

5.1 Calibration

We calibrate a stationary version of the banking problem, since it can be proved that all variables grow at a constant rate on the balanced-growth path. We report the details of the stationarization and of the numerical algorithm in Appendix B. Here, we instead report how we calibrate the parameters of the problem.

We choose a standard Cobb-Douglas production function $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$, with $L_t = 1$ for every t by construction. We impose $\alpha = .4$, and assume that the exogenous technological process A_t grows exponentially at a rate γ, that is $A_t = A_0(1+\gamma)^t$. Since, on a balanced-growth path, output and capital (as we show in Appendix B) must grow at the same rate, the production function in steady state pins down γ from $(1+g_Y)^{1-\alpha} = (1+\gamma)$, where g_Y is the growth rate of GDP that we observe in the data. By taking $g_Y = .02$ (roughly equal to the average growth rate of the U.S. GDP in the period 1970-2010), we find that $\gamma = .014$. The initial point A_0 is just a scale parameter, and we normalize it to 1.[12] We find the capital depreciation rate d from the investment equation in steady state, i.e.:

$$1 + g_Y = 1 - d + \frac{I_t}{K_t}. \tag{83}$$

With the investment-to-capital ratio I_t/K_t approximately equal to .076, as it is common in the literature, we get $d = .056$. Finally, we assume that the recovery rate r of the liquidation

[12] In every exercise, we find that $R_{t+1} > 1$, as assumed in the theory. However, if that was not the case, we could have altered the scale to ensure that the condition was still satisfied.

technology is .4, approximately equal to the average recovery rate on bank loans according to Moody's (2009), and not far from the structural estimate proposed by Chen (2010).

As far as the felicity function is concerned, we need to combine the standard CRRA, which is the only functional form consistent with balanced growth, with $u(0) = 0$ and the fact that the coefficient of relative risk aversion is larger than 1. To this end, we define our arbitrary-precise zero to be equal to the tolerance level $\epsilon = 10^{-6}$, and write the felicity function as $u(c) = (c^{1-1/\sigma} - x)/(1 - 1/\sigma)$. We set $x = \epsilon^{1-1/\sigma}$, so that $u(\epsilon) = 0$ and the utility is always positive for positive consumption.[13] Moreover, in our benchmark calibration, we assume that the coefficient of relative risk aversion is 1.2, but also run a series of robustness checks with relative risk aversion equal to 2 and 3.

From the steady-state Euler equation of the no-run banking equilibrium, we calibrate the value of the discount factor β:

$$(1 + g_Y) = \beta R = \beta(F_K + 1 - \delta) = \beta \left(\alpha \frac{Y}{K} + 1 - d \right). \tag{84}$$

With the output-to-capital ratio Y/K approximately equal to .30, and with $d = .056$, we obtain that $\beta = .96$.

The last parameter left to calibrate is the probability π of the realization of the idiosyncratic shock θ_t, that makes the agents willing to consume at night. To find the appropriate value, we take the liquidity constraint of the no-run banking problem, and divide both sides by Y_t to get:

$$\frac{\ell_t}{Y_t} = \frac{\pi c_{2t}}{Y_t} = \frac{\pi c_{1t}}{Y_t} = \pi \frac{c_{1t}}{K_t} \frac{K_t}{Y_t}, \tag{85}$$

where we use the fact that, in the no-run banking equilibrium, $c_{2t} = c_{1t}$. From the resource constraint of the economy, we know that it must be the case that $c_{1t} = Y_t - I_t$. Hence, dividing both sides by K_t and plugging the result into (85), we obtain:

$$\pi = \frac{\frac{\ell_t}{Y_t} \frac{Y_t}{K_t}}{\frac{Y_t}{K_t} - \frac{I_t}{K_t}}. \tag{86}$$

We set the ratio of liquid assets to GDP to be .21, equal to the average liquidity (as a percentage of GDP) of the U.S. financial business in the period 1970-2010, that we draw from the U.S. financial accounts.[14] From here, we find that π is approximately equal to .28.[15]

[13] Without this normalization, the CRRA utility function with $u(0) = 0$ would violate both continuity and monotonicity around zero.

[14] Financial Businesses are defined as the sum of: private depository institutions (U.S.-chartered depository institutions, excluding credit unions; foreign banking offices in the U.S.; banks in U.S.-affiliated areas; credit unions); property-casualty insurance companies; life insurance companies; private and public pension funds; money market mutual funds; mutual funds; closed-end and exchange-traded funds; government-sponsored enterprises; agency- and GSE-backed mortgage pools; issuers of asset-backed securities; finance companies; real estate investment trusts; security brokers and dealers; holding companies; funding corporations (source: Flow of Funds of the United States).

[15] This value is quite different from the one in Gertler and Kiyotaki (2013), who assume that $\pi = .03$, but is close to the one in Ennis and Keister (2003), who instead assume that $\pi = .25$.

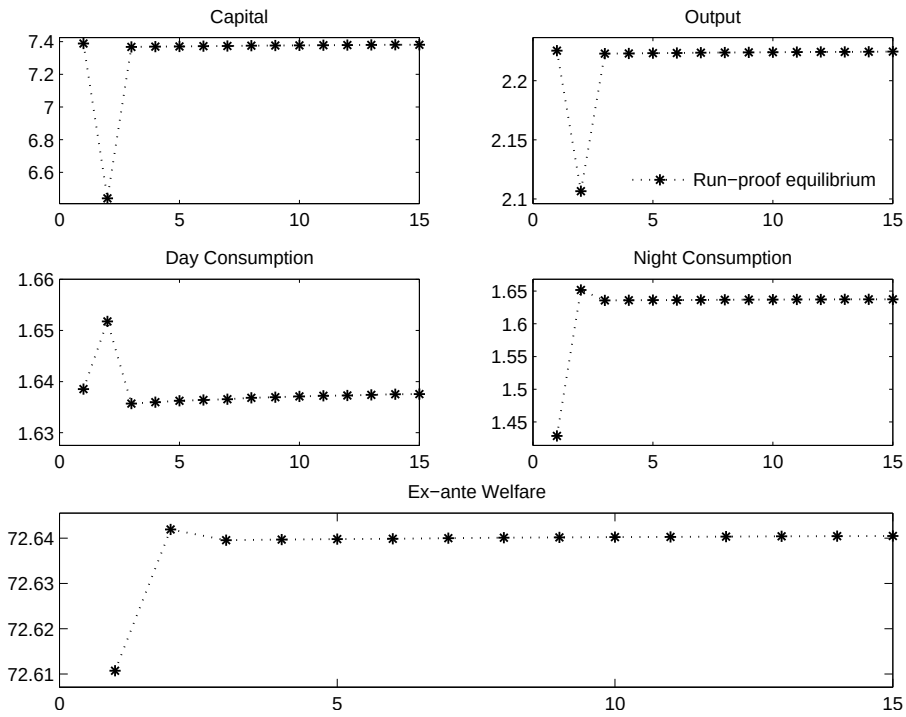

Figure 1: The impulse-response functions of run-proof equilibrium.

Finally, we need to provide some reference numbers for the probability of the realization of the sunspot q. In this respect, we take a conservative approach, and compare different values, without any prior about what the true one is. Specifically, we show the numerical results for a set of probabilities in the interval $[0, .05]$, because that is the upper bound beyond which the representative bank, in our simulations, chooses the run-proof contract. Hence, only for values of q lower than that, the economy exhibits runs in equilibrium with some positive probability. In the considered interval, the conditions in (53) is always satisfied, hence $z_t = 0$ in the run equilibrium with sequential service, and $z_{t+1} > 0$ in the run equilibrium with equal service.

5.2 Results

We start our analysis with the qualitative characterization of the behavior of the three equilibria. Figure 1 shows the impulse-response functions of the run-proof equilibrium: when the probability of the realization of the sunspot increases, the bank avoids a run by increasing the amount of liquidity held in portfolio, i.e. by tightening credit, and this generates a considerable drop in output. As we argued in the previous section, in order for this to be an equilibrium, the marginal utility of night consumption must be larger than the marginal utility of day consumption. Thus, night consumption drops on impact, while day consumption jumps, and this generates an overall negative impact on welfare of around .04 percentage points, as it is clear from the bottom panel of figure 1. In the period following the run, the economy goes

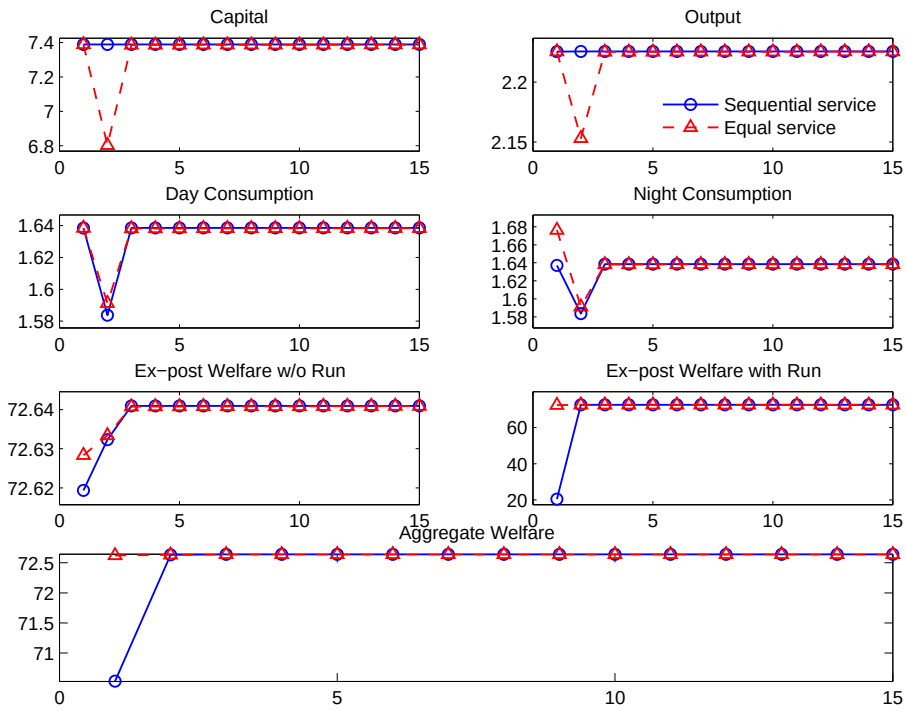

Figure 2: The impulse-response functions of the run banking equilibria with $q = .04$.

back to the no-run equilibrium, where $c_{2t+1} = c_{1t+1}$, hence night consumption increases and welfare recovers. The amount of credit available in the economy is low, but a big part of the liquidity held in portfolio in the previous period is rolled over, so it recovers quickly. However, since the amount of current consumption to finance is relatively high, future credit k_{t+2} is still lower than its steady-state value, and future day consumption c_{1t+2} drops, bringing down the future night consumption c_{2t+2}, as well as the future welfare. Then, the economy goes back to the steady state following its transition path, where credit, consumption, and welfare slowly recover.

In figure 2, we instead show the impulse-response functions of the two run equilibria. With sequential service, the bank reacts to a shock to the probability of the realization of the sunspot by increasing liquidity and tightening credit. Moreover, as we proved in the theory, day consumption decreases, and night consumption increases consistently (see the middle right panel of figure 2). This is due to the willingness of the bank to counteract the welfare loss of a run, consequence of the fact that, if the run is realized, only a fraction $\delta_t(r) = \pi = .28$ of agents is served, while the remaining fraction $1 - \pi = .72$ consumes zero, and have zero utility. However, the depositors of the banks that are ex post hit by a run still suffer a massive welfare drop that, in all our calibrations, is of around 71 percentage points. Due to the reallocation of resources between night consumption and day consumption, the ex-post welfare of the banks that are not hit ex post by a run also decreases. All in all, as it is clear from the bottom panel of the figure, the aggregate ex ante welfare (the linear combinations of the run and no-run

Table 1: The effect of a bank run on the banking equilibria

Panel A: Welfare costs (%)

	q=0.01	q=0.10	q=1.00	q=2.00	q=3.00	q=4.00	q=5.00
RP	0.1744	0.1744	0.1744	0.1744	0.1744	0.1744	0.1744
SEQ	0.0232	0.2331	2.3028	4.5422	6.7202	8.8390	10.9003
EQ	0.0006	0.0057	0.0562	0.1073	0.1415	0.1641	0.1785

Panel B: Output drop on impact (%)

	q=0.01	q=0.10	q=1.00	q=2.00	q=3.00	q=4.00	q=5.00
RP	5.6429	5.6429	5.6429	5.6429	5.6429	5.6429	5.6429
SEQ	0.0008	0.0024	0.0202	0.0407	0.0615	0.0823	0.1036
EQ	0.0011	0.0076	0.0710	0.8865	2.1202	3.2494	4.3008

welfare) exhibits a consistent drop on impact (of around 1.8 percentage points in the graph). In the period following the run, the economy goes back to the no-run equilibrium, hence night consumption drops, and credit, consumption and welfare slowly recover back to their steady states.

The impulse-response functions of the equilibrium with equal service are similar to those with sequential service, but with a key difference: in equilibrium, the bank lowers credit more than with sequential service, in order to hold the excess liquidity necessary to pay the depositors if the run equilibrium is selected. This means that the welfare drop on impact is lower than with sequential service, and, since part of the liquidity is rolled over to the following period, also the recovery is faster, as in the run-proof equilibrium.

To sum up, from a qualitative perspective, we find that the run-proof equilibrium avoids bank runs, but at the cost of a massive credit tightening, that leads to an equally massive drop in output. However, the shock is temporary, and the recovery is fast, because the excess liquidity held to rule out the run is rolled over, and soon employed to increase credit. In the run equilibrium with equal service, the bank behaves similarly, but the credit tightening is smaller, as the bank is not forced to hold a sufficient amount of liquidity to be able to serve all depositors. In the run equilibrium with sequential service, instead, a smaller credit tightening comes at the cost of a large welfare drop, and of a long-lasting recovery.

The question of which equilibrium the representative bank is going to choose depends on how quantitatively large these effects are, and how they counterbalance each other. In turn, this depends on the probability of the realization of the sunspot so, in table 1, we provide a quantitative comparison for different values of q. In panel A, we report the welfare costs of the three equilibria, in percentage of consumption equivalents, with respect to the no-run equilibrium of section 3.[16] In the run-proof equilibrium, the bank is forced to hold a sufficient amount of liquidity to pay night consumption to all depositors, no matter what the probability

[16] We report the details of these calculations in Appendix B.3.

Table 2: Robustness checks on the banking equilibrium

Panel A: Welfare costs (%)

	q=0.01	q=0.10	q=1.00	q=2.00	q=3.00	q=4.00	q=5.00
RRA = 1.2	0.0006	0.0057	0.0562	0.1073	0.1415	0.1641	0.1744
RRA = 2	0.0010	0.0102	0.0883	0.1324	0.1582	0.1744	0.1789
RRA = 3	0.0024	0.0233	0.1140	0.1494	0.1691	0.1811	0.1832

Panel B: Output drop on impact (%)

	q=0.01	q=0.10	q=1.00	q=2.00	q=3.00	q=4.00	q=5.00
RRA = 1.2	0.0011	0.0076	0.0710	0.8865	2.1202	3.2494	5.6428
RRA = 2	0.0024	0.0148	1.0355	2.4146	3.4592	4.3311	6.1549
RRA = 3	0.0054	0.0377	2.2819	3.4860	4.3287	4.9991	6.4305

of the realization of the sunspot is, so the welfare cost is invariant with respect to q. Conversely, the welfare costs of the two run equilibria are increasing in q, as expected. The welfare costs with sequential service increase at a very fast pace, and are higher than 10 per cent for q equal to 5 per cent. That is because, as we mentioned above, with sequential service a large fraction of agents runs but is not served, and gets zero utility. In contrast, the welfare costs with equal service are lower than those of the run-proof equilibrium for values of q below 5 per cent. This is an interesting result for two reasons. First, it allows us to fully characterize the equilibrium of definition 2: for values of q below 5 per cent, the banking equilibrium is equivalent to the run equilibrium with equal service; for values of q larger than or equal to 5 per cent, it is instead equivalent to the run-proof equilibrium. The second reason of interest is that, in this way, we quantitatively characterize this threshold, whose existence was proved by Cooper and Ross (1998) in a static environment.

The welfare comparison of panel A further allows us to study the equilibrium response of the banking equilibrium to a one-period shock to the probability of a run. In panel B, we report the negative response of GDP on impact: it is increasing in the probability of the realization of the sunspot, and ranges from 0.0011 per cent when q is 0.01 per cent, to around 3.25 per cent when q is 4 per cent. For q larger than or equal to 5 per cent, instead, the bank chooses the run-proof equilibrium over the run equilibrium with equal service, so bank runs are endogenously ruled out, but this comes at the cost of a drop in GDP of around 5.6 percentage points.

We conclude our analysis by checking the robustness of our results to changes in the other exogenous parameter of our calibration, namely the coefficient of relative risk aversion. In table 2, we report the welfare costs and the output drop on impact of the banking equilibrium (i.e. the run equilibrium with equal service for q lower than or equal to 4 per cent, and the run-proof equilibrium with q equal to 5 per cent or above) for relative risk aversion equal to 2 and 3, together with our baseline calibration. Both panel A and B show that our conclusions

are not qualitatively altered in any way: for given relative risk aversion, both the welfare costs and the output drop are increasing in the probability of the realization of the sunspot. Ceteris paribus, the higher the risk aversion is, the higher is the distortion of the banking equilibrium with respect to the no-run equilibrium. This is a consequence of the fact that the more risk averse the agents are, the less they tolerate any difference in their ex post consumption profiles, hence the more the bank distorts the economy when the probability of the realization of the sunspot is positive.

6 Concluding Remarks

In the present paper, we contribute to the literature on the economics of banking and crises by studying the macroeconomic effects of systemic bank runs, in a neoclassical growth model with a fully-microfounded banking system. Our main contribution regards the quantitative evaluation of these effects. In particular, we are the first ones, to the best of our knowledge, to calibrate the probability of the idiosyncratic liquidity shock, which is a standard parameter in many theories of banking. Our findings show that the model works remarkably well from a qualitative perspective: it predicts that a shock to the probability of a bank run leads to a credit tightening, followed by a drop in GDP and a recovery, replicating the behavior of many real economies in some recent crises. Moreover, from a quantitative point of view, the outcomes of the model show that the macroeconomic effects of bank runs can be substantial, both on GDP and on welfare. However, our findings represent only a lower bound for the effects that systemic bank runs have on the real economy. In that sense, we need to extend it in different directions, particularly for it to be able to replicate the kind of long-lasting recoveries that we observe in the data.

On a more general point, we contribute to the literature by defining and characterizing a general-equilibrium theory of banking, that can appeal to both microeconomists and macroeconomists, as it bridges the microeconomic literature on banks and bank runs to a standard macroeconomic model. This also means that we can use this setting as a starting point for future research in many different directions. For example, we could modify the informational structure of the banking system, to analyze the general-equilibrium dynamic effects of moral hazard, adverse selection and asymmetric information. In a similar way, we could also extent the macroeconomic environment, to study how the banking system interacts with the labor market, with business-cycle fluctuations, or with monetary policy.

Finally, our work has some interesting policy implications. From the comparison between the two service protocols, in the case when a run is realized, it is clear that stating what the banks can or cannot do during a systemic run is key. In particular, allowing them to choose equal service over sequential service, while generating a larger drop in GDP on impact, seems to have a beneficial effect on the economy, in terms of a faster recovery, and of lower intertemporal welfare costs. Moreover, in the environment proposed here, the first theorem of

welfare economics holds, and any banking equilibrium, with a internally-funded macroprudential scheme that rules out systemic runs, would be equivalent to the run-proof equilibrium. Therefore, our welfare comparisons show that ruling out systemic runs via macroprudential policy can be costly, both in terms of a distorted allocation of capital and of intertemporal welfare, and is worthwhile implementing only when the probability of the realization of this event is sufficiently high.

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Appendices

A Microfoundations of the Banking Equilibrium

In this section, we compare the no-run banking equilibrium of section 3 with an "autarkic" equilibrium, where the agents do not form banking coalitions to hedge against the idiosyncratic shocks, but independently choose their portfolios of liquidity and credit to the production sector, and adjust them, after the realization of the idiosyncratic shocks, in a secondary asset market. More formally, the program reads:

$$\max_{\{x_{2t},x_{1t+1},\ell_t^M,f_{t+1}^M,k_{t+1}^M\}} \sum_{t=0}^{\infty} \beta^t \left[\pi u(x_{2t}) + \beta(1-\pi)u(x_{1t+1})\right], \tag{87}$$

subject to the budget constraints:

$$x_{1t} + \ell_t^M + f_{t+1}^M = w_t + R_t k_t, \tag{88}$$

$$\ell_t^M + p_t(f_{t+1}^M - k_{t+1}^M) = x_{2t}. \tag{89}$$

The agents maximize their expected intertemporal welfare, subject to their budget constraints: they invest the total per-period resources $w_t + R_t k_t$ in current day consumption x_{1t}, which is chosen in the previous period, and in financial investment, divided in liquidity ℓ_t^M and financial assets f_{t+1}^M. However, differently from what happens in the banking equilibrium, the amount of night consumption x_{2t} is equal to the available liquidity plus the amount of financial assets $f_{t+1}^M - k_{t+1}^M$ that are not saved for the future period, and are sold in the secondary market at price p_t.

The characterization of the equilibrium starts from the equilibrium price on the secondary market p_t. It is easy to argue, following Allen and Gale (2004), that it must be the case that $p_t = 1$ at every point in time. The rationale for this result is based on the fact that the agents must be ex ante indifferent between holding liquidity or not, otherwise the secondary market does not clear. Assume, for example, that $p_t > 1$. Then, credit yields a higher return than liquidity, and everyone invests in it. However, if that is the case, the night consumers would like to sell their assets in the secondary market and consume, but there would be no buyers. Thus, the price p_t would go to zero, which is a contradiction. Similarly, if $p_t < 1$, liquidity dominates credit, and everybody invests in it. If that is the case, the future day consumers would like to buy assets in the secondary market, but there would be no sellers, thus the price would go to $R_t > 1$, leading again to a contradiction.

The fact that the only possible equilibrium in the secondary market is one where the agents are ex ante indifferent between holding liquidity and credit implies that their asset portfolios are undetermined. Nevertheless, we can still use the budget constraints to solve for

the equilibrium allocation. Using (89) into (88), with $p_t = 1$, we get:

$$x_{1t} + x_{2t} + k^M_{t+1} = w_t + R_t k_t. \tag{90}$$

Attach to this constraint the Lagrange multiplier $\beta^t \lambda_t$. The first-order conditions are:

$$x_{1t+1}: \qquad (1-\pi)u'(x_{1t+1}) = \lambda_{t+1}, \tag{91}$$
$$x_{2t}: \qquad \pi u'(x_{2t}) = \lambda_t, \tag{92}$$
$$k^M_{t+1}: \qquad \lambda_t = \beta R_{t+1}\lambda_{t+1}, \tag{93}$$

Rearranging, we get:

$$u'(x_{1t}) = \frac{\pi}{1-\pi}u'(x_{2t}), \tag{94}$$
$$\pi u'(x_{2t}) = (1-\pi)\beta R_{t+1}u'(x_{1t+1}). \tag{95}$$

It is evident that the market equilibrium in (94)-(95) and the banking equilibrium in (18)-(19) are different. In a static environment, Diamond and Dybvig (1983) formally prove that the banking equilibrium is efficient, i.e. that there is no other feasible allocation that provides higher welfare. We claim that this result holds in our dynamic framework, too, and we show that in a simple two-period economy.

Assume that $t = 0, 1$. The initial capital K_0 is given, and $u(c) = \log(c)$. The production function takes the form $Y_t = ZK_t^\alpha \bar{K}_t^{1-\alpha}$, where $\bar{K}_t$ is the total capital in the economy and represents an externality like in Romer (1990), Z is a scale parameter, and capital fully depreciates after one period. Thus, in equilibrium we have $w_t = (1-\alpha)ZK_t$ and $R_t = \alpha Z$. Assume that $Z > 1/\alpha$, so that $R_t > 1$.

We first solve for the no-run banking equilibrium. From the budget constraint at $t = 1$, we find that $c_{11} = c_{21} = ZK_1$. The Euler equation and the budget constraint at $t = 0$ give:

$$\frac{1}{ZK_0 - K_1} = \frac{\alpha\beta Z}{ZK_1}, \tag{96}$$

hence:

$$K_1 = \frac{\alpha\beta Z K_0}{1+\alpha\beta}, \tag{97}$$
$$c_{10} = c_{20} = \frac{ZK_0}{1+\alpha\beta}, \tag{98}$$
$$c_{11} = c_{21} = \frac{\alpha\beta Z^2 K_0}{1+\alpha\beta}. \tag{99}$$

We now solve for the autarkic equilibrium. From the intratemporal condition (94), we have that $x_{21} = \frac{\pi}{1-\pi}x_{11}$. Plug this in the budget constraint at $t = 1$ to obtain $x_{11} = (1-\pi)ZK_1$. Finally, from the Euler equation at $t = 0$ and the budget constraint, we get:

$$\frac{\pi}{\pi(ZK_0 - K_1)} = \frac{\alpha\beta Z(1-\pi)}{(1-\pi)ZK_1}, \tag{100}$$

which clearly gives the same allocation of capital as in the banking problem. While this is a consequence of the assumption of log-utility, and would disappear with a CRRA felicity function with relative risk aversion larger than 1, it is clear that the market equilibrium provides a worse allocation of intratemporal resources than the banking equilibrium. This is an interesting result, because is in contrast with what happens in the static environment, where the banking equilibrium is equivalent to the market equilibrium, if we assume log-utility.

B Numerical Analysis

In this section, we provide the details of the algorithms that we use for the numerical analysis of section 5. We let the economy reach the steady state of the no-run equilibrium of section 3, and hit it with a one-period shock to the probability of the realization of a the sunspot q. We evaluate the evolution of the economy on impact, as well as on its trajectory back to the steady state. In order to solve the model, we use the fact that the no-run banking equilibrium is efficient (Diamond and Dybvig, 1983), and focus on the stationary representation of the social planner problem.

B.1 Stationary Planner Problem With No Runs

From the resource constraint of the economy:

$$(1-\pi)c_{1t} + \pi c_{2t} + K_{t+1} = Y_t + (1-d)K_t, \tag{101}$$

it is easy to argue that, on a balanced-growth path, all variables must growth at the same rate g, that we back up from the production function[17]. Knowing that the physical capital and the output must grow at the same rate, we find that $1+g = (1+\gamma)^{1/(1-\alpha)}$, where γ is the growth rate of the exogenous technological process A_t.

We divide all variables in the problem by their own growth rate, and use the tilde to indicate the stationary variables. If we assume a CRRA felicity function of the form $u(c) = \frac{c^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}}$, with $\sigma < 1$ (so that the coefficient of relative risk aversion $1/\sigma$ is larger than 1, as we assumed), the objective function becomes:

$$\sum_{t=0}^{\infty} \beta^t \left[\pi \frac{\left(\frac{c_{2t}(1+g)^t}{(1+g)^t}\right)^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}} + \beta(1-\pi) \frac{\left(\frac{c_{1t+1}(1+g)^{t+1}}{(1+g)^{t+1}}\right)^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}} \right] =$$

$$= \sum_{t=0}^{\infty} \beta^t \left[\pi \frac{\tilde{c}_{2t}^{1-\frac{1}{\sigma}}(1+g)^{t(1-\frac{1}{\sigma})}}{1-\frac{1}{\sigma}} + \beta(1-\pi) \frac{\tilde{c}_{1t+1}^{1-\frac{1}{\sigma}}(1+g)^{(t+1)(1-\frac{1}{\sigma})}}{1-\frac{1}{\sigma}} \right] - \frac{\pi + \beta(1-\pi)}{(1-\beta)\left(1-\frac{1}{\sigma}\right)} =$$

[17] As it is common in the literature, here we use K_t to indicate the amount of capital in the social planner problem.

$$= \sum_{t=0}^{\infty} \tilde{\beta}^t \left[\pi \frac{\tilde{c}_{2t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \tilde{\beta}(1-\pi) \frac{\tilde{c}_{1t+1}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right] - \frac{\pi + \beta(1-\pi)}{(1-\beta)\left(1-\frac{1}{\sigma}\right)}, \tag{102}$$

where $\tilde{\beta} = \beta(1+g)^{1-\frac{1}{\sigma}}$, and $\tilde{c}_t = \frac{c_t}{(1+g)^t}$. Notice that the second part of the objective function is a constant, and does not affect the equilibrium conditions. In a similar way, the resource constraint becomes:

$$(1-\pi)\tilde{c}_{1t} + \pi\tilde{c}_{2t} + (1+g)\tilde{K}_{t+1} = F(\tilde{K}_t) + (1-d)\tilde{K}_t. \tag{103}$$

The planner problem is to maximize (102) subject to (103). Attach the Lagrange multiplier $\tilde{\beta}^t \lambda_t$ to the resource constraint. The first-order conditions are:

$$\tilde{c}_{2t}: \quad \tilde{c}_{2t}^{-\frac{1}{\sigma}} = \lambda_t, \tag{104}$$

$$\tilde{c}_{1t+1}: \quad \tilde{c}_{1t+1}^{-\frac{1}{\sigma}} = \lambda_{t+1}, \tag{105}$$

$$\tilde{K}_{t+1}: \quad (1+g)\lambda_t = \tilde{\beta}\left[F'(\tilde{K}_{t+1}) + 1 - d\right]\lambda_{t+1}. \tag{106}$$

Thus, we derive the equilibrium conditions:

$$\tilde{c}_{2t} = \tilde{c}_{1t}, \tag{107}$$

$$(1+g)\tilde{c}_{2t}^{-\frac{1}{\sigma}} = \tilde{\beta}\left[F'(\tilde{K}_{t+1}) + 1 - d\right]\tilde{c}_{1t+1}^{-\frac{1}{\sigma}}, \tag{108}$$

$$\tilde{c}_{1t} + (1+g)\tilde{K}_{t+1} = F(\tilde{K}_t) + (1-d)\tilde{K}_t. \tag{109}$$

Notice that $\tilde{c}_{1t}$ and $\tilde{K}_t$ are both state variables of the problem at time t. As a consequence, the resource constraint in (109), where we use (107) to simplify the left-hand side, characterizes the value of future capital $\tilde{K}_{t+1}$. With this in hand, we derive $\tilde{c}_{1t+1}$ from the Euler equation in (108).

B.1.1 Recursive Formulation

To solve the planner problem, we often find it useful to take its recursive representation:

$$V(\tilde{K}_t, \tilde{c}_{1t}) = \sum_{s=0}^{\infty} \tilde{\beta}^s \left[\pi u(\tilde{c}_{2t+s}) + \tilde{\beta}(1-\pi)u(\tilde{c}_{1t+1+s})\right] = \tag{110}$$

$$= \pi u(\tilde{c}_{2t}) + \tilde{\beta}(1-\pi)u(\tilde{c}_{1t+1}) + \tilde{\beta}\pi u(\tilde{c}_{2t+1}) + \tilde{\beta}^2(1-\pi)u(\tilde{c}_{1t+2}) + \cdots = \tag{111}$$

$$= \pi u(\tilde{c}_{2t}) + \tilde{\beta}(1-\pi)u(\tilde{c}_{1t+1}) + \tilde{\beta}V(\tilde{K}_{t+1}, \tilde{c}_{1t+1}). \tag{112}$$

In this way, we can write the following recursive formulation, where we drop the time underscripts for simplicity, and use the apex to indicate future values:

$$V(\tilde{K}, \tilde{c}_1) = \pi u(\tilde{c}_2) + \tilde{\beta}(1-\pi)u(\tilde{c}_1') + \tilde{\beta}V(\tilde{K}', \tilde{c}_1'). \tag{113}$$

Then, the Bellman equation for the social planner problem reads:

$$V(\tilde{K}, \tilde{c}_1) = \max_{\tilde{c}_2, \tilde{c}_1', \tilde{K}'} \pi u(\tilde{c}_2) + \tilde{\beta}(1-\pi)u(\tilde{c}_1') + \tilde{\beta}V(\tilde{K}', \tilde{c}_1'), \tag{114}$$

subject to:

$$(1-\pi)\tilde{c}_1 + \pi\tilde{c}_2 + (1+g)\tilde{K}' \leq F(\tilde{K}) + (1-d)\tilde{K}. \tag{115}$$

Attach the Lagrange multiplier λ to the resource constraint. The first-order conditions of the program are:

$$\tilde{c}_2: \quad u'(\tilde{c}_2) = \lambda, \tag{116}$$

$$\tilde{c}_1': \quad (1-\pi)u'(\tilde{c}_1') + V_2(\tilde{K}', \tilde{c}_1') = 0, \tag{117}$$

$$\tilde{K}': \quad \tilde{\beta}V_1(\tilde{K}', \tilde{c}_1') = (1+g)\lambda, \tag{118}$$

where $V_j(\tilde{K}', \tilde{c}_1')$ is the derivative of the value function with respect to its j-th element. The envelope conditions read:

$$\tilde{c}_1: \quad V_2(\tilde{K}, \tilde{c}_1) = (1-\pi)\lambda, \tag{119}$$

$$\tilde{K}: \quad V_1(\tilde{K}, \tilde{c}_1) = \lambda\big[F'(\tilde{K}) + 1 - d\big]. \tag{120}$$

Thus, we derive the equilibrium conditions:

$$u'(\tilde{c}_2) = u'(\tilde{c}_1), \tag{121}$$

$$(1+g)u'(\tilde{c}_2) = \tilde{\beta}\big[F'(\tilde{K}') + 1 - d\big]u'(\tilde{c}_1'). \tag{122}$$

The fact that, at every point in time, the equilibrium night consumption is equal to the day consumption greatly simplifies the quantitative analysis, because it implies that the planner problem with no runs is observationally equivalent to the neoclassical growth model. Accordingly, we solve the model using a standard value-function-iteration procedure.

B.2 Stationary Planner Problem With Runs

Following the same steps as before, we also define the stationary objective function of the planner problem with runs. In doing this, we assume that the planner, as the banks, take as given the available service protocols in the case that the run equilibrium is selected. Hence, she chooses between sequential service and equal service, too. The objective function of the stationary planner problem with sequential service reads:

$$\sum_{t=0}^{\infty} \tilde{\beta}^t \left[(1-q)\left(\pi \frac{\tilde{c}_{2t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \tilde{\beta}(1-\pi)\frac{\tilde{c}_{1t+1}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right) + q\tilde{\delta}_t(r)\big[\pi + \beta(1-\pi)\big]\frac{\tilde{c}_{2t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right] - \frac{[\pi + \beta(1-\pi)](1-q+q\tilde{\delta}_t(r))}{(1-\beta)\left(1-\frac{1}{\sigma}\right)}, \tag{123}$$

where $\tilde{\delta}_t(r) = (\ell_t + r\tilde{D}_t)/\tilde{c}_{2t}$. The budget constraints, instead, are:

$$(1-\pi)\tilde{c}_{1t} + \ell_t + (1+g)\tilde{K}_{t+1} + \tilde{D}_t \leq F(\tilde{K}_t) + (1-d)\tilde{K}_t, \tag{124}$$
$$\ell_t \geq \pi\tilde{c}_{2t} \tag{125}$$
$$\tilde{D}_t \geq 0, \tag{126}$$
$$\tilde{K}_{t+1} \geq 0. \tag{127}$$

The first-order conditions of this problem are similar to those in the text, and, with $\tilde{\delta}_t(r) = \pi$ and $\ell_t = \pi\tilde{c}_{2t}$, yield the equilibrium conditions:

$$\tilde{c}_{2t}^{-\frac{1}{\sigma}}\left(1 + \frac{q}{(1-q)}\big[\pi + \beta(1-\pi)\big]\right) = \tilde{c}_{1t}^{-\frac{1}{\sigma}}, \tag{128}$$
$$(1+g)\tilde{c}_{1t}^{-\frac{1}{\sigma}} = \tilde{\beta}\left[F'(\tilde{K}_{t+1}) + 1 - d\right]\tilde{c}_{1t+1}^{-\frac{1}{\sigma}}, \tag{129}$$
$$q\big[\pi + \beta(1-\pi)\big]r\frac{\tilde{c}_{2t}^{-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \eta_t = (1-q)\tilde{c}_{1t}^{-\frac{1}{\sigma}}. \tag{130}$$

In a similar way, the objective function of the stationary planner problem with equal service reads:

$$\sum_{t=0}^{\infty}\tilde{\beta}^t\left[(1-q)\left(\pi\frac{\tilde{c}_{2t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \tilde{\beta}(1-\pi)\frac{\tilde{c}_{1t+1}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\right) + q\big[\pi + \beta(1-\pi)\big]\frac{(\tilde{\ell}_t + r\tilde{D}_t)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\right] \tag{131}$$
$$-\frac{\pi + \beta(1-\pi)}{(1-\beta)\left(1-\frac{1}{\sigma}\right)}. \tag{132}$$

The bank maximizes the objective function, subject to the budget constraints:

$$(1-\pi)\tilde{c}_{1t} + \tilde{\ell}_t + (1+g)\tilde{f}_{t+1} \leq F(\tilde{K}_t) + (1-d)\tilde{K}_t + \tilde{z}_t, \tag{133}$$
$$\pi c_{2t} \leq \ell_t, \tag{134}$$
$$(1+g)\tilde{K}_{t+1} \leq (1+g)\tilde{f}_{t+1} - \tilde{D}_t, \tag{135}$$
$$(1+g)\tilde{z}_{t+1} \leq \tilde{\ell}_t - \pi\tilde{c}_{2t}, \tag{136}$$
$$\tilde{D}_t \geq 0, \tag{137}$$
$$\tilde{K}_{t+1} \geq 0, \tag{138}$$
$$\tilde{z}_{t+1} \geq 0. \tag{139}$$

The first-order conditions of the problem yield the equilibrium conditions:

$$\tilde{c}_{1t}^{-\frac{1}{\sigma}} - \tilde{c}_{2t}^{-\frac{1}{\sigma}} = \frac{q}{1-q}\big[\pi + \beta(1-\pi)\big]\tilde{\ell}_t^{-\frac{1}{\sigma}}, \tag{140}$$
$$(1+g)\tilde{c}_{1t}^{-\frac{1}{\sigma}} = \tilde{\beta}\left[F'(\tilde{K}_{t+1}) + 1 - d\right]\tilde{c}_{1t+1}^{-\frac{1}{\sigma}}, \tag{141}$$
$$(1+g)\tilde{c}_{2t}^{-\frac{1}{\sigma}} = \tilde{\beta}\tilde{c}_{1t+1}^{-\frac{1}{\sigma}}, \tag{142}$$

where we used the fact that, in equilibrium, $\tilde{c}_{2t}^R = \tilde{\ell}_t$.

The algorithm that we use to solve the simulation is the following:[18]

[18] We use Matlab for all numerical simulations, and a standard nonlinear solver to find the solution of the

1. Find $\tilde{c}_{ss}$ and $\tilde{K}_{ss}$ from the steady-state equilibrium conditions of the no-run problem;
2. Solve for the transition path to the steady state of the no-run problem, using value function iteration;
3. Use $\{\tilde{K}_{ss}, \tilde{c}_{ss}\}$ as state variables, and derive $\tilde{c}_{2t}$, $\tilde{c}_{1t+1}$, $\tilde{f}_{t+1}$, $\tilde{K}_{t+1}$ and $\tilde{D}_t$ for the three banking problems with runs, from the steady-state versions of the first order conditions reported above;
4. With $\{\tilde{K}_{t+1}, \tilde{c}_{1t+1}\}$ from the previous step as state variables, use value function iteration to solve for the new transition path back to the steady state of the no-run banking problem.

B.3 Welfare Costs

As already mentioned in the text, we define the welfare costs in terms of consumption equivalents, i.e. as the constant percentage drop in consumption that we have to impose in order to make the no-run banking equilibrium equivalent, in terms of welfare, to the run equilibrium. More formally:

$$W\left(\{(1-\chi)c_{2t}^{NR}, (1-\chi)c_{1t+1}^{NR}\}\right) = W\left(\{c_{2t}^{i}, c_{1t+1}^{i}\}\right), \tag{143}$$

where the superscript $i \in \{RP, SEQ, EQ\}$ refers to the three equilibria that we characterized in section 4. To simplify the notation, we define:

$$W^{NR} = \sum_{t=0}^{\infty} \tilde{\beta}^t \left[\pi \frac{\tilde{c}_{2t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \tilde{\beta}(1-\pi)\frac{\tilde{c}_{1t+1}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\right]. \tag{144}$$

Notice that W^{RP} is equal to W^{NR}, except for the fact that we calculate the felicity function using $\{\tilde{c}_{2t}^{RP}, \tilde{c}_{1t+1}^{RP}\}$. In a similar way, we also define:

$$W^{SEQ} = \sum_{t=0}^{\infty} \tilde{\beta}^t \left[(1-q)\left(\pi \frac{\tilde{c}_{2t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \tilde{\beta}(1-\pi)\frac{\tilde{c}_{1t+1}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\right) + q\tilde{\delta}_t(r)\big(\pi + \beta(1-\pi)\big)\frac{\tilde{c}_{2t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\right]. \tag{145}$$

Moreover:

$$W^{EQ} = \sum_{t=0}^{\infty} \tilde{\beta}^t \left[(1-q)\left(\pi \frac{\tilde{c}_{2t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \tilde{\beta}(1-\pi)\frac{\tilde{c}_{1t+1}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\right) + q\big[\pi + \beta(1-\pi)\big]\frac{(\tilde{\ell}_t + r\tilde{D}_t)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\right]. \tag{146}$$

With these in hand, we find that, for the run equilibrium with sequential service, (143) becomes:

$$(1-\chi^{SEQ})^{1-\frac{1}{\sigma}} W^{NR} - \frac{\pi + \beta(1-\pi)}{(1-\beta)\left(1-\frac{1}{\sigma}\right)} = W^{SEQ} - \frac{[\pi + \beta(1-\pi)][1-q+q\tilde{\delta}_t(r)]}{(1-\beta)\left(1-\frac{1}{\sigma}\right)}, \tag{147}$$

run equilibria. All codes are available upon request.

which gives:

$$\chi^{SEQ} = 1 - \left[\frac{W^{SEQ} + \frac{q(1-\tilde{\delta}_t(r)])[\pi+\beta(1-\pi)]}{(1-\beta)\left(1-\frac{1}{\sigma}\right)}}{W^{NR}} \right]^{\frac{\sigma}{\sigma-1}} . \tag{148}$$

For the run equilibrium with equal service and for the run-proof equilibrium, instead, we get:

$$\chi^{EQ} = 1 - \left[\frac{W^{EQ}}{W^{NR}} \right]^{\frac{\sigma}{\sigma-1}} , \tag{149}$$

$$\chi^{RP} = 1 - \left[\frac{W^{RP}}{W^{NR}} \right]^{\frac{\sigma}{\sigma-1}} . \tag{150}$$

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